# Exceptional Field Theories & Applications


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# duality symmetries in supergravity

- upon toroidal reduction on $T^d$, eleven-dimensional supergravity exhibits the global exceptional symmetry group $E_{d(d)}$

D=11 supergravity $\xrightarrow{T^7}$ D=4 supergravity

maximal supersymmetry, global $E_{7(7)}$

[Cremmer, Julia 1979]

IIB supergravity $\xrightarrow{T^5}$ D=5 supergravity

maximal supersymmetry, global $E_{6(6)}$

after proper dualization/reorganisation of the fields

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after proper dualization/reorganisation of the fields

- the compact subgroup $SU(8) \subset E_{7(7)}$ can be made visible already in eleven dimensions [de Wit, Nicolai 1986]

$$SO(1,10) \longrightarrow SO(1,3) \times SO(7) \longrightarrow SO(1,3) \times SU(8)$$

- to which extent are (remnants of) these symmetries present in D=11 ?

# exceptional field theory

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  [Hull, Tseytlin, Duff, Siegel, Hillmann, Hohm, Zwiebach, Berman, Godazgar, Godazgar, Perry, West, Musaev, Coimbra, Strickland-Constable, Waldram, Pacheco, Kwak, Jeon, Lee, Park, Suh, Blair, Malek, Cederwall, Kleinschmidt, Thompson, Edlund, Karlsson, Aldazabal, Grana, Marques, Rosabal, Geissbühler, ... , ...]

double field theory

generalized geometry

exceptional geometry

gauged supergravity

$\longrightarrow$ exceptional field theory

## plan

- exceptional field theory
  - exceptional geometry & tensor hierarchy
  - invariant action functionals
- applications
  - generalized Scherk-Schwarz reductions
  - consistent truncations and AdS vacua

based on work with Olaf Hohm,
Arnaud Baguet, Hadi Godazgar, Mahdi Godazgar, Hermann Nicolai, Gianluca Inverso, Emanuel Malek, Marc Magro, Edvard Musaev, Mario Trigiante, Guillaume Bossard, Martin Cederwall, Franz Ciceri, Axel Kleinschmidt, Jakob Palmkvist, Dan Butter, Ergin Sezgin

**example:** $E_{6(6)}$ exceptional field theory (ExFT)

## D=5 maximal supergravity

after reduction of D=11 supergravity on $T^6$ and proper dualization of the dof's,
the D=5 bosonic Lagrangian takes the $E_{6(6)}$ invariant form

$$\mathcal{L} \;=\; R + \frac{1}{24}\,\partial_\mu \mathcal{M}_{MN} \partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\, F_{\mu\nu}{}^M F^{\mu\nu\,N} \;+ e^{-1}\,\mathcal{L}_{\rm top}$$

[Cremmer, 1980]

$g_{\mu\nu}$ : 5 x 5 external metric

$\mathcal{M}_{MN}$ : 27 x 27 internal metric (scalars), parametrizing the coset $E_{6(6)}/USp(8)$

$A_\mu{}^M$ : 27 vector fields $\longleftrightarrow$ 27 two-form fields $B_{\mu\nu\,M}$

with $\mathcal{L}_{\rm top} \;=\; d_{KMN}\, F^M \wedge F^N \wedge A^K$

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**exceptional field theory:**

– same Kaluza-Klein reorganisation of the higher-dimensional fields

– keeping the dependence on all internal coordinates (non-abelian gauge structure)

# $E_{6(6)}$ exceptional field theory

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[Hohm, HS]

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[Hohm, HS]

- non-abelian gauge structure: generalized diffeomorphisms

$$\mathcal{D}_\mu \;=\; \partial_\mu - \mathcal{L}_{\mathcal{A}_\mu} \qquad\qquad \mathcal{L}_\Lambda V^M \;=\; \Lambda^N \partial_N V^M - \kappa \left[\partial_N \Lambda^M\right]_{\rm adj} V^N$$

[Coimbra, Strickland-Constable, Waldram]

> combining into a single vector parameter $\Lambda^M \;\in\; \mathbf{27}$

$$\Lambda^M \;=\; \begin{cases} \Lambda^m & \text{internal diffeomorphisms} \\ \Lambda_{mn} & \text{internal 3-form gauge transformations} \\ \Lambda_{klmnp} & \text{internal 6-form gauge transformations} \end{cases}$$

> by construction $E_{6(6)}$ covariant: $\mathcal{M}^{-1}\mathcal{L}_\Lambda \mathcal{M} \;\in\; \mathfrak{e}_{6(6)}$ $\qquad \mathcal{L}_\Lambda d^{KMN} \;=\; 0$

# $E_{6(6)}$ exceptional field theory

$$\mathcal{L} = R + \frac{1}{24}\partial_\mu \mathcal{M}_{MN}\partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\mathcal{M}_{MN}\, F_{\mu\nu}{}^{M} F^{\mu\nu\, N} + e^{-1}\mathcal{L}_{\rm top}$$

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[Coimbra, Strickland-Constable, Waldram]

- embedding $\partial_m \longrightarrow \partial_M$ subject to the section constraint

$$d^{KMN}\,\partial_M \otimes \partial_N = 0 \qquad \left\{ \begin{array}{l} d^{KMN}\,\partial_M \partial_N f = 0 \\ d^{KMN}\,\partial_M f\,\partial_N g = 0 \end{array} \right.$$

[Berman, Godazgar, Perry, West, Cederwall, Kleinschmidt, Thompson]

covariant restriction down to 6(5) coordinates

IID: $\partial_M \rightarrow \{\partial_m, \cancel{\partial^{mn}}, \cancel{\partial^{mnpqr}}\}$

IIB: $\partial_M \rightarrow \{\partial_i, \cancel{\partial^{ijk}}, \cancel{\partial^{\alpha}}, \cancel{\partial^{i\alpha}}\}$

# $E_{6(6)}$ exceptional field theory

$$\mathcal{L} \;=\; R + \frac{1}{24}\,\partial_\mu \mathcal{M}_{MN}\partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,F_{\mu\nu}{}^{M}F^{\mu\nu\,N} \;+ e^{-1}\,\mathcal{L}_{\rm top}$$

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[Coimbra, Strickland-Constable, Waldram]

- non-associative gauge algebra $\longrightarrow$ modified YM field strengths

$$\mathcal{F}_{\mu\nu}{}^{M} \;=\; 2\,\partial_{[\mu}\mathcal{A}_{\nu]}{}^{M} - [\mathcal{A}_\mu, \mathcal{A}_\nu]_{\rm E}^{M} + 10\, d^{MNK}\,\partial_K\,\mathcal{B}_{\mu\nu\,N}$$

with **27** two-forms $\mathcal{B}_{\mu\nu\,M}$

and topological term $d\mathcal{L}_{\rm top} \;=\; d_{KMN}\,\mathcal{F}^K \wedge \mathcal{F}^M \wedge \mathcal{F}^N - 40\, d^{KMN}\,\mathcal{H}_K \wedge \partial_M \mathcal{H}_N$

# $E_{6(6)}$ exceptional field theory

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[Hohm, HS]

- "potential"

$$\begin{aligned} V(\mathcal{M},e) \;=\; & \frac{1}{24}\,\mathcal{M}^{MN}\partial_M \mathcal{M}^{KL}\,(12\,\partial_L \mathcal{M}_{NK} - \partial_N \mathcal{M}_{KL}) \\ & -\frac{1}{2}\,g^{-1}\partial_M g\,\partial_N \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}^{MN} g^{-1}\partial_M g\, g^{-1}\partial_N g - \frac{1}{4}\mathcal{M}^{MN}\partial_M g^{\mu\nu}\,\partial_N g_{\mu\nu} \end{aligned}$$

— invariant under generalized diffeomorphisms

— generalized (internal) curvature scalar

# $E_{6(6)}$ exceptional field theory

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

- unique two-derivative action with generalized diffeomorphism invariance
  - modulo section constraints
  - internal $\Lambda^M$ & external $\xi^\mu$ diffeomorphisms
  - uniquely fixed by bosonic symmetries (but can be supersymmetrized)

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- unique two-derivative action with generalized diffeomorphism invariance
  - modulo section constraints
  - internal $\Lambda^M$ & external $\xi^\mu$ diffeomorphisms
  - uniquely fixed by bosonic symmetries (but can be supersymmetrized)

- section constraint admits two inequivalent solutions $d^{KMN} \partial_M \otimes \partial_N = 0$

IID: $\partial_M \to \{\partial_m, \cancel{\partial^{mn}}, \cancel{\partial^{mnpqr}}\}$

IIB: $\partial_M \to \{\partial_i, \cancel{\partial^{ijk}}, \cancel{\partial^{\alpha}}, \cancel{\partial^{i\alpha}}\}$

together with proper dictionary of ExFT fields into IID/IIB supergravity

$$\mathcal{M}_{11\text{D}} = \begin{pmatrix} \mathcal{M}_{kn} & \mathcal{M}_k{}^{mn} & \mathcal{M}_k{}^{mnpqr} \\ \mathcal{M}^{kl}{}_n & \mathcal{M}^{kl,mn} & \mathcal{M}^{kl,mnpqr} \\ \mathcal{M}^{hijkl}{}_n & \mathcal{M}^{hijkl,mn} & \mathcal{M}^{hijkl,mnpqr} \end{pmatrix} \qquad \mathcal{M}_{\text{IIB}} = \begin{pmatrix} \mathcal{M}_{im} & \mathcal{M}_{i\alpha}{}^{mnp} & \mathcal{M}_{i\alpha}{}^{\beta} & \mathcal{M}_i{}^{m\beta} \\ \mathcal{M}^{ijk}{}_m & \mathcal{M}^{ijk,mnp} & \mathcal{M}^{ijk,\beta} & \mathcal{M}^{ijk,m\beta} \\ \mathcal{M}^{\alpha}{}_m & \mathcal{M}^{\alpha,mnp} & \mathcal{M}^{\alpha\beta} & \mathcal{M}^{\alpha,m\beta} \\ \mathcal{M}^{i}{}_m & \mathcal{M}^{i,mnp} & \mathcal{M}^{i,\beta} & \mathcal{M}^{i,m\beta} \end{pmatrix}$$

the ExFT equations of motion reproduce full IID/IIB supergravity

# $E_{6(6)}$ exceptional field theory

manifestly duality covariant formulation of maximal supergravity

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

dictionary

dictionary

**D=11 sugra**

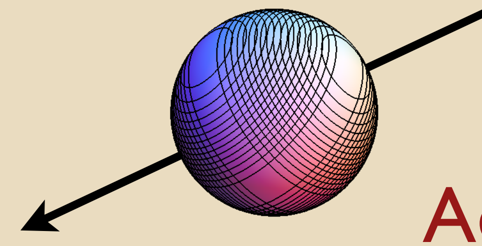
**IIB sugra**

**applications:** **consistent truncations**

## consistent truncations

IIB sugra

- $AdS_5 \times S^5$ background of IIB
  - KK towers of linearized fluctuations



$AdS_5 \times S^5$

D=5 maximal sugra
gauge group SO(6)

[Gunaydin,Romans,Warner]

- $AdS_5 \times S^5$ : lowest KK-multiplet $\longrightarrow$ D=5 maximal supergravity
  - non-linear embedding in IIB such that any D=5 solution defines a IIB solution

$$ds^2 \;=\; \Delta^{-2/3}(x,y)\, g_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right)\left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right)$$

$$G^{mn}(x,y) = \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x)$$

etc.

# consistent truncations

- $AdS_5 \times S^5$ background of IIB
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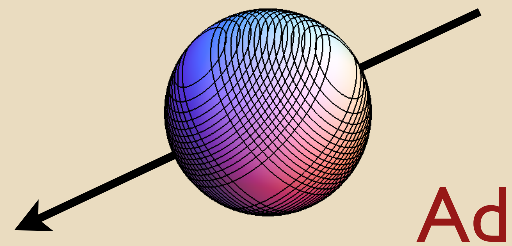
**IIB sugra**



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$$G^{mn}(x,y) = \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x)$$

etc.

- construction of IIB solutions
- holography: trust D=5 supergravity calculations
- used to be scarce (only few examples until recently)

$AdS_4 \times S^7$ : [de Wit, Nicolai] 1987
$AdS_7 \times S^4$ : [Nastase, van Nieuwenhuizen, Vaman] 1999

# consistent truncations from ExFT

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

dictionary

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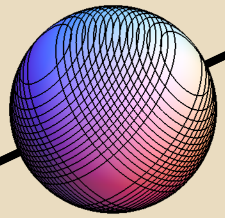

AdS$_5$ x S$^5$

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captured by a generalized Scherk-Schwarz reduction of ExFT

$$\mathcal{M}_{MN}(x,Y) = U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y)$$

$$\mathcal{A}_\mu{}^M(x,Y) = \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x)$$

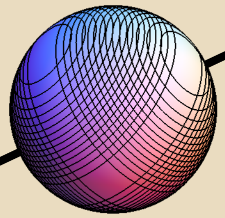
$$\mathcal{B}_{\mu\nu\, M}(x,Y) = \rho^{-2}(Y)\, U_M{}^K(Y)\, B_{\mu\nu\, K}(x)$$

GSS

$U(Y) \in E_{6(6)}$

dictionary

**IIB sugra**



AdS$_5$ x S$^5$

**D=5 maximal sugra**

gauge group SO(6)

[Kaloper, Myers, Dabholkar, Hull, Reid-Edwards, Dall'Agata, Prezas, HS, Trigiante, Hohm, Kwak, Aldazabal, Baron, Nunez, Marques, Geissbuhler, Grana, Berman, Musaev, Thompson, Rosabal, Lee, Strickland-Constable, Waldram, Dibitetto, Roest, Malek, Blumenhagen, Hassler, Lust, Cho, Fernández-Melgarejo, Jeon, Park, Guarino, Varela, Inverso, Ciceri, ...]

# consistent truncations from ExFT

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$$\begin{aligned}
\mathcal{M}_{MN}(x,Y) &= U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y) \\
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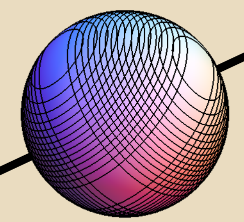


AdS$_5$ x S$^5$

**D=5 maximal sugra**

gauge group SO(6)

in terms of an E$_{6(6)}$–valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$

- system of consistency equations $\left[(U^{-1})_M{}^P (U^{-1})_N{}^L\, \partial_P U_L{}^K\right]_{\mathbf{351}} \overset{!}{=} \rho\, X_{MN}{}^K$
- generalized (Leibniz) parallelizability $\mathcal{L}_{\mathcal{U}_M} \mathcal{U}_N = X_{MN}{}^K\, \mathcal{U}_K$

# AdS$_5$ x S$^5$

- twist matrix for AdS$_5$ x S$^5$ $\quad U = \begin{pmatrix} g^{-1/2}\partial_i \mathcal{Y}^A \\ \mathcal{Y}^A - 2\,\zeta^i \partial_i \mathcal{Y}^A \end{pmatrix} \in \mathrm{SL}(6) \subset \mathrm{E}_{6(6)}$

- e.g. metric (standard Kaluza-Klein form)

$$ds^2 \;=\; \Delta^{-2/3}(x,y)\,\mathrm{g}_{\mu\nu}(x)\,dx^\mu dx^\nu + G_{mn}(x,y)\left(dy^m + \mathcal{K}_{[ab]}{}^m(y)A_\mu^{ab}(x)dx^\mu\right)\left(dy^n + \mathcal{K}_{[cd]}{}^n(y)A_\nu^{cd}(x)dx^\nu\right)$$

$$G^{mn}(x,y) = \Delta^{2/3}(x,y)\,\mathcal{K}_{[ab]}{}^m(y)\mathcal{K}_{[cd]}{}^n(y)\,M^{ab,cd}(x)$$

- e.g. 4-form (after reconstructing all components, in Kaluza-Klein basis)

$$\begin{aligned}
C_{klmn} &= \tilde{C}_{klmn} + \frac{1}{16}\tilde{\omega}_{klmnp}\Delta^{4/3}m_{\alpha\beta}\tilde{G}^{pq}\partial_q(\Delta^{-4/3}m^{\alpha\beta}),\\
C_{\mu kmn} &= \frac{\sqrt{2}}{4}\mathcal{Z}_{[ab]kmn}A_\mu{}^{ab},\\
C_{\mu\nu mn} &= \frac{\sqrt{2}}{4}\mathcal{K}_{[ab]}{}^k\mathcal{Z}_{[cd]kmn}A_{[\mu}{}^{ab}A_{\nu]}{}^{cd},\\
C_{m\mu\nu\rho} &= -\frac{1}{32}\mathcal{K}_{[ab]m}\left(2\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}M_{ab,N}F^{\sigma\tau N} + \sqrt{2}\varepsilon_{abcdef}\Omega_{\mu\nu\rho}^{cdef}\right) - \frac{1}{4}\sqrt{2}\mathcal{K}_{[ab]}{}^k\mathcal{K}_{[cd]}{}^l\mathcal{Z}_{[ef]mkl}(A_{[\mu}{}^{ab}A_\nu{}^{cd}A_{\rho]}{}^{ef}),\\
C_{\mu\nu\rho\sigma} &= -\frac{1}{16}\mathcal{Y}_a\mathcal{Y}^b\left(\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}D^\tau M_{bc,N}M^{Nca} + 2\sqrt{2}\varepsilon_{cdefgb}F_{[\mu\nu}{}^{cd}A_\rho{}^{ef}A_{\sigma]}{}^{ga}\right)\\
&\quad + \frac{1}{4}\left(\sqrt{2}\mathcal{K}_{[ab]}{}^k\mathcal{K}_{[cd]}{}^l\mathcal{K}_{[ef]}{}^n\mathcal{Z}_{[gh]kln} - \mathcal{Y}_h\mathcal{Y}^j\varepsilon_{abcegj}\eta_{df}\right)A_{[\mu}{}^{ab}A_\nu{}^{cd}A_\rho{}^{ef}A_{\sigma]}{}^{gh} + \Lambda_{\mu\nu\rho\sigma}(x).
\end{aligned}$$

$$\begin{aligned}
D_{[\mu}\Lambda_{\nu\rho\sigma\tau]} &= -\frac{1}{80}\mathcal{Y}_a\mathcal{Y}^b\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}D_\lambda(M^{Nac}D^\lambda M_{bc,N})\\
&\quad + \frac{1}{40}\mathcal{Y}_a\mathcal{Y}^b\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}F^{\kappa\lambda N}\left(M_{bc,N}F_{\kappa\lambda}{}^{ac} - \frac{1}{2}\sqrt{10}\varepsilon_{\alpha\beta}\eta_{db}M^{d\alpha}{}_N B_{\kappa\lambda}{}^{a\beta}\right)\\
&\quad + \frac{1}{100}\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}\mathcal{Y}_a\mathcal{Y}^b(10M^{ac,fd} + \mathcal{X}^{(af)ec,d}{}_e)\eta_{cd}\eta_{bf}\\
&\quad + \frac{1}{32}\sqrt{2}\varepsilon_{abcdef}F_{[\mu\nu}{}^{ab}F_{\rho\sigma}{}^{cd}A_{\tau]}{}^{ef} + \frac{1}{16}F_{[\mu\nu}{}^{ab}A_\rho{}^{cd}A_\sigma{}^{ef}A_{\tau]}{}^{gh}\varepsilon_{abcdeh}\eta_{fh}\\
&\quad + \frac{1}{40}\sqrt{2}A_{[\mu}{}^{ab}A_\nu{}^{cd}A_\rho{}^{ef}A_\sigma{}^{gh}A_{\tau]}{}^{ij}\varepsilon_{abcegi}\eta_{df}\eta_{hj},
\end{aligned}$$

- **proves the consistent truncation of IIB on AdS$_5$ x S$^5$**

## consistent truncations from ExFT

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

GSS $\tilde{U}(Y) \in E_{6(6)}$ → **D=5 maximal sugra** gauge group SO(p,q)

GSS $U(Y) \in E_{6(6)}$ → **D=5 maximal sugra** gauge group SO(6)

GSS $\hat{U}(Y) \in E_{6(6)}$ → **D=5 maximal sugra** gauge group CSO(p,q,r)

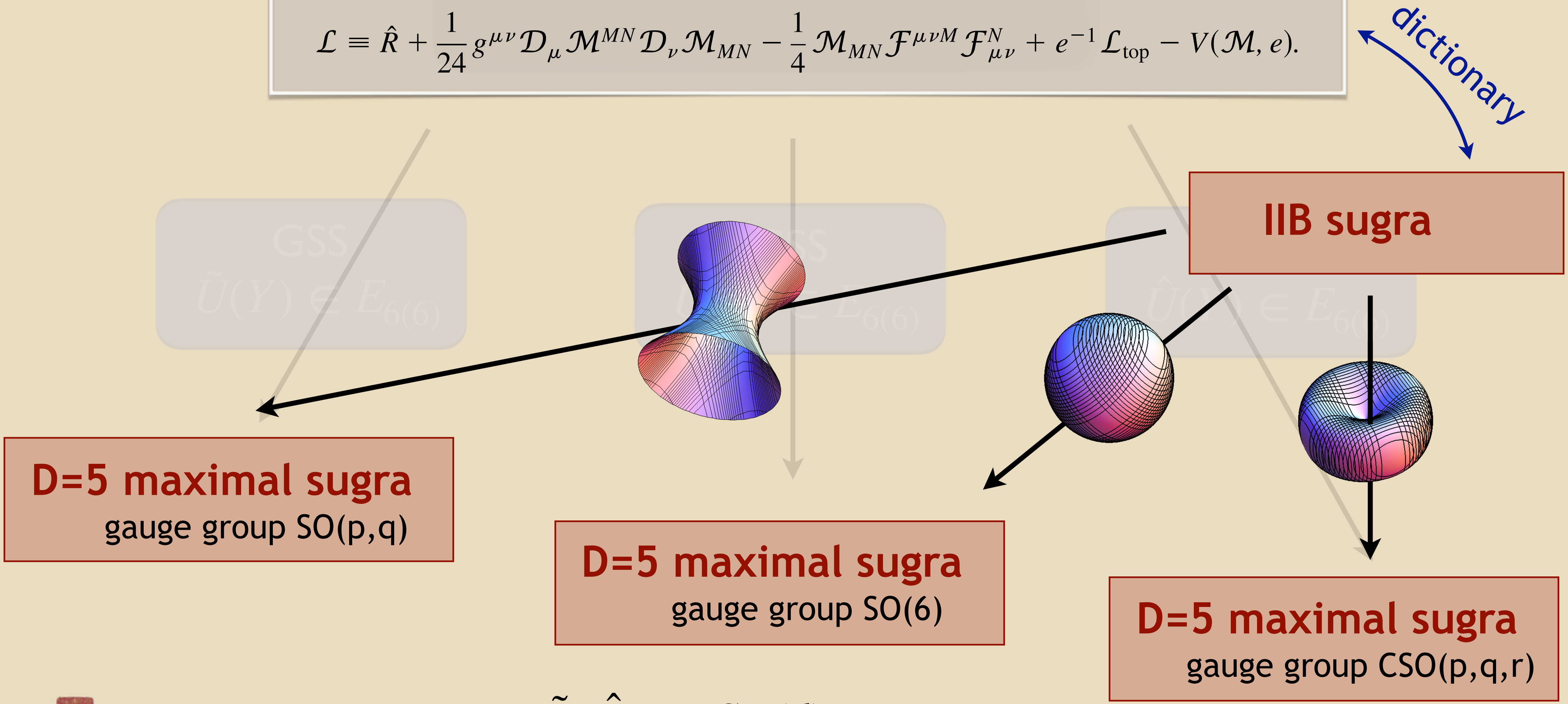
- similar: twist matrices $\tilde{U}, \hat{U} \;\in\; \mathrm{SL}(6)$
  associated to $\mathrm{SO}(p,q)$ and $\mathrm{CSO}(p,q,r)$

  built from sphere harmonics on $\mathrm{SO}(p,q)/\mathrm{SO}(p,q-1)$ $\quad \tilde{U} = \begin{pmatrix} g^{-1/2}\partial_i \mathcal{Y}^A \\ \mathcal{Y}^A - 2\,\zeta^i \partial_i \mathcal{Y}^A \end{pmatrix} \;\in\; \mathrm{SL}(6)$

# consistent truncations from ExFT

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$




- similar: twist matrices $\tilde{U}, \hat{U} \in \mathrm{SL}(6)$ associated to $\mathrm{SO}(p,q)$ and $\mathrm{CSO}(p,q,r)$
  - background: (warped) hyperboloids [Hull, Warner] [Baron, Dall'Agata]
  - in general no IIB solutions, still consistent truncations!

# other examples of consistent truncations

- consistent truncations with smaller isometry groups [Inverso, HS, Trigiante, Malek]

  products of spheres and hyperboloids $S^p \times S^q$ , $S^p \times H^q$

  specific D=4 construction, based on electric/magnetic split of internal coordinates

  inducing dyonic gaugings $(\mathrm{SO}(p,q) \times \mathrm{SO}(p',q')) \ltimes N$ [Dall'Agata, Inverso]

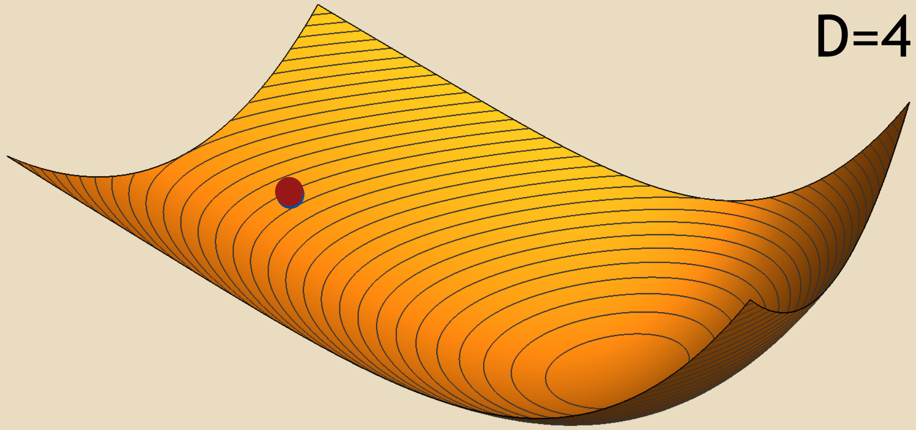
**IIB sugra**

$\Big\downarrow$ $AdS_4 \times S^5 \times H^1$

**D=4 maximal sugra**

gauge group

$[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$



D=4 scalar potential

● SO(6) : not stationary $AdS_4 \times S^5 \times H^1$

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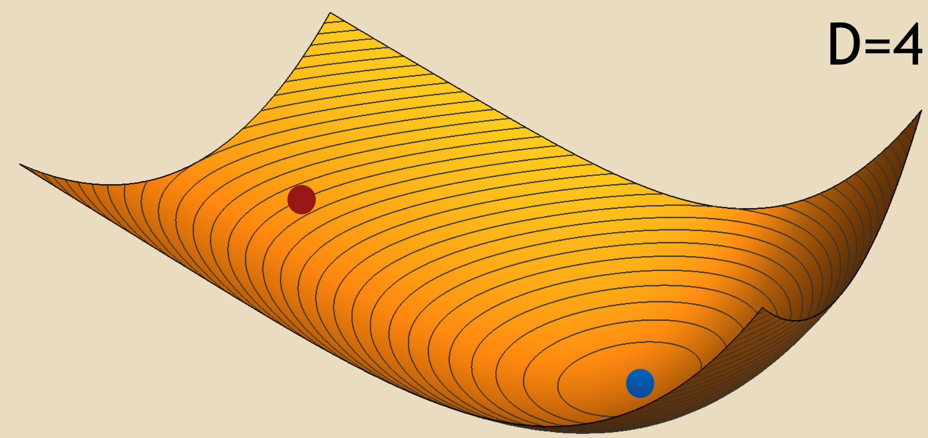

- $\mathrm{SO}(6)$ : not stationary $AdS_4 \times S^5 \times H^1$
- $\mathrm{SO}(4)$ : $\mathcal{N} = 4$, AdS$_4$ vacuum $AdS_4 \times S^2 \times S^2 \times \Sigma$

$$
\begin{aligned}
ds^2 \;&=\; \Delta^3 \sin^2 x \,(1+2\cos^2 x)\, d\mathcal{Y}_1^p d\mathcal{Y}_1^p + \Delta^3\,(1+2\sin^2 x)\cos^2 x \, d\mathcal{Y}_2^p d\mathcal{Y}_2^p \\
&\quad + \Delta^{-1}\,(dx\,dx + d\eta\, d\eta) + \frac{1}{2}\,\Delta^{-1}\, ds^2_{\mathrm{AdS}_4} \;,
\end{aligned}
$$

Janus solution [D'Hoker, Estes, Gutperle]
[Assel, Bachas, Estes, Gomis][Assel, Tomasiello]

with a maximally supersymmetric consistent truncation around

# other applications / developments

- ExFT for all finite-dimensional exceptional groups $E_{d(d)}$ , d<9
  - based on the different splits external/internal coordinates

  [Hohm, HS] [Abzalov, Bakhmatov,Musaev, Hohm, Wang, Berman,Blair, Malek,Rudolph]

- ExFT for the affine Kac-Moody algebra $E_{9(9)}$
  - infinite-dimensional highest-weight representations

  [Bossard, Cederwall, Ciceri, Inverso, Kleinschmidt, Palmkvist, HS]

- fermions & superspace
  - $E_{7(7)}$ : super-diffeomorphisms in $(4+56|32)$

  [Butter, HS, Sezgin] ⟶ [Howe, Lindström 1981]

- embedding of massive IIA theory
  - by deformations of ExFT
  - by Scherk-Schwarz reduction violating the section constraints

  [Ciceri, Guarino, Inverso] [Cassani, de Felice, Petrini, Strickland-Constable, Waldram]

- embedding of 'generalized IIB' theory
  - background from $\eta$-deformed $AdS_5 \times S^5$ sigma model
  - T-dual of IIA with non-isometric dilaton

  [Baguet, Magro, HS] [Sakatani , Uehara, Yoshida]

## other applications / developments

- consistent truncations with less supersymmetry in ExFT (in type II sugra)
  - construction and classification of supersymmetric AdS vacua [Malek]
- unifying framework for brane solutions
  - organisation of exotic branes

    [Berman, Rudolph, Bakhmatov, Kleinschmidt, Musaev, Otsuki, Fernandez-Melgarejo, Kimura, Sakatani]
- orbifolds and orientifolds in ExFT
  - unified approach in terms of generalized orbifolds (O-folds) [Blair, Malek, Thompson]
- exceptional string sigma model
  - string sigma model with ExFT background fields [Arvanitakis, Blair]
- ExFT loop calculations
  - duality covariant graviton amplitudes [Bossard, Kleinschmidt]
- underlying mathematical structures
  - $L_\infty$-algebras, Borchers superalgebras, tensor hierarchy algebras

    [Cederwall, Palmkvist][Hohm, Kupriyanov, Lüst, Traube]
    [Cagnacci, Codina, Marques][Arvanitakis][Hohm, Zwiebach]

## conclusions

- **exceptional field theory**
  - manifestly duality covariant formulation of maximal supergravity
  - based on generalized diffeomorphisms in exceptional geometry
  - <u>unique</u> theory with generalized diffeomorphism invariance
  - upon an explicit solution of the section constraints
    the theory reproduces full D=11 supergravity <u>and</u> full D=10 IIB supergravity

- powerful tools for construction & analysis of vacua & consistent truncations

# conclusions

## exceptional field theory

- manifestly duality covariant formulation of maximal supergravity
- based on generalized diffeomorphisms in exceptional geometry
- unique theory with generalized diffeomorphism invariance
- upon an explicit solution of the section constraints
  the theory reproduces full D=11 supergravity and full D=10 IIB supergravity

powerful tools for construction & analysis of vacua & consistent truncations

## challenges

- higher order corrections [Hohm, Zwiebach]
- decrease number of external dimensions $\longrightarrow$ unifying picture
- weaken / relax section constraints

[Bossard, Kleinschmidt, Sezgin]

new variables for supergravity

— or hints towards a more fundamental structure ..?