Sewing Entanglement Wedges

Jan de Boer, Amsterdam

Based on work with Bartek Czech, Dongsheng Ge, and Lampros Lamprou, arXiv:1903.04493 + work in progress with Lampros Lamprou

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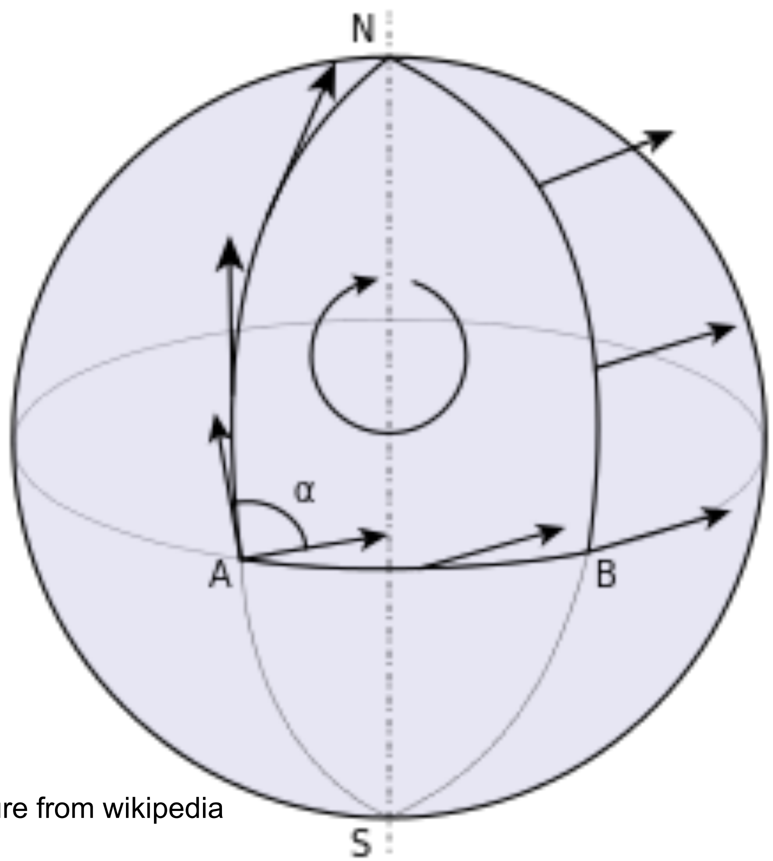
Recent work has uncovered interesting relations between:

- Various notions of quantum information theory
- Algebraic QFT
- Chaos
- Shock waves
- Energy Conditions
- Bootstrap methods
- Bulk reconstruction
- Constraints on semiclassical gravitational backgrounds

Here, by thinking about a notion of parallel transport for entanglement wedges/modular Hamiltonians we will run into another set of such connections..

Parallel transport – adiabatic process – low energy – different approach to EFT?

In general relativity, parallel transport provides a way to compare tangent vectors or frames at different points.



picture from wikipedia

Freely falling observers accomplish parallel transport.

Important: local symmetry group = Lorentz group.

Well-known example in quantum mechanics: Berry phase.

Parallel transport from entanglement in quantum mechanics:

$$|\psi\rangle_{AB} = \sum_{ij} W_{ij} |i\rangle_A |j\rangle_B$$

Assume for simplicity ρ_A, ρ_B are maximally mixed.

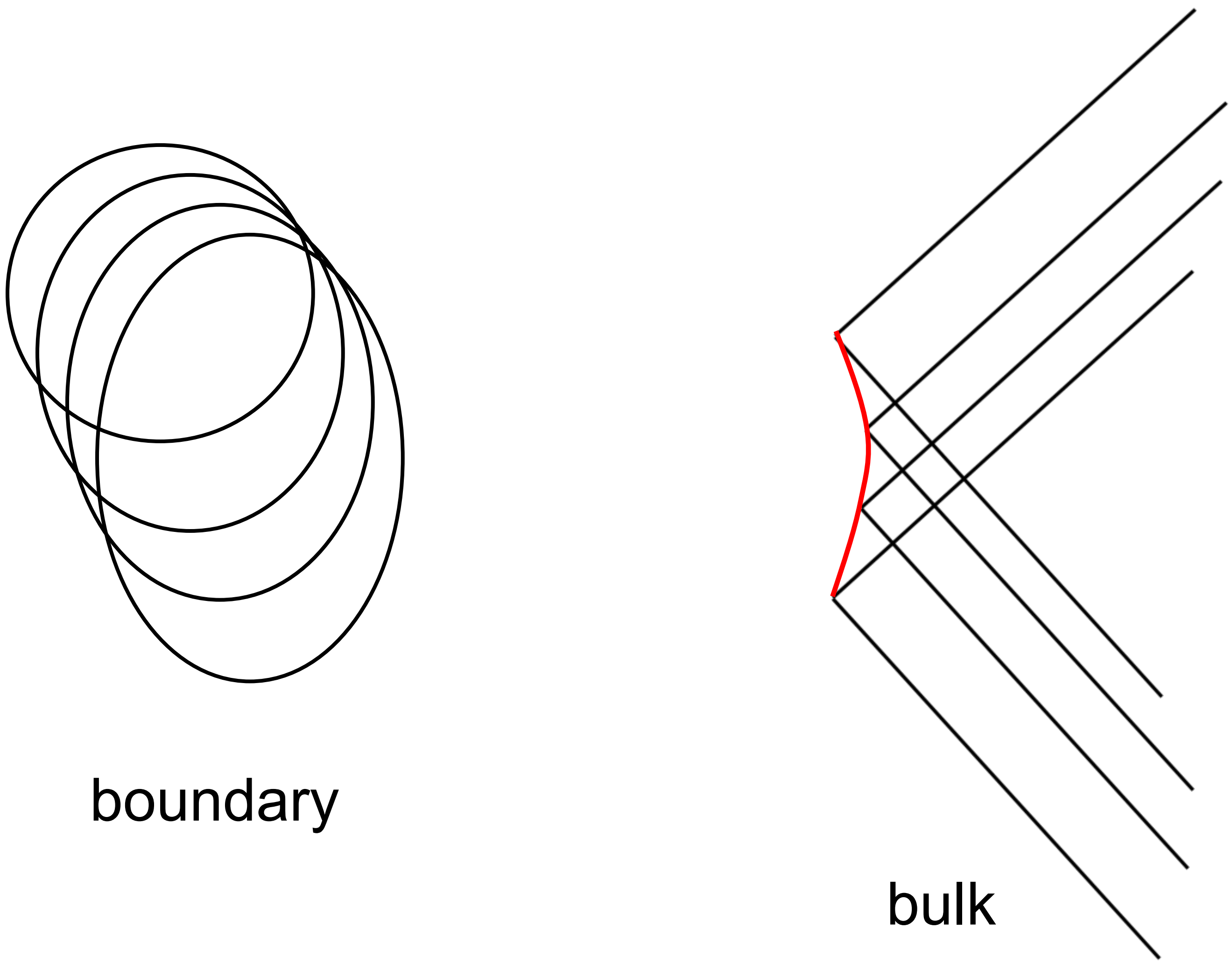
$$\langle U_A^\dagger \mathcal{O}_A U_A \rangle = \langle \mathcal{O}_A \rangle \qquad \langle U_B^\dagger \mathcal{O}_B U_B \rangle = \langle \mathcal{O}_B \rangle$$

But

$$\langle U_A^\dagger \mathcal{O}_A U_A U_B^\dagger \mathcal{O}_B U_B \rangle \neq \langle \mathcal{O}_A \mathcal{O}_B \rangle$$

unless we correlate the two unitaries in a precise way. → Symmetry group of entangled systems

Plan: use these ideas to study parallel transport for a family of entanglement wedges in AdS/CFT.



What are the symmetries?

Boundary: Unitaries that preserve the state, or equivalently, the modular Hamiltonian ($H_{\rm mod} = -\log \rho_A$)

$$U_A H_{{\rm mod},A} U_A^\dagger = H_{{\rm mod},A}$$

Using JLMS (Jafferis, Lewkowycz, Maldacena, Suh 15)

$$P_{\rm code} H_{{\rm mod}A} P_{\rm code} = \frac{A}{4G_N} + H_{\rm bulk}$$

Bulk: Diffeomorphisms that preserve the entanglement wedge, the area, and leave $H_{\rm bulk}$ invariant.

Boundary

$$H_{\rm mod}(\lambda) = U^\dagger(\lambda)\Delta(\lambda)U(\lambda)$$

Δ diagonal

U is ambiguous up to unitaries that leave Δ invariant. Generically $U(1)^d$. Under infinitesimal change

$$\dot{H}_{\rm mod} = [\dot{U}^\dagger U, H_{\rm mod}] + U^\dagger \dot{\Delta} U$$

Parallel transport = particular choice of $U(\lambda)$ (flatness condition)

$$\dot{H}_{\rm mod} = [\dot{U}^\dagger U, H_{\rm mod}] + U^\dagger \dot{\Delta} U$$

Proposal: parallel transport is defined by requiring that $\dot{U}^\dagger U$ does not possess a *modular zero mode*

$$P_0^\lambda[V] \equiv \lim_{\Lambda\to\infty} \frac{1}{2\Lambda} \int_{-\Lambda}^{\Lambda} ds\ e^{iH_{\rm mod}(\lambda)s}\, V e^{-iH_{\rm mod}(\lambda)s}$$

$$P_0^\lambda[\dot{U}^\dagger U] = 0$$

$$\dot{H}_{\rm mod} = [\dot{U}^\dagger U, H_{\rm mod}] + U^\dagger \dot{\Delta} U$$

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zero mode

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zero mode   zero mode

These equation may look peculiar, but are nothing but a generalization of the Berry phase:

Take $\rho(\lambda) = |\psi(\lambda)\rangle\langle\psi(\lambda)|$ then

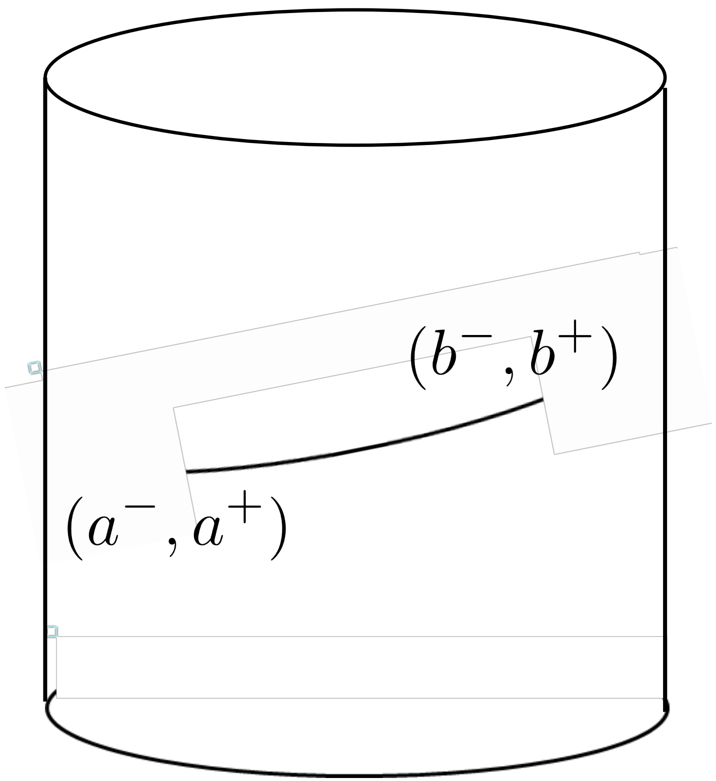
$$|\dot{\psi}\rangle\langle\psi| + |\psi\rangle\langle\dot{\psi}| = [\ |\dot{\psi}\rangle\langle\psi| - |\psi\rangle\langle\dot{\psi}|\, ,\ |\psi\rangle\langle\psi|\ ]$$

project

$$\langle\psi\ \ |\dot{\psi}\rangle\langle\psi| - |\psi\rangle\langle\dot{\psi}|\ \ \psi\rangle = \underbrace{\langle\psi|\dot{\psi}\rangle - \langle\dot{\psi}|\psi\rangle}_{\text{usual Berry connection}}$$

usual Berry connection

Example 1: 2d CFT with intervals



$$H_{\rm mod} = s_1 L_1 + s_0 L_0 + s_{-1} L_{-1} + t_1 \bar{L}_1 + t_0 \bar{L}_0 + t_{-1} \bar{L}_{-1}$$

$$s_1 = \frac{2\pi \cot(b^+ - a^+)/2}{e^{ia^+} + e^{ib^+}} \qquad s_0 = -2\pi \cot(b^+ - a^+)/2$$ etc

$$\frac{\partial H_{\mathrm{mod}}}{\partial \lambda} = \frac{\partial a^+}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial a^+} + \frac{\partial b^+}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial b^+} + \frac{\partial a^-}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial a^-} + \frac{\partial b^-}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial b^-}$$

$$2\pi i V = \frac{\partial a^+}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial a^+} - \frac{\partial b^+}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial b^+} - \frac{\partial a^-}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial a^-} + \frac{\partial b^-}{\partial \lambda}\frac{\partial H_{\mathrm{mod}}}{\partial b^-}$$

then

$$\partial_\lambda H_{\mathrm{mod}} = [V, H_{\mathrm{mod}}]$$

This reproduces the results of Czech, Lamprou, McCandlish and Sully 17. In particular, the holonomy is related to differential entropy.

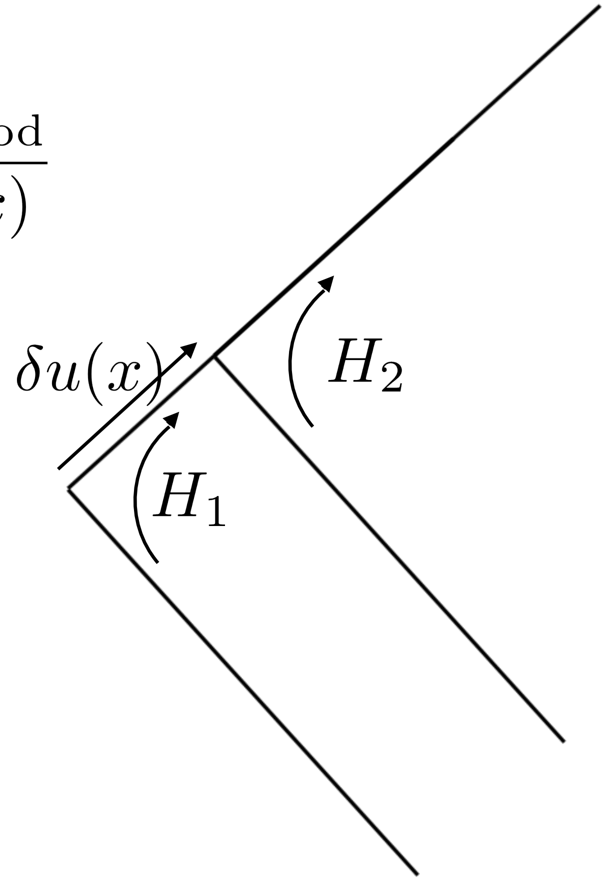
Example 2: modular inclusions (Casini, Teste, Torroba 17)

$$[H_2, H_1] = 2\pi i (H_2 - H_1)$$

$$\left[\frac{\delta H_{\rm mod}}{\delta u(x)}, H_{\rm mod}\right] = 2\pi i \frac{\delta H_{\rm mod}}{\delta u(x)}$$

↓

connection



Bulk picture

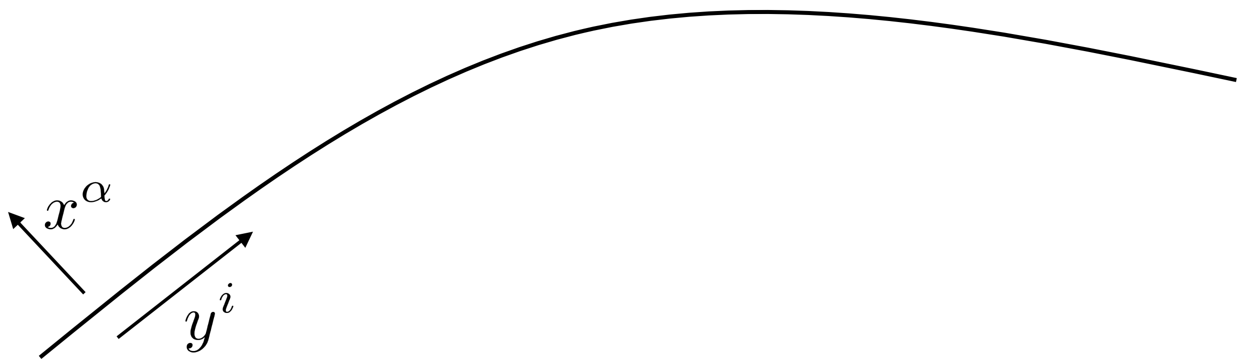
$$P_{\text{code}} H_{\text{mod}\,A} P_{\text{code}} = \frac{A}{4G_N} + H_{\text{bulk}}$$

What are the relevant symmetries? The bulk modular Hamiltonian becomes approximately a boost near the minimal surface.

Will take the point of view of “surface symmetries”, “edge modes”, “asymptotic symmetry group”, etc (cf Donnelly, Freidel 16; Speranza 17; Camps 18).

Surface symmetries are diffeomorphisms with a non-trivial Noether charge on the minimal surface. Related to non-factorization of bulk gravitational Hilbert space.

Consider a minimal surface



Choose coordinates

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta + \gamma_{ij} dy^i dy^j + \mathcal{O}(x)$$

Modular Hamiltonian is boost in transversal plane

$$\begin{aligned} \zeta^\alpha_{\rm mod} &= 2\pi \epsilon^{\alpha\beta} x_\beta + \mathcal{O}(x^2) \\ \zeta^i_{\rm mod} &= \mathcal{O}(x^2) \end{aligned}$$

$$
\begin{aligned}
\zeta^{\alpha}_{\text{mod}} &= 2\pi\epsilon^{\alpha\beta}x_{\beta} + \mathcal{O}(x^2) \\
\zeta^{i}_{\text{mod}} &= \mathcal{O}(x^2)
\end{aligned}
$$

Symmetries:

$$
\begin{aligned}
\zeta^{\alpha}_{\text{sym}} &= \omega(y)\epsilon^{\alpha\beta}x_{\beta} + \mathcal{O}(x^2) \\
\zeta^{i}_{\text{sym}} &= \zeta^{i}_{0}(y) + \mathcal{O}(x^2)
\end{aligned}
$$

Preserve asymptotic form of metric, have non-trivial charges, and commute with modular Hamiltonian vector field.

Two components: diffeomorphisms along surface and location-dependent frame rotations.

Microscopic interpretation? Only visible in code subspace?

Parallel transport should provide:

- a relation between the coordinates y of nearby minimal surfaces
- a relation between choices of orthonormal bases for the normal plane

One can repeat analysis:

$$\frac{d\zeta_{\rm mod}}{d\lambda} = [\xi, \zeta_{\rm mod}]$$

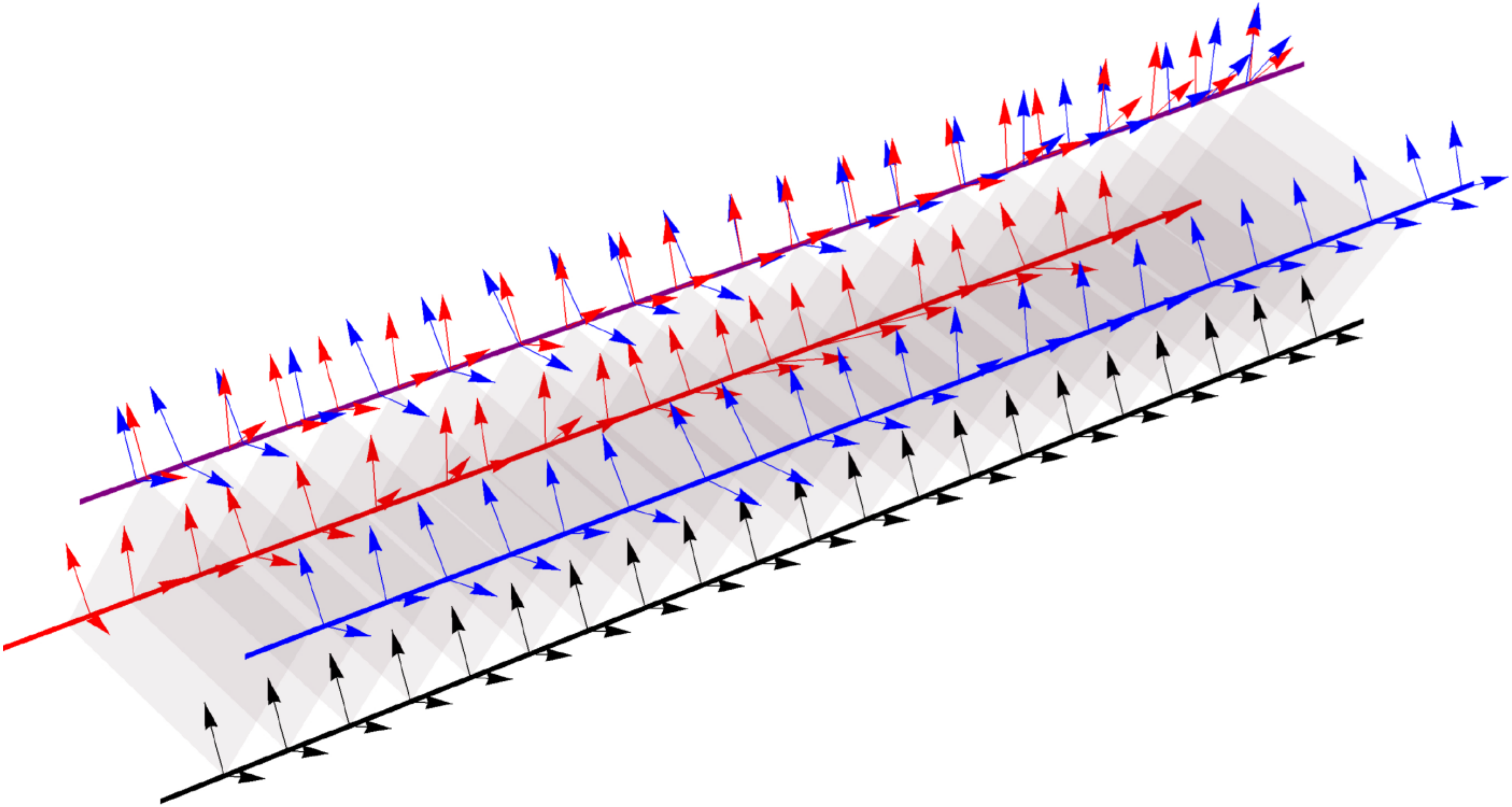
where on minimal surface ξ should not have modular zero modes:

$$\begin{aligned} \epsilon^{\alpha\beta}\partial_\alpha \xi_\beta &= 0 \\ \xi^i &= 0 \end{aligned}$$

Qualitative picture:

- To transport a point on a minimal surface to a point on a nearby minimal surface: separation should be orthogonal to the minimal surfaces

- To transport normal frame: parallel transport in orthogonal direction and project into new normal plane



Holonomy: surface diffeomorphism + frame rotation

Surface diffeomorphism illustrated

Endpoint of line gets displaced by length of path. Can use this to reconstruct length of curves.

(Balasubramanian, Chowdhury, Czech, JdB, Heller 13)

Interpretation of frame rotation? Closely related to bulk curvature.

Consider e.g. 2d case where minimal surfaces are points.

Procedure reduces to ordinary parallel transport of tangent vectors, and curvature of connection = geometric curvature.

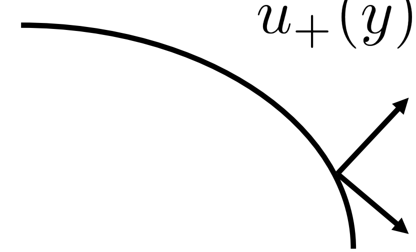
Connection to curvature only clear in case $K^2, \partial K \ll R$

For AdS3, boundary discussion can be directly translated into bulk (symmetries=Killing vectors) and agrees with above picture.

Examples show that there are special situations in which

$$\pm 2\pi i \, \delta H_{\mathrm{mod}} = [\delta H_{\mathrm{mod}}, H_{\mathrm{mod}}]$$

In the bulk, these correspond to deformations in light cone directions. One can think of these vector fields as “shock waves”



$$u_+(y)\frac{\partial}{\partial x^+} + \ldots$$

$$u_-(y)\frac{\partial}{\partial x^-} + \ldots$$

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta + \gamma_{ij} dy^i dy^j + \mathcal{O}(x)$$

In the bulk, these modes always exist. But what about the boundary theory?

Conjecture:

If $[H_{\rm mod}, \delta H_{\rm mod}] = i\lambda \delta H_{\rm mod}$ then $|\lambda| \le 2\pi$. This is closely related to the bound on chaos. If there is a semiclassical gravitational dual the bound is saturated.

cf modular chaos of
Faulkner, Li, Wang 18

Can be a bit more precise:

If $\mathcal{N} \subset \mathcal{M}$ then $\Delta_{\mathcal{M}}^{it}\Delta_{\mathcal{N}}^{-it}$ is analytic for $0 < \mathrm{Imt} < 1/2$ and obeys $|\Delta_{\mathcal{M}}^{it}\Delta_{\mathcal{N}}^{-it}| \leq 1$. (Borchers 99)

If $\phi \leq \psi$ then $\Delta_{\phi,\psi}^{-it}\Delta_{\psi}^{it}$ is analytic for $0 < \mathrm{Imt} < 1/2$ and obeys $|\Delta_{\phi,\psi}^{-it}\Delta_{\psi}^{it}| \leq 1$ (Araki 76)

$$\log(\Delta + \delta\Delta) - \log\Delta = \frac{\pi}{2}\int_{-\infty}^{\infty}\frac{dt}{\cosh^2 \pi t}\Delta^{-it-1/2}\delta\Delta\Delta^{it-1/2} + \dots$$

(Sarosi, Ugajin 17; Lashkari, Liu, Rajagopal 18)

$$[H_{\rm mod}, \delta H_{\rm mod}] = i\lambda \delta H_{\rm mod}$$

Example: take a 2d CFT on a circle and as subspace half of the circle.

$$H_{\rm mod} = \pi i (L_1 - L_{-1})$$

$$[H_{\rm mod}, L_1 + L_{-1} \pm 2L_0] = \pm 2\pi i (L_1 + L_{-1} \pm 2L_0)$$

These are the shockwave operators corresponding to ANEC operators in the CFT.

ANEC is the statement that $L_1 + L_{-1} + 2L_0 \geq 0$ which indeed holds in 2d CFT…

- Conjecture also holds for Virasoro deformations of AdS3
- does not hold in higher spin theories in AdS3 – maximal value of λ agrees with result of Perlmutter 16

$$\delta H_{\mathrm{mod}} = W_{-2} + 4W_{-1} + 6W_0 + 4W_1 + W_2$$

$$[H_{\mathrm{mod}}, \delta H_{\mathrm{mod}}] = 4\pi i\, \delta H_{\mathrm{mod}}$$

The “shock wave” vector fields have an interesting commutator

$$Q([V_+, V_-]) \sim \int_\Sigma \sqrt{\gamma}(\frac{1}{2}\gamma^{ij}\partial_i u_+ \partial_j u_- - 2R_{-+-+}u_+u_-)$$

which is somewhat similar to the expression for Planckian scattering found by ‘t Hooft 90; Verlinde, Verlinde 90…

Key in this computation is to pick the right vector fields, i.e. where the modular zero mode has been projected out.

This is also an explicit expression for part of the Berry curvature.

Have not used Einstein equations or extremality of surface in above computation yet, still missing some ingredients… (cf Lewkowycz, Parrikar 18)

Many questions:

- General interpretation of the curvature/holonomy?
- (dis)prove conjecture
- Clarify the precise relation to chaos and shockwaves
- Connect to recent work on shock waves (Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov 19; Belin, Hofman, Mathys 19)
- Connection to other recent appearances of the Berry connection? (Belin, Lewkowycz, Sarosi 18)
- Connection to bulk reconstruction? (Faulkner, Li, Wang 18; Kabat, Lifschytz 17,18)
- Useful to get a handle on the code subspace?
- Connection to recent discussions of soft hair? (…)
- Include other fields (e.g. gauge fields) with non-trivial edge modes?
- Generalize to other spacetimes?
- Can we get dynamics in this framework?
- Applications to two-sided case?