Some thoughts on bulk reconstruction

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Introduction

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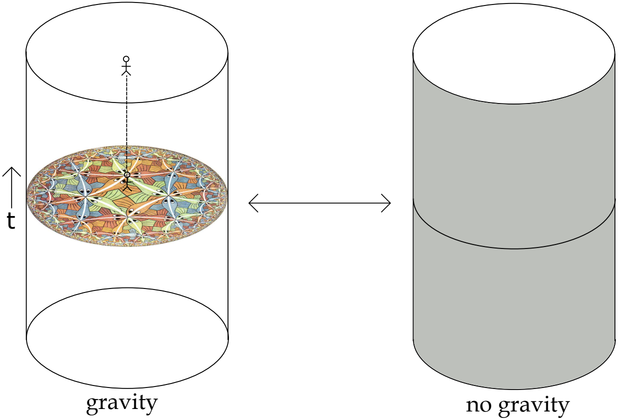

It tells us that non-perturbative quantum gravity in (A)AdS is really just quantum field theory in disguise!

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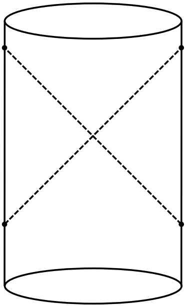
(To match all of the CFT local operators this way, in the bulk we need to include "quasi-local operators" which create bound states, black holes, etc, see Harlow/Ooguri)

The extrapolate dictionary is sufficient to describe all "scattering" type experiments in the bulk theory:



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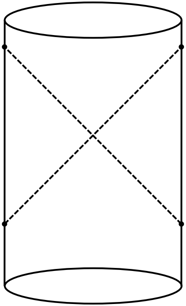

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In particular it is sufficient (in principle) to describe the formation and evaporation of small black holes, as well as (more conveniently) the "two-point function" version of the black hole information problem Maldacena.

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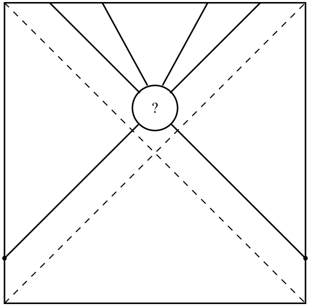

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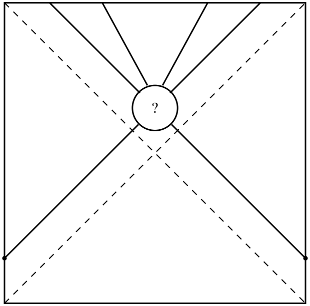

For example, how can we predict the outcome of a scattering experiment which happens behind a black hole horizon?

Bulk reconstruction

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- Such representations will always have some inherent imprecision, due to the approximate nature of the emergence of bulk effective field theory.
- At least in ordinary situations this imprecision seems to be quite small, for example quantum field theory in curved spacetime describes LHC physics quite well.
- Thus what we would like to understand is how a theory in d spacetime dimensions can "pretend" to be one in $d+1$ spacetime dimensions, at least in a restricted set of situations.

A first attempt

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$$\begin{aligned}\phi(x) = &\int_{\partial M} dX K(x,X)\mathcal{O}(X)\\ &+ g\int_{\partial M} dX dX' \int_M dx' G(x,x')K(x',X)K(x',X')\mathcal{O}(X)\mathcal{O}(X')\\ &+ \dots,\end{aligned}$$

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which can be shown to obey the algebra and reproduce the correlators of bulk EFT for $O(1)$ numbers of operators on the vacuum.
It would be a pity however if this were all there were to holography, and indeed we will see momentarily that this method is not sufficient to describe regions of spacetime which are behind causal horizons.

In the last few years a considerable advance in bulk reconstruction has come from considering the idea of *subregion duality*, which is the proposal that each spatial subregion of the boundary CFT is separately dual to some particular subregion of the bulk. Bousso, Leichenauer, Rosenhaus, Czech, Karczmarek, Nogueira, Van Raamsdonk, Freivogel, Zukowski

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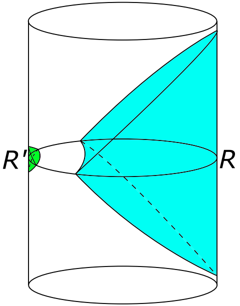
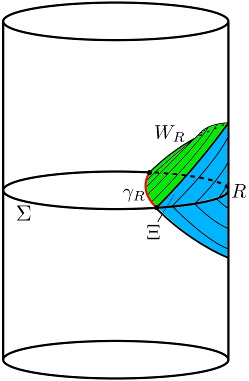


The natural bulk subregion to consider from the point of view of solving the bulk equations is the *causal wedge*:

$$C_R \equiv J^+[D[R]] \cap J^-[D[R]].$$

The idea that C_R is the appropriate bulk subregion dual to R is called into question however by the Ryu-Takayanagi formula:



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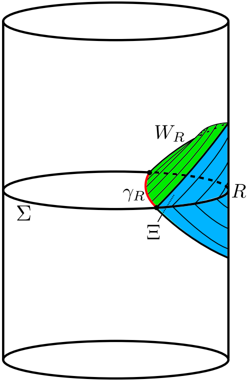

$$S[\rho_R] = \frac{\langle A(\gamma_R) \rangle}{4G} + S(\rho_{W_R}).$$

It is a general result that $C_R \subset W_R$, Wall, Headrick/Hubeny/Lawrence/Rangamani so apparently the RT formula is able to access information beyond the causal wedge.

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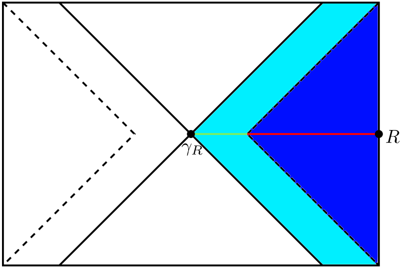

There is a very important distinction between C_R and W_R: C_R can never contain points behind an event horizon, while W_R can. So if entanglement wedge reconstruction is true, this tells us that it is possible to reconstruct bulk degrees of freedom which are inside of a black hole as operators in the dual CFT!

In 2014 Almheiri, Dong, and I pointed out that subregion duality in AdS/CFT was an example of a more general phenomenon called *quantum error correction*, and we argued that this should lead to a resolution of the question of entanglement wedge reconstruction. In the following years the pieces have come together, leading to the following general results:

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- Entanglement wedge reconstruction is a direct consequence of the Ryu-Takayanagi formula, and is in fact equivalent to it. Since this formula has been independently established by Lewkowycz and Maldacena, entanglement wedge reconstruction is a true property of AdS/CFT. Jafferis/Lewkowycz/Maldacena/Suh, Dong/Harlow/Wall, Harlow

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- The emergence of the bulk radial direction from the boundary degrees of freedom can be quantitatively explained in terms of the degree of protection of logical degrees of freedom from erasures, and this picture can be explicitly confirmed in exactly soluble models. I believe we now understand it to a similar degree as we understood the confinement of quarks after the 1974 paper of Wilson.

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In quantum gravity it is possible for decoupled quantum systems to describe a bulk geometry in which there is a shared interior that can be accessed by observers from either system.
This claim was implicit in Maldacena’s paper of 2001, but it is only in the last few years that we have begun to appreciate its incredible consequences. I personally do not feel that I have really absorbed it yet, and I suspect there are many surprises left.

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- At absurdly long times, the thermofield double state undergoes quantum recurrences and becomes short again. How does this happen?

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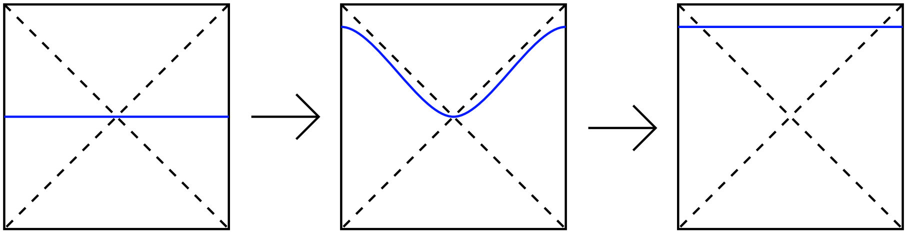

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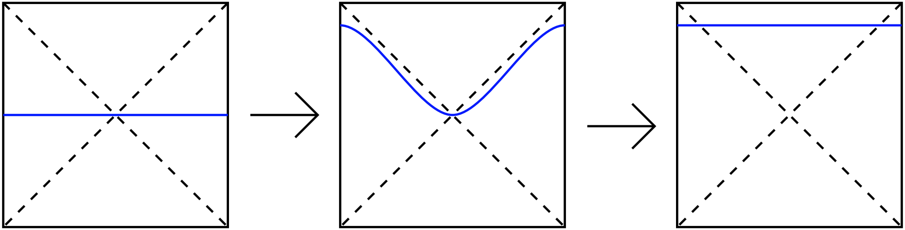

You can evolve all you want on the boundary but never get into the black hole interior unless you learn how to move the bulk slice up independently in the bulk, but this is done by a gauge transformation which acts trivially on the Hilbert space. I suspect we need to learn how to describe the interior directly using diffeomorphism-invariant observables.

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- Thirdly, I think the formalism of approximate quantum error correction gives a promising avenue to explore the imprecision inherent in the emergence of bulk effective field theory, and in particular I would like to mention the "alpha-bit" work of Hayden/Penington, which I think can be used to study how "small" errors in bulk effective field theory can accumulate as we consider larger and larger sets of states, eventually qualitatively changing the nature of the physics.

- Finally I would like to mention the "quantum extremal surface" proposal of Engelhardt/Wall, which is a generalization of the RT formula to include higher-order quantum effects. Very recently this has been used, together with ideas from approximate quantum error correction, to study the evaporation of a AdS black hole coupled to an external system Penington,Almheiri/Engelhardt/Marolf/Maxfield. It seems a powerful synthesis is brewing here, I suspect we will hear quite a bit more about it in Strings 2020.

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Thanks for listening!