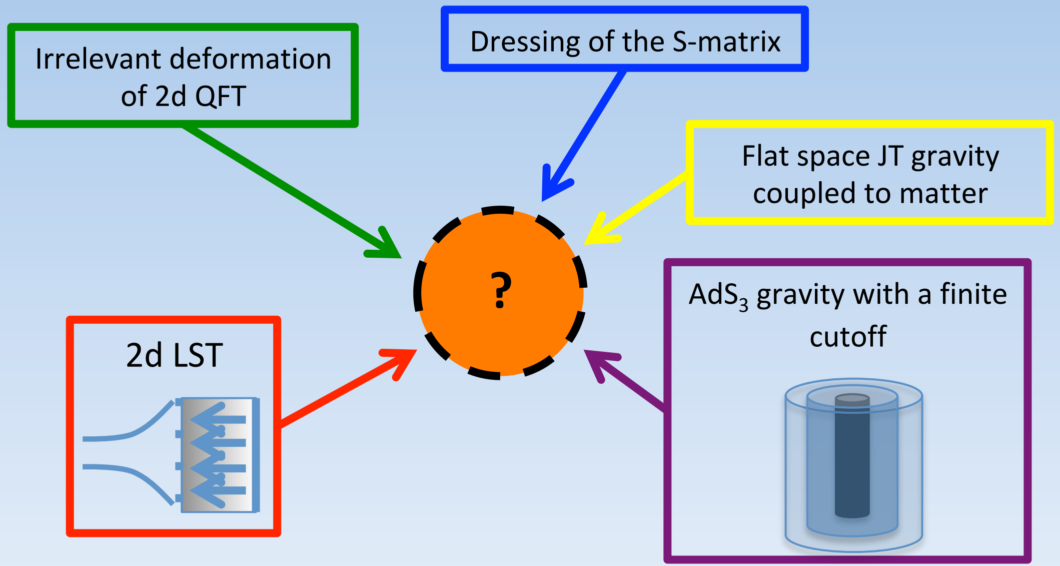
The $T\bar{T}$ deformation



Márk Mezei

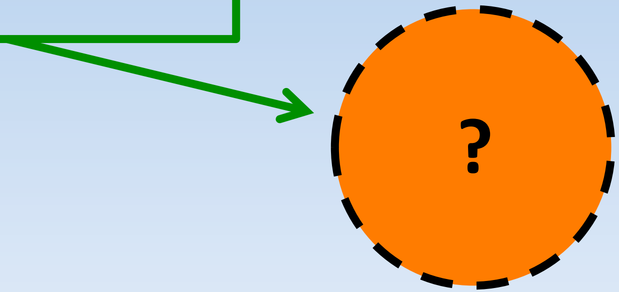
Simons Center for Geometry and Physics, SUNY, Stony Brook

Strings 2019, 07/10/2019

There is a new theory out there!

Irrelevant deformation of
2d QFT

$$\frac{d\mathcal{L}}{d\lambda} \propto T\bar{T}$$



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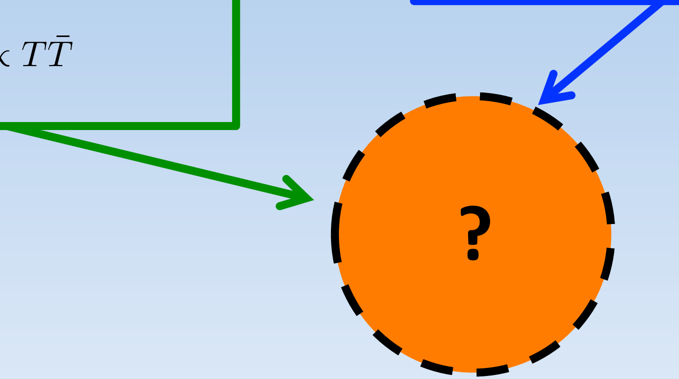
Dressing of the S-matrix

$$\hat{S}(s) = e^{i\ell_s^2\, s/4}\, S(s)$$



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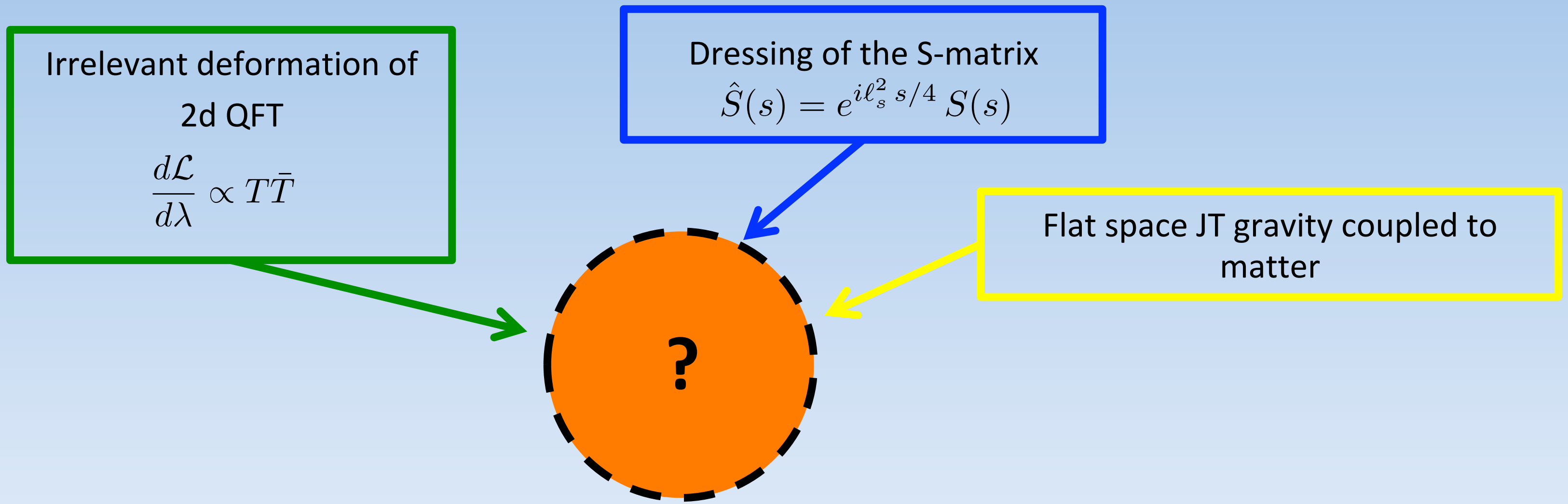

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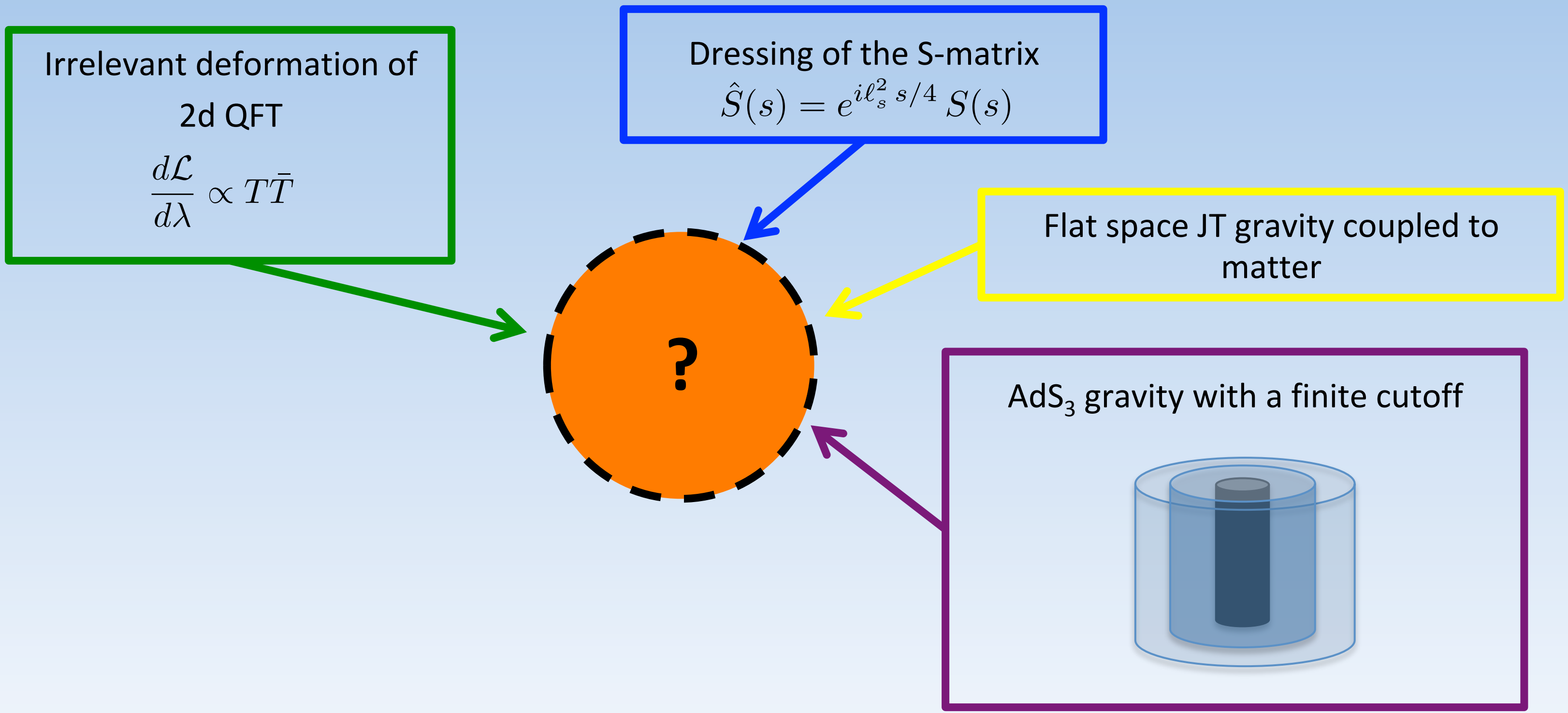

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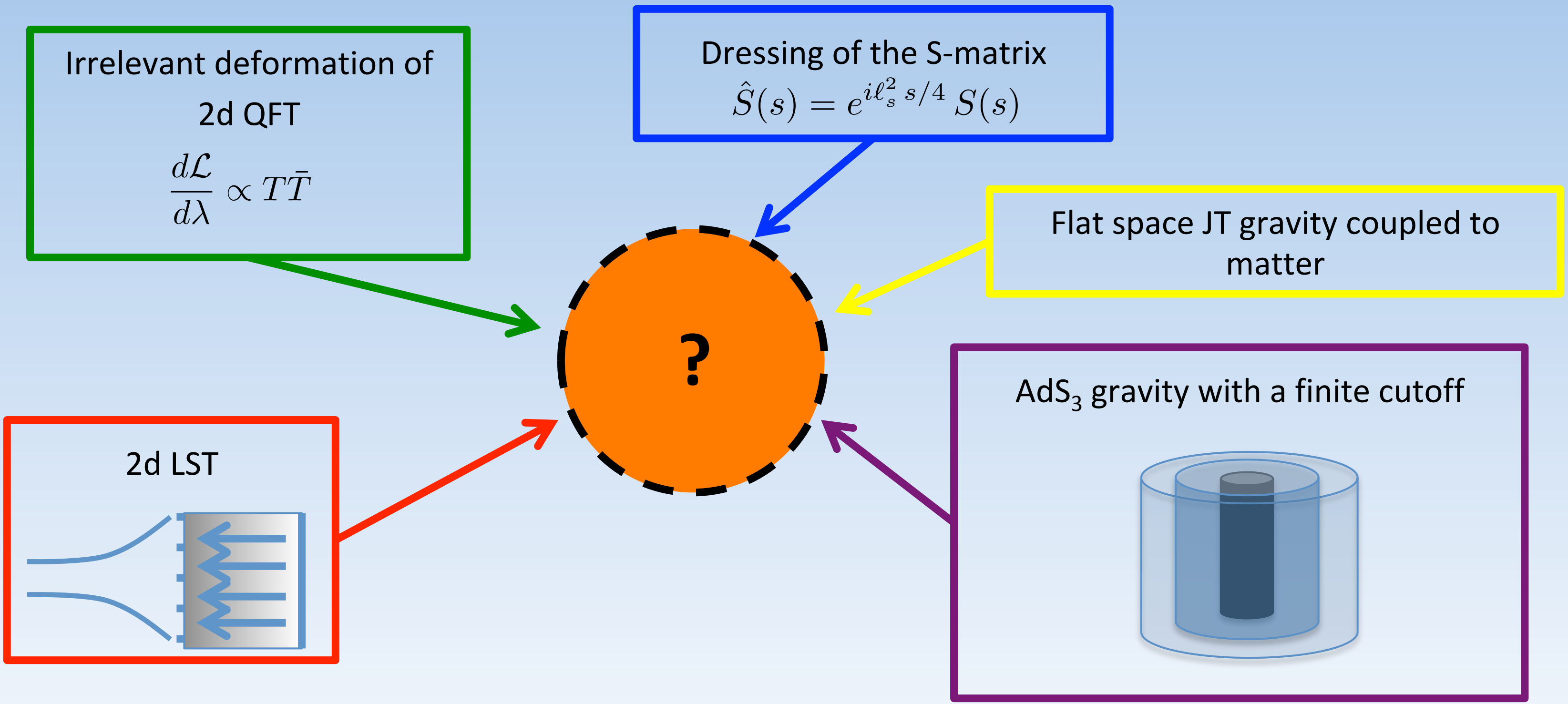


- Different formulations allow to compute different quantities (easily)
- They provide hints for future developments
- Not unfamiliar state of affairs in a **string theory** conference…

There is a new theory out there!

Irrelevant deformation of 2d QFT teaches us how to flow up the RG.

Dressing of the S-matrix makes CDD factors relevant in the nonintegrable context.

Flat space JT gravity coupled to matter description highlights expected nonlocality.

AdS_3 gravity with a finite cutoff resolves old puzzle in holographic RG.

2d LST: prospect of understanding LST from field theory.

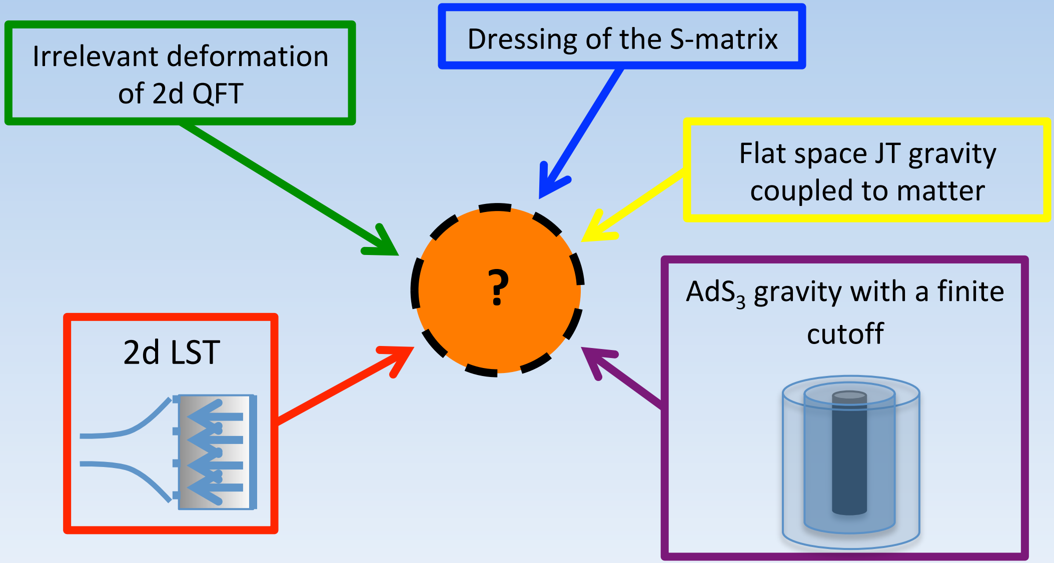
?

Outline

Different formulations
- QFT approach
- JT gravity
- Holography

Generalizations
- Higher dimensions
- Single trace
- Related deformations

Open questions



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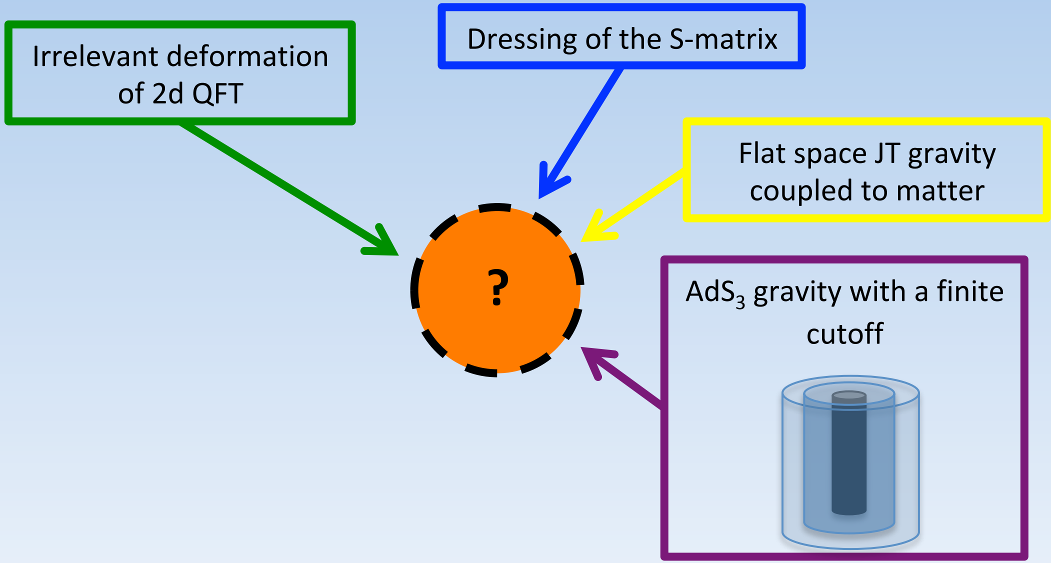

QFT approach

In any 2d QFT, exists the composite operator $T\bar{T}$ with remarkable properties:

- Defined by point splitting

$$\text{``}T\bar{T}\text{''}(x) \equiv -\pi^2 \lim_{x' \to x} \left(\epsilon^{aa'} \epsilon^{bb'} T_{a'b'}(x')\, T_{ab}(x) \right) + (\text{tot. der.})$$

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[Zamolodchikov]

- Consider the theory on $S^1_L \times \mathbb{R}$. Obeys factorization in eigenstates:

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[Smirnov, Zamolodchikov]

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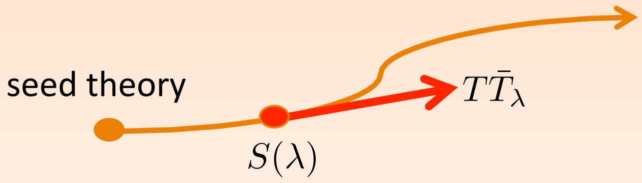

Irrelevant deformation, questions about UV completeness. Preserves almost all symmetries of seed theory. New class of integrable theories. In superspace the deformation is (supercurrent)^2.

[Smirnov, Zamolodchikov; Le Floch, MM]

[Baggio et al.; Chang et al.; Jiang et al.; Coleman et al.]

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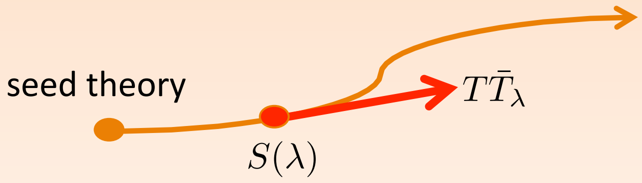

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- Burgers equation for the spectrum − **QFT question turned into PDE problem**

$$\frac{\partial}{\partial\lambda} E_n = E_n \partial_L E_n + \frac{P_n^2}{L} \quad \xleftrightarrow{\lambda = -t,\ E_n = u,\ L = x} \quad \partial_t u + u\, \partial_x u = f(x)$$

[Smirnov, Zamolodchikov; Cavaglia, Negro, Szecsenyi, Tateo]

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- If seed theory had higher spin charges, they ride the Burgers flow (passive scalar)

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- Initial conditions

$$E(\lambda=0,L) \overset{\text{CFT}}{=} \frac{e}{L} \quad \longleftrightarrow \quad u(t=0,x)\,, \quad P_s(t=0,x)$$

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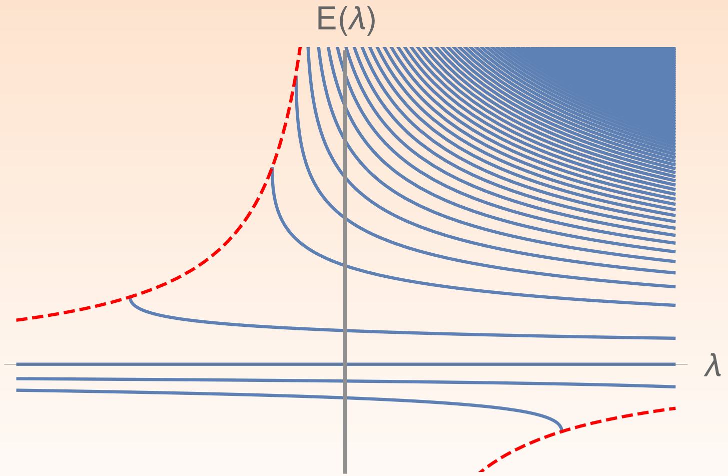
- Sketch of spectrum for $P = 0$



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In any 2d QFT, exists the composite operator $T\bar{T}$ with remarkable properties:

- Z_{T^2} function can be obtained from this data by summing over states. Instead Hubbard-Stratonovich transformation and summing over random metrics: [Cardy]

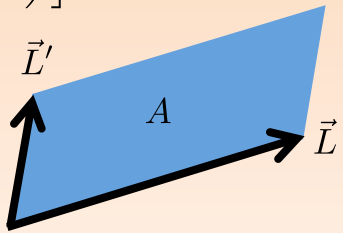
$$\exp\left[-\delta\lambda \int d^2x\; \epsilon^{aa'}\epsilon^{bb'} T_{a'b'}\, T_{ab}\right]$$

$$= \int Dh\;\exp\left[\int d^2x\;\left(\frac{1}{2} h_{ab}\, T^{ab} + \frac{1}{16\delta\lambda}\,\epsilon^{aa'}\epsilon^{bb'} h_{a'b'}\, h_{ab}\right)\right]$$

Gives diffusion equation:

$$\partial_\lambda \left(Z_{T^2}/A\right) = -\partial_L \wedge \partial_{L'} \left(Z_{T^2}/A\right)$$

Modularity of Z_{T^2} is guaranteed.



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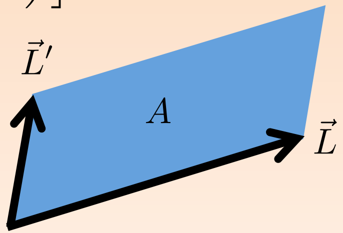

- Energy eigenvalues evolve independently. Z_{T^2} (with CFT seed) is the unique modular covariant partition function with this property. [Aharony, Datta, Giveon, Jiang, Kutasov]

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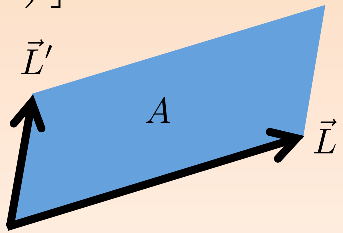

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- The classical Lagrangian starting from a seed of $D_\perp$ free massless bosons is Nambu-Goto in static gauge: [Cavaglia, Negro, Szecsenyi, Tateo]

$$\mathcal{L} = \frac{1}{\ell_s^2}\left(\sqrt{\det(\delta_{ab} + \ell_s^2\, \partial_a \vec{X} \partial_b \vec{X})} - 1\right)$$

S-matrix approach

Dressing phase for the S-matrix:

- Scattering on the worldsheet of the NG string is an integrable theory:

[Dubovsky, Flauger, Gorbenko]

$$\hat{S}_{2\to 2}(s) = \exp\left(i\frac{\ell_s^2}{4}\,s\right)\,, \qquad \Delta t = \frac{\ell_s^2}{2}\,E = \lambda\,E$$

Gravitational phase shift. For $\lambda > 0$ healthy theory, likely no local observables. Superluminarity for $\lambda < 0$, but S-matrix is well-defined (though not polynomially bounded).

[Cooper, Dubovsky, Mohsen]

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- S-matrix dressing in generic 2d theory:

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$$\hat{S}_n(\{p_\alpha\}) = \exp\left(i\frac{\ell_s^2}{4}\sum_{\alpha<\beta}\epsilon_{ab}\, p^a_\alpha p^b_\beta\right) S_n(\{p_\alpha\})$$

Unitary, crossing symmetric, analytic S-matrix.

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  Unitary, crossing symmetric, analytic S-matrix.

- In the integrable case can go between spectrum and S-matrix using TBA.

$$E(\lambda, L) = \frac{L}{\ell_s^2}\left(\sqrt{1+\frac{2\ell_s^2}{L^2}\,e+\frac{\ell_s^4}{L^4}\,p^2}-1\right)$$

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Coupling matter to flat space JT gravity: $\mathcal{L}_{JT} = \mathcal{L}_{\text{seed}}(g, \phi_m) + \frac{1}{\lambda} + \varphi R$

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$$g_{ab} = \eta_{ab}\,, \qquad \varphi = -\frac{1}{4\lambda}\, x^+ x^-$$

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$$X^{\pm} \equiv -4\lambda\, \partial_{\mp}\varphi \equiv x^{\pm} + Y^{\pm}$$

$$\partial_a Y^b = \frac{\lambda}{2}\, \epsilon_a{}^c \epsilon^{bd}\, T_{cd}$$

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$$A^\dagger_{\text{in}}(p) = a^\dagger_{\text{in}}(p)\, \exp\left(i\, p_a Y^a(\{p_\alpha\})\right)$$

leads to dressed S-matrix.

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- The S-matrix can be understood as the flat space limit of AdS JT gravity boundary correlators. The X^a are analogs of the reparametrization mode of SYK.

- Z_{T^2} can be reproduced, but to get measure right, need to use first order formalism. X^a needs to have torus topology.

[Dubovsky, Gorbenko, Hernandez-Chifflet]

JT gravity approach

Coupling matter to flat space JT gravity: $\mathcal{L}_{JT} = \mathcal{L}_{\text{seed}}(g, \phi_m) + \frac{1}{\lambda} + \varphi R$

- Constructing Lagrangians and conserved charges using dynamical coordinates

$$\partial_a X^b(x) = \delta_a^b + \frac{\lambda}{2}\,\epsilon_a{}^c \epsilon^{bd}\, T_{cd}(x)$$

- Conserved charges

[Conti, Negro, Tateo]

$$P_s = \int_C \mathcal{P}_s$$

$$\mathcal{P}_s = \tau_{s+1}(x)\,dz + \theta_{s-1}(x)\,d\bar{z}$$
$$= \tau^{(\lambda)}_{s+1}(X)\,dZ + \theta^{(\lambda)}_{s-1}(X)\,d\bar{Z}$$

Provides explicit expressions with little effort, provides hints at generalizations.

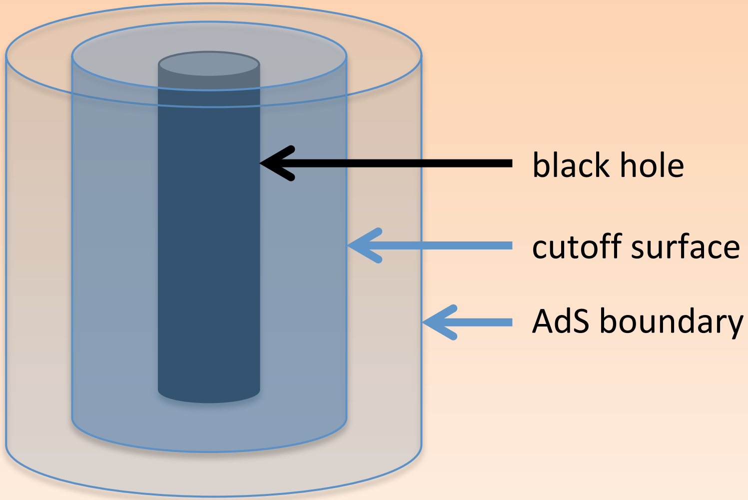
- Mapping of solutions, Lax pair for $T\bar{T}$ - deformed sine-Gordon model.

[Conti, Iannella, Negro, Tateo]

AdS/CFT approach

The analogy with SYK makes it natural to expect a relation with AdS_3 with a finite cutoff.

[McGough, MM, Verlinde]



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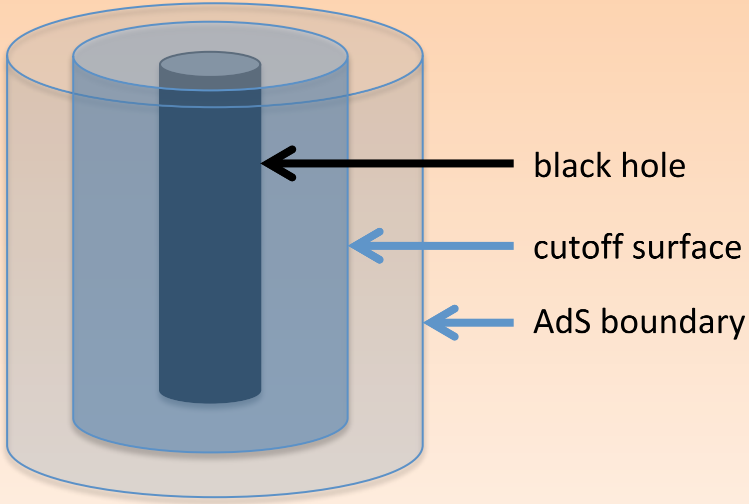
- BTZ black holes geometry

$$ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 \left(d\theta - \omega(r)\, dt\right)^2$$

$$f(r) = 1 - \frac{4G}{\pi r^2}\, e + \frac{4G^2}{\pi^2 r^4}\, p^2 \,, \qquad \omega(r) = \frac{2G}{\pi r^2}\, p$$

Quasilocal energy

$$E(r_c) = \frac{r_c}{4G}\left(1 - \sqrt{f(r_c)}\right) \quad \Longrightarrow \quad \lambda = -\frac{4\pi G}{r_c^2} \quad (L = 2\pi)$$



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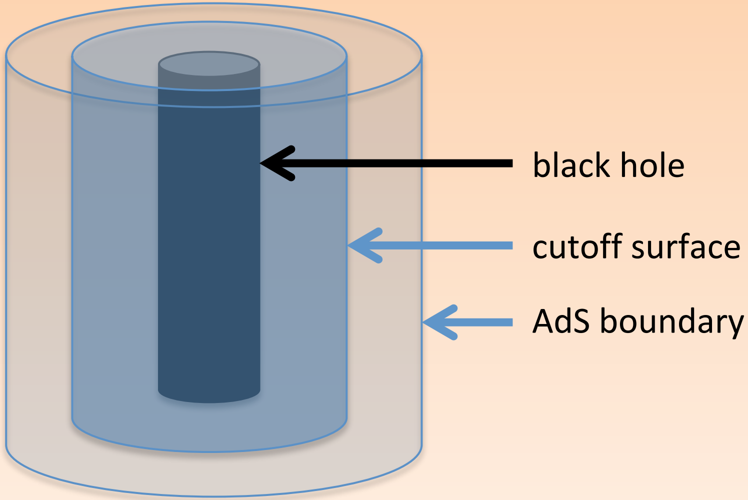

- Superluminality – boundary gravitons propagate along

$$\frac{d\theta}{dt} = \pm 1 \quad \Longrightarrow \quad \frac{d\theta_c}{dt_c} = \frac{1 \pm \omega(r_c)}{\sqrt{f(r_c)}}$$

[Marolf, Rangamani; Cooper, Dubovsky, Mohsen; Cardy; McGough, MM, Verlinde]

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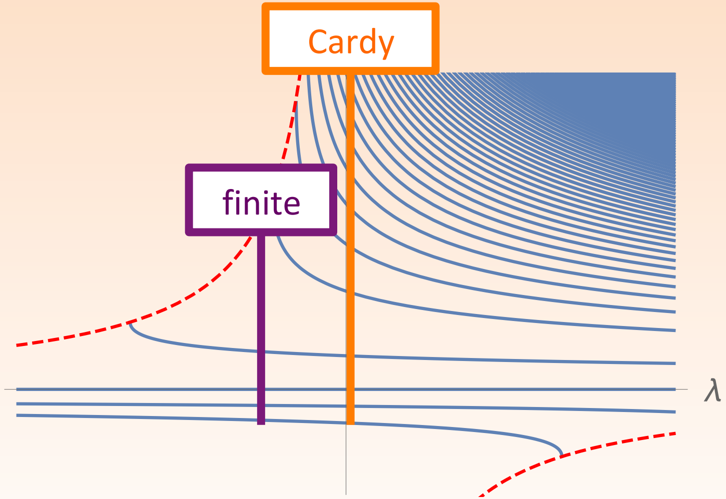
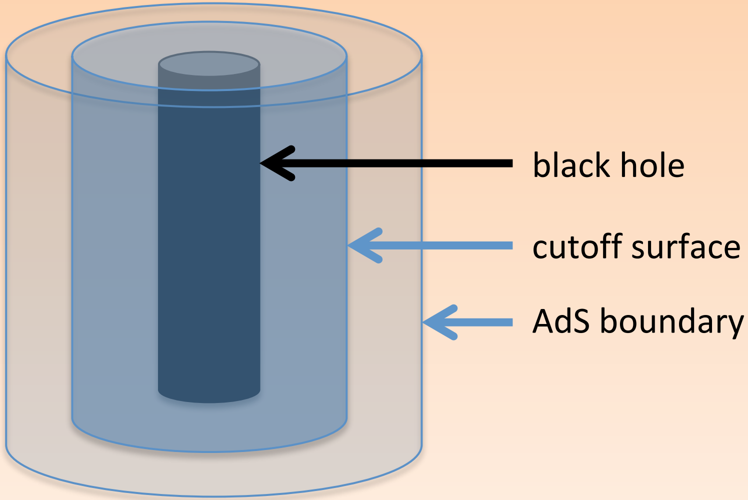
[McGough, MM, Verlinde]

- Finite number of states – the largest black hole that fits inside the cutoff has finite entropy.

$$E(r_c) = \frac{r_c}{4G}\left(1 - \sqrt{f(r_c)}\right) \quad \Longrightarrow \quad \lambda = -\frac{4\pi G}{r_c^2} \quad (L = 2\pi)$$





AdS/CFT approach

The analogy with SYK makes it natural to expect a relation with AdS_3 with a finite cutoff.

- Holographic RG can be rewritten in the form [McGough, MM, Verlinde]

$$ds^2 = \frac{dr^2}{r^2} + r^2 g_{ab}(x,r)\, dx^a dx^b \,, \qquad \lambda = -\frac{4\pi G}{r_c^2}$$

Flow equation:

$$\frac{dS[g,J;\lambda]}{d\lambda} = -\frac{1}{2}\int d^2x\,\sqrt{g}\,X$$

$$X = T^{ab}T_{ab} - (T^a_a)^2 - \frac{c}{24\pi\lambda}\,R$$

[Shyam; Taylor; Hartman, Kruthoff, Shaghoulian, Tajdini]

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- Deviations from $T\bar{T}$ upon including matter fields. [Guica’s talk] Pure gravity story provides compelling interpretations of some features of $T\bar{T}$ deformed theories.

$$\frac{dS[g,J;\lambda]}{d\lambda} = -\frac{1}{2}\int d^dx\, \sqrt{g}\left(X_d + \#\lambda^{-2(d-\Delta)/d}\,\mathcal{O}^2 + \#\lambda^{2(d-\Delta+1)/d}\,(\partial_a J)^2\right)$$

[Kraus, Liu, Marolf; Hartman, Kruthoff, Shaghoulian, Tajdini; Guica, Monten]

Outline

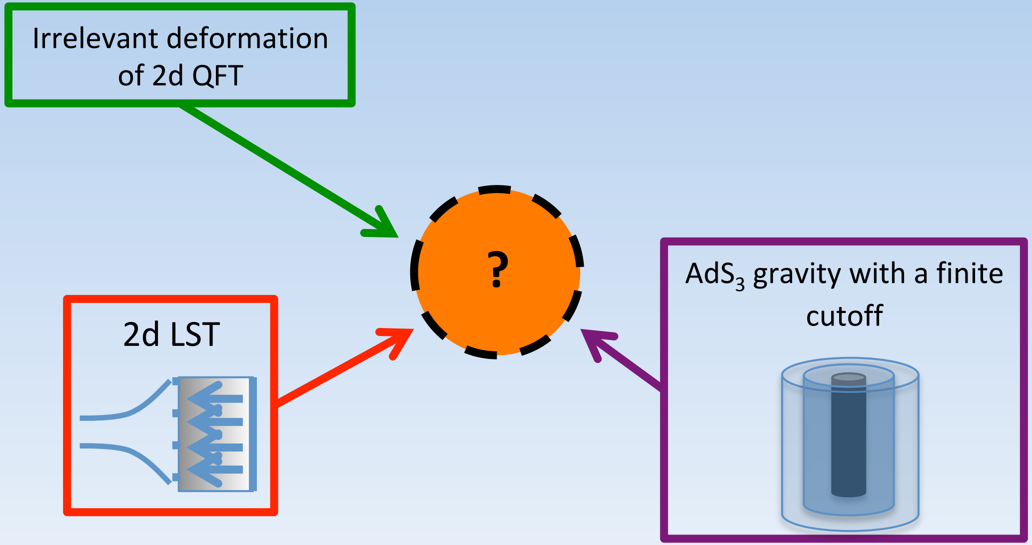
Different formulations

- QFT approach
- JT gravity
- Holography

Generalizations

- Higher dimensions
- Single trace
- Related deformations

Open questions



Holographic generalizations

Holographic generalizations:

- Rewrite holographic RG in terms of field theory. Using large-N factorization match between field theory and gravity.

[Taylor; Hartman, Kruthoff, Shaghoulian, Tajdini; Shyam; Caputa, Datta, Shyam]

$$\lambda = -\frac{8\pi G}{d\, r_c^d}$$

$$X_4 = T^{ab}T_{ab} - \frac{1}{3}(T_a^a)^2 + \sqrt{\frac{-C_T}{4\pi\lambda}}\left(G^{ab}T_{ab} - \frac{1}{3}G_a^a T_b^b\right) - \frac{C_T}{16\pi\lambda}\left(G^{ab}G_{ab} - \frac{1}{3}(G_a^a)^2\right)$$

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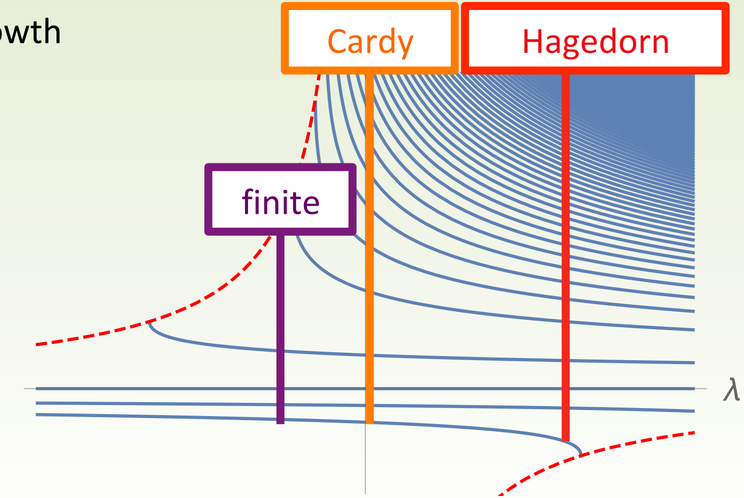
- For $\lambda > 0$ Hagedorn growth from Cardy growth of the density of states:

$$E(\lambda, L) \approx \sqrt{\frac{e}{\lambda}}$$

$$S_{\text{Cardy}} \approx \sqrt{\frac{2\pi c}{3}}e \approx \sqrt{\frac{2\pi c\lambda}{3}}\,E$$

Can be anticipated from Nambu-Goto.



[Dubovsky, Flauger, Gorbenko; Giveon, Itzhaki, Kutasov]

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- Black holes in linear dilaton backgrounds produce this density of states, $T\bar{T}$ deformed theories share features of 2d Little String Theories. [Giveon, Itzhaki, Kutasov]

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[Giveon, Itzhaki, Kutasov]

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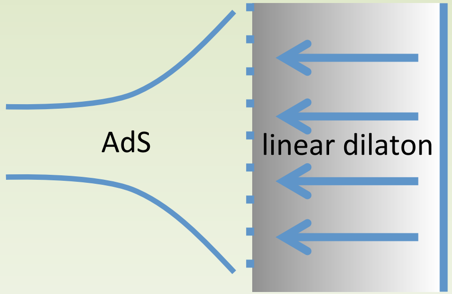

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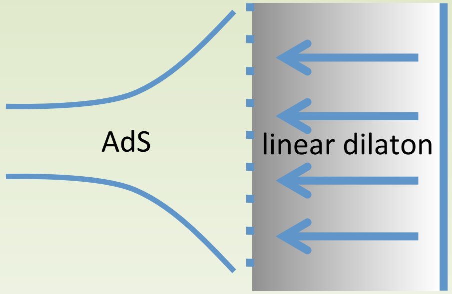

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- The dual CFT is of the form $\mathcal{M}^N/S_N$ [Eberhardt's talk], deformed long string spectrum reproduced by the **single trace** deformation: [Eberhardt, Gaberdiel] [Giveon, Itzhaki, Kutasov]

$$\lambda \sum_{i=1}^{p} T_i \bar{T}_i$$

Related deformations

There exist many factorizing quadratic operators, can deform by them, the deformed theory preserves many symmetries.

[Smirnov, Zamolodchikov]

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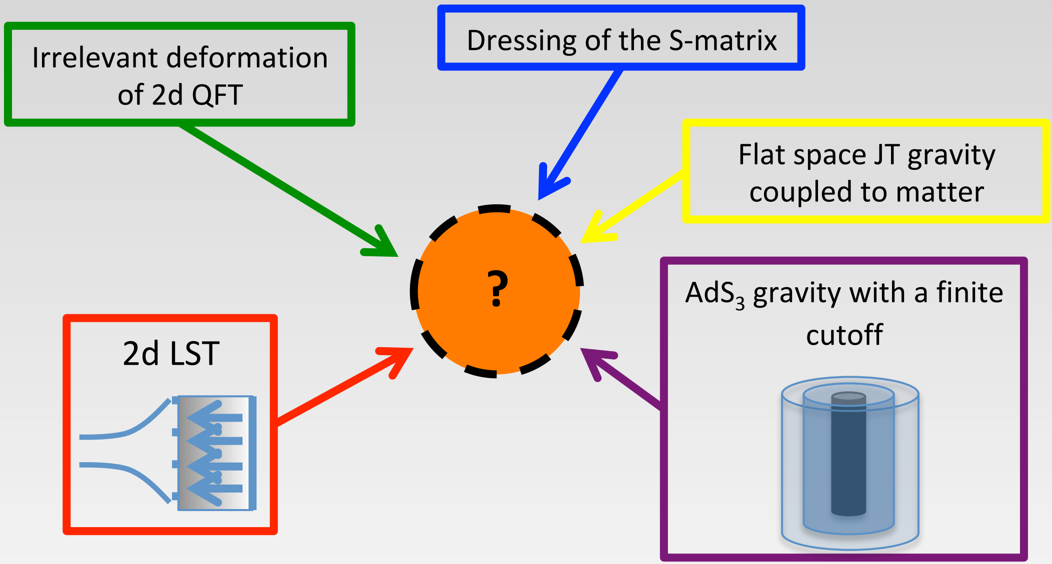
- In dS/dS correspondence another variant is important: [Gorbenko, Silverstein, Torroba]

$$\frac{d}{d\lambda} S(\lambda) = -\frac{1}{\pi^2} \int d^2x \, \left(T\bar{T}_\lambda(x) - \frac{\#}{\lambda^2} \right)$$

Nontrivial change in the spectrum.

Open questions

- Applications to phase transitions and QCD? [Cardy; Zamolodchikov; Dubovsky et al.]
- Putting the theory in curved space? [Jiang]
- What generalizations of $T\bar{T}$ are solvable? New possible UV behaviors?
- How is the $\lambda < 0$ theory best defined? Does string theory in cutoff AdS_3 make sense?
- How non-local is the theory? Observables?



Reference material: $T\bar{T}$ and Other Solvable Deformations of Quantum Field Theories (SCGP video library), Jerusalem Lectures by Giveon, Kutasov, and Zamolodchikov (youtube), [Jiang]