Near-Extremal Black Holes and the Jackiw-Teitleboim Model

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Outline

- Introduction
- Thermodynamics
- Low frequency response
- Concluding Comments

Based on:

1) P. Nayak, A. Shukla, R. Soni, V. Vishal and ST, hep-th/1802.09547

2) U. Moitra, M. Vishal and ST hep-th/1808.08239

3) U. Moitra, S. Sake, ST and V. Vishal hep-th/1905.10378

M. V. Vishal

Upamanyu Moitra

Sunil Sake

Introduction

- Extremal Black holes have been a rich and rewarding subject of study.
- Success in State counting: Susy and Non-Susy
- Attractor Mechanism: Near horizon geometry: AdS_2
- Moduli and L_{AdS_2} drawn to fixed values at horizon determined by charges.
- Similar to Flux Compactfications

Introduction

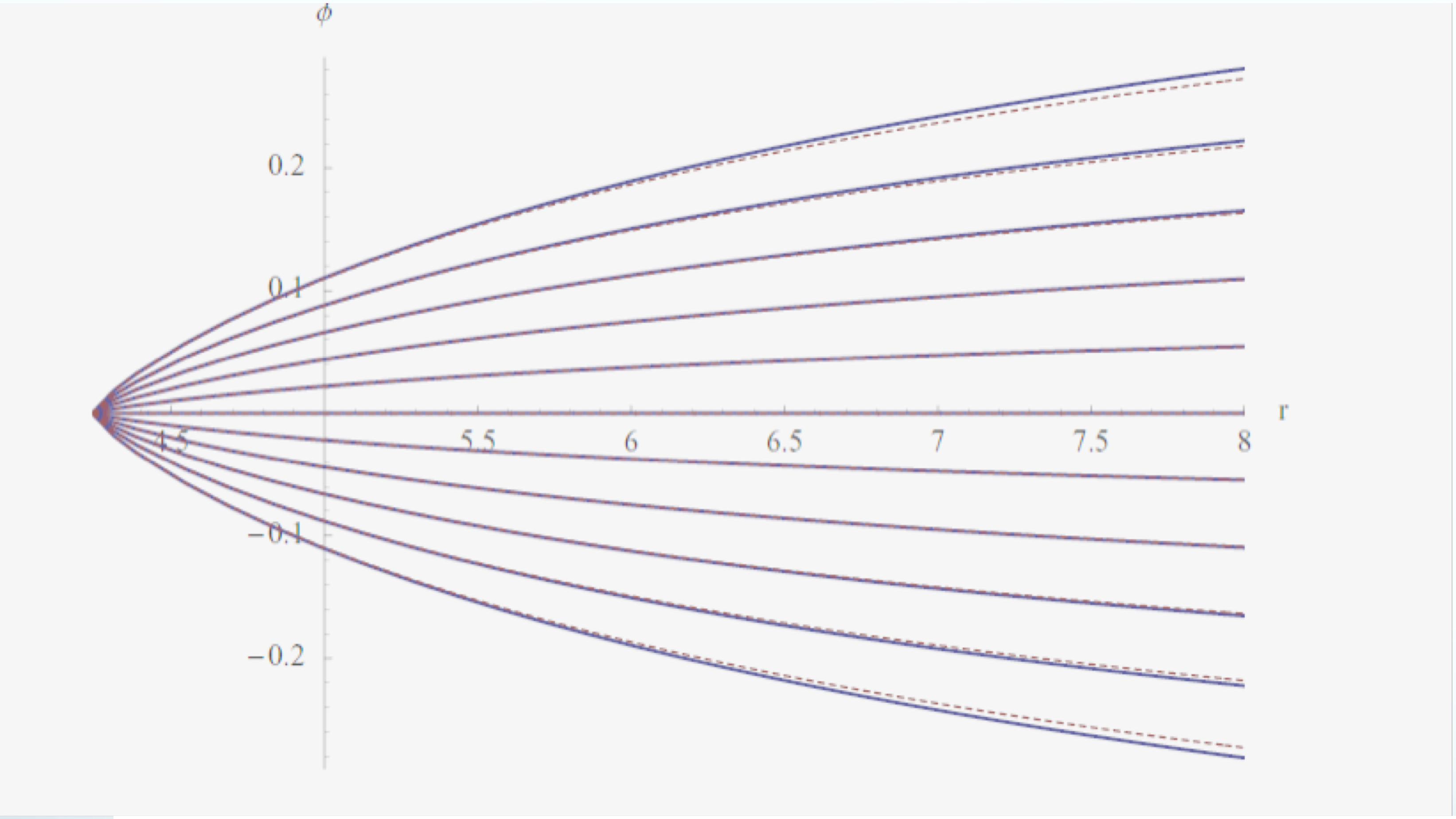
Can be understood by extremising an Effective potential or Entropy Function

Works for Susy and Non-susy cases.

Ferrara, Kallosh, Strominger Phys. Rev. D 52, R5412 (1995), …,

A. Sen, *JHEP* **09** (2005) 038

Goldstein, Iizuka, Jena and ST, PRD 72 (2005)



Goldstein, Iizuka, Jena and ST, PRD 72 (2005) 124021

Introduction

E.g.: Extremal Kerr in 4- Dim

Near-horizon geometry can be understood as an attractor obtained by extremising the entropy function.

$$ds^2 = \frac{1+\cos^2\theta}{2}\left(\frac{r^2}{2r_h^2}dt^2 + \frac{2r_h^2}{r^2}dr^2 + 2r_h^2 d\theta^2\right) + \frac{4r_h^2\sin^2\theta}{(1+\cos^2\theta)}\left(d\phi - \frac{r}{2r_h^2}dt\right)^2$$

Bardeen Horowitz, PRD 60 (1999) 104030;
Astefanesci, Goldstein Jena, Sen, ST, JHEP 10 (2006) 058

Beyond Extremality?

- Here the situation for black holes, unlike extended black branes, has been less clear until recently.
- For black branes at small temperatures or low frequencies the response can be understood as arising from the near horizon AdS_{d+1} region.
- The celebrated AdS/CFT correspondence

Beyond Extremality?

- For Black Holes situation it has been known for some time that deviations from the attractor solution must be included.

Physically this is because :

The backreaction in the near-horizon geometry to any probe is singular since internal volume is fixed at a finite size (e.g. $AdS_2 \times S^{d-1}$)

The energy does not have ``enough space" to dissipate in. (analogy: Electromagnetism)

But how to incorporate the corrections in a precise way was not understood until recently.

(Internal space: $S^{d-1}, S^n \times \mathcal{M}$, etc).

In a seminal paper, Maldacena, Stanford and Yang (MSY) showed in a particular model of 2 dimensional gravity, the Jackiw-Teitleboim (JT) model, that once the leading corrections to the near horizon solution is incorporated, the finite energy response is well controlled.

Teitleboim, Phys. Lett., 126B (1983) 41.
Jackiw, NPB *252 (1985) 343*,
Almheiri, Polchinski, JHEP 11 (2015) 014,
Maldacena Stanford and Yang, *Phys. Rev.* **D94** (2016) 106002

- They were motivated by earlier work on condensed matter systems by Sachdev, Ye and Kitaev (many subsequent follow-ups by now).

- *Sachdev and Ye, Phys. Rev. Lett.* **70** (1993) 3339

- A. Y. Kitaev, Talk at KITP

- ………………………………………………………………
- ………………………………………………………………
- ………………………………………………………………..
- (Many important follow ups !).

Beyond Extremality?

- The physical argument suggests that corrections to the attractor value of the Volume of the internal space must be incorporated

- But other components of the metric also deviate from their attractor values at the same order, so at first sight it is not clear how to truncate to a near horizon region consistently.

Introduction

- The aim of our work was to investigate whether the JT model correctly reproduces the behaviour of higher dimensional extremal and near-extremal black holes.

- At low temperatures and low frequencies.

MSY studied the Jackiw- Teitleboim (JT) model of 2 dim gravity.

$$I = \frac{1}{16\pi\tilde{G}}[\int d^2x\sqrt{-g}\ \phi(R + \frac{2}{L_2^2}) + 2\int_\partial dx\sqrt{-\gamma}\phi(K - \frac{1}{L_2})]$$

ϕ - Dilaton can be thought of as volume of internal space

Boundary terms are important.

System admits an AdS_2 solution:

$$ds^2 = \frac{L_2^2}{z^2}(-dt^2 + dz^2)$$

$$\phi = \frac{1}{Jz}$$

Boundary at $\phi = \phi_B$

The ``rolling" dilaton breaks the SL(2,R) symmetry of AdS_2 .

This breaking, characterised by scale J, is extremely important.

Model has three parameters:

$\tilde{G}$: Two Dimensional Newton's Constant

J : Scale of breaking of conformal invariance

L_{AdS_2} : Radius of AdS_2

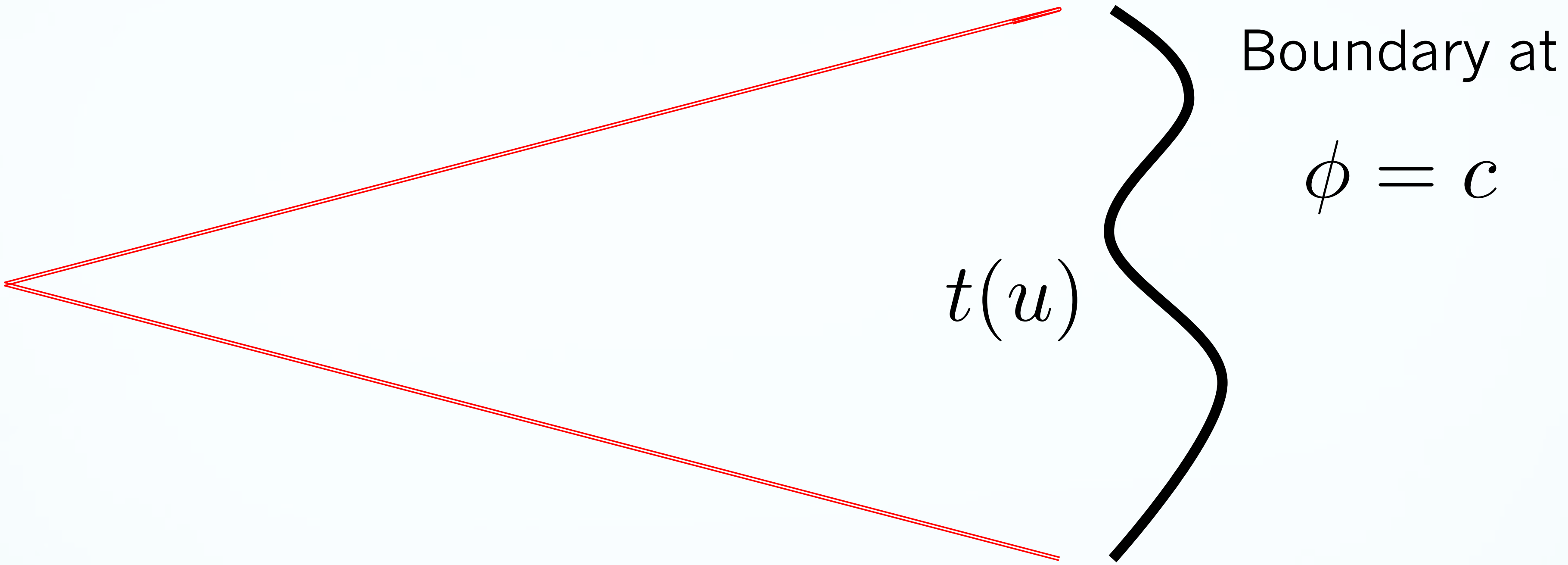
The result for the dynamics of this system that MSY got was very elegant.

The response of the system arises due to the fluctuations of the boundary. These fluctuations can be characterised in terms of time reparametrisations: $t(u)$

u – proper time along boundary.

With action involving Schwarzian derivative:

$$I = \frac{1}{8\pi\tilde{G}J}\int du\{Schw(t(u),u\}$$



The fluctuations can be thought of as arising from diffeomorphisms which are asymtotically isometries of AdS_2 . These diffeomorphisms do not leave the boundary invariant.

Thermodynamics:

Correctly reproduced by this action near extremality.

Entropy:

$$S = S_0 + \frac{\pi}{2\tilde{G}} \frac{T}{J}$$

Response to a probe :

Arises by coupling Schwarzian action to the probe. The coupling is determined by symmetries.

$$(\nabla^2 + m^2)\psi(z,t) = 0$$

$$\Delta = \sqrt{\frac{1}{4} + m^2 L_2^2}$$

Gives rise to a four point function :

$$S_{4pt} \sim \int \prod_{i=1}^{4} d\omega_i \psi(\omega_1)\psi(\omega_2)\psi(\omega_3)\psi(\omega_4) < O(\omega_1)O(\omega_2)O(\omega_3)O(\omega_4) >$$

$$< O(\omega_1)O(\omega_2)O(\omega_3)O(\omega_4) \sim J \omega^{4\Delta-3} \ \delta(\sum \omega_i)$$

Breaking of conformal invariance important.

Enhancement by a factor of $\frac{J}{\omega}$

Our Analysis: Central Result

- The JT model, with a suitable identification of parameters, is a good approximation for a wide class of extremal and near-extremal black holes
- At low temperatures and small frequencies

$$\frac{T}{J} \ll 1 \qquad \frac{\omega}{J} \ll 1$$

- True for spherically symmetric black holes: Thermodynamics and Response to probes.

- Shown to be true for the thermodynamics of rotating black holes as well.

Our Analysis: Some Key Points

The dilaton is related to the volume of the internal space (more correctly the deviation of this volume from its attractor value).

E.g.:

$$ds^2 = \frac{L_2^2}{z^2}(-dt^2 + dz^2) + \Phi^2 d\Omega_{d-1}^2$$

$$\Phi = \Phi_0(1 + \phi)$$

Deviation from attractor

Our Analysis: Some Key Points

- A key point is that many other fields (e.g., components of the metric) besides the dilaton, also deviate from their attractor values, at the same order in $\frac{1}{J}$
- These deviations are not captured correctly in the JT model.
- However these deviations lead to sub-leading contributions at low-temperatures and low-frequency response, suppressed by powers
- of $\frac{T}{J}$, $\frac{\omega}{J}$ and can therefore be neglected.

Our analysis

E.g., In spherically symmetric case, dimensional reduction to 2 dim. Gives rise to additional term at quadratic order:

$$I = \frac{1}{16\pi\tilde{G}}[\int d^2x\sqrt{-g}\ \phi(R+\frac{2}{L_2^2}) + 2\int_\partial dx\sqrt{-\gamma}\phi(K-\frac{1}{L_2})]$$

$$+\int d^2x\sqrt{-g}[(\nabla\phi)^2 + c\ \phi^2]$$

Removed by change of frame

Goes like $\frac{1}{J^2}$

Our analysis

Equation of Motion of Dilaton:

$$R = -\frac{2}{L_2^2} - 2c\,\phi$$

Geometry Deviates From AdS_2 at order $\frac{1}{J}$

Our analysis

But resulting change in action is O($\frac{1}{J^2}$)

Therefore suppressed.

$$I = \frac{1}{16\pi\tilde{G}}\left[\int d^2x\sqrt{-g}\ \phi(R + \frac{2}{L_2^2}) + 2\int_\partial dx\sqrt{-\gamma}\phi(K - \frac{1}{L_2})\right]$$

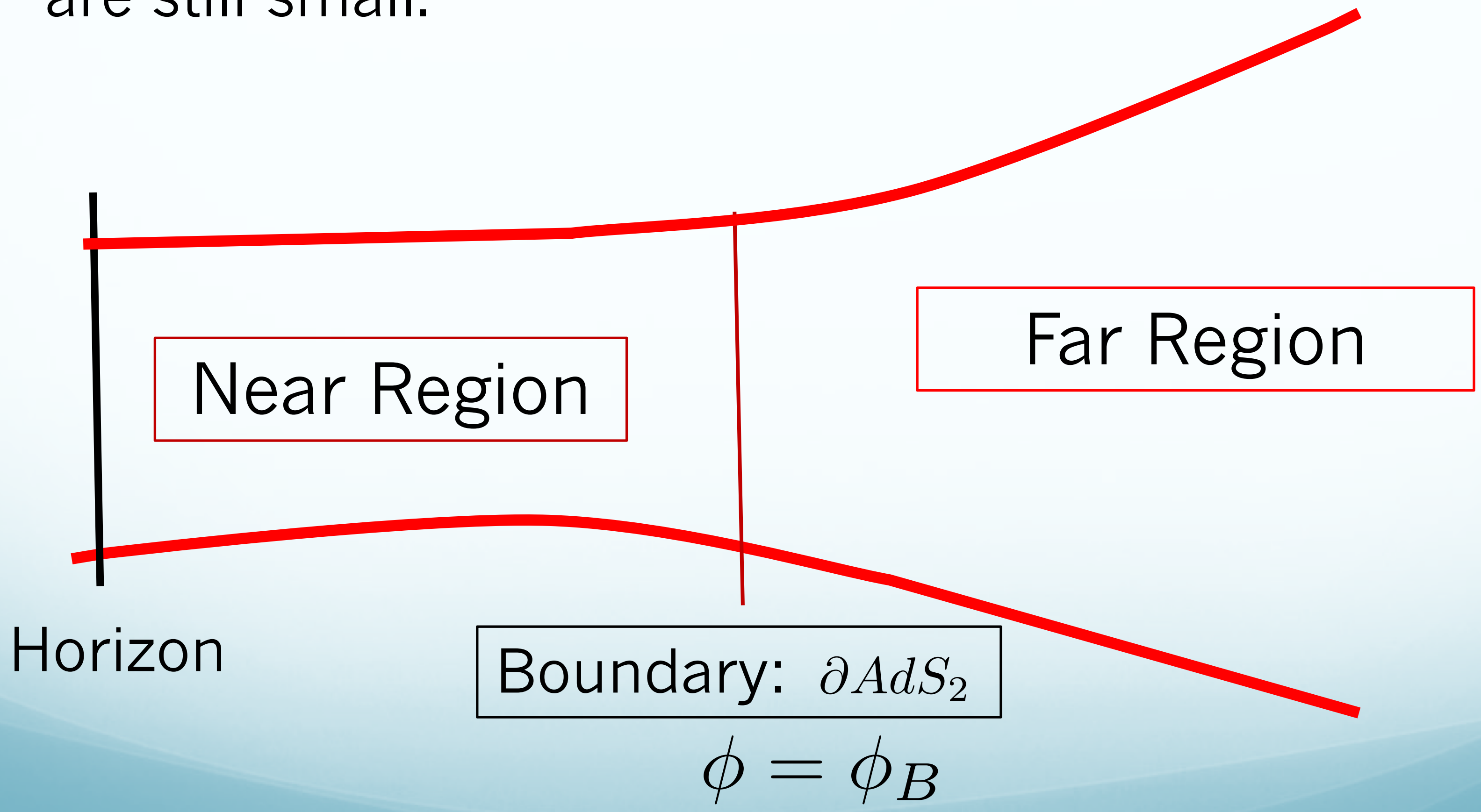
$$+ \int d^2x\sqrt{-g}\ c\ \phi^2$$

Goes like $\frac{1}{J^2}$

$$\phi = \frac{1}{Jz}$$

It is important that the boundary needs to be located in the ``Asymptotic AdS_2 region”

Where the effects of finite temperature have died away, while the deviation from the attractor solution are still small.



Our Analysis: Summary

- The final conclusion is therefore very simple and elegant.
- We expected the deviations in the volume of the internal space to be important to allow the back reaction to be tamed.
- This turns out to be the only deviation we need to include at leading order. Other corrections are not as important.
- The JT model keeps track of this deviation, via the dilaton, and is therefore describes, universally, extremal and near-extremal black holes with a near-horizon AdS2 (more checks needed in rotating case).

Thermodynamics

Why is the JT Model a good approximation for low temperatures?

Consider the temperature dependence of the Free energy (at fixed charge)

The Free energy is given by the on-shell action.

Start with extremal solution and consider the change in metric due to ``turning on" a small temperature

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \Delta g_{\mu\nu}$$

Thermodynamics

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \Delta g_{\mu\nu}$$

Small perturbation in far region

Since $\bar{g}_{\mu\nu}$ is a solution to the equations of motion, which extremise the action, the first order change in on-shell action vanishes in far region

Remaining contribution in near region correctly given by the JT model.

More correctly, action in far region vanishes after adding boundary terms at AdS_2 boundary (locus of constant dilaton)

JT Model

$$\frac{1}{16\pi G}[\int_{H\to\partial AdS_2]} d^{d+1}x\sqrt{g}[R-\Lambda_{d+1}+\cdots] - \frac{1}{8\pi G}\int_{\partial AdS_2} d^d x\sqrt{\gamma}K$$

$$+\frac{1}{16\pi G}[\int_{[\partial AdS_2\to\infty]} d^{d+1}x\sqrt{g}(R-\Lambda_{d+1})+\cdots] + \frac{1}{8\pi G}\int_{\partial AdS_2} d^d x\sqrt{\gamma}K$$

Vanishes

This is true of rotating case as well.

In near region, after carrying out integral over angular coordinates one gets the JT model in remaining 2 dimensions.

We have checked explicitly in many cases, 4 and 5 dim. Rotating black holes, including Kerr in 4 Dim.

$$F = -\frac{\pi}{\tilde{G} J} T^2$$

Determined by the deviation of the internal volume from attractor value

$$\tilde{G} = \frac{G_{d+1}}{V_{d-1}}$$

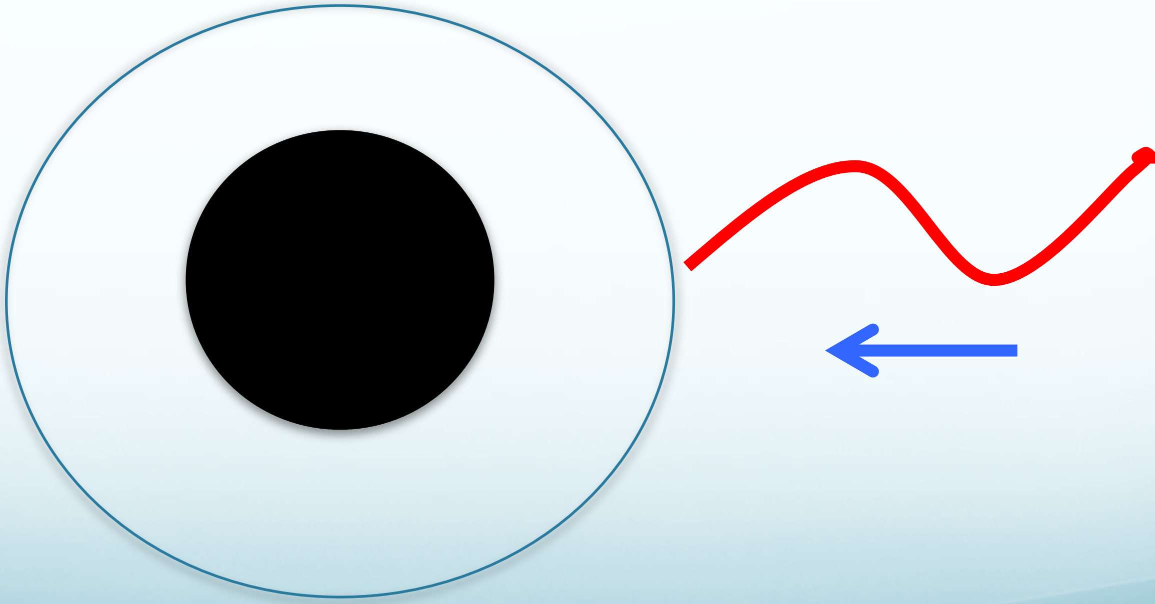
Horizon area

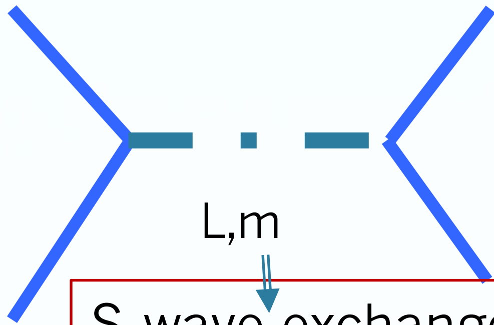
Response to Low-frequency probes:

Worked out for Non-rotating case so far.

Response arises from Near-horizon region.

Break up Analysis into different partial waves.





S-wave exchange, when allowed, dominates

- Gravity effects in S-wave arise as analogue of Coulomb effects in Electromagnetism. Described by Schwarzian action. Dominate at low energies.

- Agreement with Schwarzian contribution checked by direct calculation in AdS_4

- Higher partial waves suppressed at low energies. Their effects can be cumulatively significant at intermediate energies. Breaking of Conformal invariance not important for these.

Higher partial waves suppressed at low energies. Their effects can be cumulatively significant at intermediate energies. Breaking of Conformal invariance not important for these.

Physically this is because for higher partial waves the effects, when averaged over the internal sphere, cancel out. So the back reaction is controlled even without letting the volume of the internal space grow.

Four Point Function

S-wave

$$< O(\omega_1)O(\omega_2)O(\omega_3)O(\omega_4) >\sim J\omega^{4\Delta-3}\delta(\sum_i \omega_i)$$

Higher Partial Wave

$$< O(\omega_1)O(\omega_2)O(\omega_3)O(\omega_4) >\sim \omega^{4\Delta-2}\delta(\sum_i \omega_i)$$

(Notice S wave contribution blows up when $J \to \infty$)

Important Comment:

The S-wave exchange due to a gauge field also give an enhanced contribution, due to Coulomb effects.

One can show that an extra phase mode arises from a gauge field.

$$S_{phase} = \frac{c}{\hat{J}} \int dt(\dot{\theta})^2$$

Arises due to deviation of scalars, which determine the gauge coupling, from their attractor values

This phase mode can be intuitively thought of as a Wilson line which extends from the horizon to the boundary of the near AdS_2 -in a spherically symmetric way. (Maldacena)

Its coupling to the matter field can be determined from gauge invariance.

More generally for a general isometry group G:

$$S_{phase} = \frac{c}{\hat{J}} \int dt Tr(g^{-1}\dot{g} g^{-1}\dot{g})$$

Phase Mode in SYK models and JT:
Davison, Fu, Georges, Gu, Jensen, Sachdev, Phys. Rev. B95 (2017).

Cherns Simons theory:
Gaikwad, Joshi, Mandal,Wadia, arXiv:1802.07746.

(Many more references)

- More generally for a general isometry group G, the phase mode action is given by :

$$S_{phase} = \frac{c}{\hat{J}} \int dt Tr(g^{-1}\dot{g}g^{-1}\dot{g})$$

- Also, if there is a background charge, the phase mode couples to time reparametrisations: $t \to t + \epsilon(t)$

$$S_{phase} \sim \frac{1}{\hat{J}} \int d\tau (\dot{\theta} + \dot{\bar{\theta}}\dot{\epsilon})^2$$

$$Q \propto \dot{\bar{\theta}}$$

Background charge

- This coupling needed to produce the correct 4-point function for charged matter.

Response to low frequency probes

- The final Result is thus quite elegant. The S-wave sector dominates at low frequencies.
- The JT model, along with extra phase modes, incorporate the effects of this sector and therefore gives the leading low frequency response.
- For higher partial wave exchange breaking of conformal invariance is not important. These can therefore be included in standard ways based on AdS/CFT.

Response to low frequency probes

- For rotating case, details (beyond thermodynamics) are still being worked out. Isometries of S^{d-1} give rise to gauge fields in . AdS_2.

- Suggests that rotation can be incorporated also in a near horizon description based on the JT model with corresponding phase modes.

- Super radiant modes etc remain to be understood.

Kerr/CFT:

M. Guica, T. Hartman, W. Song and A. Strominger, PRD 80 (2009)

T. Hartman, K. Murata, T. Nishioka and A. Strominger, *JHEP* **04** (2009)

....

A. Castro, F. Larsen and I.Papadimitriou,   JHEP 1810, (2018)

A. Castro and V.Godet, arXiv:1906.09083

Concluding Comments

1. Behaviour of extremal and non-extremal black holes is correctly described by the JT model at low temperatures and low frequencies.

2. Leading corrections only require that the internal volume be allowed to change from its attractor value. (Plus phase modes).

Concluding Comments:

3. Near AdS2 Near CFT1:

- Our results suggest that the microscopic theory describing a near-extremal black hole should flow at low energies to a near-CFT1 governed by a Schwarzian action and phase modes.
- This should be universally true.
- It would be good to check this on the field theory side directly.

Summary and Conclusions

4. New-Extremal Fluid Mechanics?

The low frequency response was studied in the probe approximation.

- What about large amplitude slowly varying situations? Can a fluid-gravity correspondence for the near-extremal fluid be formulated ? Valid when

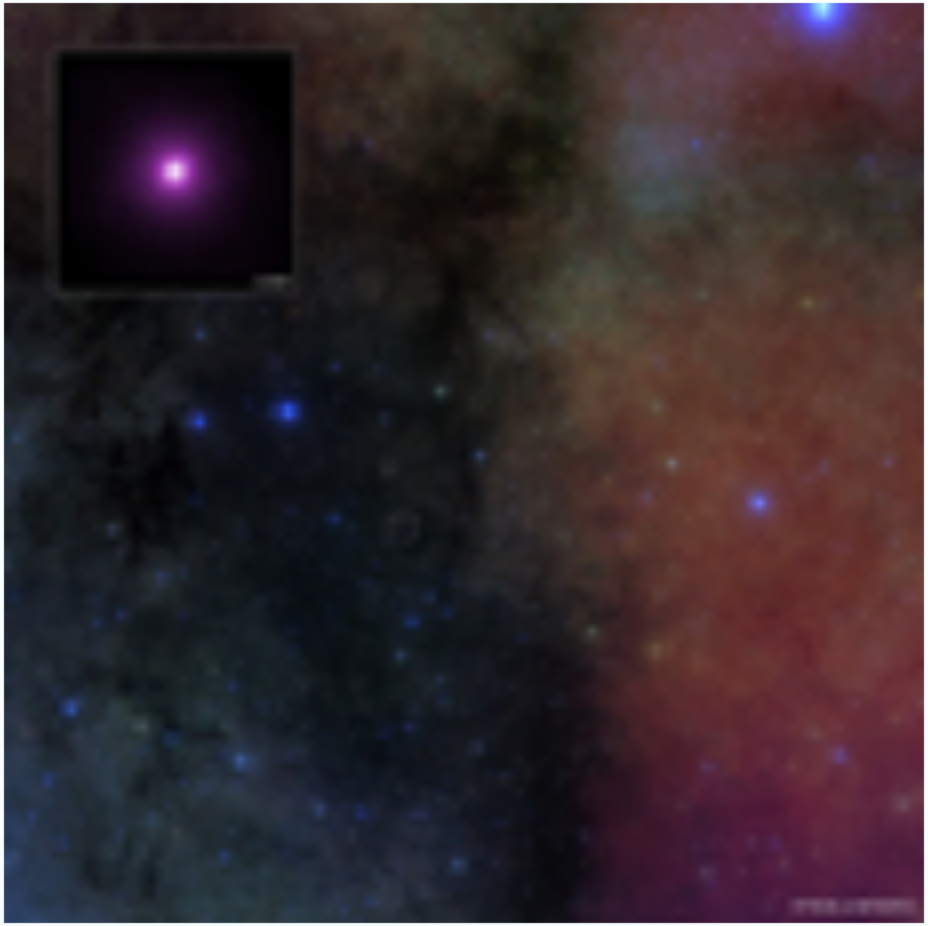
$$T \ll \frac{1}{L} \ll \mu$$

- L: length scale of variation, μ : Chemical Potential
- (Home work assignment given by Prof. Shiraz !)

Concluding Comments

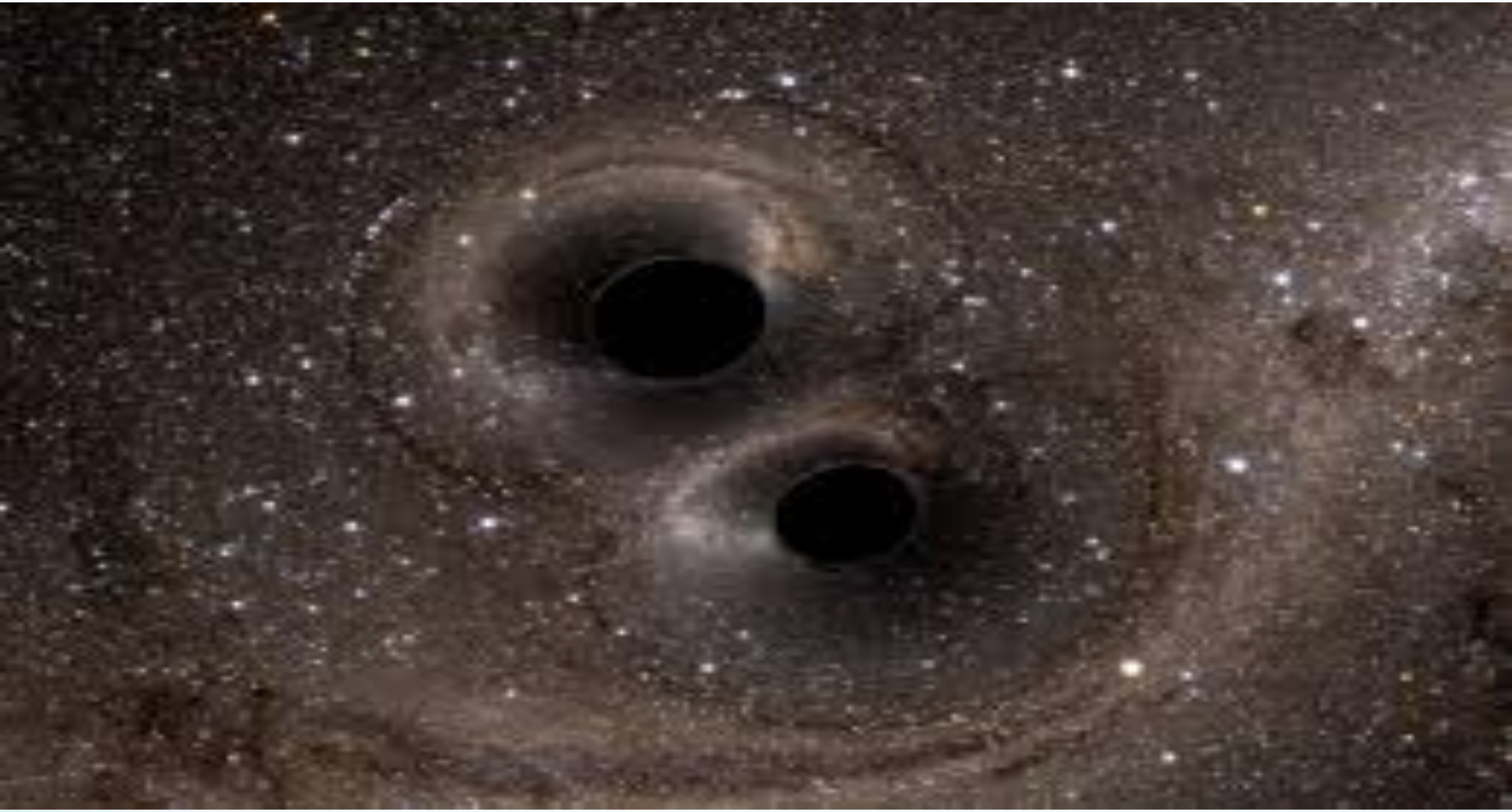
5. Near Extremal Kerr in the Sky?

- Results suggest that the JT model could be useful in studying near-extremal Kerr black holes found in nature (The approximation would be valid at time scales longer than the horizon light crossing time).



(GRS 1915+105, Chandra Observatory)

Thank You!



Thank you

Next consider a background gauge field.
In this case the phase mode which arises also couples to the time reparametrisation mode.

$$S_{phase} \sim \frac{1}{\hat{J}} \int d\tau (\dot{\theta} + \dot{\bar{\theta}} \dot{\epsilon})^2$$

R.~A.~Davison, W.~Fu, A.~Georges, Y.~Gu, K.~Jensen and S.~Sachdev, Phys. Rev. B95 no. 15, 155131 (2017)

Cumulative contribution:

- No. of KK modes $n_{KK} \sim (\frac{R_h}{L_2})^d$
- For $\omega < J\ n_{KK}$
  Higher partial wave channels can be neglected.

- For $J n_{KK} < \omega < \omega_{max}$
  Higher partial waves are important. Their contribution can be obtained from AdS_2 region neglecting breaking of conformal invariance

- ω_{max} determined by requirement that the response arises from near horizon geometry.

Introduction

The low-energy behaviour of near-extremal systems has proved to be a rich and rewarding area of study.

For black branes (as opposed to black holes) the situation is now quite well understood.

The near-horizon region is often $AdS_d \times S^D$ with $d \geq 3$

$$S_{coup} \sim \int dt\ dt'\ \frac{\psi^\dagger(t)\psi(t)}{|t-t'|^{2\Delta_+}}\ [\theta(t)-\theta(t')]$$

$$S_{4pt} = \int \prod_{i=1}^{4} d\omega_i \psi(\omega_1)\psi(\omega_2)\psi^\dagger(\omega_3)\psi^\dagger(\omega_4) < O(\omega_1)O(\omega_2)O(\omega_3)^\dagger)O(\omega_4)^\dagger >$$

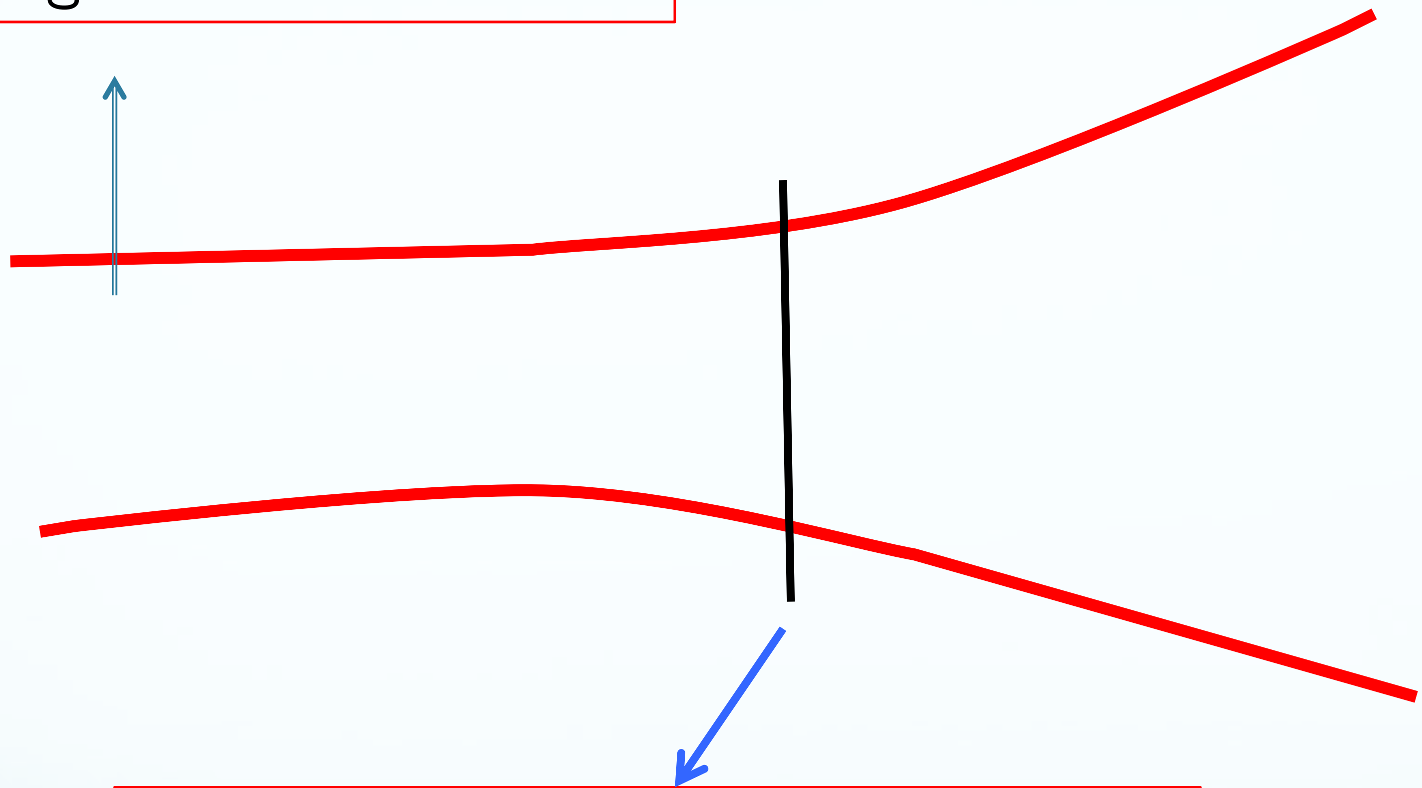
$$< OOO^\dagger O^\dagger) > \sim \hat{J}\ \omega^{(4\Delta-3)}\ \delta(\sum_i \omega_i)$$

Near AdS2 Near CFT1

- The behaviour of near-extremal black holes, at low frequencies, arises from the near horizon AdS_2 region after incorporating some effects due to the breaking of conformal invariance.

- The bulk dynamics itself reduces to that of the boundary.

long throat



Boundary fluctuations described by Schwarzian action, along with phase modes, give rise to response.