$$y^{-1}+2+y-(2y^{-3}+2y^{-2}-2y^{-1}+4-2y+2y^2+2y^3)q+(y^{-5}-2y^{-4}-6y^{-3}-4y^{-2}+5y^{-1}+12+5y-4y^2-6y^3-2y^4+y^5)q^2+\dots$$

$$y^{-1}+1+y-(y^{-4}+y^{-3}-y^{-1}-2-y+y^3+y^4)q+(-y^{-5}-2y^{-4}-2y^{-3}+3y^{-1}+4+3y-2y^3-2y^4-y^5)q^2+\dots$$

$$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\dots$$

$$y^{-1}+4+y+(y^{-3}-8y^{-2}-y^{-1}+16-y+8y^2+y^3)q+(4y^{-4}-y^{-3}-32y^{-2}+y^{-1}+56+y+32y^2-y^3+4y^4)q^2+\dots$$

$$y^{-1}+y+(-y^{-5}+y^{-1}+y-y^5)q+(-y^{-7}-y^{-5}+2y^{-1}+2y-y^5-y^7)q^2+\dots$$

Siegel paramodular forms

Applications to holography and quantum black holes

Strings 2019, Brussels

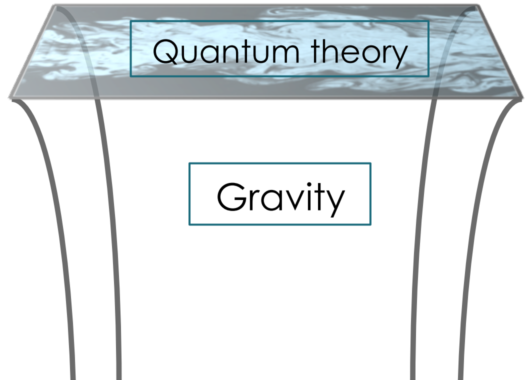
Based on
arXiv: 1611.04588 [hep-th]
arXiv: 1805.09336 [hep-th]
with Alex Belin, Joao Gomes and Christoph Keller

And work in progress with
Alex Belin, Christoph Keller and Beatrix Mühlmann

Collaborators

Our modern understanding of quantum gravity relies on Holography, in particular AdS/CFT.

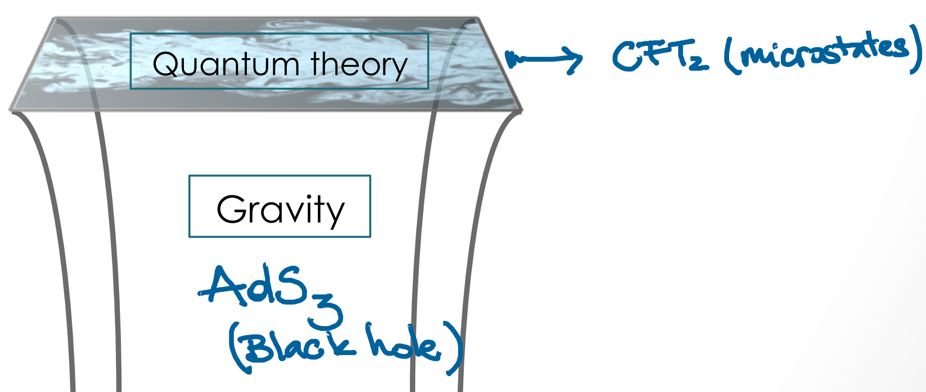
Can we build CFTs with holographic features?



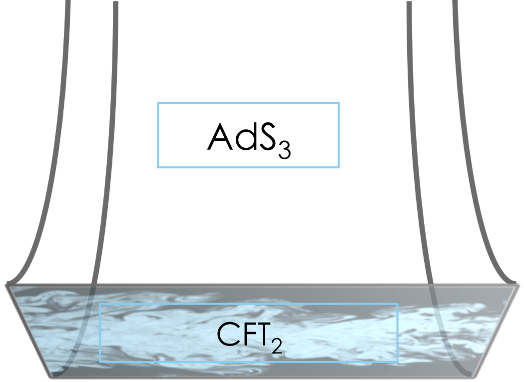
$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\ldots$

We will focus on the difficulties you encounter in AdS_3/CFT_2.
Not universal, but it illustrates the challenges.

[See also talks by Eberhardt and Mazac]



$y^{-1}+4+y+(y^{-3}-8y^{-2}-y^{-1}+16-y+8y^{2}+y^{3})q+(4y^{-4}-y^{-3}-32y^{-2}+y^{-1}+56+y+32y^{2}-y^{3}+4y^{4})q^{2}+\dots$



AdS$_3$/CFT$_2$

Supergravity description

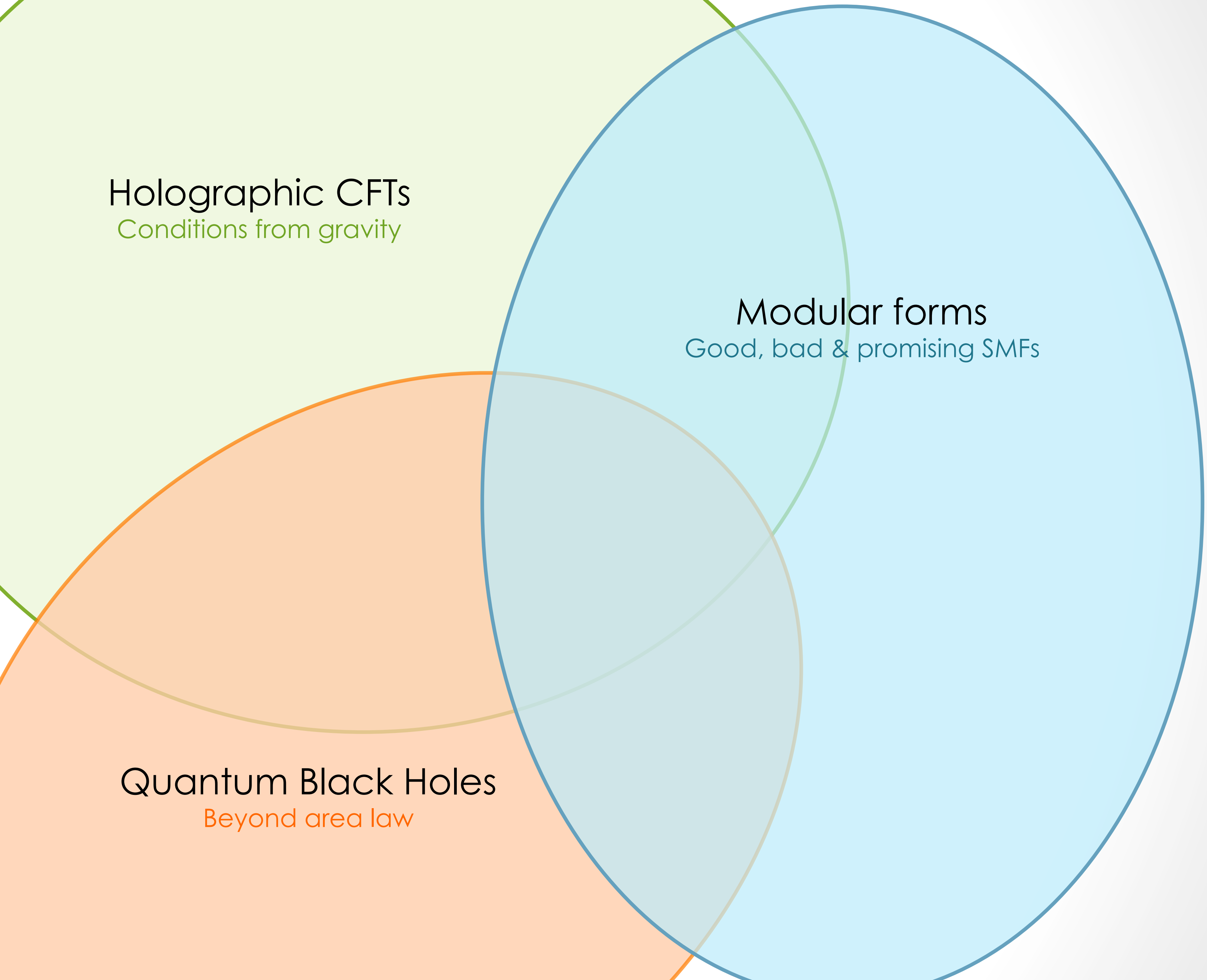
Large central charge

STRATEGY

1. Inspiration from known examples in String Theory.
2. Exploit holography.
3. Exploit number theory: crafting suitable counting formulas.

$y^{-1}+y+(-y^{-5}+y^{-1}+y-y^{5})q+(-y^{-7}-y^{-5}+2y^{-1}+2y-y^{5}-y^{7})q^{2}+\ldots$

Outline


Holographic CFTs
Conditions from gravity
Modular forms
Good, bad & promising SMFs
Quantum Black Holes
Beyond area law

Outline

Holographic CFTs

Conditions from gravity

Necessary conditions on the spectrum of CFT_2

Modular forms

Good, bad & promising SMFs

Implementing conditions on modular forms.

Restrict the analysis to supersymmetric states.

Outline

Why it works so well from the SMF perspective.

What it can teach us about black holes.

Quantum Black Holes

Beyond area law

Outline

Conditions from gravity

HOLOGRAPHIC CFTS

AdS_3 Gravity

The theory:

$$I_{3D} = \frac{1}{16\pi G_N} \int d^3x \sqrt{-g} \left(R + \frac{2}{\ell^2} \right) + \text{matter}$$

The spectrum:

1. Light States: Perturbative states
2. Heavy States: Black holes
3. Other stuff, e.g., multi-centered, conical defects (to be ignored today)

$y^{-1}+2+y-(2y^{-3}+2y^{-2}-2y^{-1}+4-2y+2y^2+2y^3)q+(y^{-5}-2y^{-4}-6y^{-3}-4y^{-2}+5y^{-1}+12+5y-4y^2-6y^3-2y^4+y^5)q^2+\ldots$

Holographic CFT_2

We will impose two conditions

1. Black hole regime

2. Perturbative regime

Holographic CFT_2

1. Black hole regime:

$$A_H \gg G_N \longrightarrow E \sim c \gg 1$$

$$S_{\rm BH} = \ln d(c,E)$$

symmetry → $E \gg c$ Cardy regime

$$= 2\pi\sqrt{\frac{cE}{6}} + \cdots$$

Holography [Strominger]

$$= \frac{A_{\rm H}}{4G} + \cdots$$

While the Cardy regime correctly accounts the entropy of very large BHs, we want CFTs with an extended Cardy regime that covers the whole range.

$y^{-1}+2+y-(2y^{-3}+2y^{-2}-2y^{-1}+4-2y+2y^2+2y^3)q+(y^{-5}-2y^{-4}-6y^{-3}-4y^{-2}+5y^{-1}+12+5y-4y^2-6y^3-2y^4+y^5)q^2+\ldots$

Holographic CFT_2

2. Perturbative regime:

Light = Energy is O(1) in Planck units.
Perturbative excitations that do not form a black hole.

- Presence of Hawking-Page transition [Keller; Hartman, Keller, Stoica]
- Extended Cardy regime for BPS BHs [Benjamin, Cheng, Kachru, Moore, Paquette; Benjamin, Kachru, Keller, Paquette]

$$\ln d(E) \sim E^{\alpha} \qquad \alpha \leq 1$$

But these are too many states for our gravitational needs.
We will require

$$\ln d(E) \sim E^{\alpha} \qquad \alpha < 1$$

Very sparse spectrum

$y^{-1}+2+y-(2y^{-3}+2y^{-2}-2y^{-1}+4-2y+2y^2+2y^3)q+(y^{-5}-2y^{-4}-6y^{-3}-4y^{-2}+5y^{-1}+12+5y-4y^2-6y^3-2y^4+y^5)q^2+\ldots$

BPS states in $SCFT_2$

Focus on protected quantities: the elliptic genera.

$$\chi(\tau, z) = \mathrm{tr}_{RR}\left((-1)^F q^{L_0 - \frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right)$$

Focus on cases when the elliptic genera is a weak Jacobi form.

Focus on symmetric product theories.

$$\mathcal{Z}(\rho, \tau, z) = \sum_r \chi(\tau, z; \mathrm{Sym}^r(M))\, e^{2\pi i \rho\, t\, r} = \prod_{\substack{n,l,r \in \mathbb{Z} \\ r>0}} (1 - q^n y^l p^{tr})^{-c(nr,l)}$$

[DMVV]

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+ \dots$

BPS states in SCFT$_2$

Focus on protected quantities: the elliptic genera.

$$\chi(\tau, z) = \mathrm{tr}_{RR}\left((-1)^F q^{L_0 - \frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right)$$

Focus on cases when the elliptic genera is a weak Jacobi form.

Focus on symmetric product theories.

$$\mathcal{Z}(\rho, \tau, z) = \sum_r \chi(\tau, z; \mathrm{Sym}^r(M))\, e^{2\pi i \rho\, t\, r} = \prod_{\substack{n,l,r\in\mathbb{Z}\\ r>0}} (1 - q^n y^l p^{tr})^{-c(nr,l)}$$

[DMVV]

Necessary condition on light states in NS sector →

$$\ln d(E) \sim E^\alpha \qquad \alpha < 1$$

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\ldots$

BPS states in $SCFT_2$

Focus on protected quantities: the elliptic genera.

$$\chi(\tau, z) = \mathrm{tr}_{RR}\left((-1)^F q^{L_0 - \frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right)$$

Focus on cases when the elliptic genera is a weak Jacobi form.

Focus on symmetric product theories.

$$\mathcal{Z}(\rho, \tau, z) = \sum_r \chi(\tau, z; \mathrm{Sym}^r(M))\, e^{2\pi i \rho\, t\, r} = \prod_{\substack{n,l,r \in \mathbb{Z} \\ r>0}} (1 - q^n y^l p^{tr})^{-c(nr,l)}$$

[DMVV]

Necessary condition on light states in NS sector →

$$\ln d(E) \sim E^{\alpha} \qquad \alpha < 1$$

Note: The partition function of symmetric product CFT_2 has $\alpha = 1$. The elliptic genus can display cancellations that capture the spectrum away from the symmetric product point.

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+ \ldots$

BPS states in $SCFT_2$

Focus on protected quantities: the elliptic genera.

$$\chi(\tau, z) = \mathrm{tr}_{RR}\left((-1)^F q^{L_0 - \frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}}\right)$$

Focus on cases when the elliptic genera is a weak Jacobi form.

Focus on symmetric product theories.

$$\mathcal{Z}(\rho, \tau, z) = \sum_r \chi(\tau, z; \mathrm{Sym}^r(M))\, e^{2\pi i \rho\, t\, r} = \prod_{\substack{n,l,r\in\mathbb{Z} \\ r>0}} (1 - q^n y^l p^{tr})^{-c(nr,l)}$$

[DMVV]

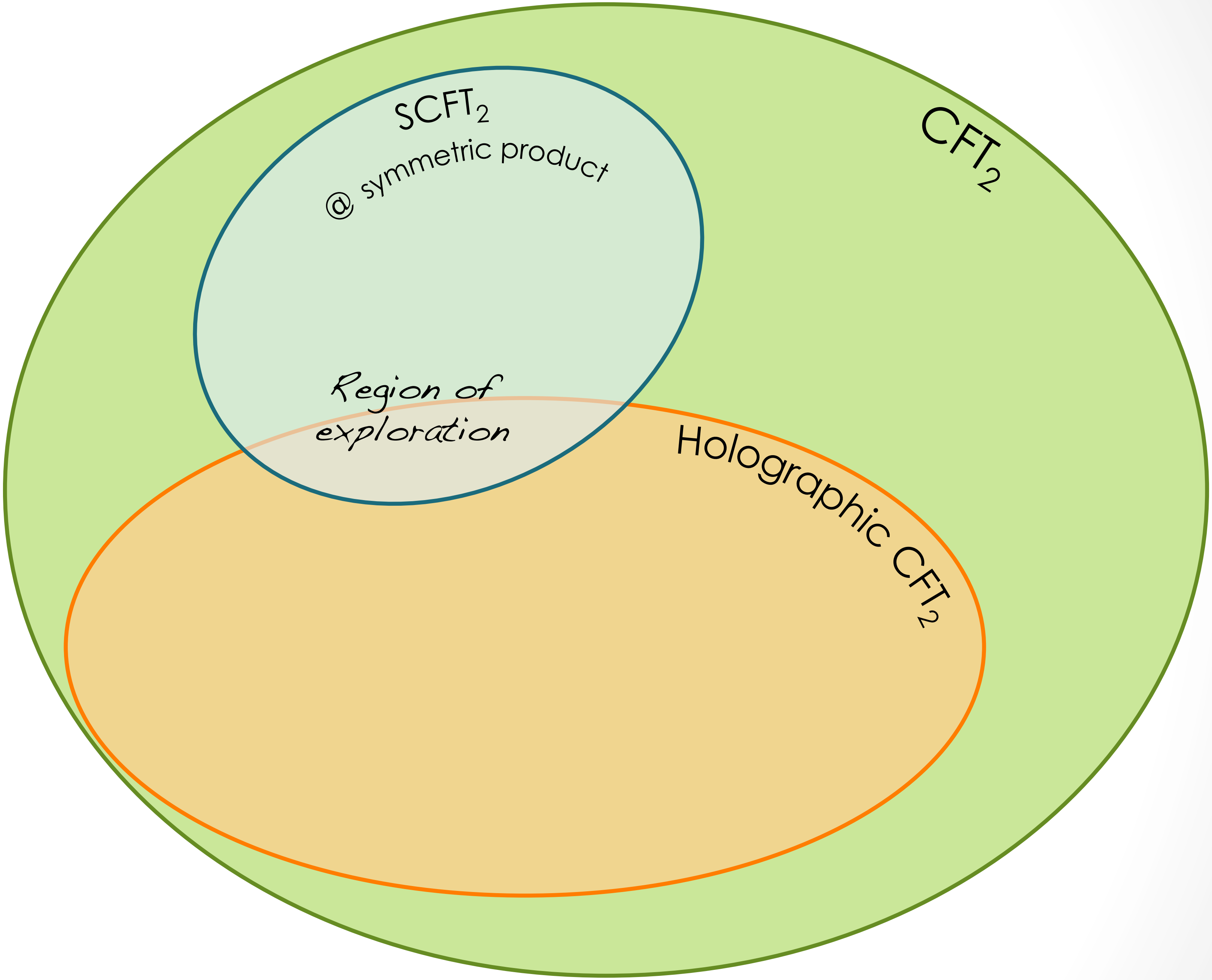
Necessary condition on light states in NS sector →

$$\ln d(E) \sim E^\alpha \qquad \alpha < 1$$

Spoiler!

We can tell you unambiguously which wJFs are holographic, i.e $\alpha<1$.
New examples are unveiled.

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\ldots$



Warning: Sizes are not meaningful.

$y^{-1}+y+(-y^{-5}+y^{-1}+y-y^{5})q+(-y^{-7}-y^{-5}+2y^{-1}+2y-y^{5}-y^{7})q^{2}+\ldots$

Good, bad & promising SMFs

MODULAR FORMS

Exploit our past

- Supersymmetry: we can evaluate at a symmetric product point, although the gravity dual is not in this regime.
- Averaging over theories: analytic continuation in central charge.
- Exchange symmetry: gives us control on the generating functional.

$y^{-1}+1+y-(y^{-4}+y^{-3}-y^{-1}-2-y+y^{3}+y^{4})q+(-y^{-5}-2y^{-4}-2y^{-3}+3y^{-1}+4+3y-2y^{3}-2y^{4}-y^{5})q^{2}+\ldots$

Siegel Paramodular Forms

Implementation: Supersymmetry + Averaging + Exchange Symmetry

$$\Phi_k(\rho,\tau,z) = \sum_m \varphi_{k,m}(\tau,z)p^m$$

τ

Jacobi Form: index m, weight k

$$p = e^{2\pi i\rho}$$
$$q = e^{2\pi i\tau}$$
$$y = e^{2\pi i z}$$

Transformation Properties: paramodular group Γ_t^+

$$\Phi_k(\rho,\tau,z) = \Phi_k(t^{-1}\tau, t\rho, z)$$

Exchange in ρ and τ

Generated by:

- SL(2,Z)
- Elliptic translations
- Exchange symmetry

Note: t=1 corresponds to $\Gamma_1^+ = Sp(4,\mathbb{Z})$

It is not (too) difficult to design a SMFs.
Exponential lifts leads to a SMF w.r.t. paramodular group.

[Gritsenko & Nikulin ('96); Gritsenko ('99)]

$$\text{Exp-Lift}(\varphi)(\Omega) = q^A y^B p^C \prod_{\substack{n,l,m\in\mathbb{Z}\\(n,l,m)>0}} (1-q^n y^l p^{tm})^{c(nm,l)}$$

Data in the SMF

$$\varphi_{0,t}(\tau,z) = \sum_{n,l} c(n,l) q^n y^l$$

SEED

$$A = \frac{1}{24}\sum_l c(0,l)\ , \quad B = \frac{1}{2}\sum_{l>0} l c(0,l)\ , \quad C = \frac{1}{4}\sum_l l^2 c(0,l)$$

$$\Omega = \begin{pmatrix} \tau & z \\ z & \rho \end{pmatrix}$$

$y^{-1}+1+y-(y^{-4}+y^{-3}-y^{-1}-2-y+y^3+y^4)q+(-y^{-5}-2y^{-4}-2y^{-3}+3y^{-1}+4+3y-2y^3-2y^4-y^5)q^2+\ldots$

It is not (too) difficult to design a SMFs.
Exponential lifts leads to a SMF w.r.t. paramodular group.

[Gritsenko & Nikulin ('96); Gritsenko ('99)]

$$\text{Exp-Lift}(\varphi)(\Omega) = q^A y^B p^C \prod_{\substack{n,l,m \in \mathbb{Z} \\ (n,l,m)>0}} (1 - q^n y^l p^{tm})^{c(nm,l)}$$

Connection to symmetric product theory

$$\text{Exp-Lift}(\varphi)(\Omega) = q^A y^B p^C \prod_{(n,l)>0} (1 - q^n y^l)^{c(0,l)} \times \prod_{\substack{n,l,m \in \mathbb{Z} \\ m>0}} (1 - q^n y^l p^{tm})^{c(nm,l)}$$

$$\mathcal{Z}(\Omega) = \sum_{r=0}^{\infty} p^{tr} \chi(\tau, z; \mathrm{Sym}^r(M)) = \frac{p^C \phi_{k,C}(\tau,z)}{\text{Exp-Lift}(\chi)(\Omega)}$$

$$\chi(\tau, z) = \mathrm{tr}_{RR}\left((-1)^F q^{L_0 - \frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}} \right)$$

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\ldots$

Gritsenko & Nikulin theorems are powerful.

$$\text{Exp-Lift}(\varphi)(\Omega) = q^A y^B p^C \prod_{\substack{n,l,m \in \mathbb{Z} \\ (n,l,m)>0}} (1 - q^n y^l p^{tm})^{c(nm,l)}$$

1. Minimal input: for any Jacobi form, we can build a SMF.
2. Zeroes and poles of SMF are known: Humbert surfaces.
3. We can systematically extract the Fourier coefficients. [Ashoke Sen: 0708.1270, 1104.1498]

$$d(m,n,l) = \oint_{p=0} \frac{dp}{2\pi i p} \oint_{q=0} \frac{dq}{2\pi i q} \oint_{y=0} \frac{dy}{2\pi i y} \frac{1}{\Phi_k(\Omega)} p^{-m} q^{-n} y^{-l}$$

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+ \ldots$

Promising SMFs

a.k.a. SMFs that meet the two necessary conditions on the BPS spectrum

Five examples of promising seeds in the exponential lift

$$\phi_{0,1} = y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^{2})q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^{2}+y^{3})q^{2}+\ldots$$

$$\phi_{0,2} = y^{-1}+4+y+(y^{-3}-8y^{-2}-y^{-1}+16-y+8y^{2}+y^{3})q+(4y^{-4}-y^{-3}-32y^{-2}+y^{-1}+56+y+32y^{2}-y^{3}+4y^{4})q^{2}+\ldots$$

$$\phi_{0,3} = y^{-1}+2+y-(2y^{-3}+2y^{-2}-2y^{-1}+4-2y+2y^{2}+2y^{3})q+(y^{-5}-2y^{-4}-6y^{-3}-4y^{-2}+5y^{-1}+12+5y-4y^{2}-6y^{3}-2y^{4}+y^{5})q^{2}+\ldots$$

$$\phi_{0,4} = y^{-1}+1+y-(y^{-4}+y^{-3}-y^{-1}-2-y+y^{3}+y^{4})q+(-y^{-5}-2y^{-4}-2y^{-3}+3y^{-1}+4+3y-2y^{3}-2y^{4}-y^{5})q^{2}+\ldots$$

$$\phi_{0,6} = y^{-1}+y+(-y^{-5}+y^{-1}+y-y^{5})q+(-y^{-7}-y^{-5}+2y^{-1}+2y-y^{5}-y^{7})q^{2}+\ldots$$

$y^{-1}+y+(-y^{-5}+y^{-1}+y-y^{5})q+(-y^{-7}-y^{-5}+2y^{-1}+2y-y^{5}-y^{7})q^{2}+\ldots$

Promising SMFs

What is special about these five examples?

- Location of poles: $H_1(1)$ Humbert surface (z=0) → Needed!
- Degree of the pole: 2
- Integral weight k

not crucial / simplifies analysis (applies to: Degree of the pole: 2; Integral weight k)

	Seed (φ)	Weight (k)	Group	A	B	C
Φ_{10}	$2\phi_{0,1}$	10	$SP(4,\mathbb{Z})$	1	1	1
Φ_4	$2\phi_{0,2}$	4	Γ_2^+	$1/2$	1	1
Φ_2	$2\phi_{0,3}$	2	Γ_3^+	$1/3$	1	1
Φ_1	$2\phi_{0,4}$	1	Γ_4^+	$1/4$	1	1
Φ_0	$2\phi_{0,6}$	0	Γ_6^+	$1/6$	1	1

$y^{-1}+y+(-y^{-5}+y^{-1}+y-y^5)q+(-y^{-7}-y^{-5}+2y^{-1}+2y-y^5-y^7)q^2+\ldots$

Promising SMFs

What is special about these five examples?

- Location of poles: $H_1(1)$ Humbert surface (z=0)
- Degree of the pole: 2
- Integral weight k

not crucial
simplifies analysis

Needed!

$$\phi_{0,t} = y^{-1}+c(0,0)+y + O(q)$$

minimal polarity wJF

Today

We have a complete classification up to index t=4.

Work in Progress

Quantify all promising SMFs based on the composition of Humbert surfaces.

$$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\dots$$

Good SMFs

a.k.a. SMFs that meet the two necessary conditions on the BPS spectrum, and we know the symm prod CFT_2 and AdS_3 supergravity theory.

1. Igusa Cusp form

$$\Phi_{10}(\Omega) = \text{Exp-Lift}(2\phi_{0,1}) \qquad \text{with} \qquad \phi_{0,1} = \frac{1}{2}\chi(\tau, z; K3)$$

$$\frac{1}{\Phi_{10}(\Omega)} = \frac{\mathcal{Z}(\Omega)}{p\,\phi_{10,1}(\tau, z)} \xrightarrow{\text{light states}} \ln d(E) \sim E^{1/2}$$

- ¼ BPS dyons in N=4 D=4 string theory [DVV]
- Quantum black hole [Sen, Dabholkar, Murthy, Gomes, …]
- $AdS_3 x S^3 x K3$ supergravity spectrum matches N=(4,4) SCFT [de Boer]
- CHL generalizations [David, Jatkar, Sen; Paquette, Volpato, Zimet]

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\ldots$

Good SMFs

a.k.a. SMFs that meet the two necessary conditions on the BPS spectrum, and we know the symm prod CFT_2 and AdS_3 supergravity theory.

2. t=4 paramodular form

Seed

$$\phi_{0,1}(\tau, 2z) + 4(1+a)\phi_{0,4}(\tau, z) = y^{\pm 2} + 4(1+a)y^{\pm 1} + 14 + 4a + \mathcal{O}(q)$$

Pole structure

Humbert surfaces are $H_4(2)$ and $H_1(1)$

Perturbative regime

$$\ln d(E) \sim E^{1/2}$$

Holography

$$\mathrm{AdS}_3 \times (S^3 \times \mathbb{T}^4)/G \longleftrightarrow$$ N=(2,2) SCFTs [Datta, Eberhardt, Gaberdiel]

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\dots$

Bad SMFs

a.k.a. they don't meet the criteria needed for a holographic CFT_2.

There are many of those. My favorite example (although not SUSY) is

$$\begin{aligned}\chi_{35} &= \text{Exp-Lift}(\varphi^{(2)}_{0,1})(\Omega) \\ &= q^3 y p^2 \prod_{(n,l,m)>0} (1 - q^n y^l p^m)^{f_1^{(2)}(4nm - l^2)}\end{aligned}$$

Seed is related to an extremal CFT

$$\varphi^{(2)}_{0,1} = (T_2 - 2)\phi_{0,1}$$

$$\begin{aligned}\varphi^{(2)}_{0,1}(\tau, 0) &= q^{-1} + 72 + 196884q + 21493760q^2 + \cdots \\ &= 72 + J(q)\ ,\end{aligned}$$

See also [de Lange, Maloney, Verlinde]

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\ldots$

Beyond area law

QUANTUM BLACK HOLES

Black holes & SMFs

The task is to extract the physics content of Fourier coefficients for large (asymptotic) values of the charges.

$$d(m,n,l) = \oint_{p=0} \frac{dp}{2\pi i p} \oint_{q=0} \frac{dq}{2\pi i q} \oint_{y=0} \frac{dy}{2\pi i y} \frac{1}{\Phi_k(\Omega)} p^{-m} q^{-n} y^{-l}$$

$$\downarrow$$

$$\Phi_k(\rho,\tau,z) = \Phi_k(t^{-1}\tau, t\rho, z)$$

$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^2)q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^2+y^3)q^2+\ \ldots$

Black holes & SMFs

The task is to extract the physics content of Fourier coefficients for large (asymptotic) values of the charges.

$$d(m,n,l) = \oint_{p=0} \frac{dp}{2\pi i p} \oint_{q=0} \frac{dq}{2\pi i q} \oint_{y=0} \frac{dy}{2\pi i y} \frac{1}{\Phi_k(\Omega)} p^{-m} q^{-n} y^{-l}$$

Asymptotic behavior of degeneracy

$$\begin{aligned} S_{\mathrm{BH}} &= \ln d(c,E) \\ &= 2\pi \sqrt{\frac{cE}{6}} + \cdots \\ &= \frac{A_{\mathrm{H}}}{4G} + \cdots \end{aligned}$$

symmetry → $d(E,c) = d(t\,E, t^{-1}c)$

Holography

Exchange symmetry gives us (almost) automatically an extended Cardy regime.

Quantum corrections

For lack of time, focus on logarithmic correction

$$S_{\rm BH} = \ln d(c,E,J) = \frac{A_H}{4G} + \#\ln(A_H/G) + \cdots$$

Scaling regime	$\tau^*_{1,2}$	A_H^2	$\ln\Lambda$
I. $E \gg 1$ $E\sim\Lambda^2,\ c\sim O(1),\ J\sim\Lambda$	Λ	Λ^2	$-(k+2)$
II. $E\sim c\gg 1$ $E\sim\Lambda,\ c\sim\Lambda,\ J\sim\Lambda$	Λ^0	Λ^2	$m_{1,1}-2$
III. $c\gg E\gg 1$ $E\sim\Lambda,\ c\sim\Lambda^2,\ J\sim\Lambda^{3/2}$	$\Lambda^{-1/2}$	Λ^3	$m_{1,1}-3-\frac{k}{2}$

Cardy regime. (Regime I)

Regime relevant for BPS BHs in 4D & 5D [Sen et al] (Regimes II and III)

To be explored

1. Logarithmic corrections

$$S_{\mathrm{BH}} = \ln d(c, E, J) = \frac{A_H}{4G} + \# \ln(A_H/G) + \cdots$$

Universality of this correction is being tested in gravity.
Less is known microscopically.

[Charles, Larsen; AC, Godet, Larsen, Zeng]
[Mukhametzhanov, Zhiboedov]

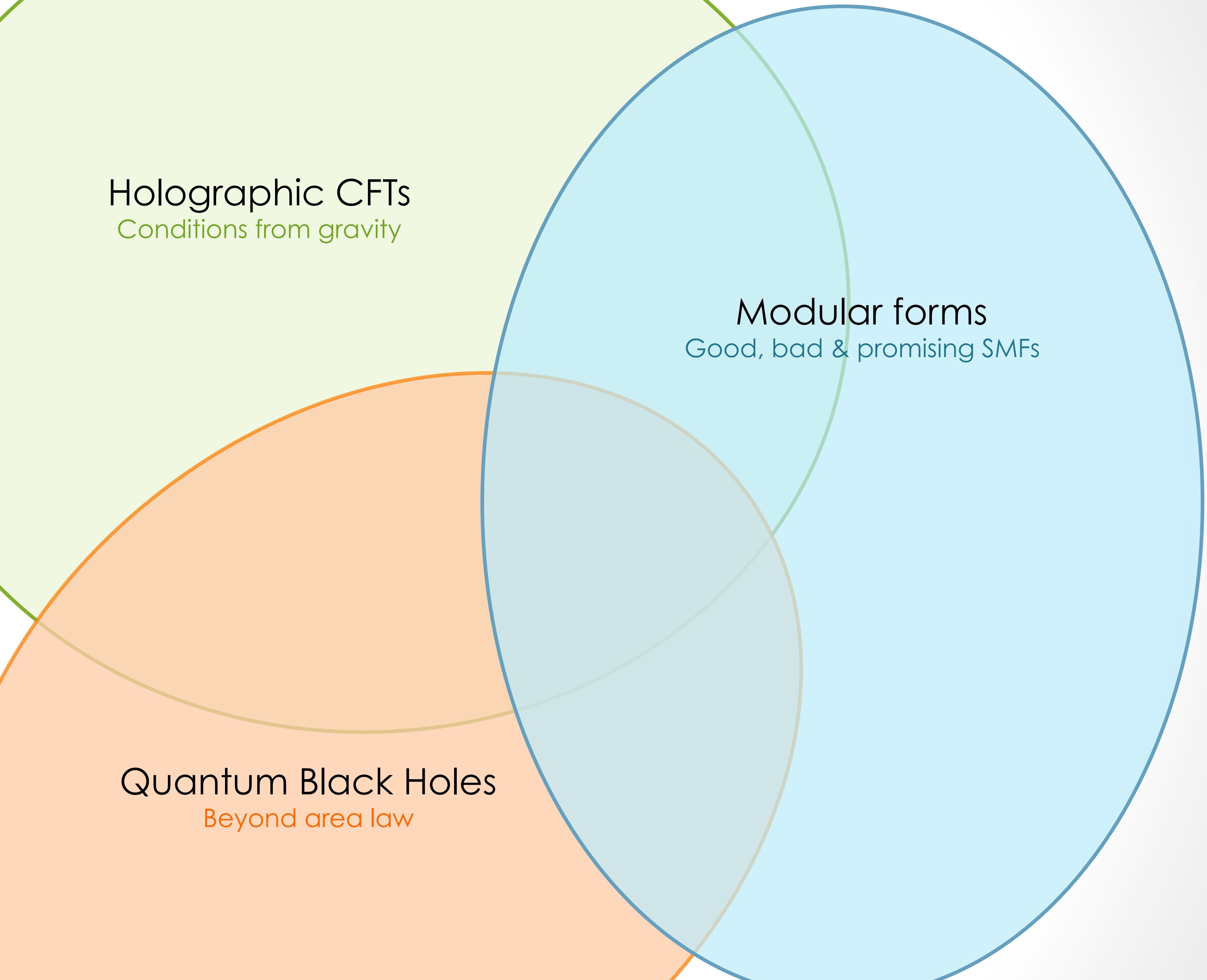
2. Mock modular forms

Decompose counting formula as quantum degeneracies of single-centered and multicentered configurations.

[Dabholkar, Murthy, Zagier]

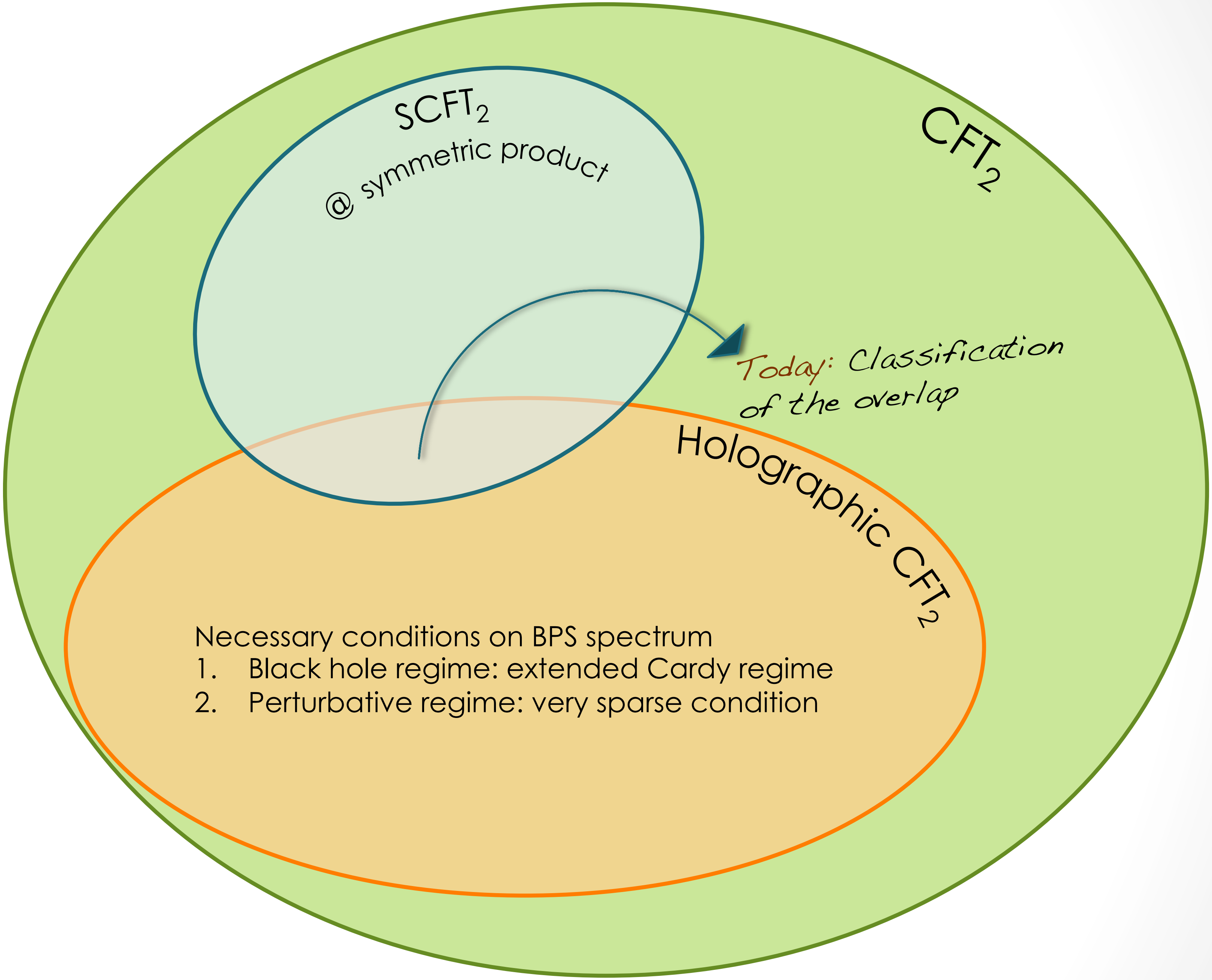
$y^{-1}+y+(-y^{-5}+y^{-1}+y-y^{5})q+(-y^{-7}-y^{-5}+2y^{-1}+2y-y^{5}-y^{7})q^{2}+\ldots$

Summary



Holographic CFTs
Conditions from gravity
Modular forms
Good, bad & promising SMFs
Quantum Black Holes
Beyond area law

Summary



Warning: Sizes are not meaningful.

THANK YOU!

$$y^{-1}+2+y-(2y^{-3}+2y^{-2}-2y^{-1}+4-2y+2y^{2}+2y^{3})q+(y^{-5}-2y^{-4}-6y^{-3}-4y^{-2}+5y^{-1}+12+5y-4y^{2}-6y^{3}-2y^{4}+y^{5})q^{2}+\dots$$

$$y^{-1}+1+y-(y^{-4}+y^{-3}-y^{-1}-2-y+y^{3}+y^{4})q+(-y^{-5}-2y^{-4}-2y^{-3}+3y^{-1}+4+3y-2y^{3}-2y^{4}-y^{5})q^{2}+\dots$$

$$y^{-1}+10+y+(10y^{-2}-64y^{-1}+108-64y+10y^{2})q+(y^{-3}+108y^{-2}-513y^{-1}+808-513y+108y^{2}+y^{3})q^{2}+\dots$$

$$y^{-1}+4+y+(y^{-3}-8y^{-2}-y^{-1}+16-y+8y^{2}+y^{3})q+(4y^{-4}-y^{-3}-32y^{-2}+y^{-1}+56+y+32y^{2}-y^{3}+4y^{4})q^{2}+\dots$$

$$y^{-1}+y+(-y^{-5}+y^{-1}+y-y^{5})q+(-y^{-7}-y^{-5}+2y^{-1}+2y-y^{5}-y^{7})q^{2}+\dots$$