

# Unitary Semiclassical Black Hole Evaporation

Ahmed Almheiri  
IAS

**AA '18**

**AA, Engelhardt, Marolf, Maxfield '19**

**Penington '19**

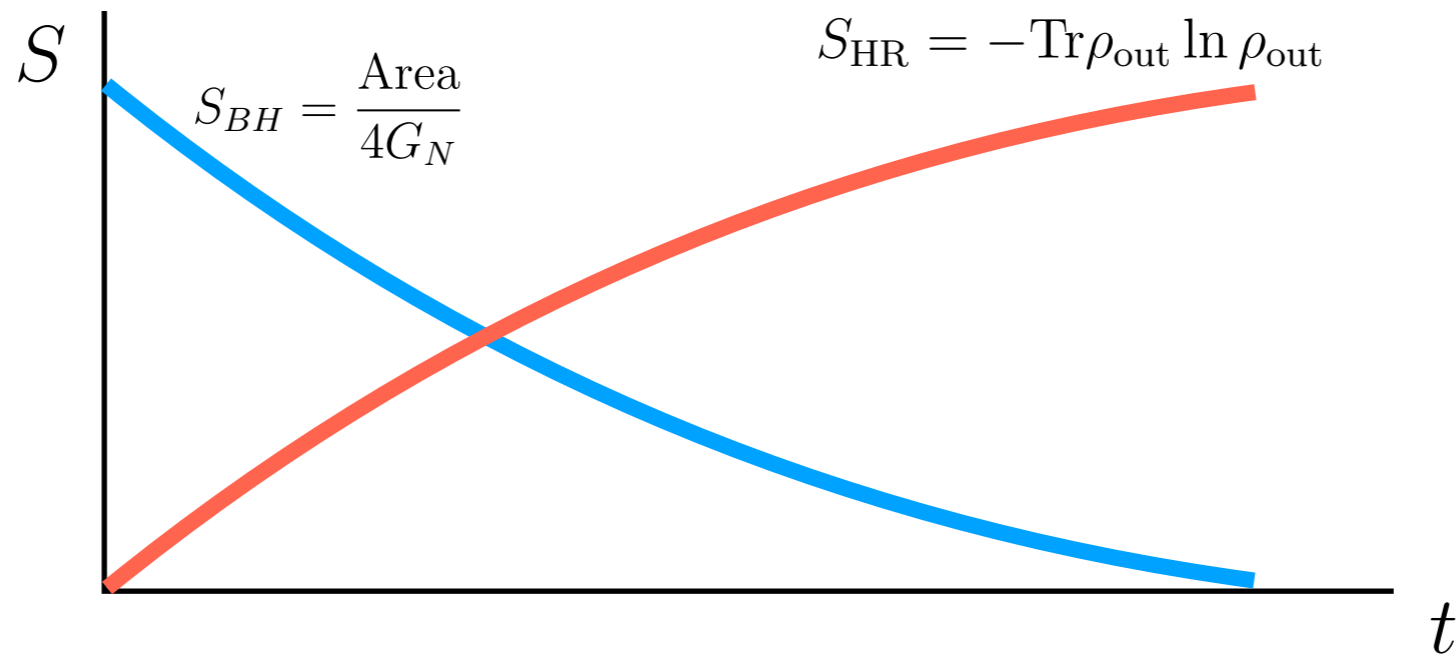
**WIP w/ Mahajan, Maldacena, Zhao**

# Information Paradox

Hawking

**Fine-Grained Entropy of Hawking Radiation:** The von Neumann entropy of the state of the Hawking radiation. [Page](#)

**Coarse-Grained Entropy of Black hole - Bekenstein-Hawking Entropy:** The area of the event horizon/ $4G$  - a measure of the dimensionality of the black hole Hilbert space.

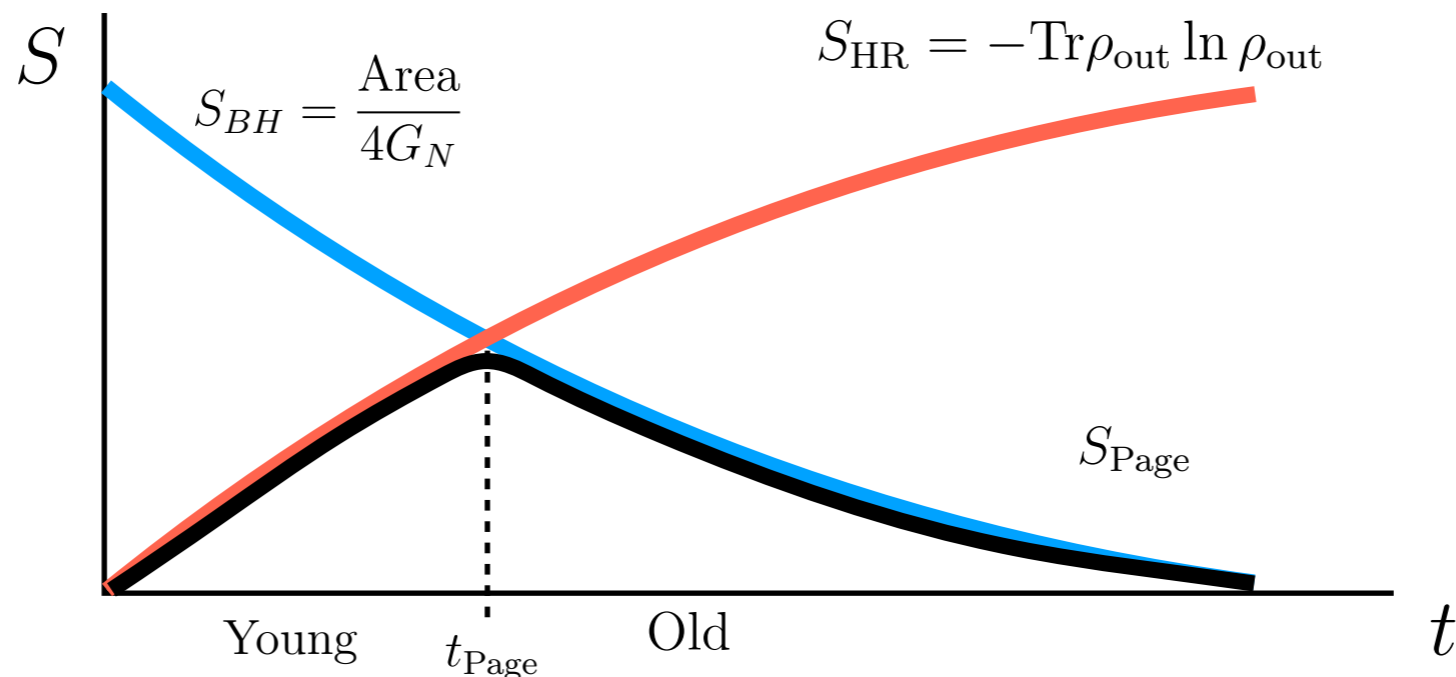


# Information Paradox

Hawking

**Fine-Grained Entropy of Hawking Radiation:** The von Neumann entropy of the state of the Hawking radiation. [Page](#)

**Coarse-Grained Entropy of Black hole - Bekenstein-Hawking Entropy:** The area of the event horizon/ $4G$  - a measure of the dimensionality of the black hole Hilbert space.



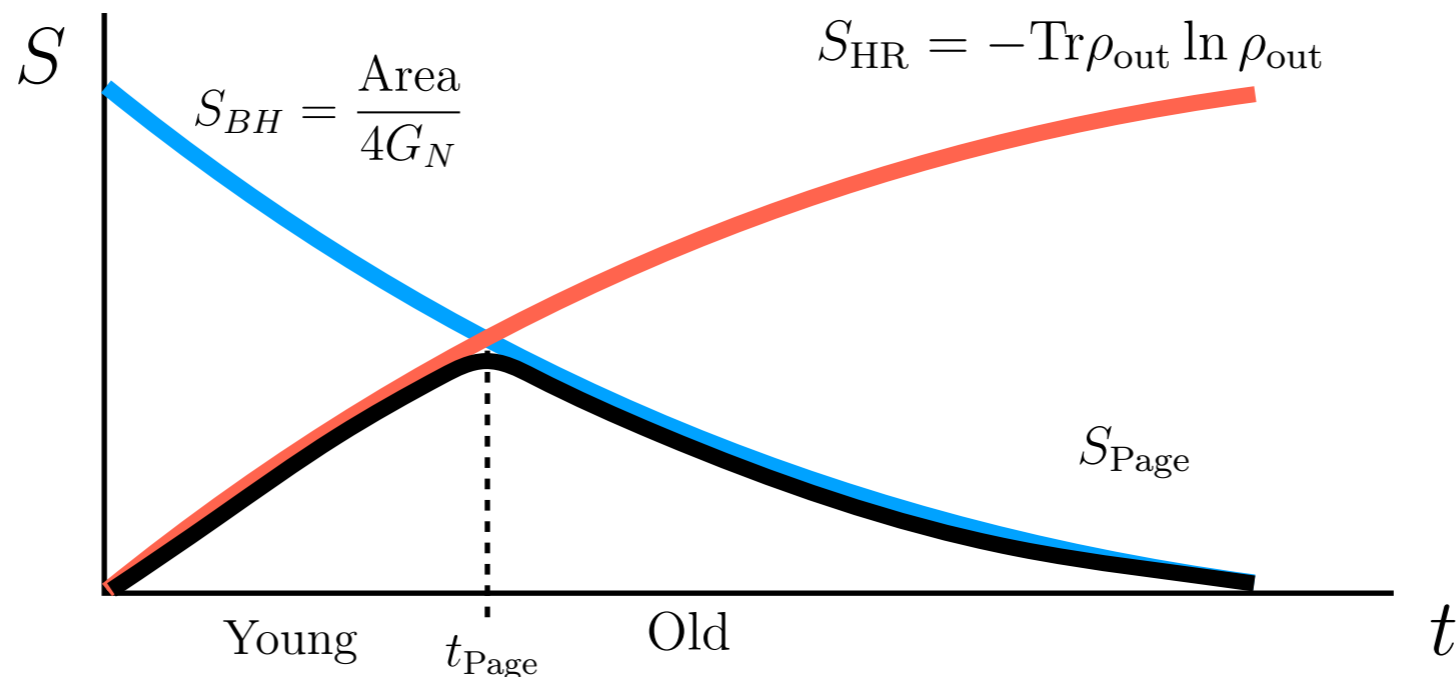
**Unitary Evolution - Page Curve:** Generic behavior of the fine-grained entropy in a unitary system governed by random Hamiltonian. Fine-grained entropy of either system. [Page](#)

# Information Paradox

Hawking

**Fine-Grained Entropy of Hawking Radiation:** The von Neumann entropy of the state of the Hawking radiation. **Page**

**Coarse-Grained Entropy of Black hole - Bekenstein-Hawking Entropy:** The area of the event horizon/ $4G$  - a measure of the dimensionality of the black hole Hilbert space.



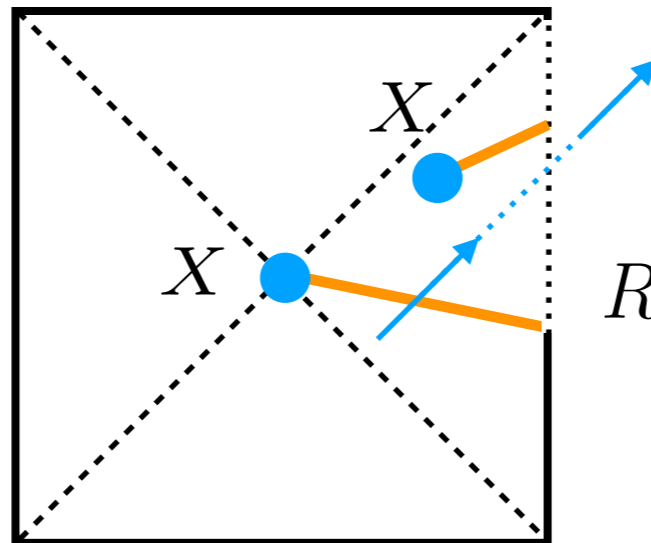
**Unitary Evolution - Page Curve:** Generic behavior of the fine-grained entropy in a unitary system governed by random Hamiltonian. Fine-grained entropy of either system. **Page**

Reproducing the Page curve **FOR BOTH SYSTEMS** is tantamount to resolving the information paradox!

# Plan for this Talk

We will consider the case of evaporating one side of the eternal black hole in JT gravity coupled to an external system.

**Plan:** To track the evolution of the von Neumann entropy of the evaporating side, and of the extracted Hawking radiation in the external system.



$$S_R(t) = \text{Min} \left[ \text{Ext} \left[ \frac{\phi(X)}{4G_N} + S_{\text{Bulk}}(\Sigma_X^R) \right] \right] \quad \text{Engelhardt, Wall}$$

Search for quantum extremal surfaces as a function of boundary time.

**Goal:** To reproduce the Page curve for  $R$  and for the extracted Hawking radiation.

# JT Gravity + Conformal Matter

MSY  
EMV  
J

We study black hole evaporation in the setting of JT gravity coupled to conformal matter.

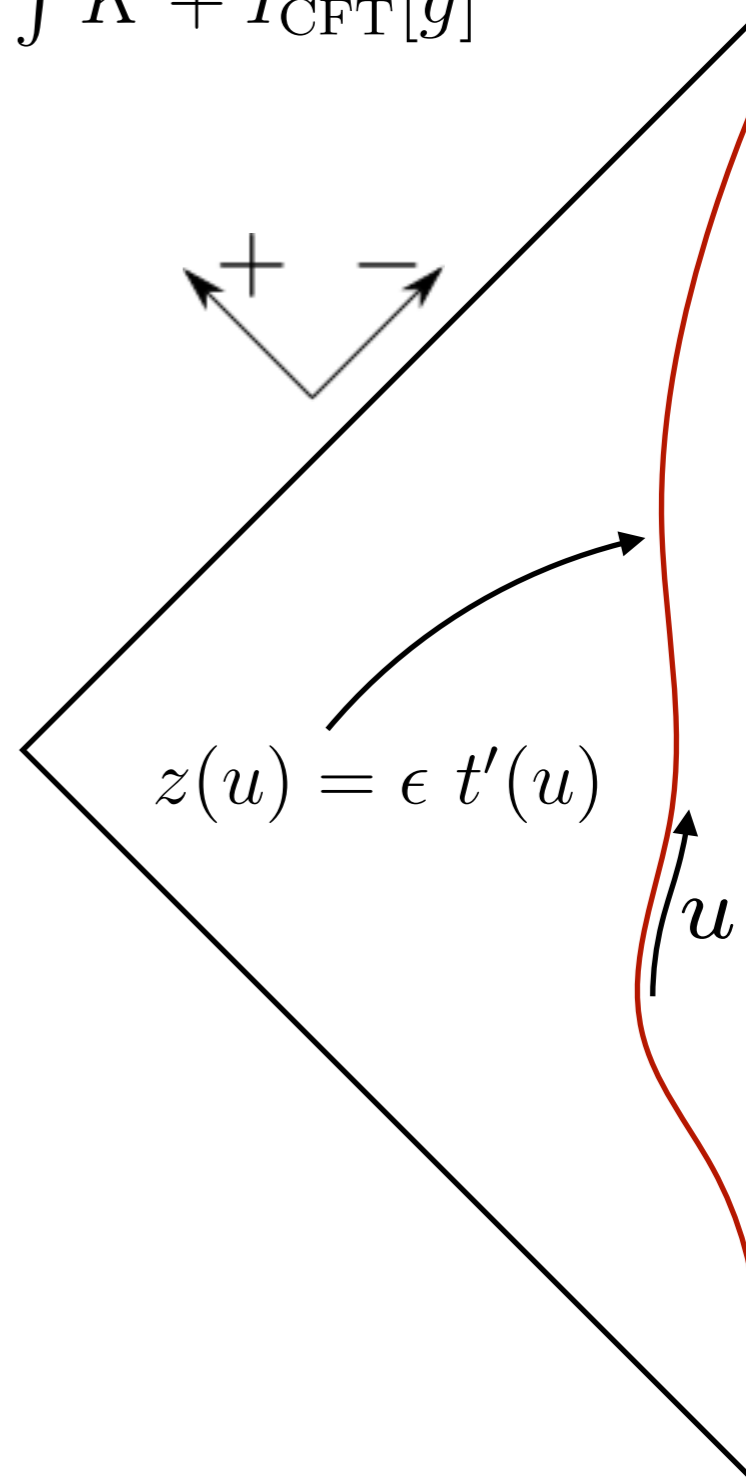
$$I = \frac{\phi_0}{G_N} \int (R + 2) + \frac{1}{G_N} \int \phi (R + 2) + \phi_b \int K + I_{\text{CFT}}[g]$$

Integral over  $\phi$  fixes the spacetime metric to be  $\text{AdS}_2$

$$ds^2 = \frac{-dt^2 + dz^2}{z^2} = \frac{-4dx^+ dx^-}{(x^+ - x^-)^2} \quad x^\pm = t \pm z$$

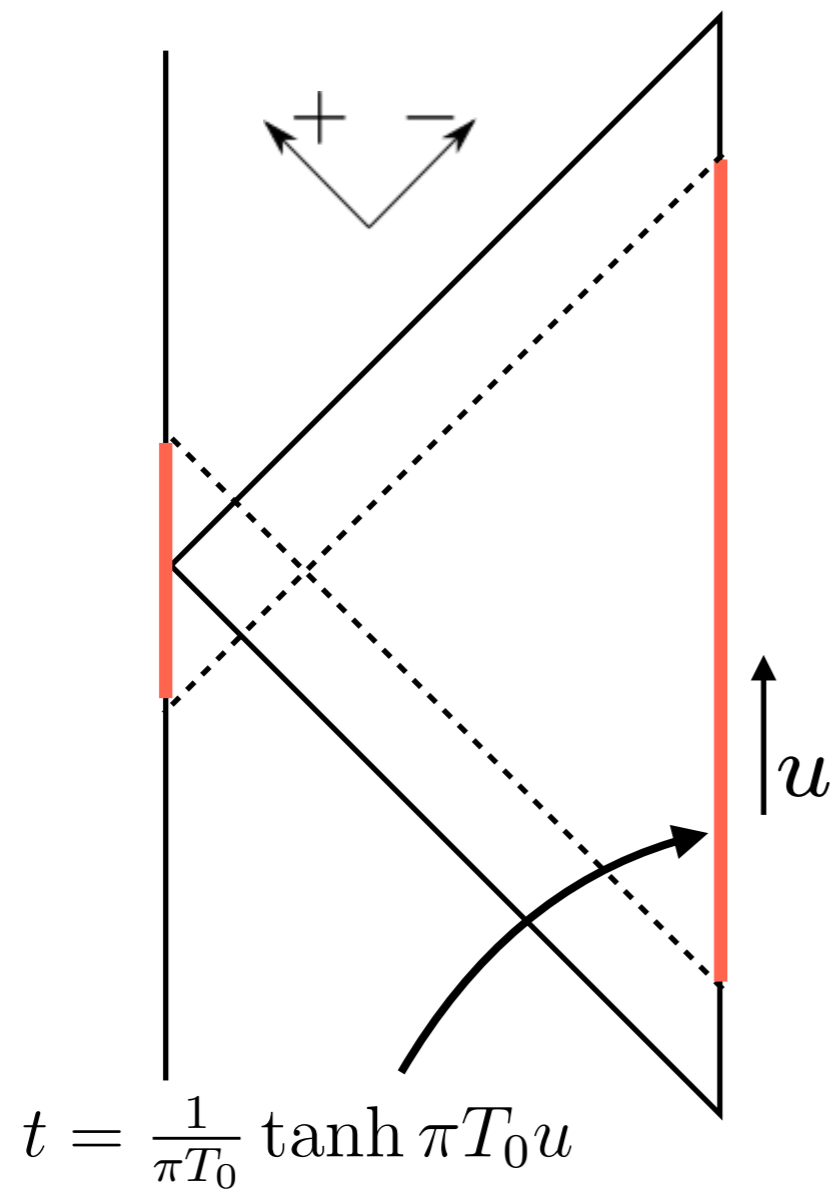
The gravitational dynamics of this theory is given entirely by that of the reparamaterization  $t(u)$  where  $u$  is the physical boundary time.

Matter sector  $I_{\text{CFT}}[g]$  is given by a BCFT - conformal field theory with boundary @  $x^+ - x^- = 0$ , with background metric being Poincare  $\text{AdS}_2$



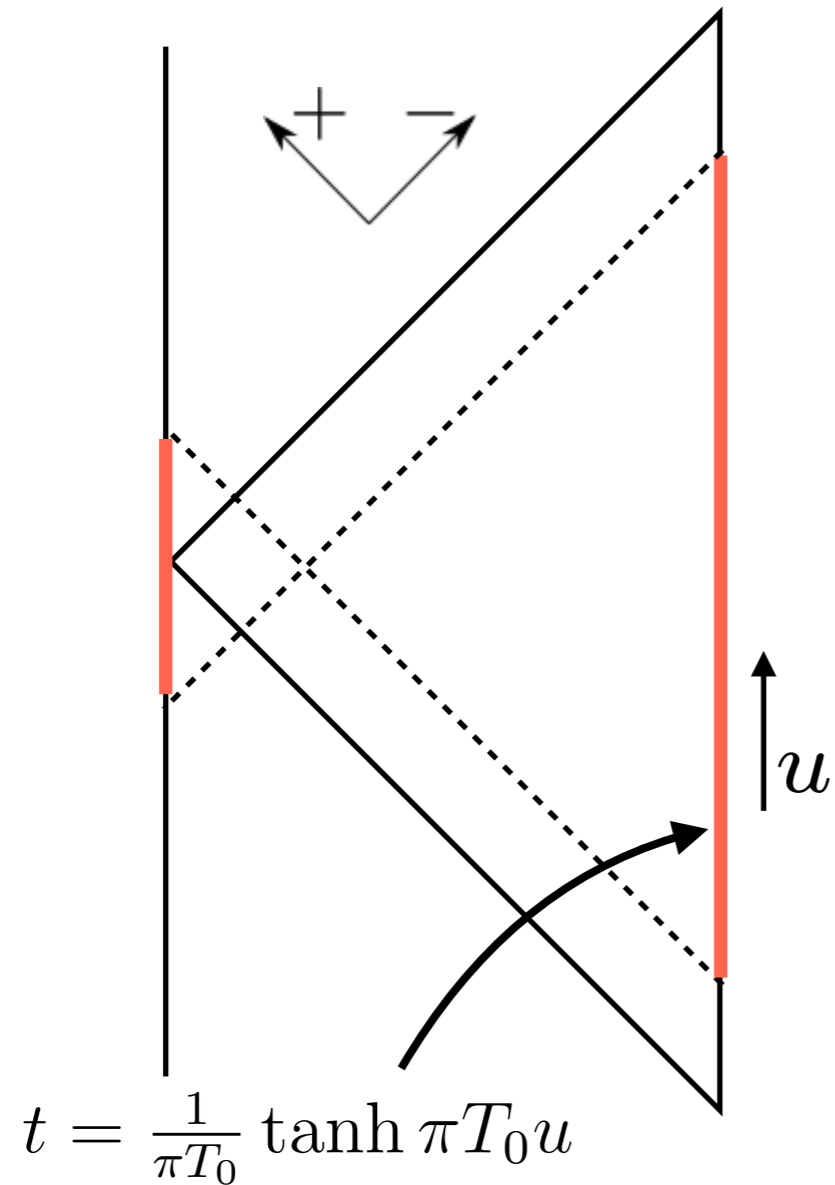
# The Eternal Blackhole in JT Gravity

The eternal black hole is a vacuum solution:  $\langle T_{x^+x^+} \rangle_{AdS_2} = \langle T_{x^-x^-} \rangle_{AdS_2} = 0$

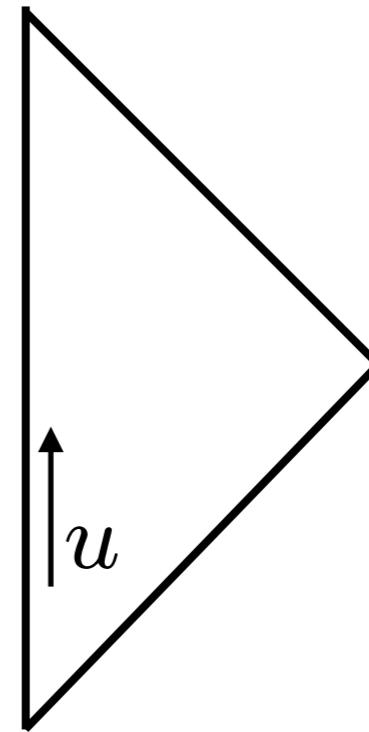


# The Eternal Blackhole in JT Gravity

The eternal black hole is a vacuum solution:  $\langle T_{x^+x^+} \rangle_{AdS_2} = \langle T_{x^-x^-} \rangle_{AdS_2} = 0$



Introduce a 'Bath'  
BCFT in the vacuum



Bath

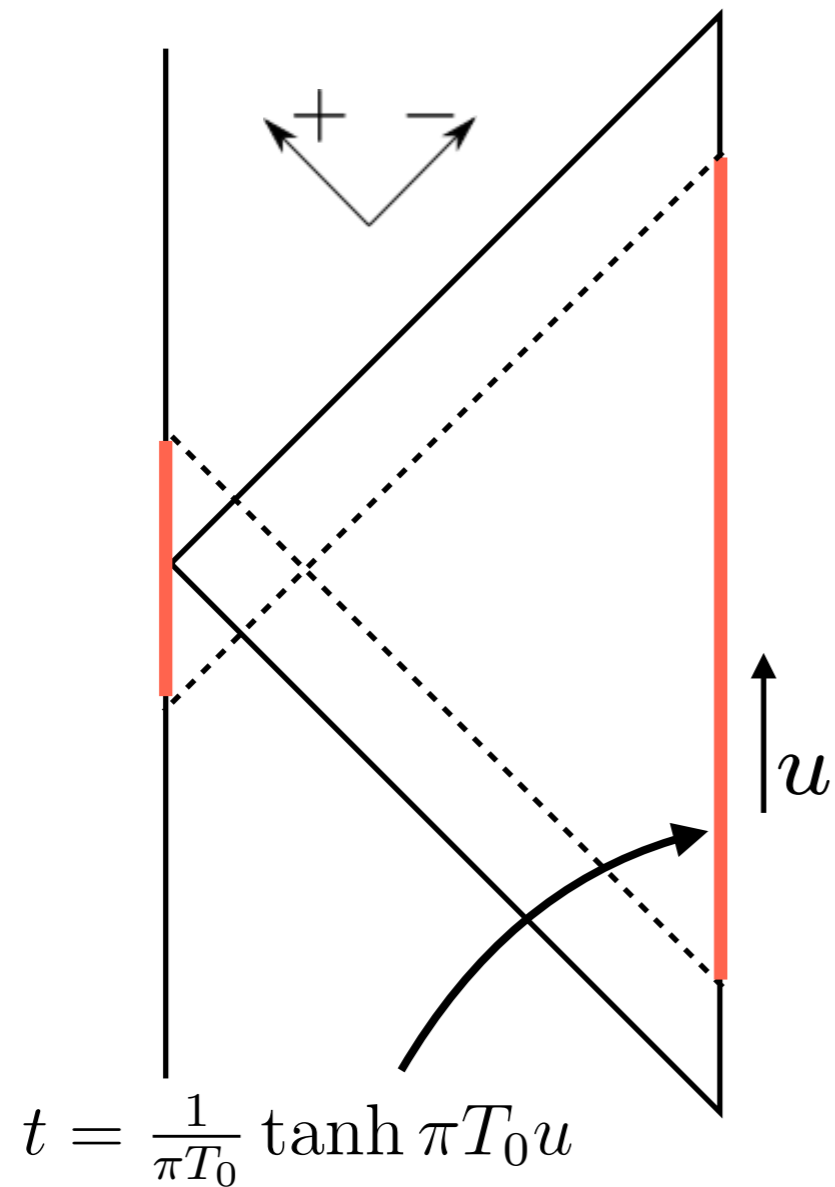
$$ds^2 = -dy^+ dy^-$$

$$y^\pm = u \pm \sigma$$



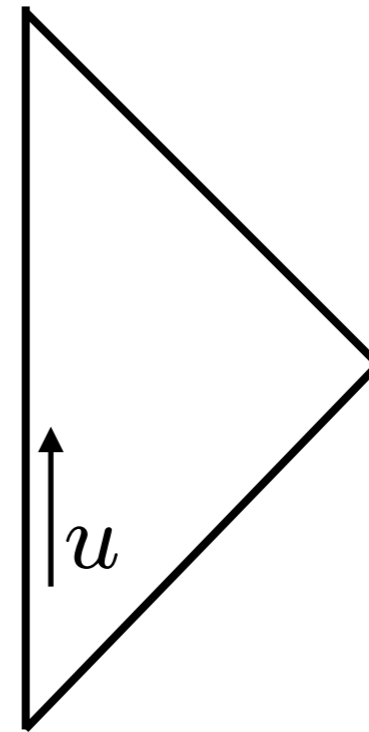
# The Eternal Blackhole in JT Gravity

The eternal black hole is a vacuum solution:  $\langle T_{x^+x^+} \rangle_{AdS_2} = \langle T_{x^-x^-} \rangle_{AdS_2} = 0$



Couple at  $u = 0$

Introduce a 'Bath'  
BCFT in the vacuum



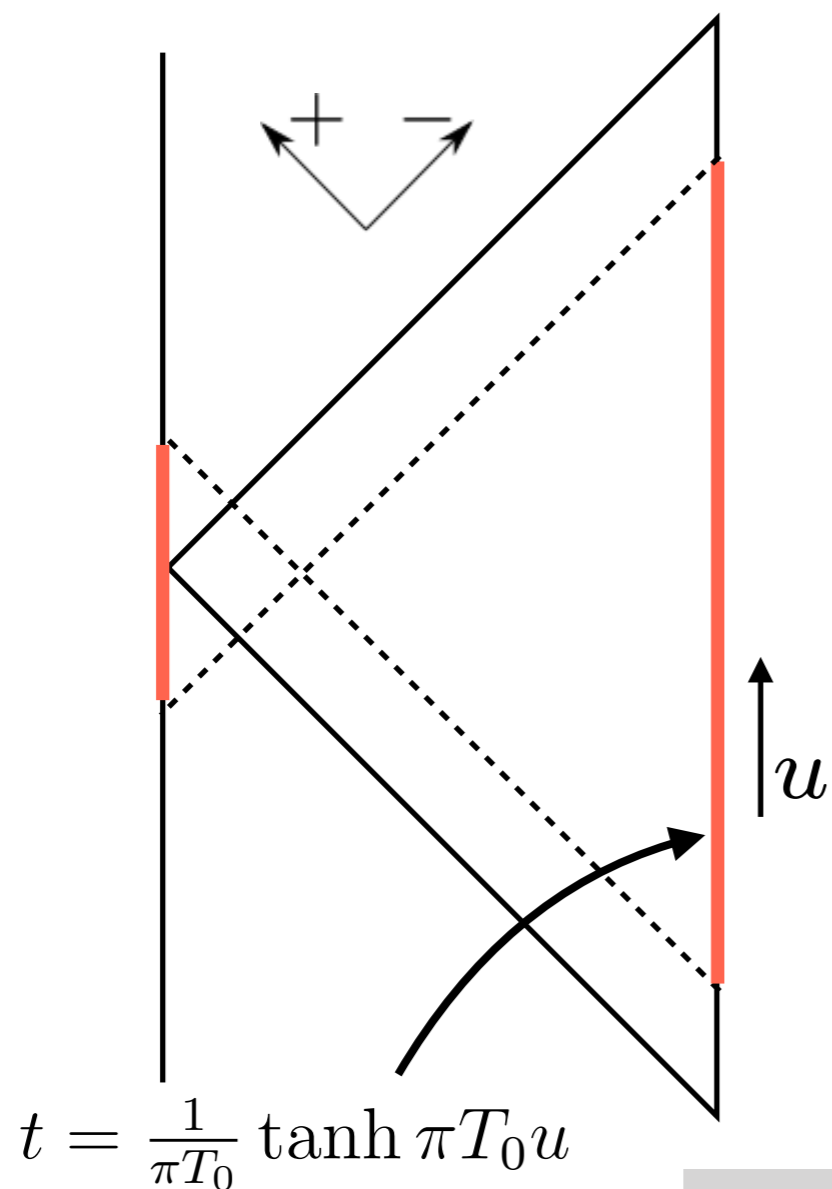
Bath

$$ds^2 = -dy^+ dy^-$$

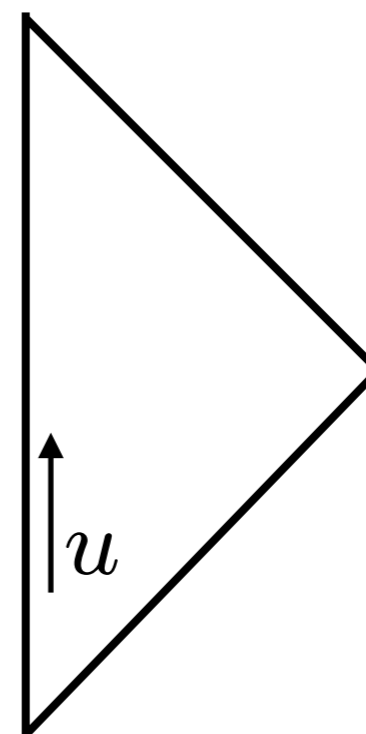
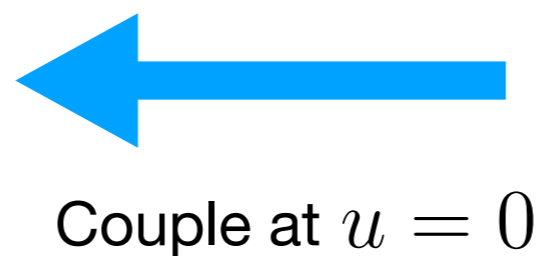
$$y^\pm = u \pm \sigma$$

# The Eternal Blackhole in JT Gravity

The eternal black hole is a vacuum solution:  $\langle T_{x^+x^+} \rangle_{AdS_2} = \langle T_{x^-x^-} \rangle_{AdS_2} = 0$



Introduce a 'Bath'  
BCFT in the vacuum



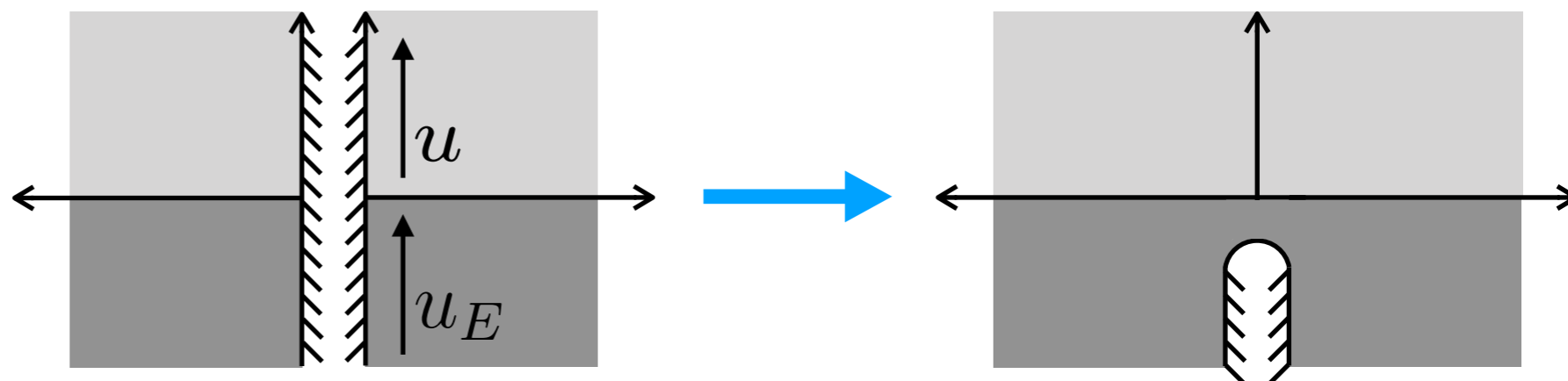
Bath

$$ds^2 = -dy^+ dy^-$$

$$y^\pm = u \pm \sigma$$

We model this as a quantum quench of two BCFTs:

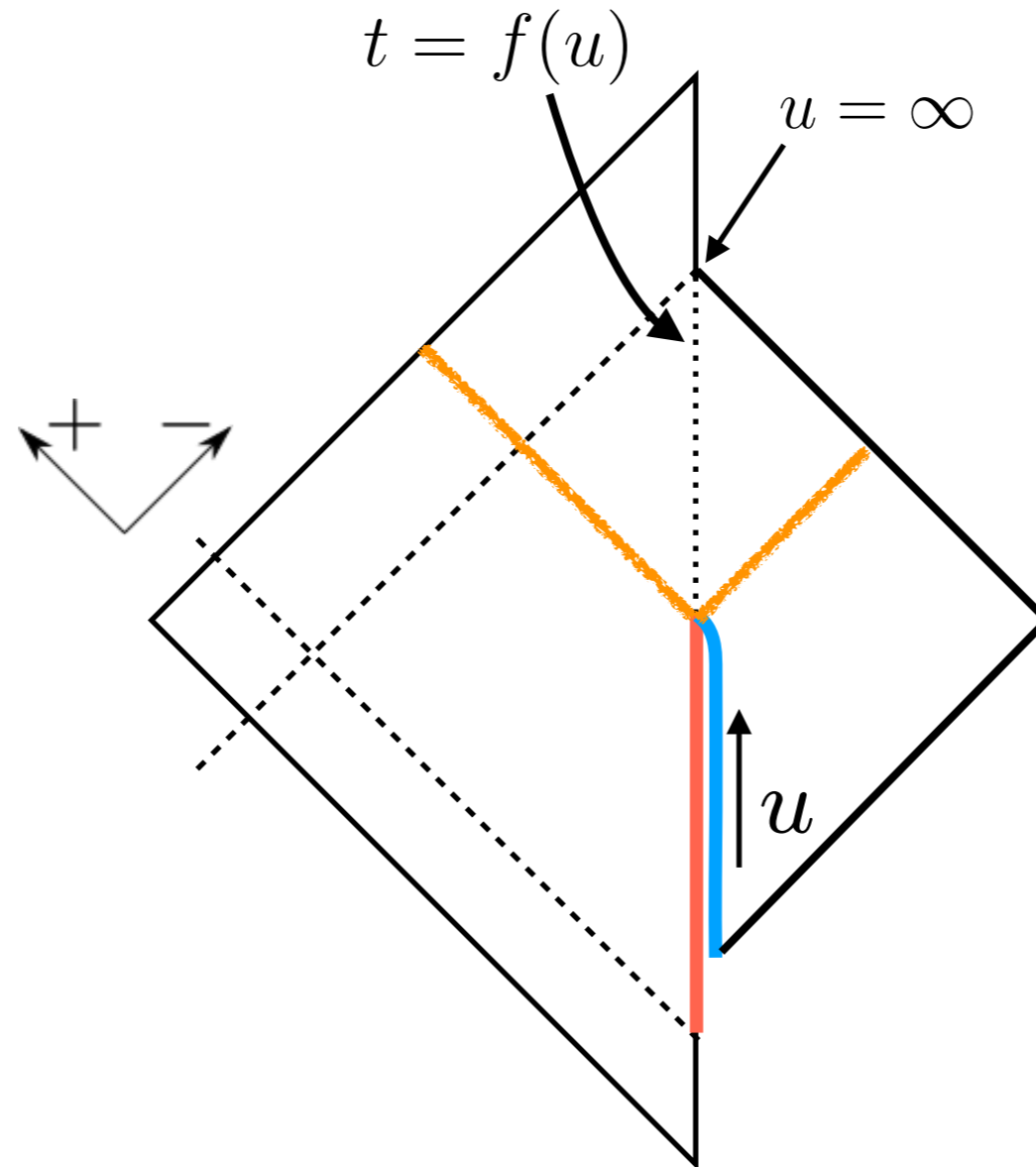
**Cardy, Calabrese**  
**Asplund, Bernamonti**



# Evaporating One Side of the Eternal Black hole

The Bath CFT is glued to the AdS<sub>2</sub> bulk along the physical boundary, identifying the physical time on the boundary with that of the bath.

Englesoy, Mertens, Verlinde



Bulk

$$ds^2 = \frac{-4dx^+ dx^-}{(x^+ - x^-)^2}$$

$$x^\pm = t \pm z$$

Bath

$$ds^2 = -dy^+ dy^-$$

$$y^\pm = u \pm \sigma$$

$$x^\pm = f(y^\pm)$$

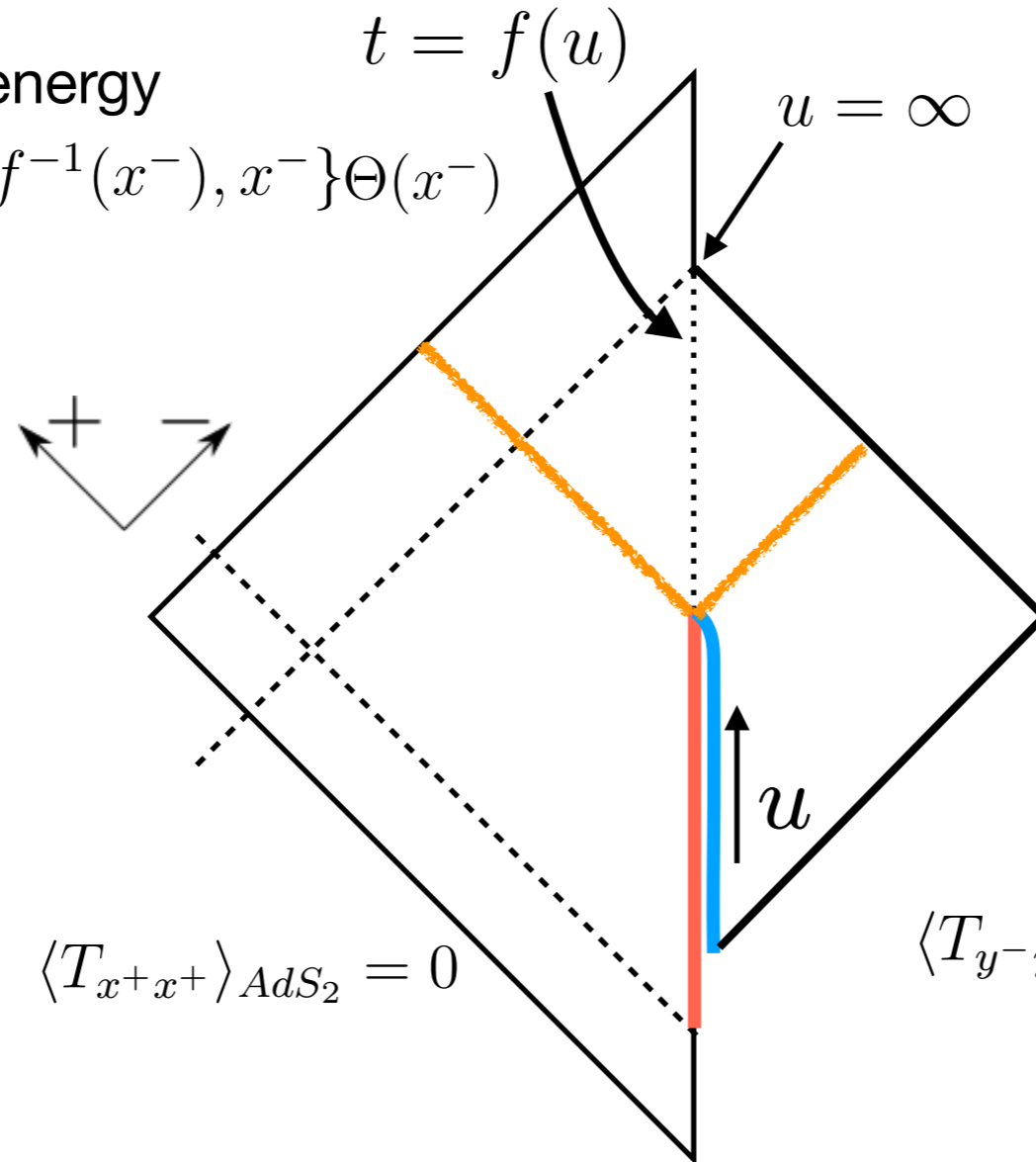
# Evaporating One Side of the Eternal Black hole

The Bath CFT is glued to the AdS<sub>2</sub> bulk along the physical boundary, identifying the physical time on the boundary with that of the bath.

Englesoy, Mertens, Verlinde

Infalling negative energy

$$\langle T_{x^-x^-} \rangle_{AdS_2} = -\frac{c}{24\pi} \{f^{-1}(x^-), x^-\} \Theta(x^-)$$



Bulk

$$ds^2 = \frac{-4dx^+ dx^-}{(x^+ - x^-)^2}$$

$$x^\pm = t \pm z$$

$$\langle T_{x^+x^+} \rangle_{AdS_2} = 0$$

$$\langle T_{y^-y^-} \rangle_{Flat} = 0$$

Bath

$$ds^2 = -dy^+ dy^-$$

$$y^\pm = u \pm \sigma$$

$$x^\pm = f(y^\pm)$$

# Evaporating One Side of the Eternal Black hole

The Bath CFT is glued to the AdS<sub>2</sub> bulk along the physical boundary, identifying the physical time on the boundary with that of the bath.

Englesoy, Mertens, Verlinde

Infalling negative energy

$$\langle T_{x^-x^-} \rangle_{AdS_2} = -\frac{c}{24\pi} \{f^{-1}(x^-), x^-\} \Theta(x^-) + E_S \delta(x^-)$$

Bulk

$$ds^2 = \frac{-4dx^+ dx^-}{(x^+ - x^-)^2}$$

$$x^\pm = t \pm z$$

$$\langle T_{x^+x^+} \rangle_{AdS_2} = 0$$

Bath

$$ds^2 = -dy^+ dy^-$$

$$y^\pm = u \pm \sigma$$

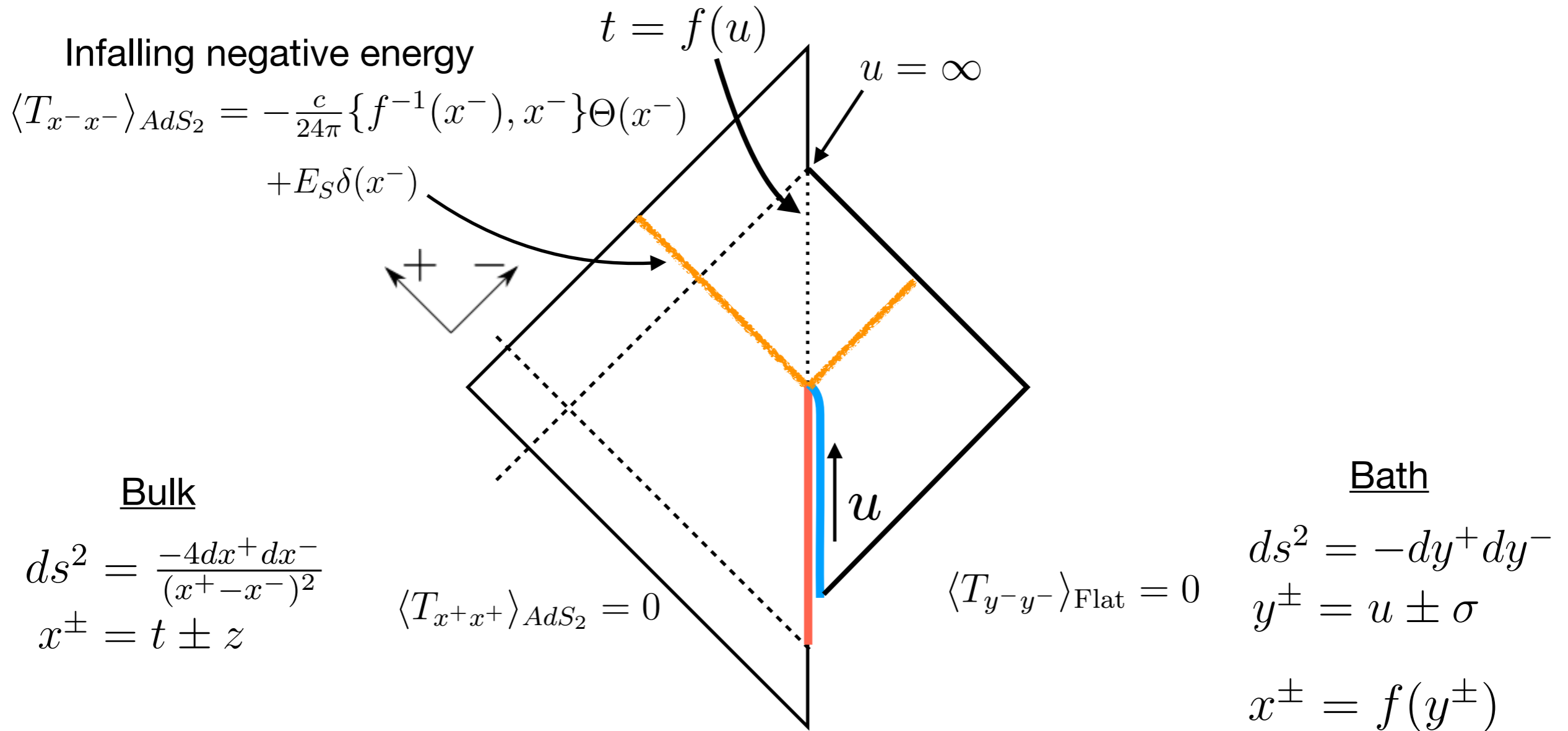
$$x^\pm = f(y^\pm)$$

$$\langle T_{y^-y^-} \rangle_{Flat} = 0$$

# Evaporating One Side of the Eternal Black hole

The Bath CFT is glued to the  $AdS_2$  bulk along the physical boundary, identifying the physical time on the boundary with that of the bath.

Englesoy, Mertens, Verlinde



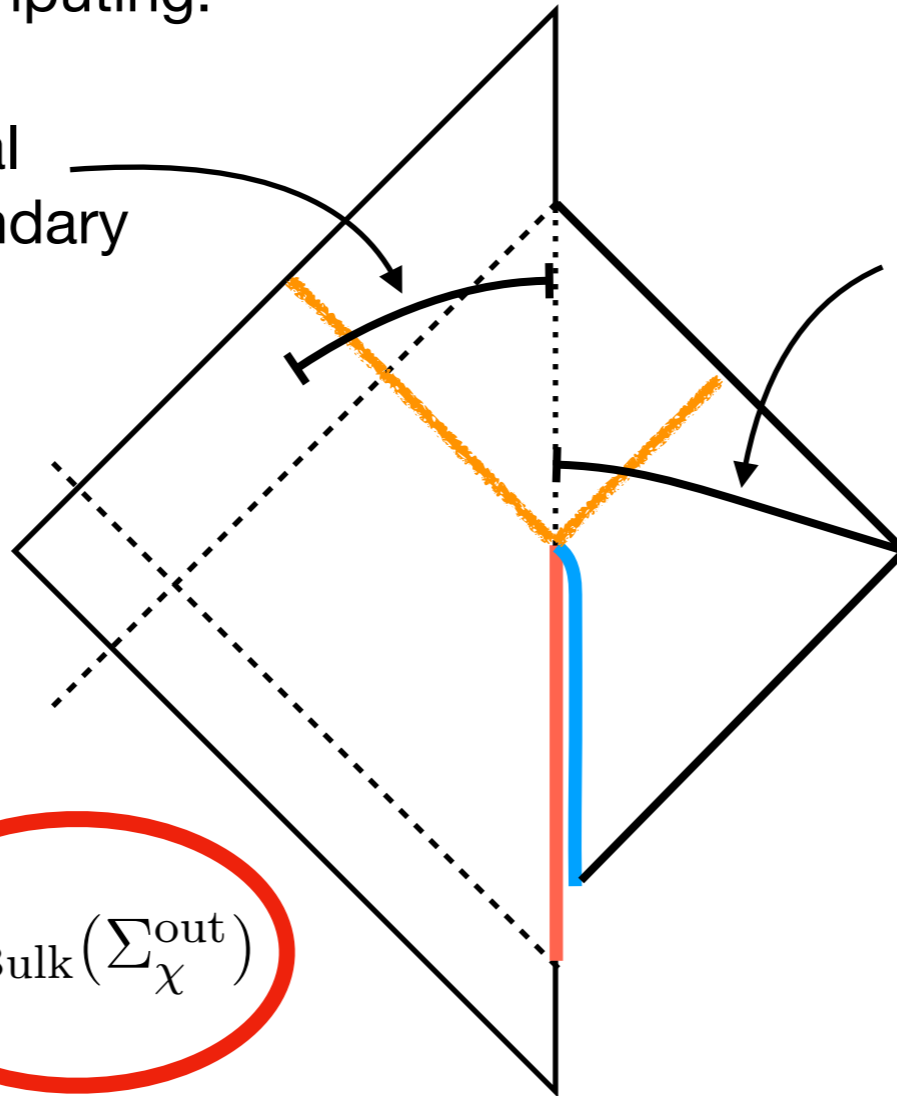
Semiclassical Limit: We source the JT equations with  $\langle T_{ab} \rangle_{AdS_2}$  to determine  $\phi$  and  $f(u)$

# Bulk Entanglement Entropy

We are interested in computing:

Entropy of interval  
anchored to boundary

Entropy of the bath or  
Hawking Radiation



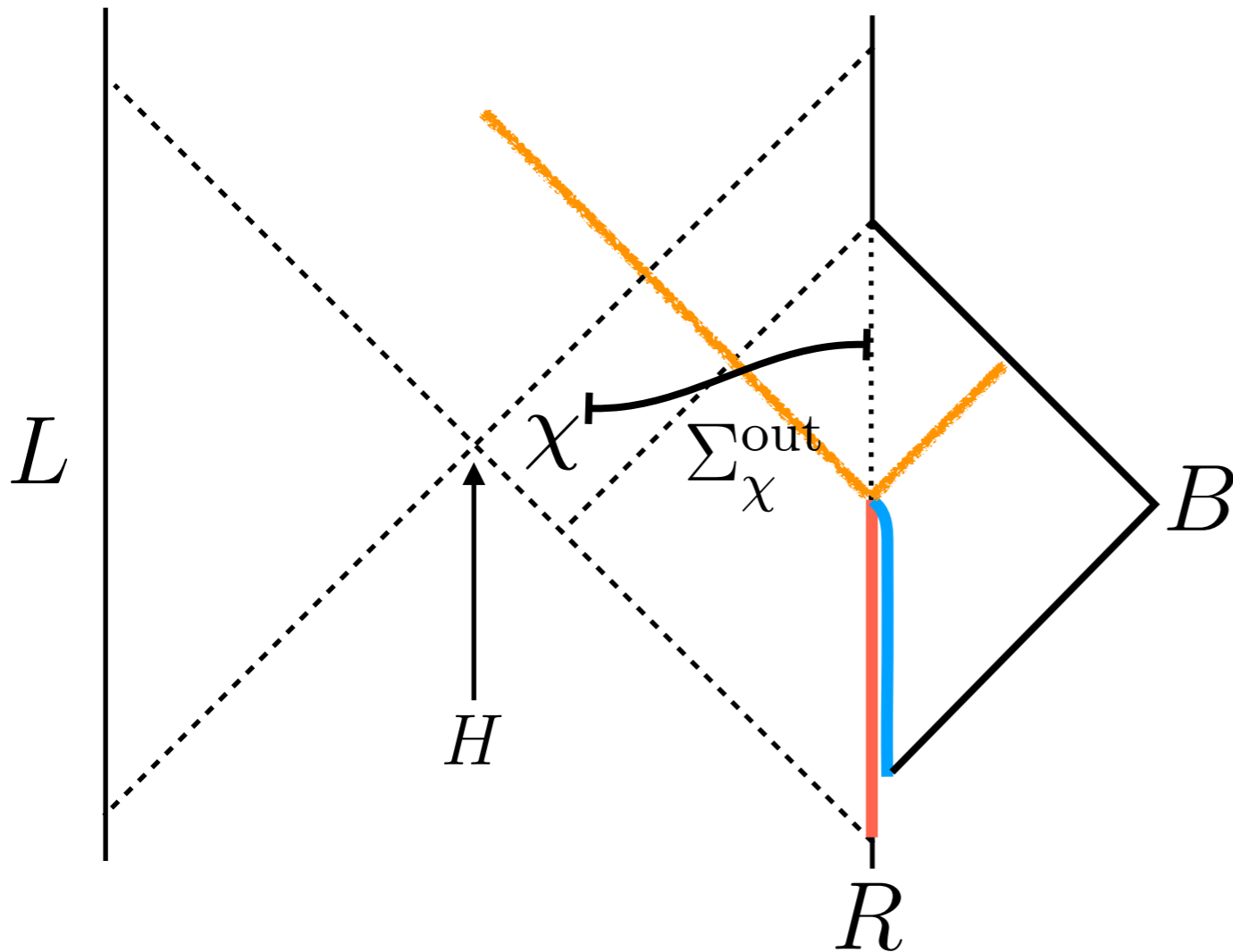
$$S_{gen} = \frac{\phi_0}{4G_N} + \frac{\phi(\chi)}{4G_N} + S_{\text{Bulk}}(\Sigma_{\chi}^{\text{out}})$$

1. Transformation to a coordinate system where stress tensor is trivial
2. Compute Entropy in that coordinate system
3. Transform back to physical coordinates and cutoffs

# Finding the Quantum Extremal Surfaces

Generalized Entanglement Entropy  $S_{gen} = \frac{\phi_0}{4G_N} + \frac{\phi(\chi)}{4G_N} + S_{\text{Bulk}}(\Sigma_\chi^{\text{out}})$

Extremize:  $\partial_\pm S_{gen} = \frac{1}{4G_N} \partial_\pm \phi(\chi) + \partial_\pm S_{\text{Bulk}}(\Sigma_\chi^{\text{out}}) = 0$

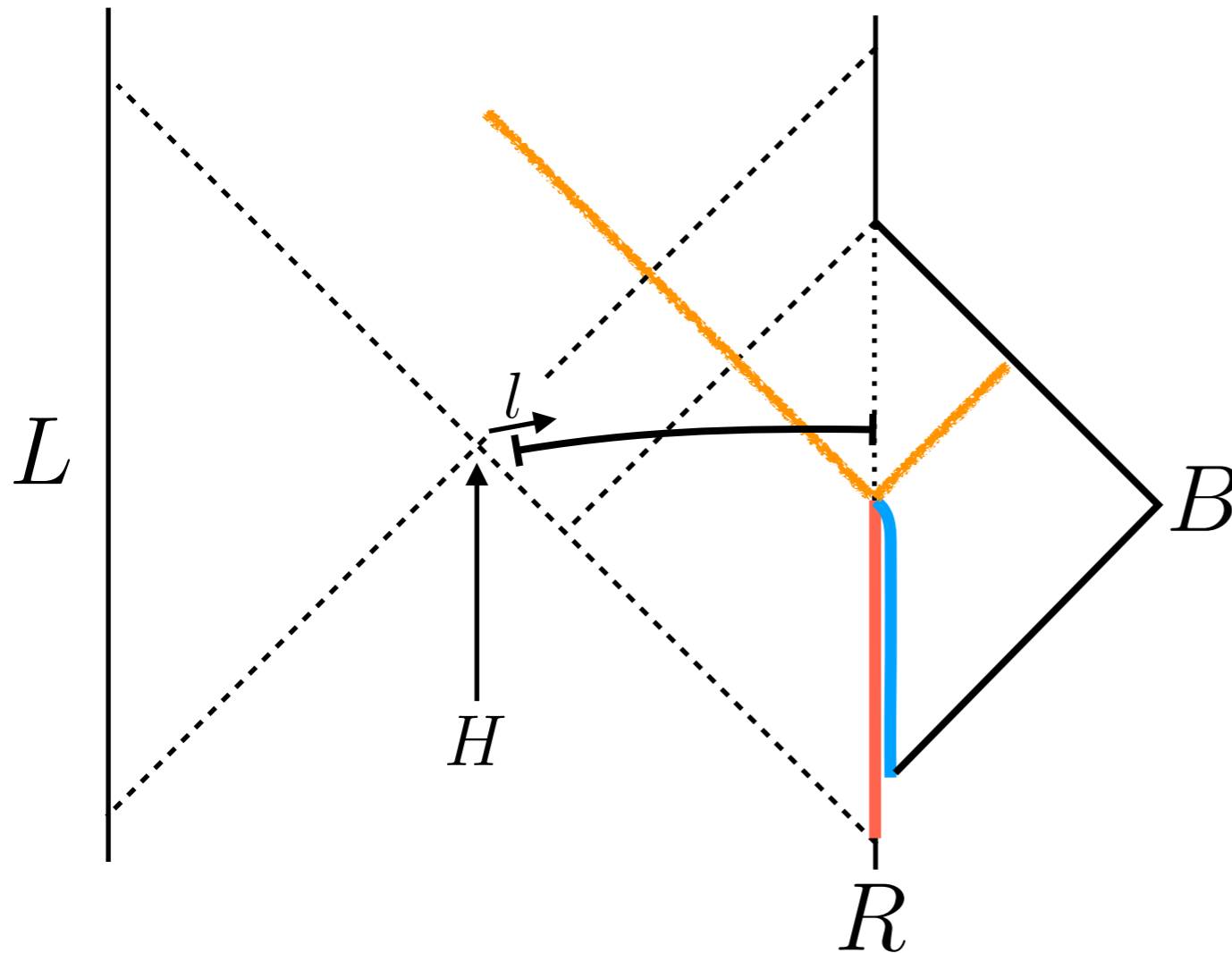




# Finding the Quantum Extremal Surfaces

Generalized Entanglement Entropy  $S_{gen} = \frac{\phi_0}{4G_N} + \frac{\phi(\chi)}{4G_N} + S_{\text{Bulk}}(\Sigma_\chi^{\text{out}})$

Extremize:  $\partial_\pm S_{gen} = \frac{1}{4G_N} \partial_\pm \phi(\chi) + \partial_\pm S_{\text{Bulk}}(\Sigma_\chi^{\text{out}}) = 0$



## Early Time Branch

Bifurcation surface is *classically* extremal but not *quantum* extremal

$$\delta \frac{\phi}{4G_N} \sim \frac{l^2}{G_N}$$

$$\delta S_{\text{Bulk}} \sim cl$$

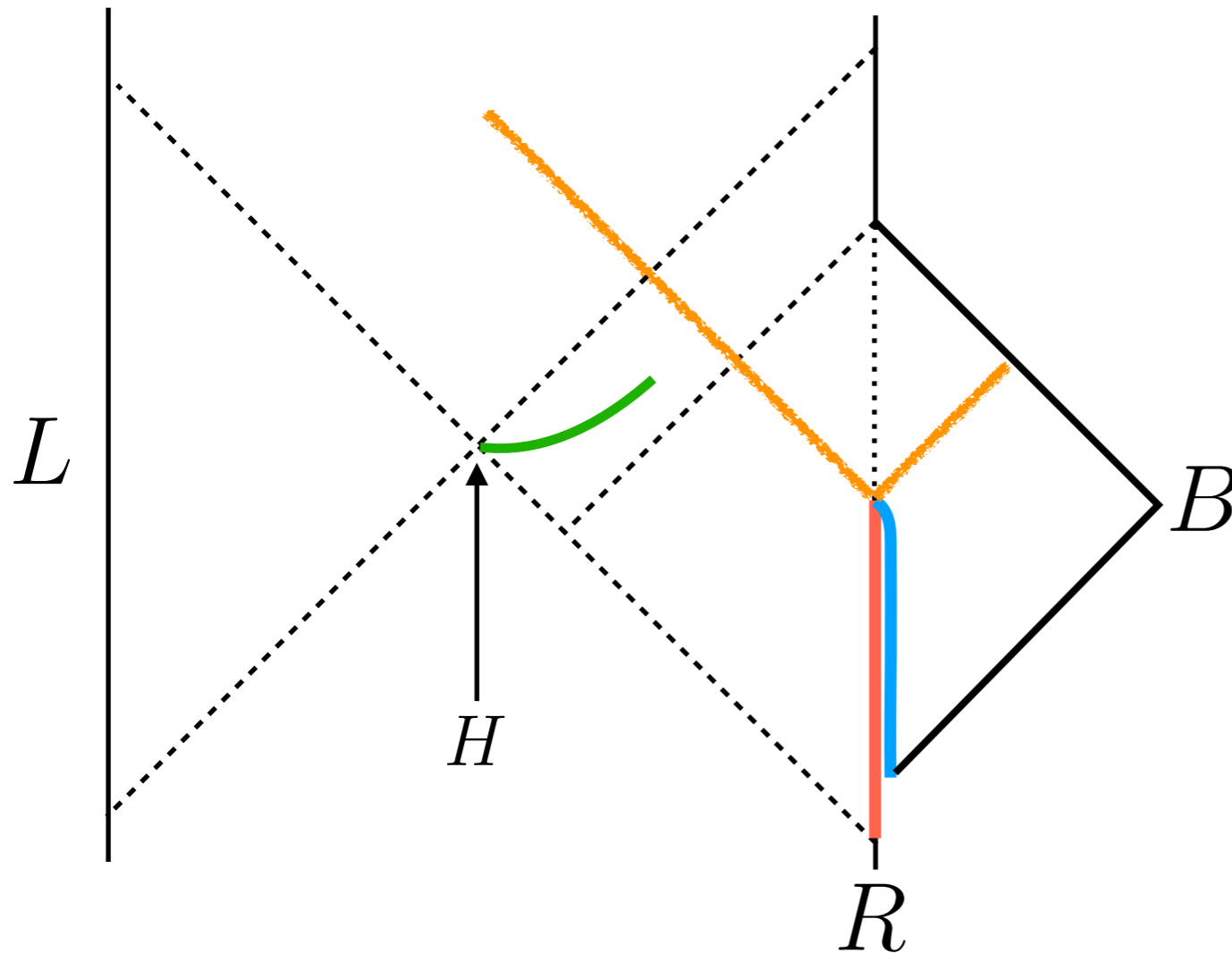
There is QES near the horizon at a distance

$$l \sim cG_N$$

# Finding the Quantum Extremal Surfaces

Generalized Entanglement Entropy  $S_{gen} = \frac{\phi_0}{4G_N} + \frac{\phi(\chi)}{4G_N} + S_{\text{Bulk}}(\Sigma_\chi^{\text{out}})$

Extremize:  $\partial_\pm S_{gen} = \frac{1}{4G_N} \partial_\pm \phi(\chi) + \partial_\pm S_{\text{Bulk}}(\Sigma_\chi^{\text{out}}) = 0$



## Early Time Branch

Bifurcation surface is *classically* extremal but not *quantum* extremal

$$\delta \frac{\phi}{4G_N} \sim \frac{l^2}{G_N}$$

$$\delta S_{\text{Bulk}} \sim cl$$

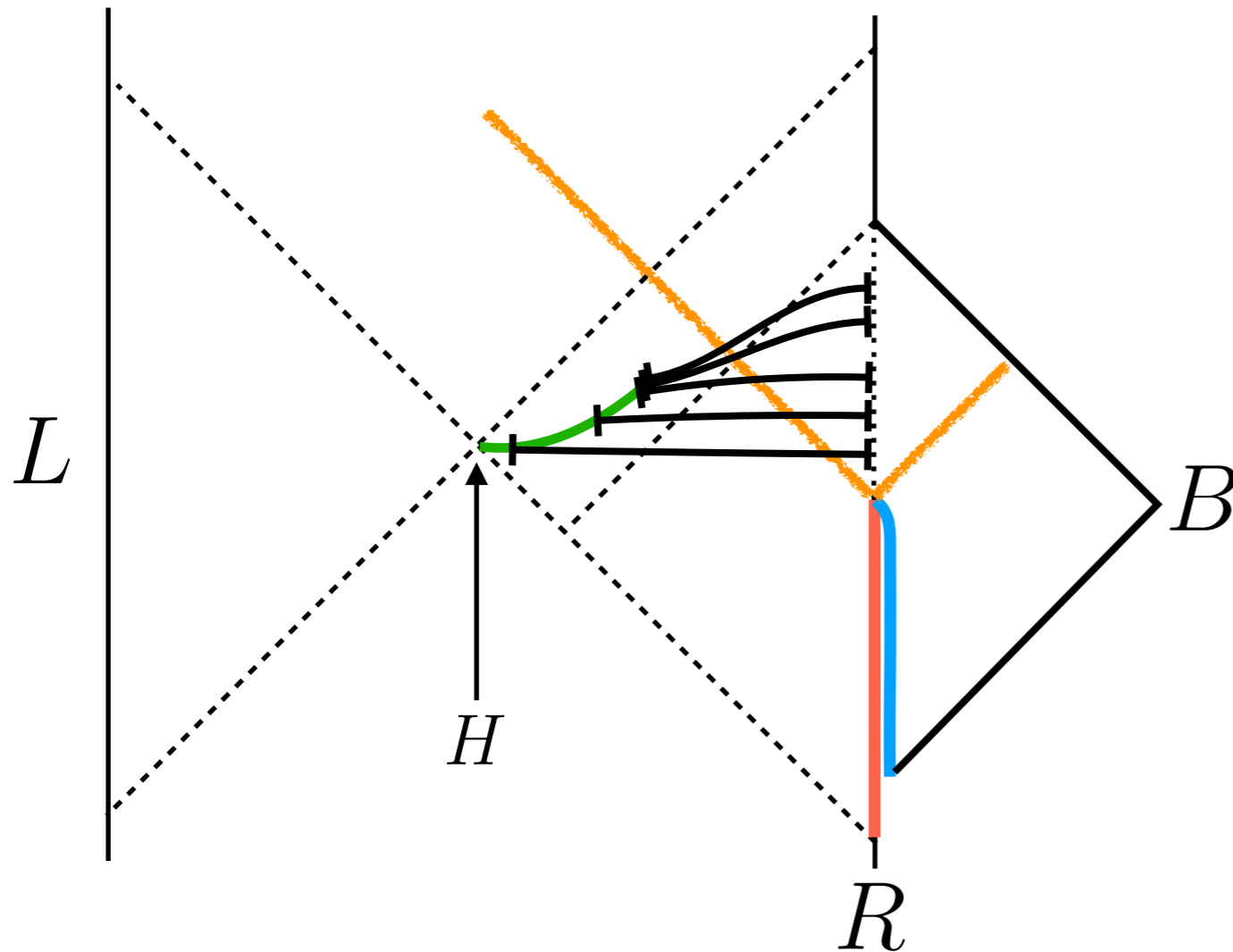
There is QES near the horizon at a distance

$$l \sim cG_N$$

# Finding the Quantum Extremal Surfaces

Generalized Entanglement Entropy  $S_{gen} = \frac{\phi_0}{4G_N} + \frac{\phi(\chi)}{4G_N} + S_{\text{Bulk}}(\Sigma_\chi^{\text{out}})$

Extremize:  $\partial_\pm S_{gen} = \frac{1}{4G_N} \partial_\pm \phi(\chi) + \partial_\pm S_{\text{Bulk}}(\Sigma_\chi^{\text{out}}) = 0$



## Early Time Branch

Bifurcation surface is *classically* extremal but not *quantum* extremal

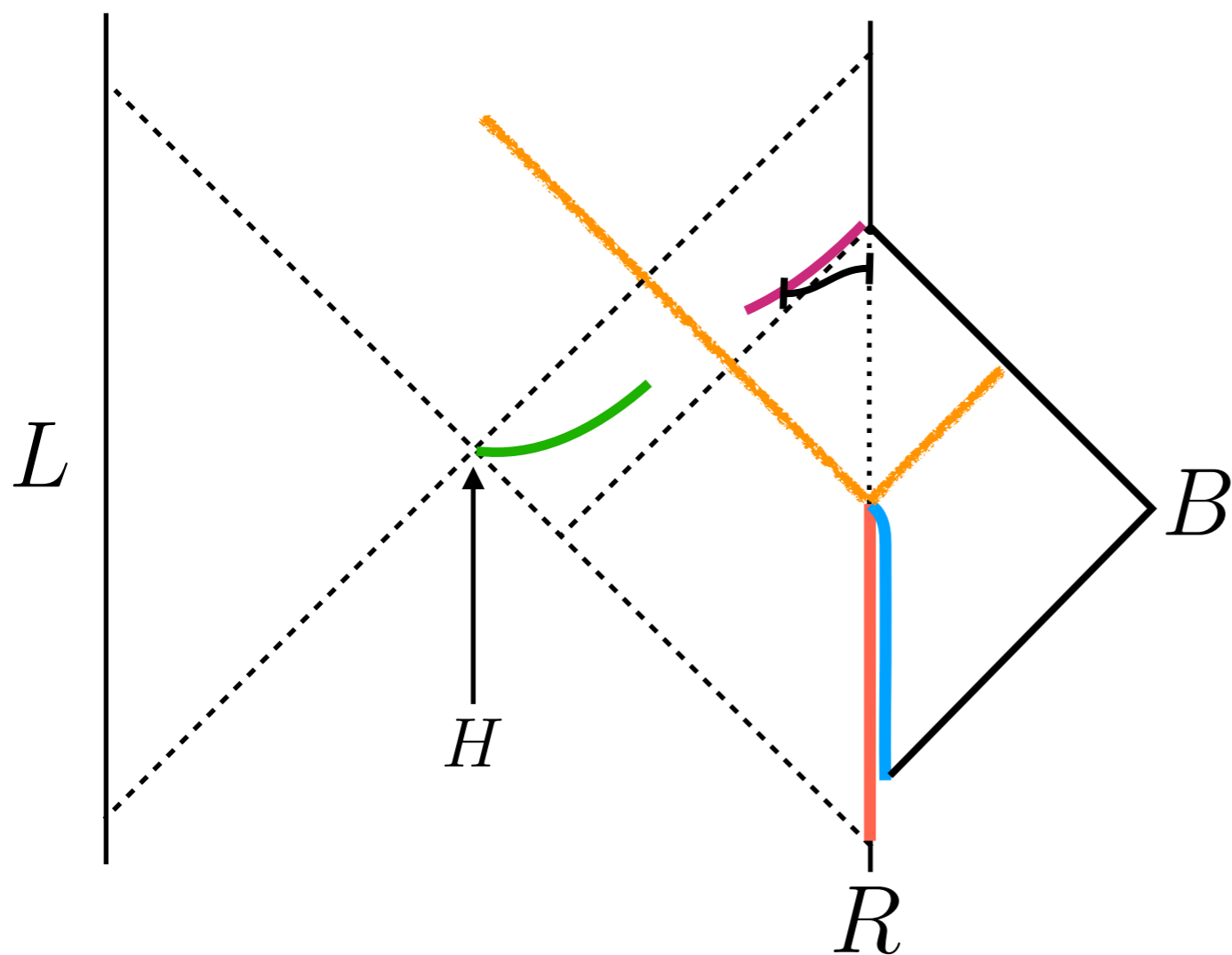
$$\delta \frac{\phi}{4G_N} \sim \frac{l^2}{G_N}$$

$$\delta S_{\text{Bulk}} \sim cl$$

There is QES near the horizon at a distance

$$l \sim cG_N$$

# Finding the Quantum Extremal Surfaces



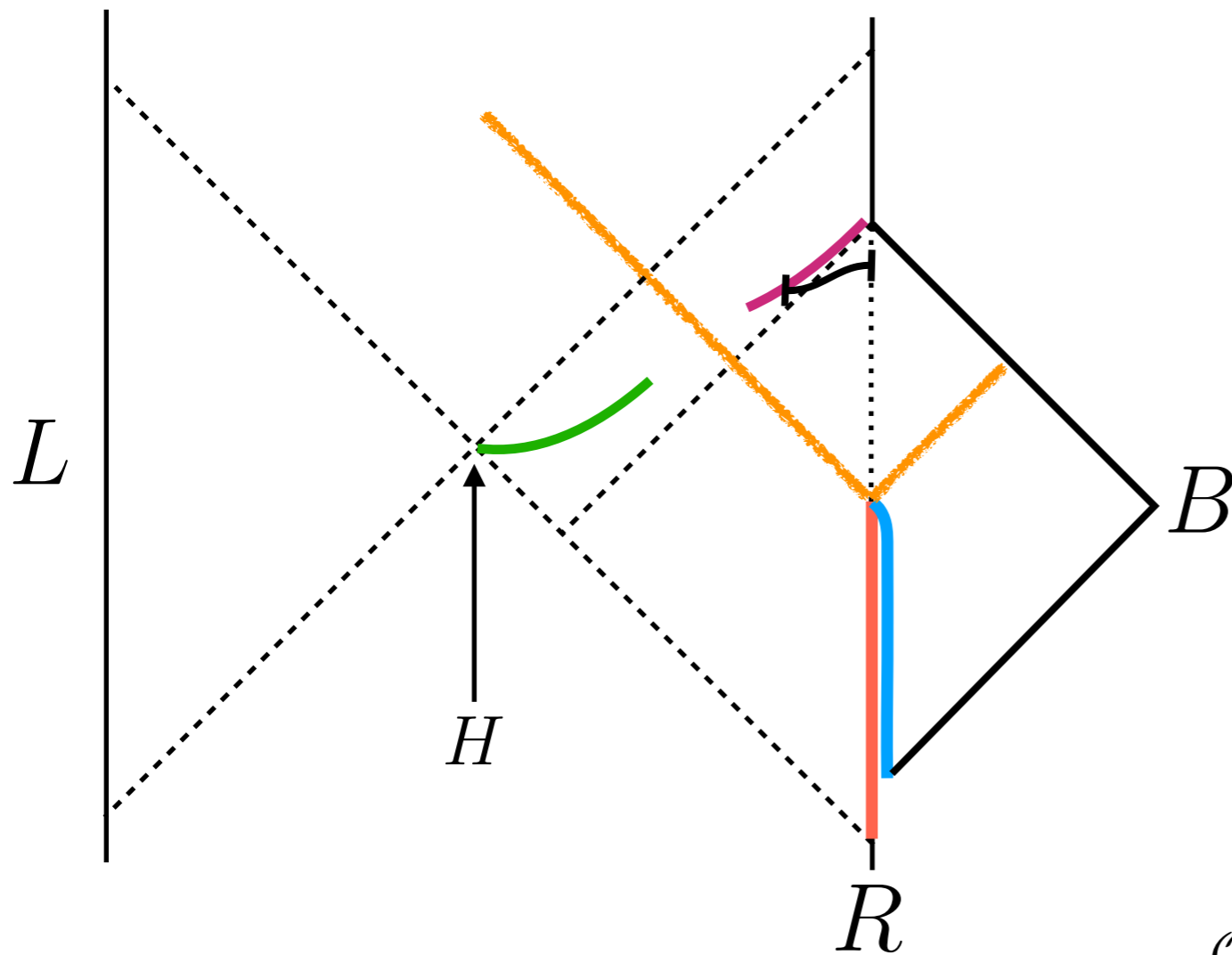
Late Time Branch

Outgoing:

$$\partial_- S_{Gen} = \frac{1}{4G_N} \partial_- \phi + \partial_- S_{Bulk} = 0$$

Search near the apparent horizon

# Finding the Quantum Extremal Surfaces



Late Time Branch

Outgoing:

$$\partial_- S_{Gen} = \frac{1}{4G_N} \partial_- \phi + \partial_- S_{Bulk} = 0$$

Search near the apparent horizon

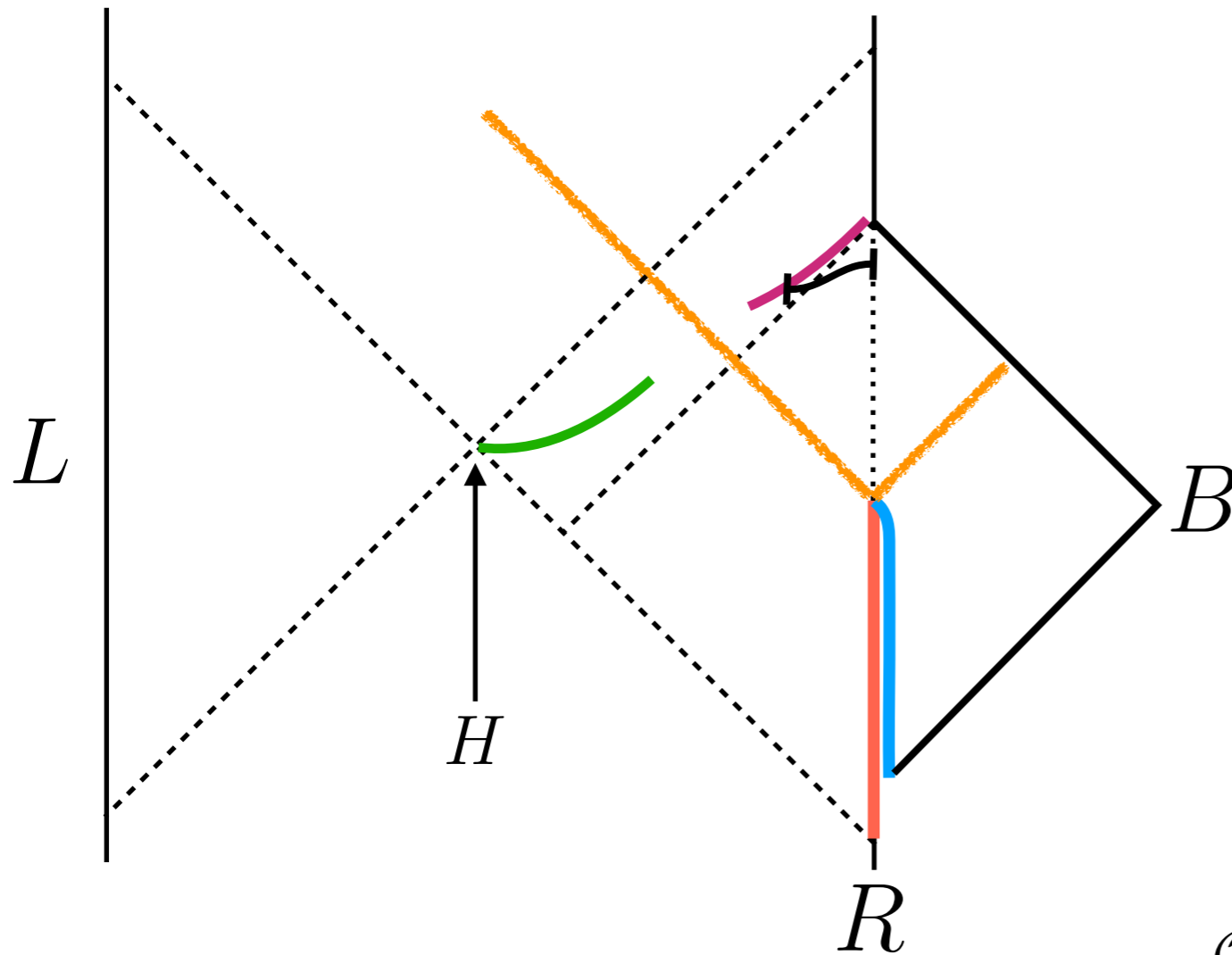
Ingoing:

$$\partial_+ S_{Gen} = \frac{1}{4G_N} \partial_+ \phi + \partial_+ S_{Bulk} = 0$$

$\mathcal{O}(1) < 0$

Must be compensated by large

# Finding the Quantum Extremal Surfaces



Late Time Branch

Outgoing:

$$\partial_- S_{Gen} = \frac{1}{4G_N} \partial_- \phi + \partial_- S_{Bulk} = 0$$

Search near the apparent horizon

Ingoing:

$$\partial_+ S_{Gen} = \frac{1}{4G_N} \partial_+ \phi + \partial_+ S_{Bulk} = 0$$

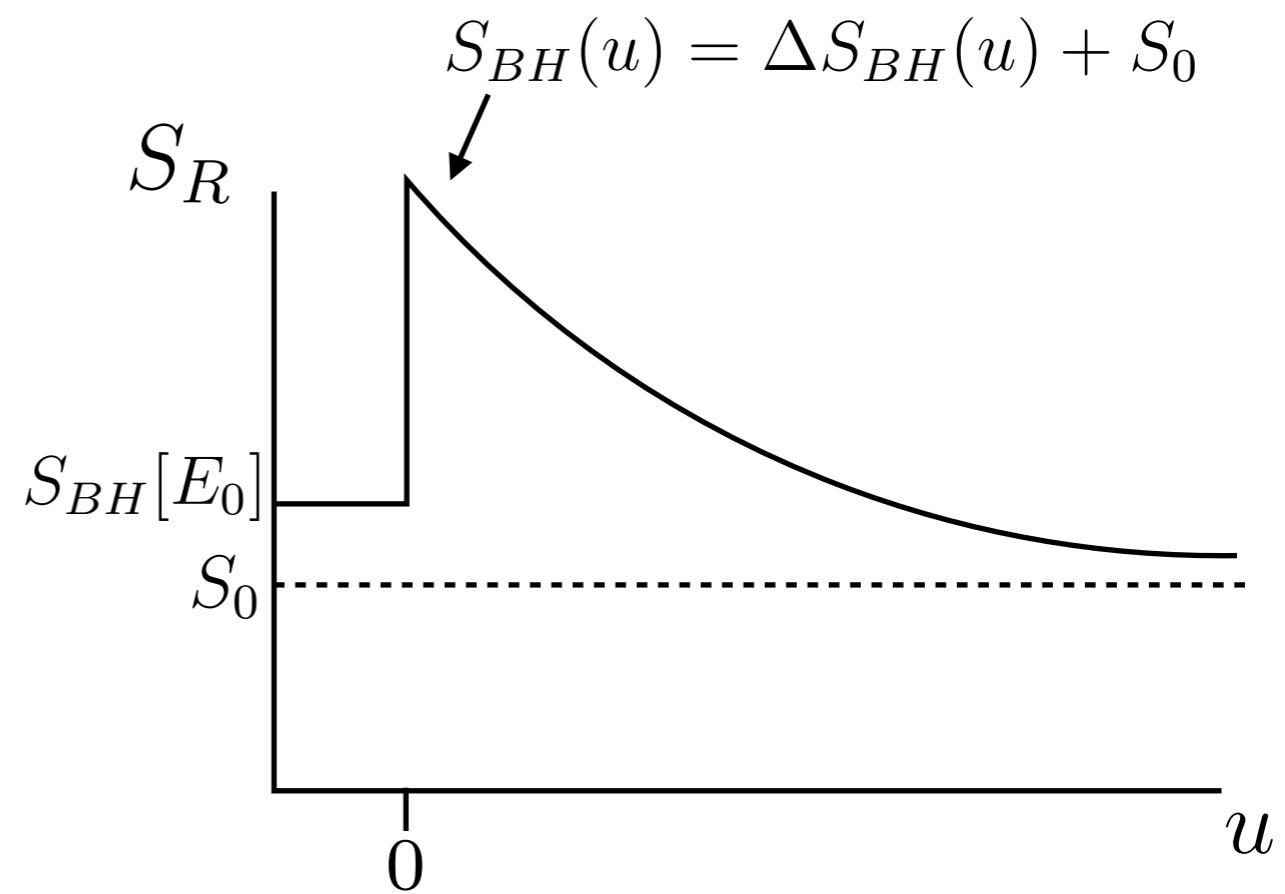
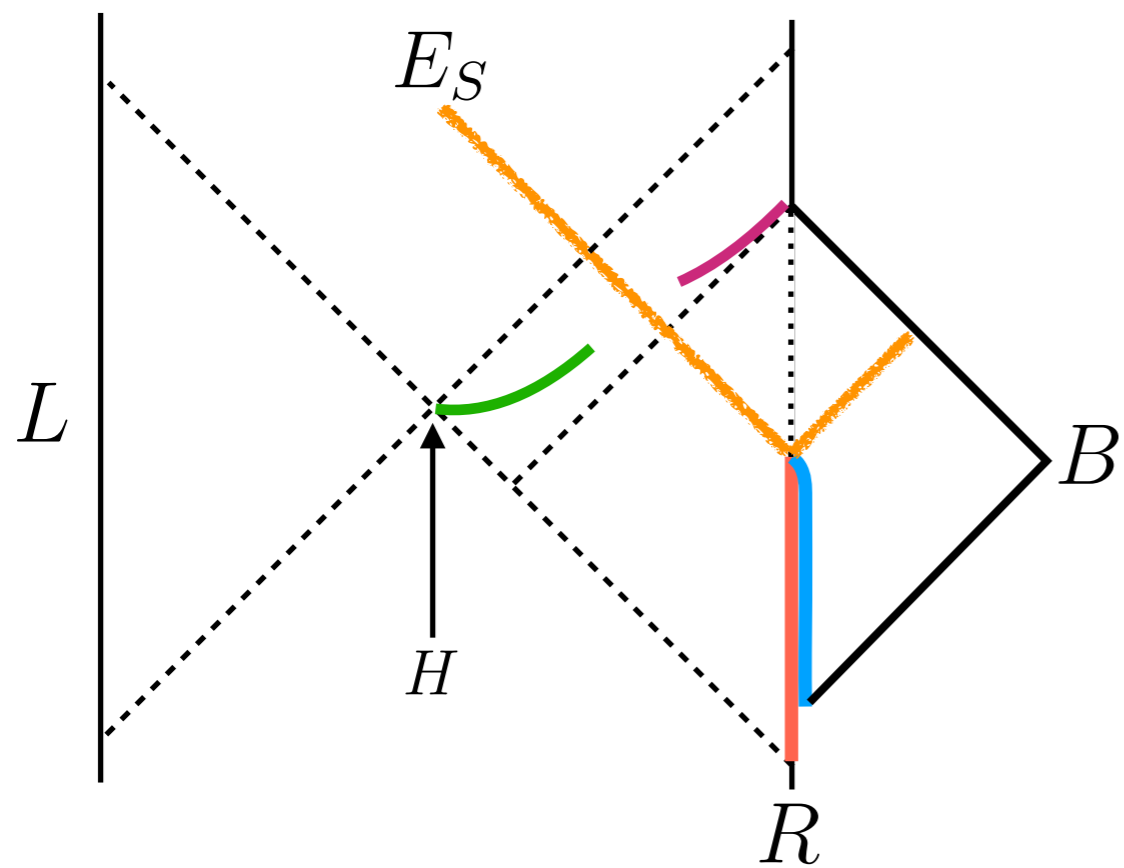
$\mathcal{O}(1) < 0$

Must be compensated by large

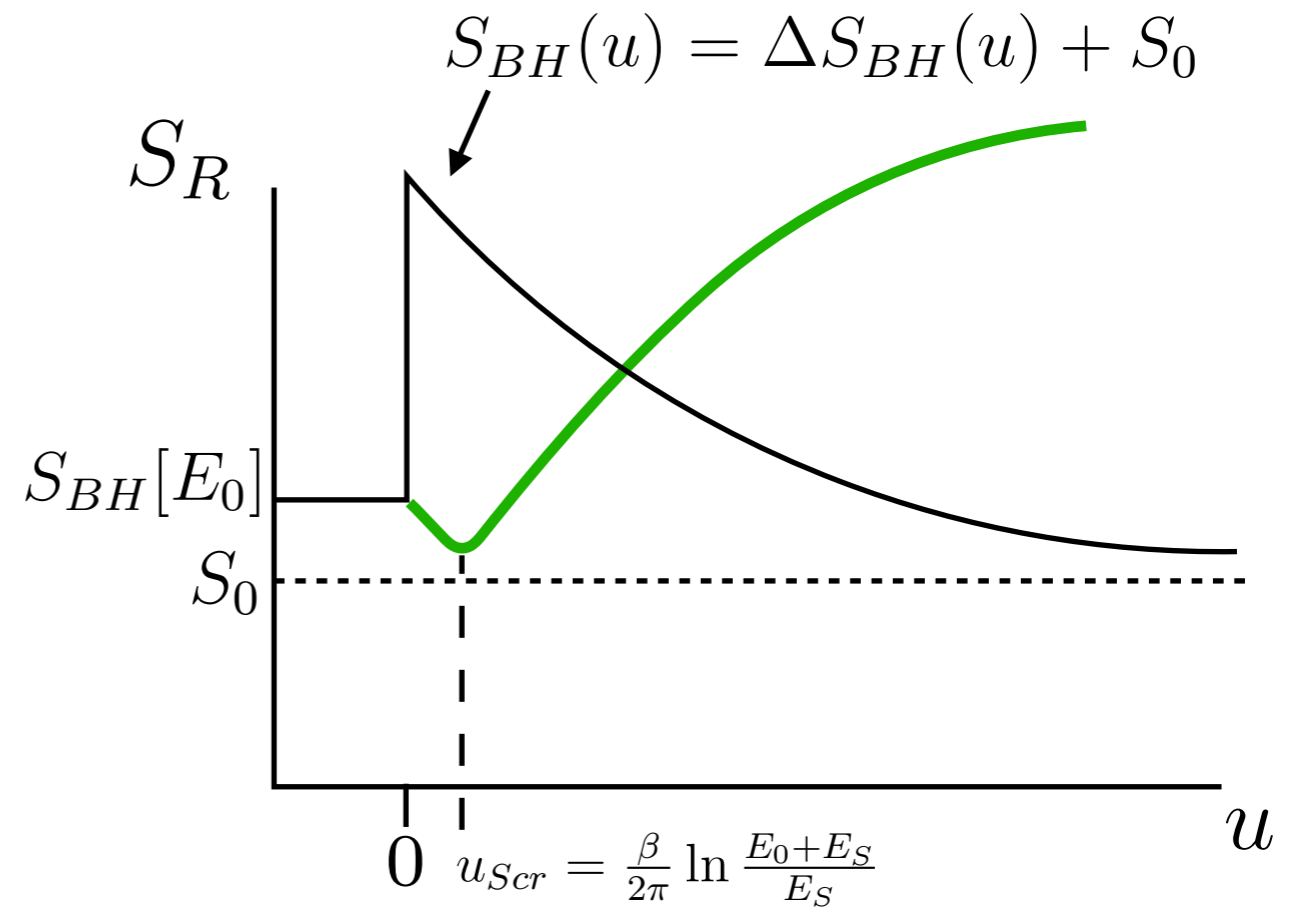
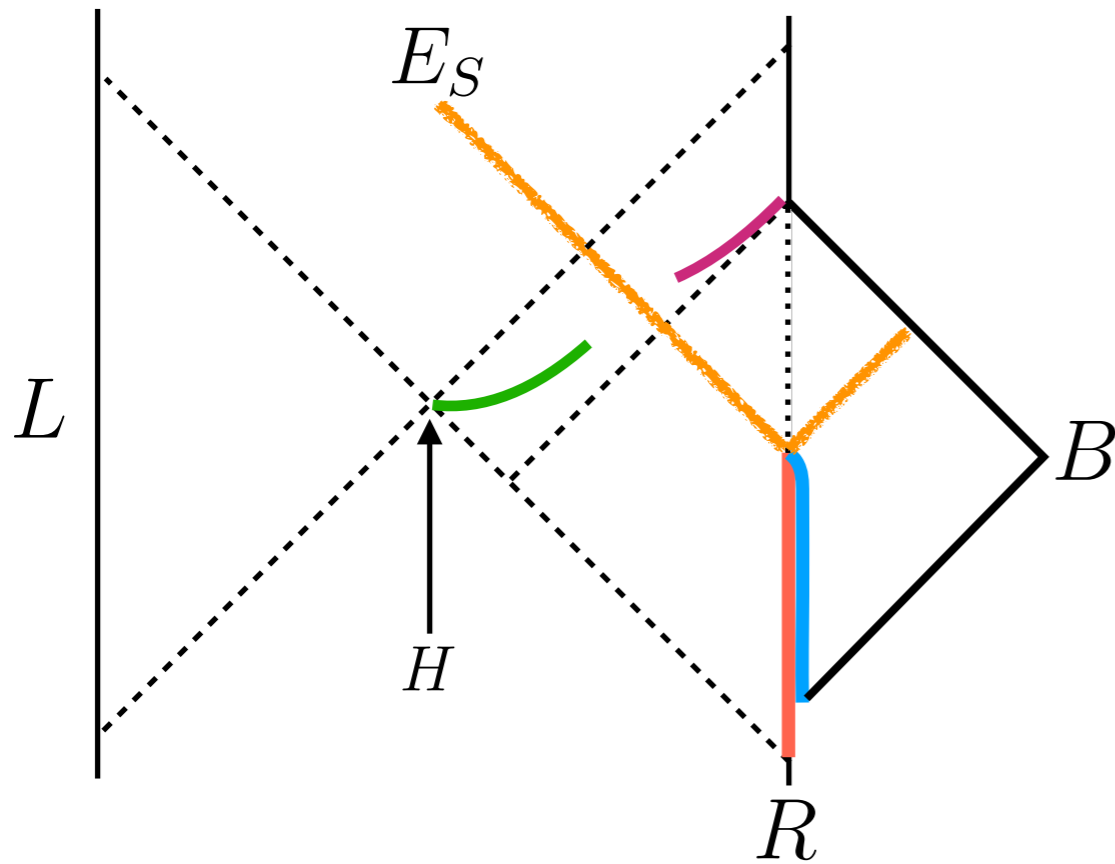
Entropy (roughly):  $S_{Bulk} \sim c \log [\text{Length}]$

Achieved in the null interval limit:  $\partial_+ S_{Bulk} \sim \frac{c}{\text{Length}} \implies \text{Length} \sim cG_N$

# Evolution of the Entropy



# Evolution of the Entropy



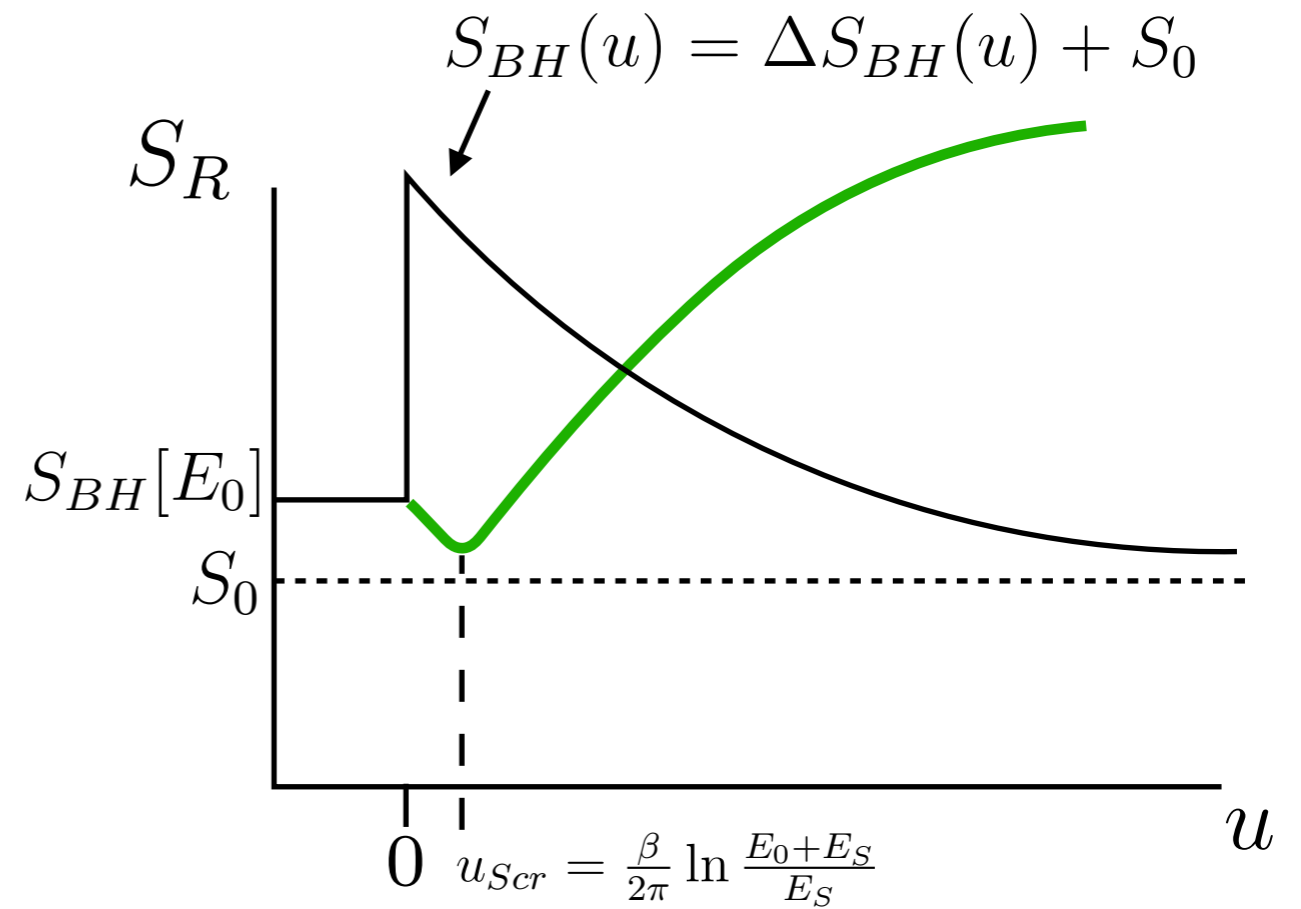
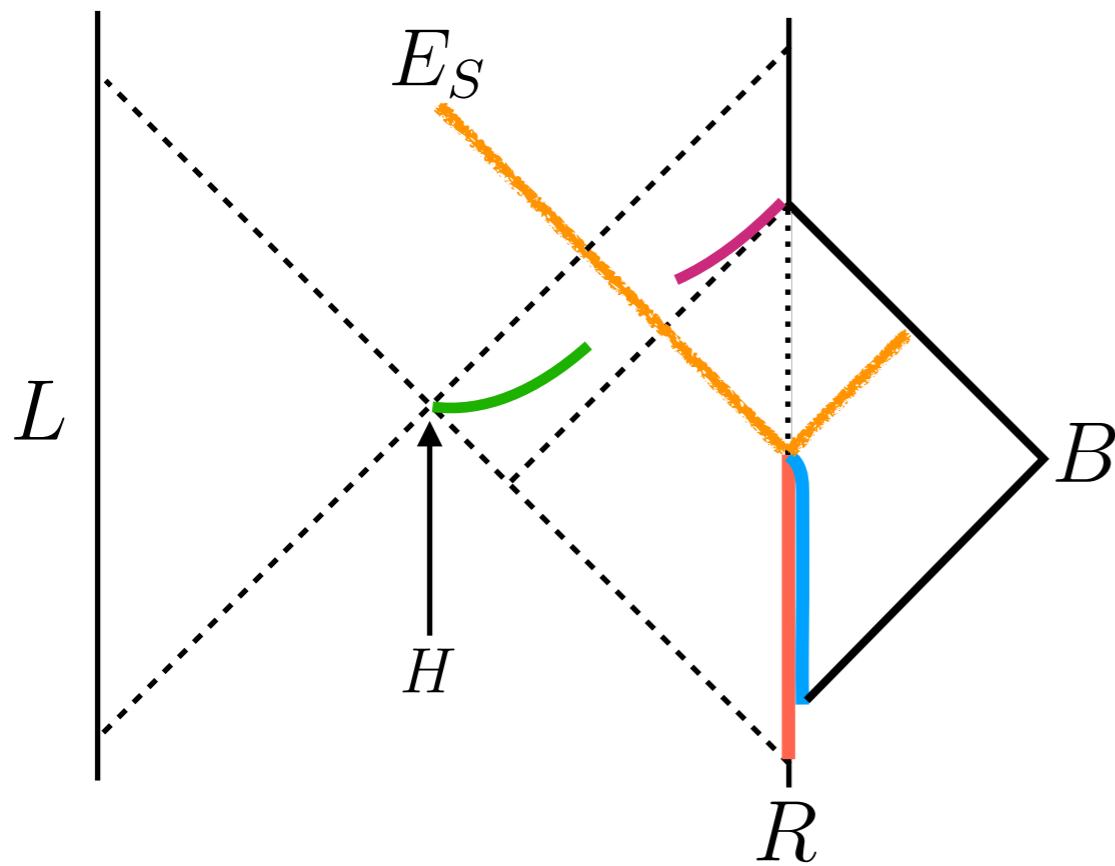
**Early time branch:** QES near original bifurcate horizon

$$S_{Gen}^R \approx \frac{\phi_0}{4G_N} + \frac{\phi(H)}{4G_N} + S_{Bulk}(u)$$

where  $S_{Bulk} \xrightarrow{\text{Late}} 2\Delta S_{BH}[E_0 + E_S] \times (1 - e^{-cG_N u/2})$



# Evolution of the Entropy



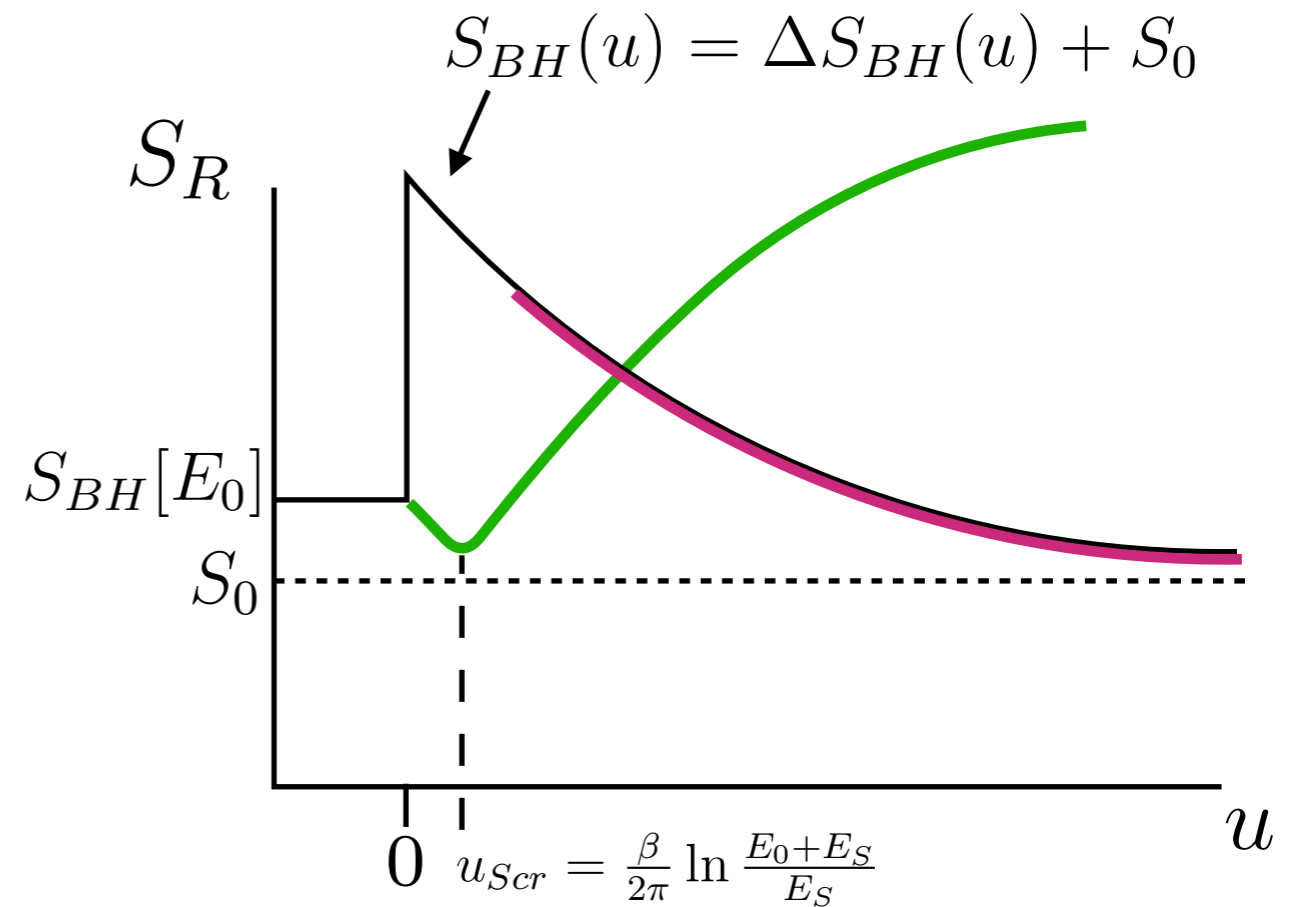
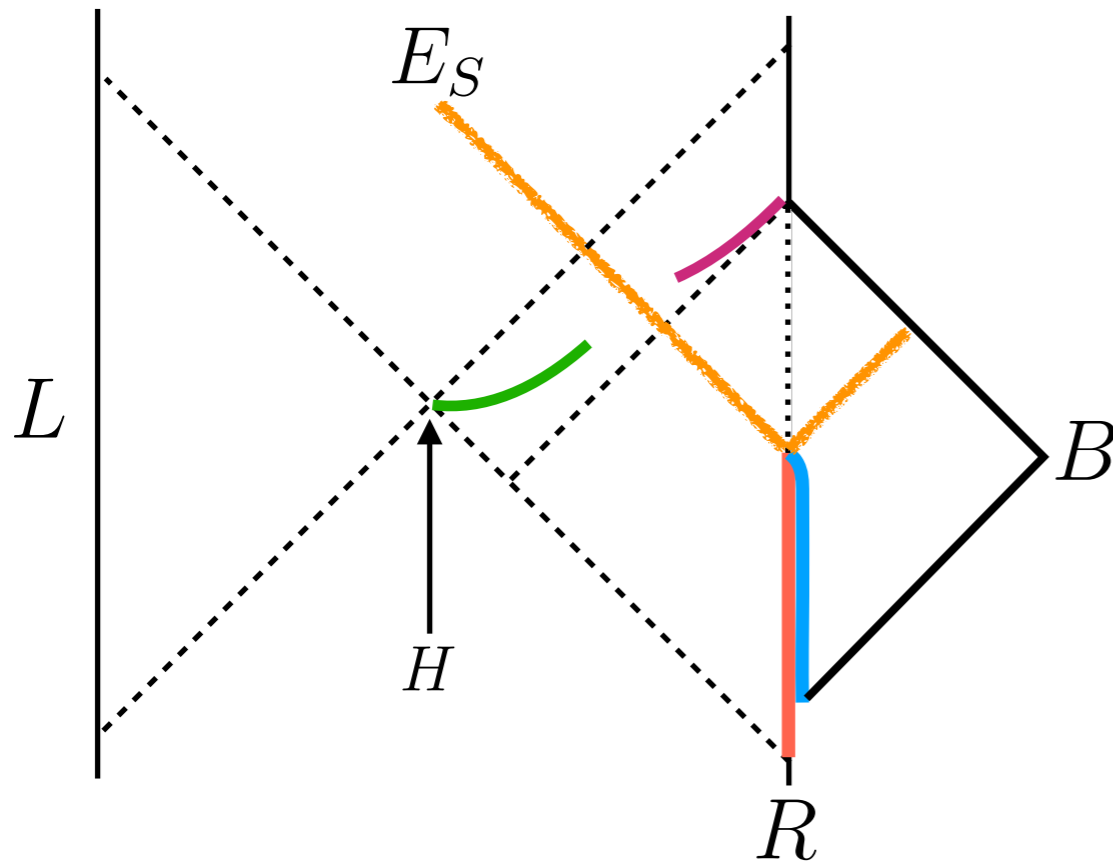
**Leichenauer**

**Early time branch:** QES near original bifurcate horizon

$$S_{Gen}^R \approx \frac{\phi_0}{4G_N} + \frac{\phi(H)}{4G_N} + S_{Bulk}(u)$$

where  $S_{Bulk} \xrightarrow{\text{Late}} 2\Delta S_{BH}[E_0 + E_S] \times (1 - e^{-cG_N u/2})$

# Evolution of the Entropy



**Leichenauer**

**Early time branch:** QES near original bifurcate horizon

$$S_{Gen}^R \approx \frac{\phi_0}{4G_N} + \frac{\phi(H)}{4G_N} + S_{Bulk}(u)$$

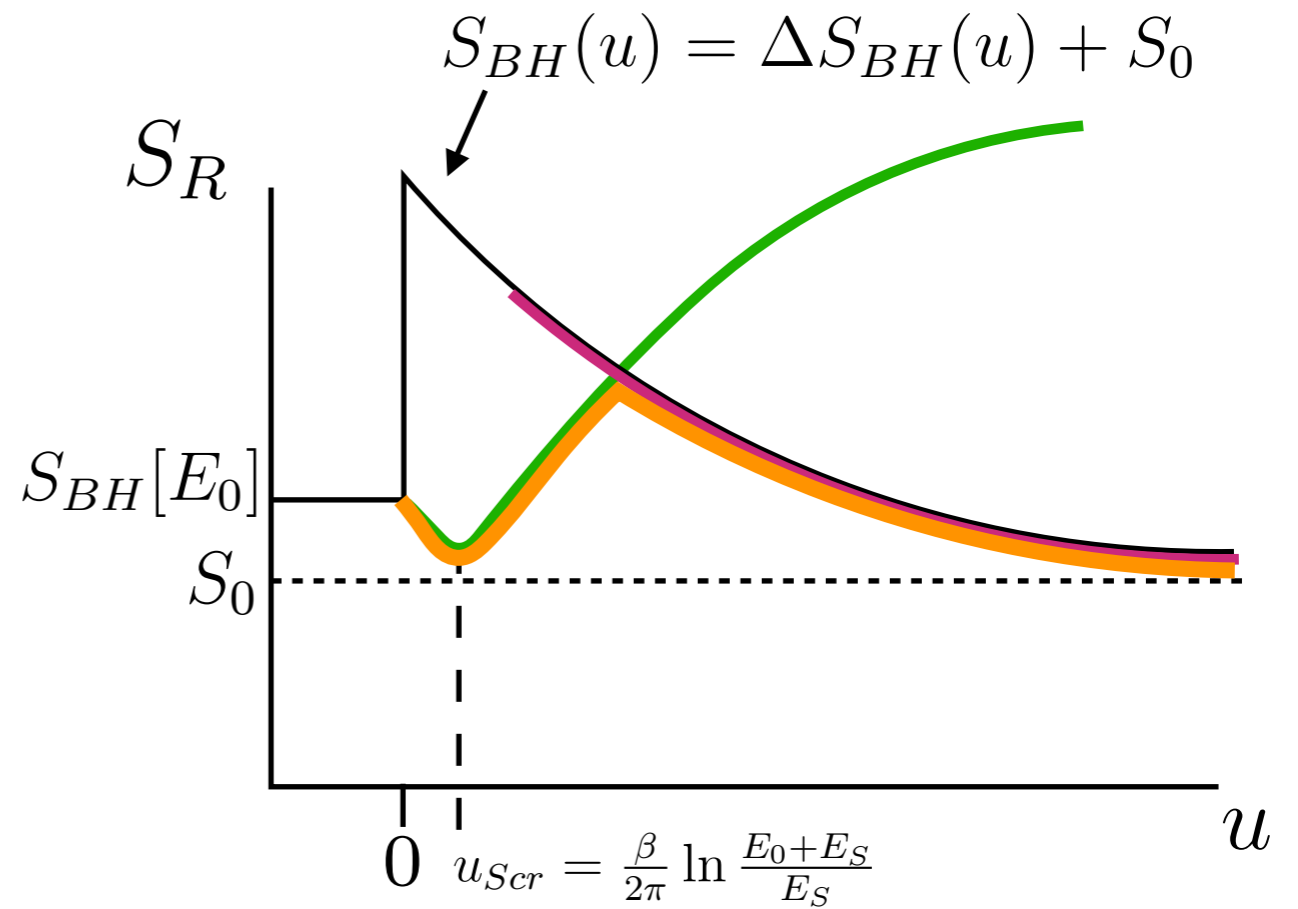
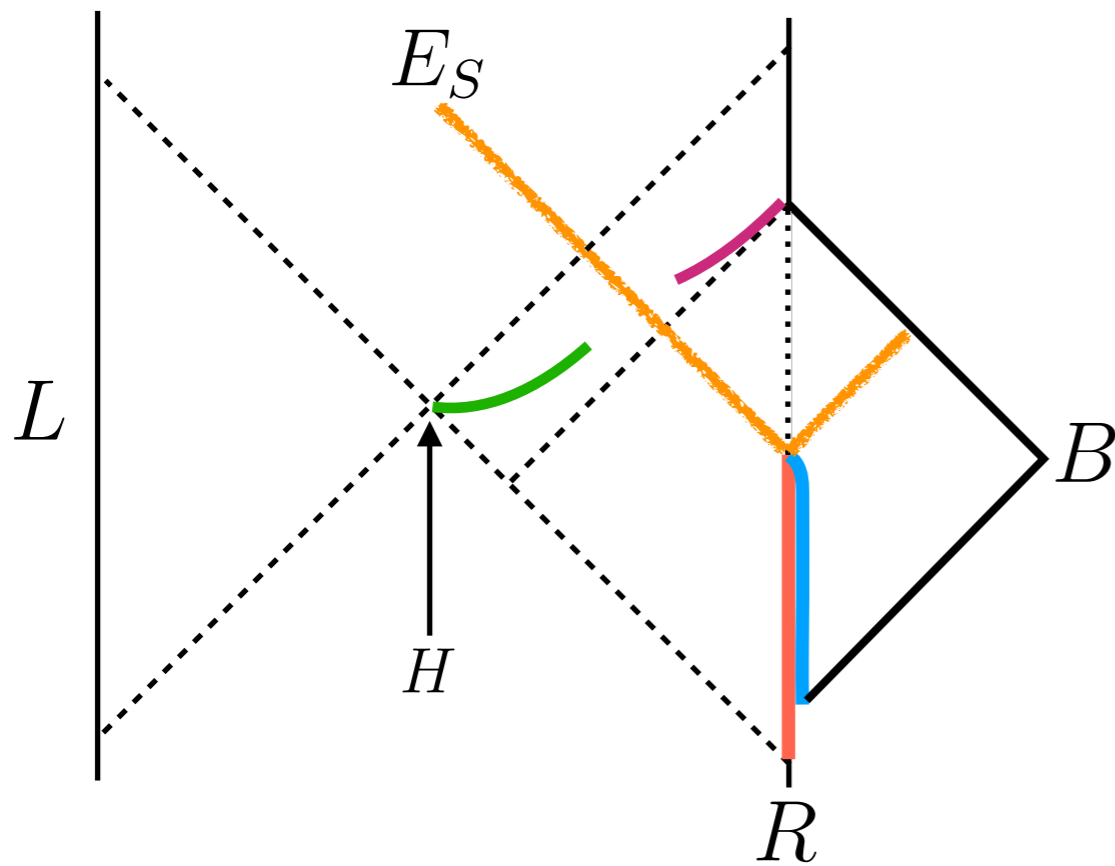
where  $S_{Bulk} \xrightarrow{\text{Late}} 2\Delta S_{BH}[E_0 + E_S] \times (1 - e^{-cG_N u/2})$

**Late time branch:** QES near the apparent horizon

$$S_{Gen}^R \approx S_{BH}[E_{ADM}(u)] - c \ln cG_N$$

$$\approx S_{BH}[E_0 + E_S]e^{-cG_N u/2} - c \ln cG_N$$

# Evolution of the Entropy



**Leichenauer**

**Early time branch:**

QES near original bifurcate horizon

$$S_{Gen}^R \approx \frac{\phi_0}{4G_N} + \frac{\phi(H)}{4G_N} + S_{Bulk}(u)$$

where  $S_{Bulk} \xrightarrow{\text{Late}} 2\Delta S_{BH}[E_0 + E_S] \times (1 - e^{-cG_N u/2})$

**Late time branch:**

QES near the apparent horizon

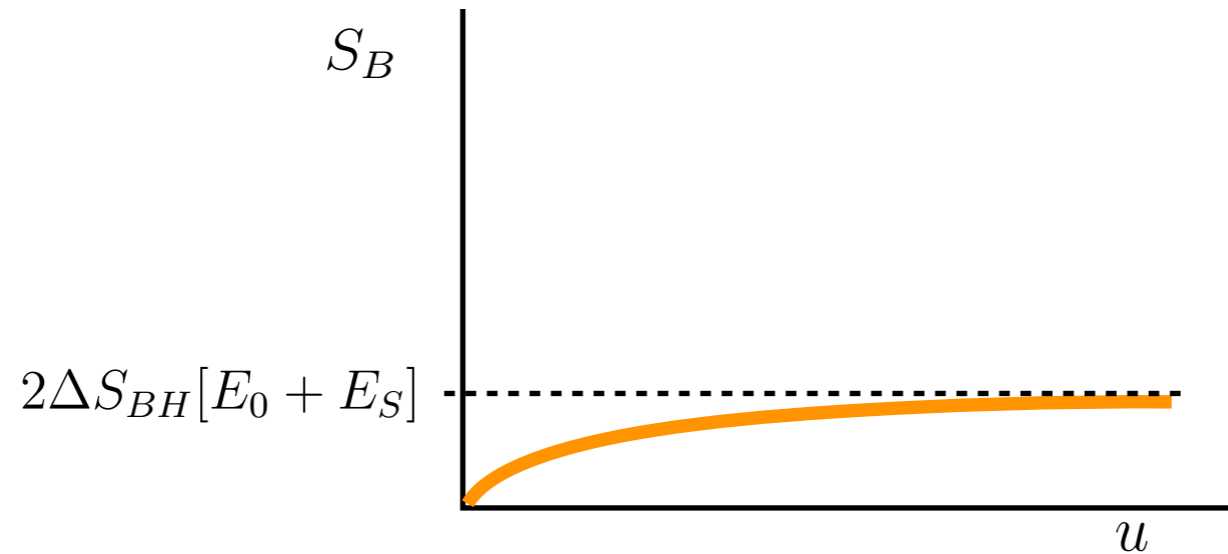
$$S_{Gen}^R \approx S_{BH}[E_{ADM}(u)] - c \ln cG_N$$

$$\approx S_{BH}[E_0 + E_S]e^{-cG_N u/2} - c \ln cG_N$$

$S_R(u) = \text{Min}$

# Does this resolve the Information Paradox?

What about the entropy of the bath?

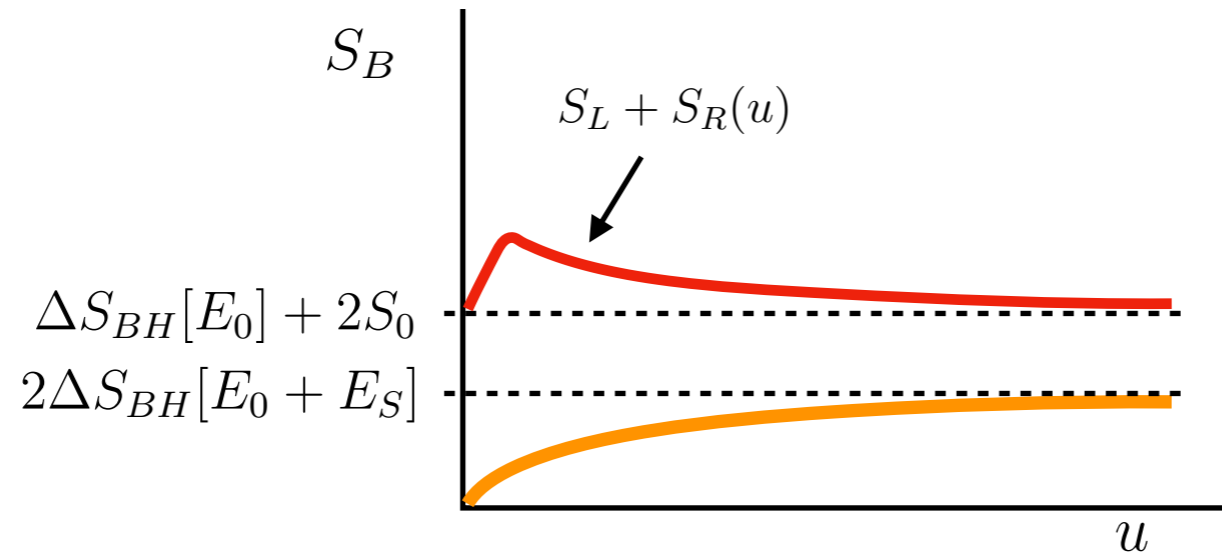


The Araki-Lieb inequality provides an upper bound on the bath entropy:

$$S_L = S_{RB} \geq |S_R - S_B| \implies S_B \leq S_L + S_R$$

# Does this resolve the Information Paradox?

What about the entropy of the bath?



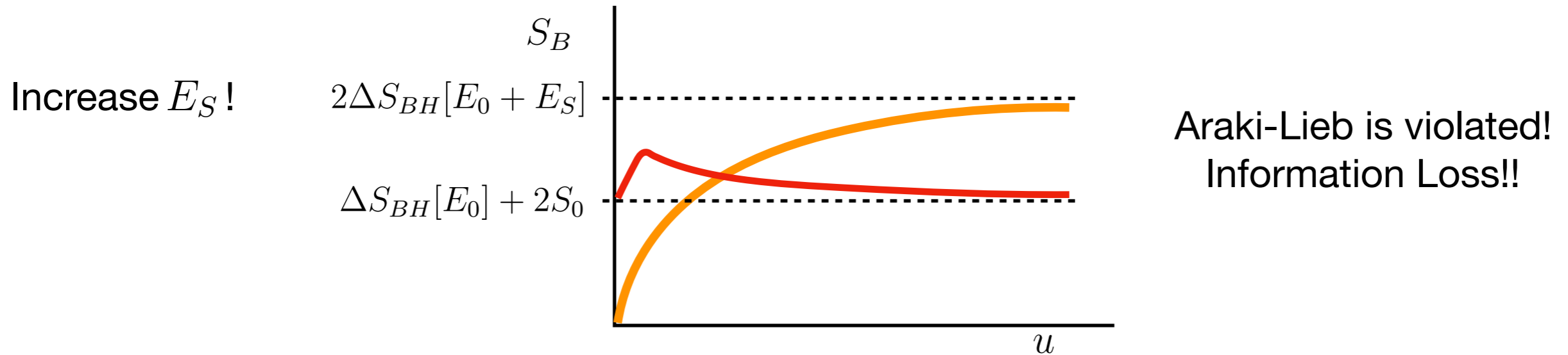
Not a problem  
when  $S_0$  is large!

The Araki-Lieb inequality provides an upper bound on the bath entropy:

$$S_L = S_{RB} \geq |S_R - S_B| \implies S_B \leq S_L + S_R$$

# Does this resolve the Information Paradox?

What about the entropy of the bath?



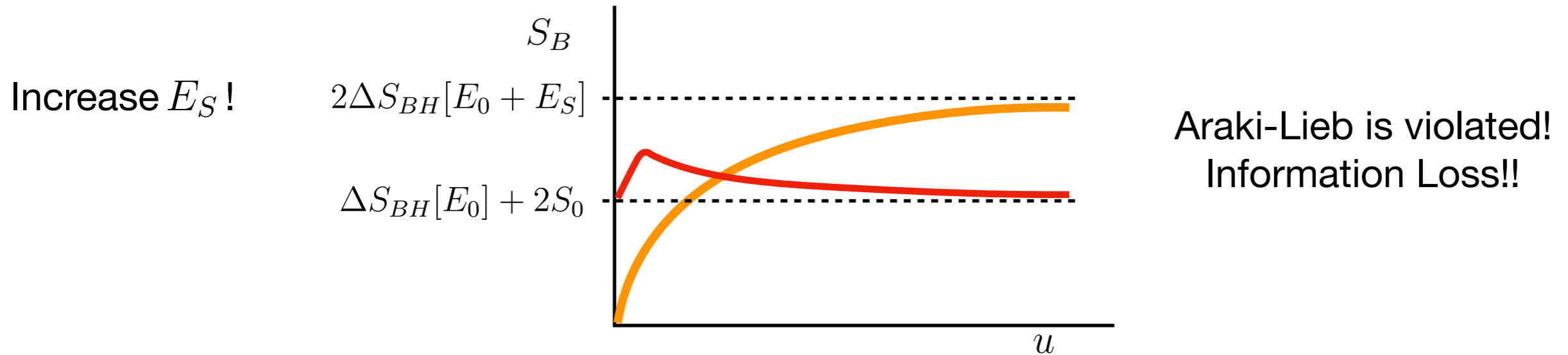
The Araki-Lieb inequality provides an upper bound on the bath entropy:

$$S_L = S_{RB} \geq |S_R - S_B| \implies S_B \leq S_L + S_R$$

We didn't find a Page curve for the bath! This is the standard Hawking result...  
This does not resolve the information paradox!

# Does this resolve the Information Paradox?

What about the entropy of the bath?



The Araki-Lieb inequality provides an upper bound on the bath entropy:

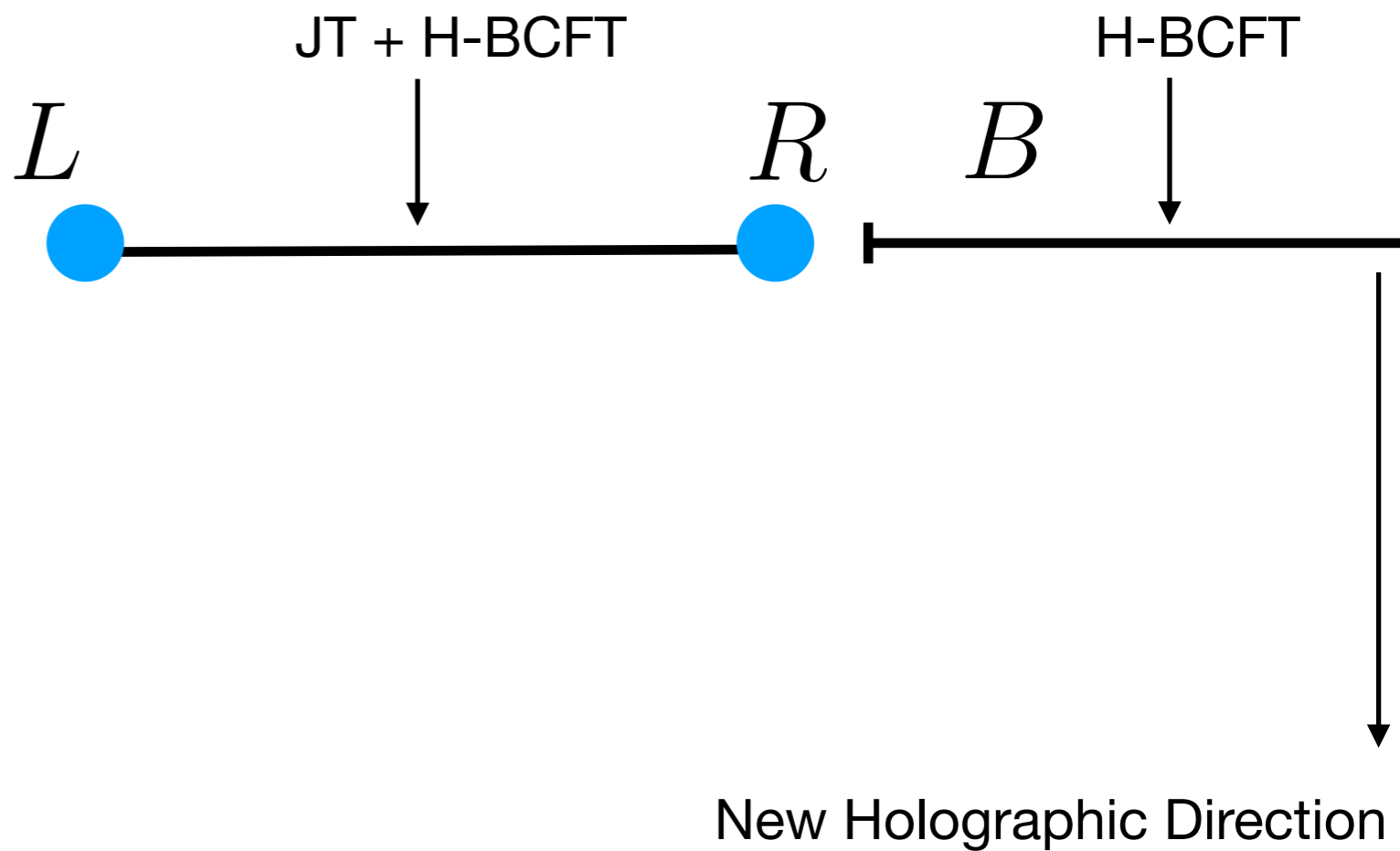
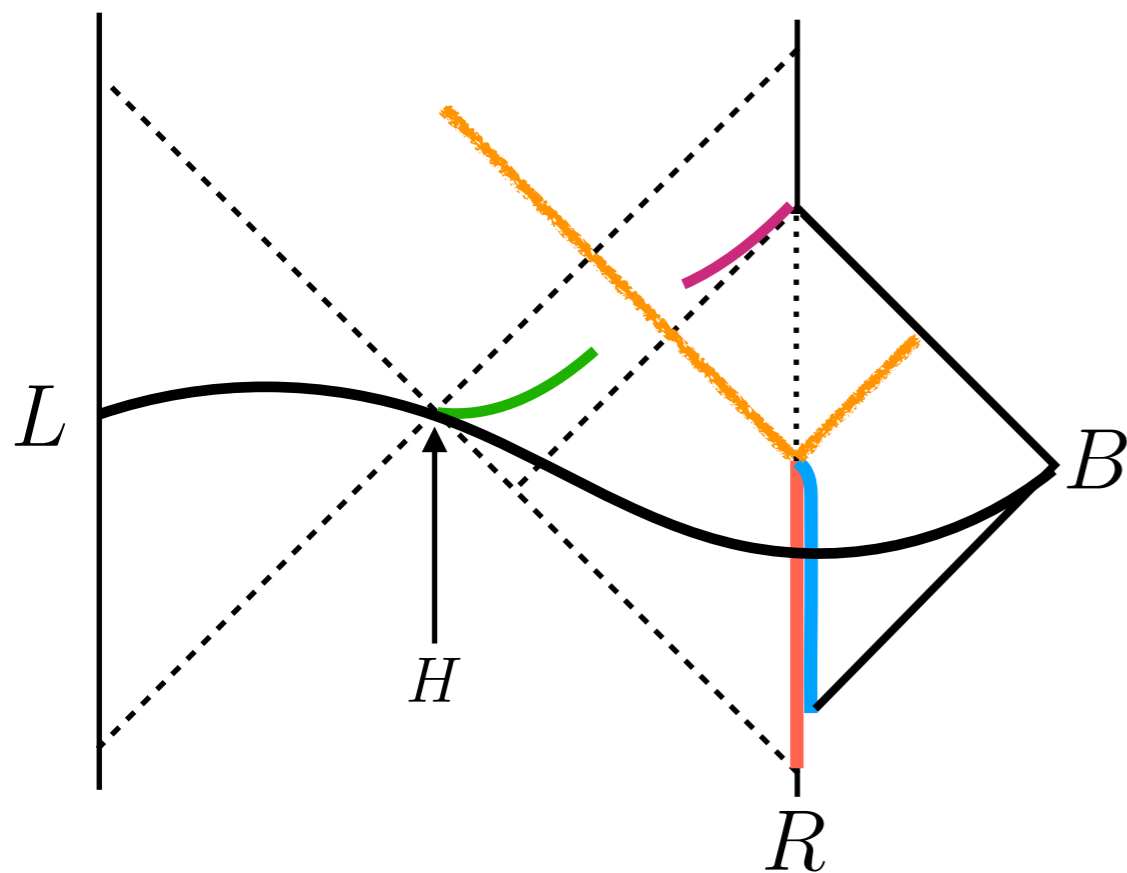
$$S_L = S_{RB} \geq |S_R - S_B| \implies S_B \leq S_L + S_R$$

We didn't find a Page curve for the bath! This is the standard Hawking result...  
This does not resolve the information paradox!

Ok, now let me tell you how to resolve it...

# What if the bulk/bath BCFT was holographic?

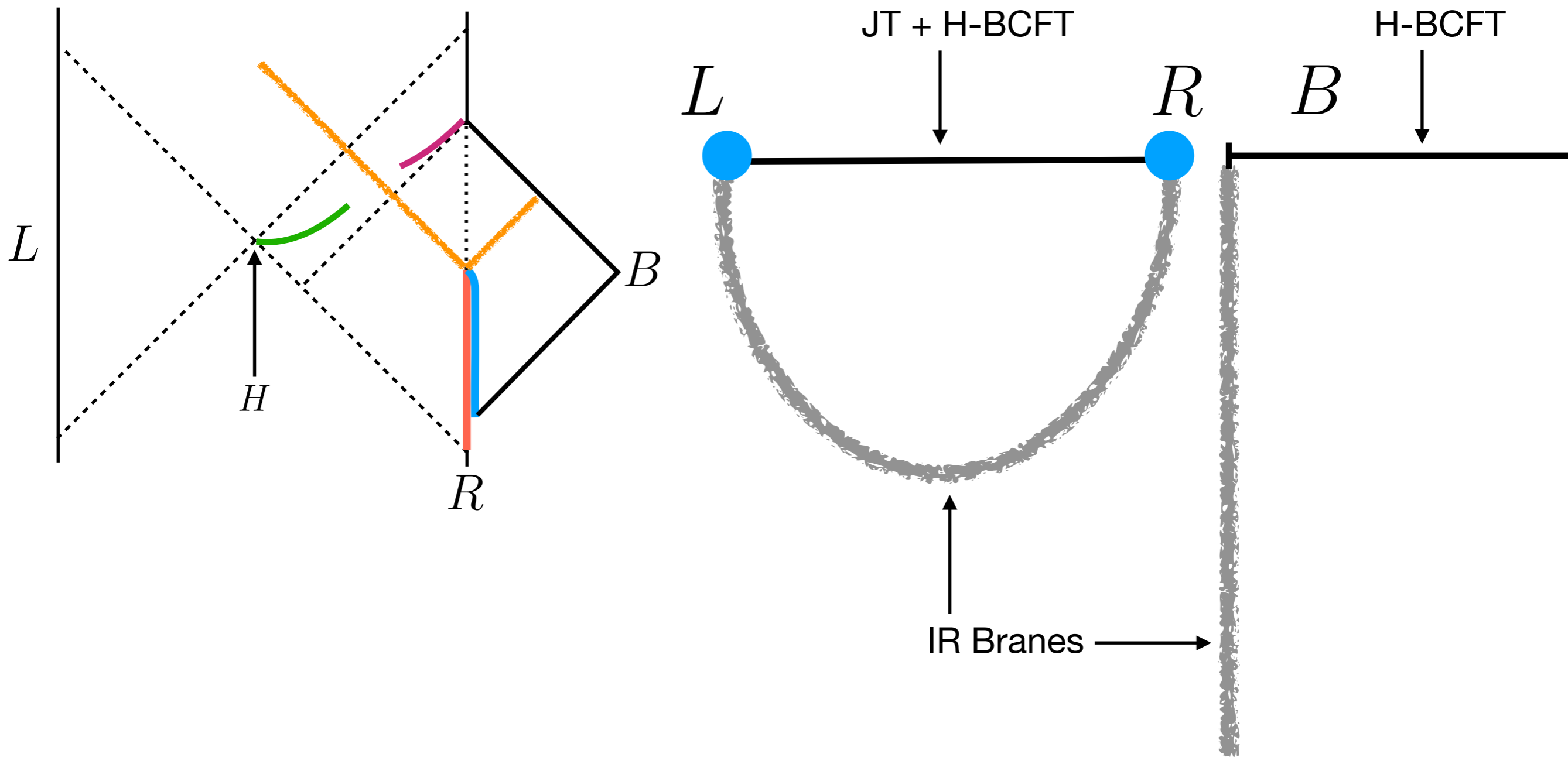
The 1+1d system will be dual to a 2+1d bulk!





# What if the bulk/bath BCFT was holographic?

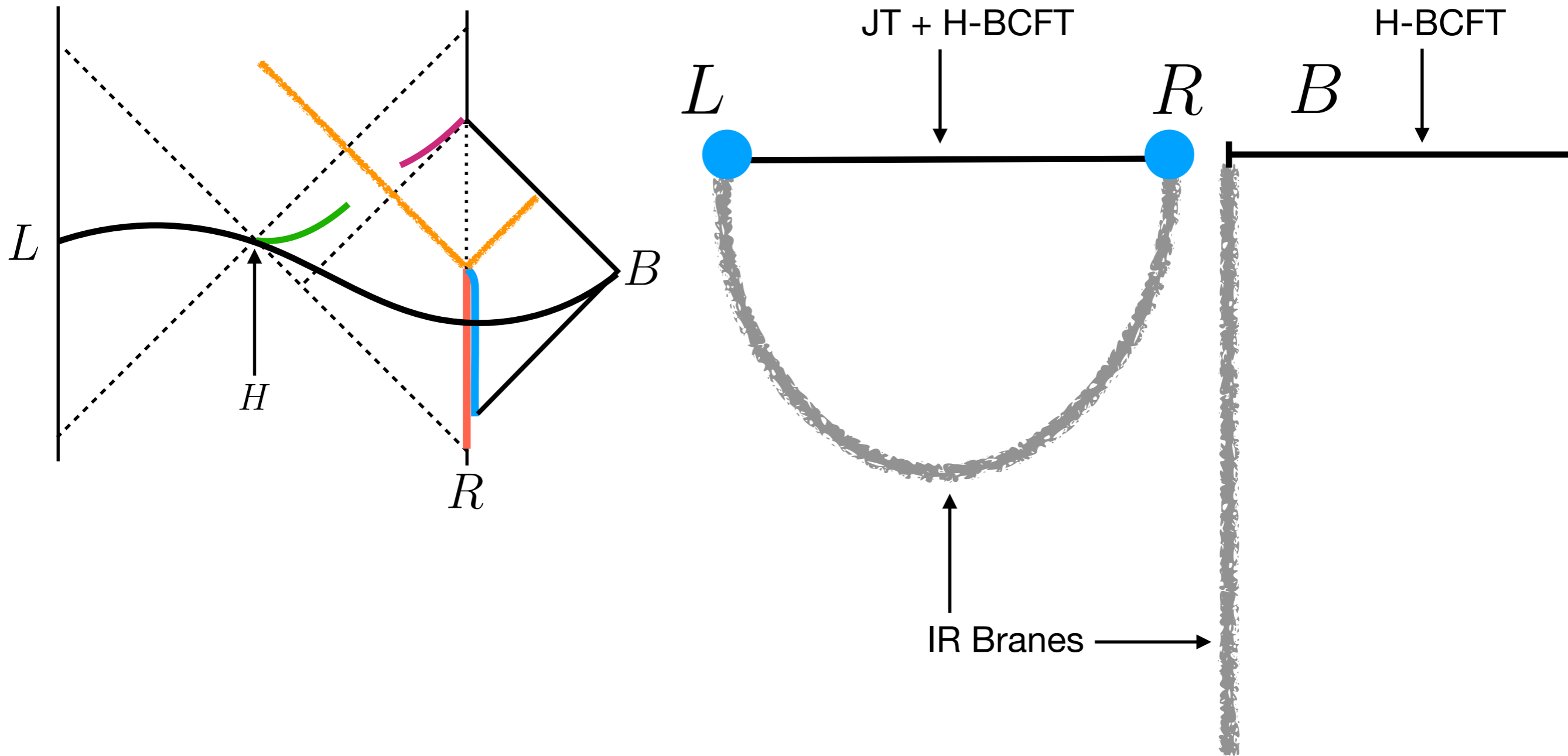
The 1+1d system will be dual to a 2+1d bulk!



# What if the bulk/bath BCFT was holographic?

The 1+1d system will be dual to a 2+1d bulk!

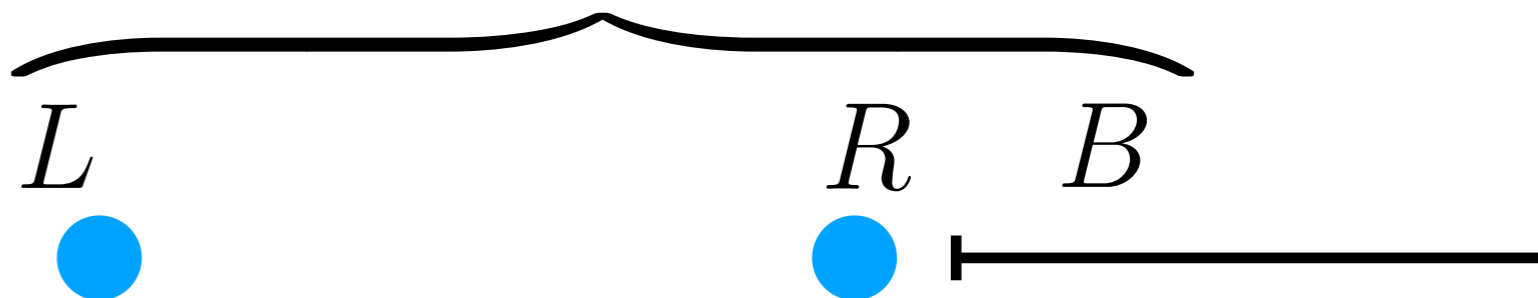
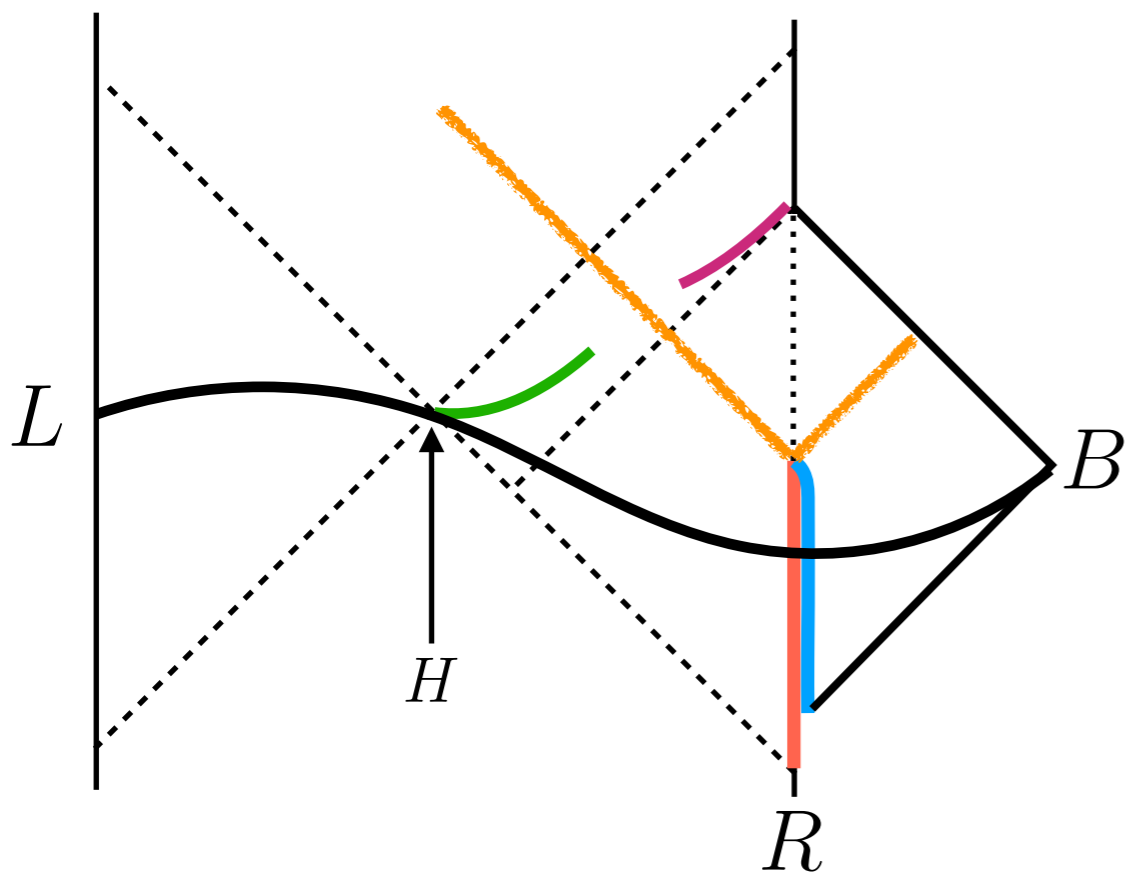
Fujita, Takayanagi, Tonni



# What if the bulk/bath BCFT was holographic?

The 1+1d system will be dual to a 2+1d bulk!

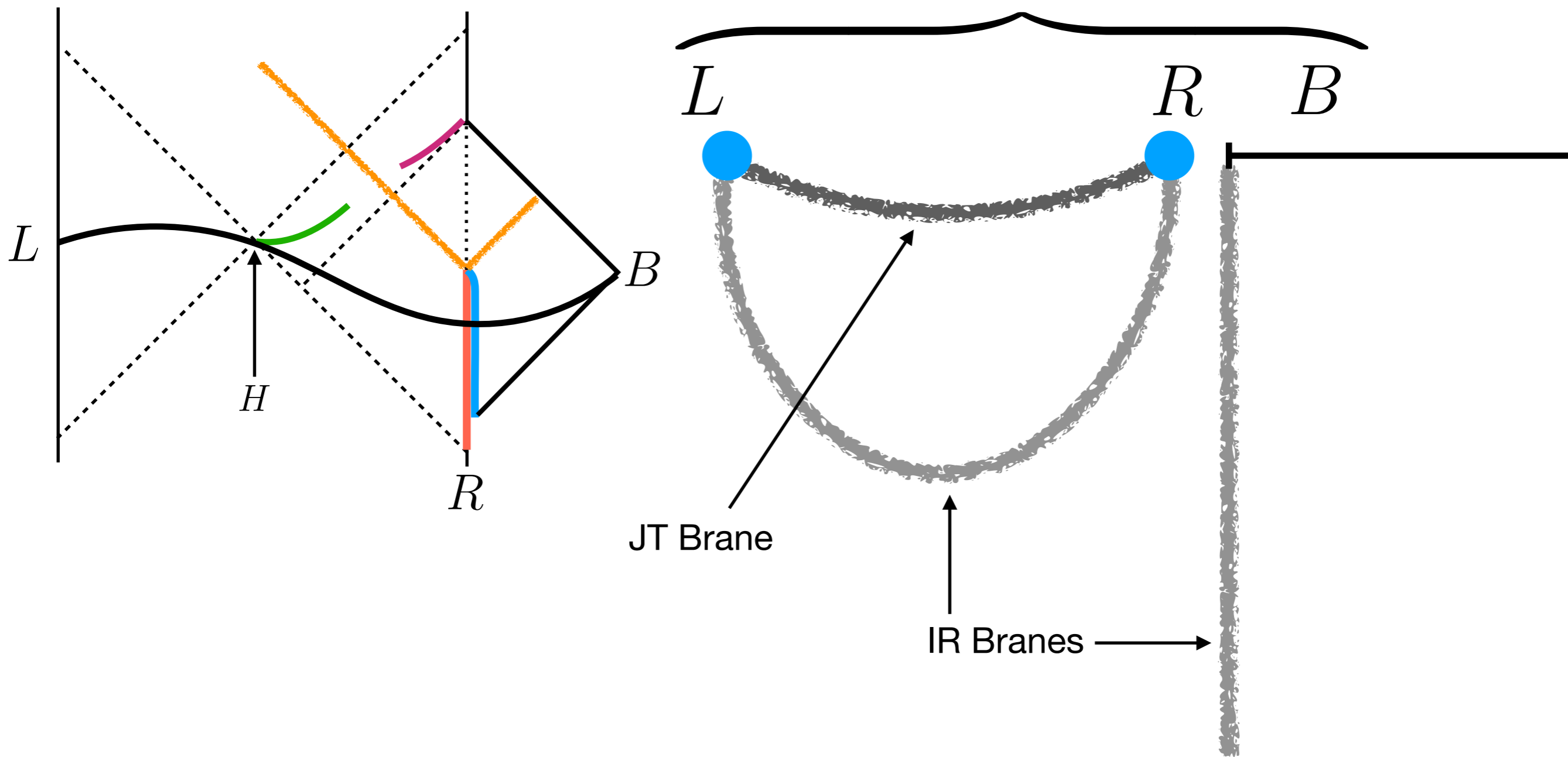
The boundary holographic system is



# What if the bulk/bath BCFT was holographic?

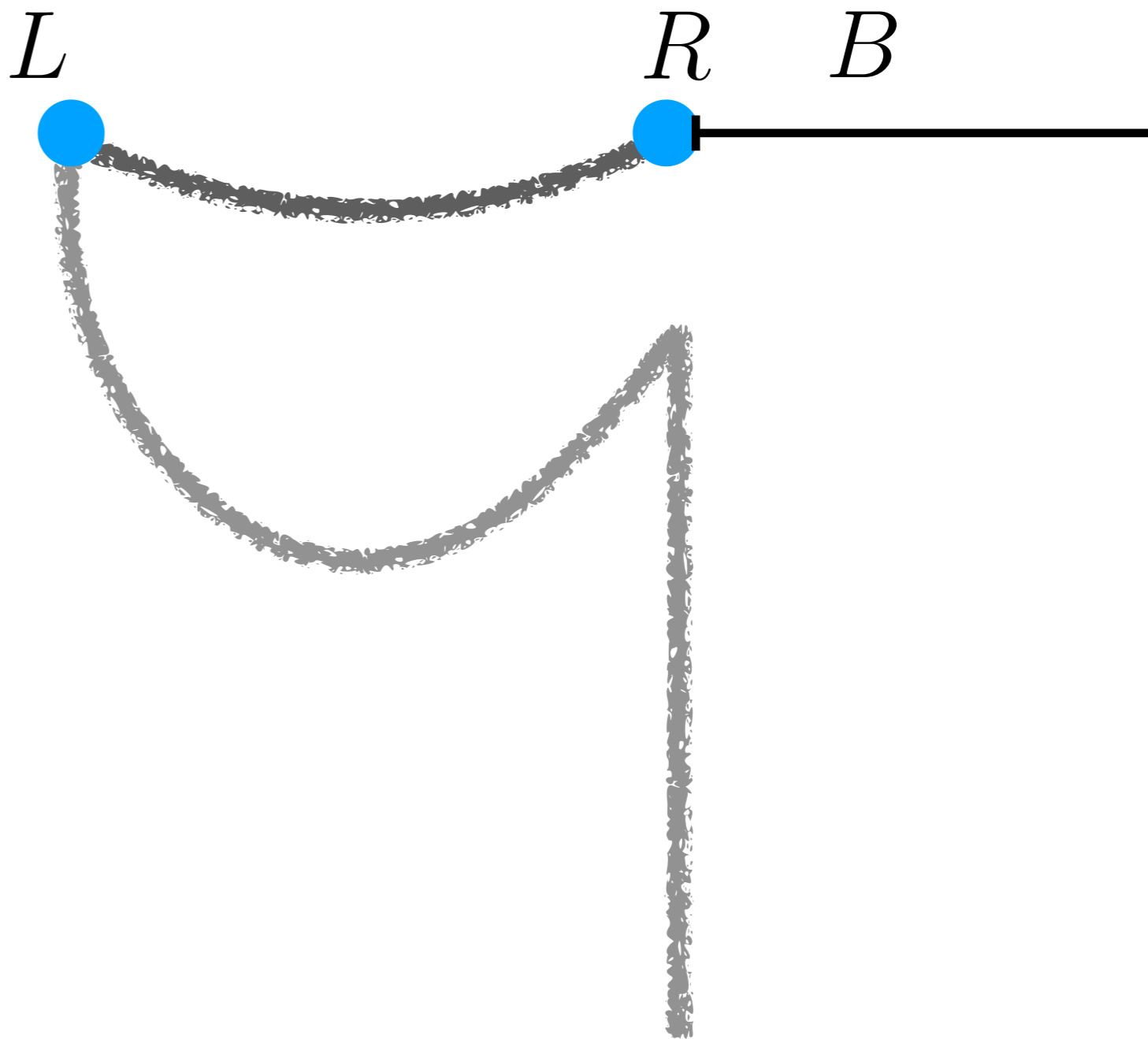
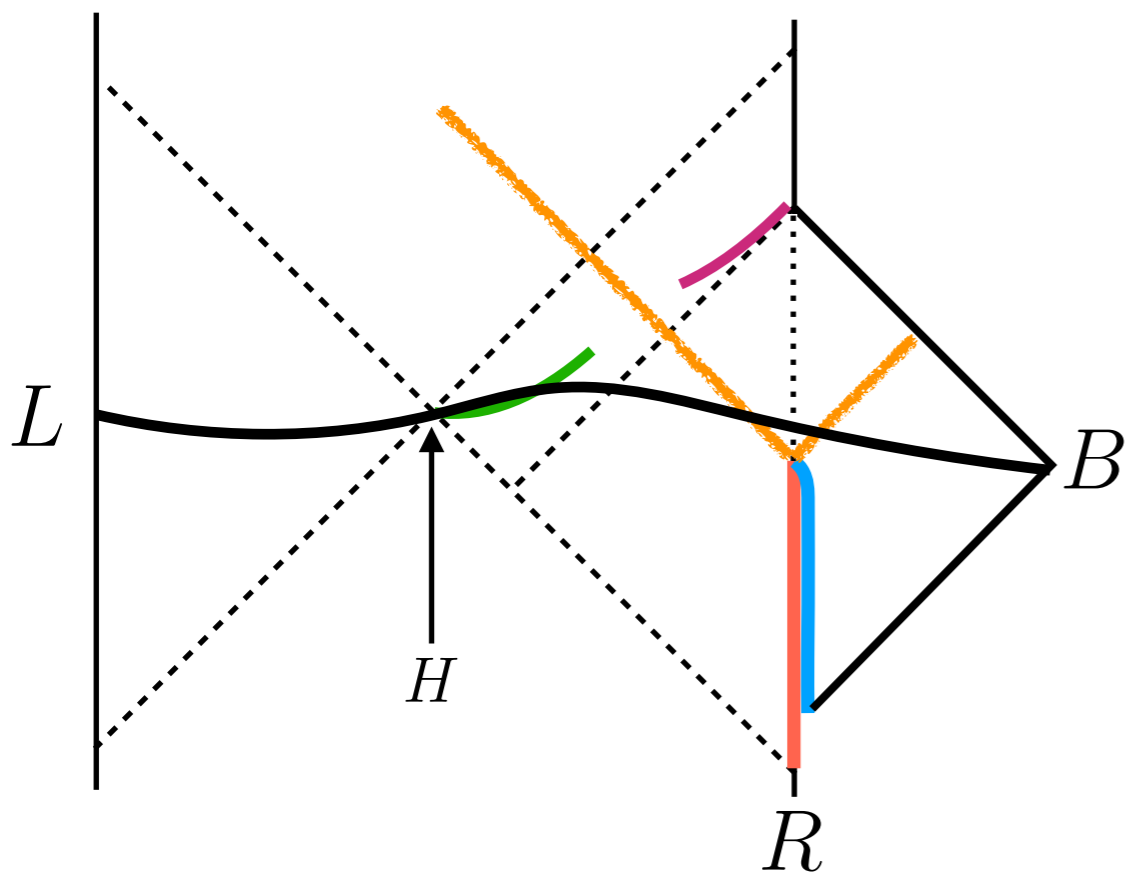
The 1+1d system will be dual to a 2+1d bulk!

The boundary holographic system is



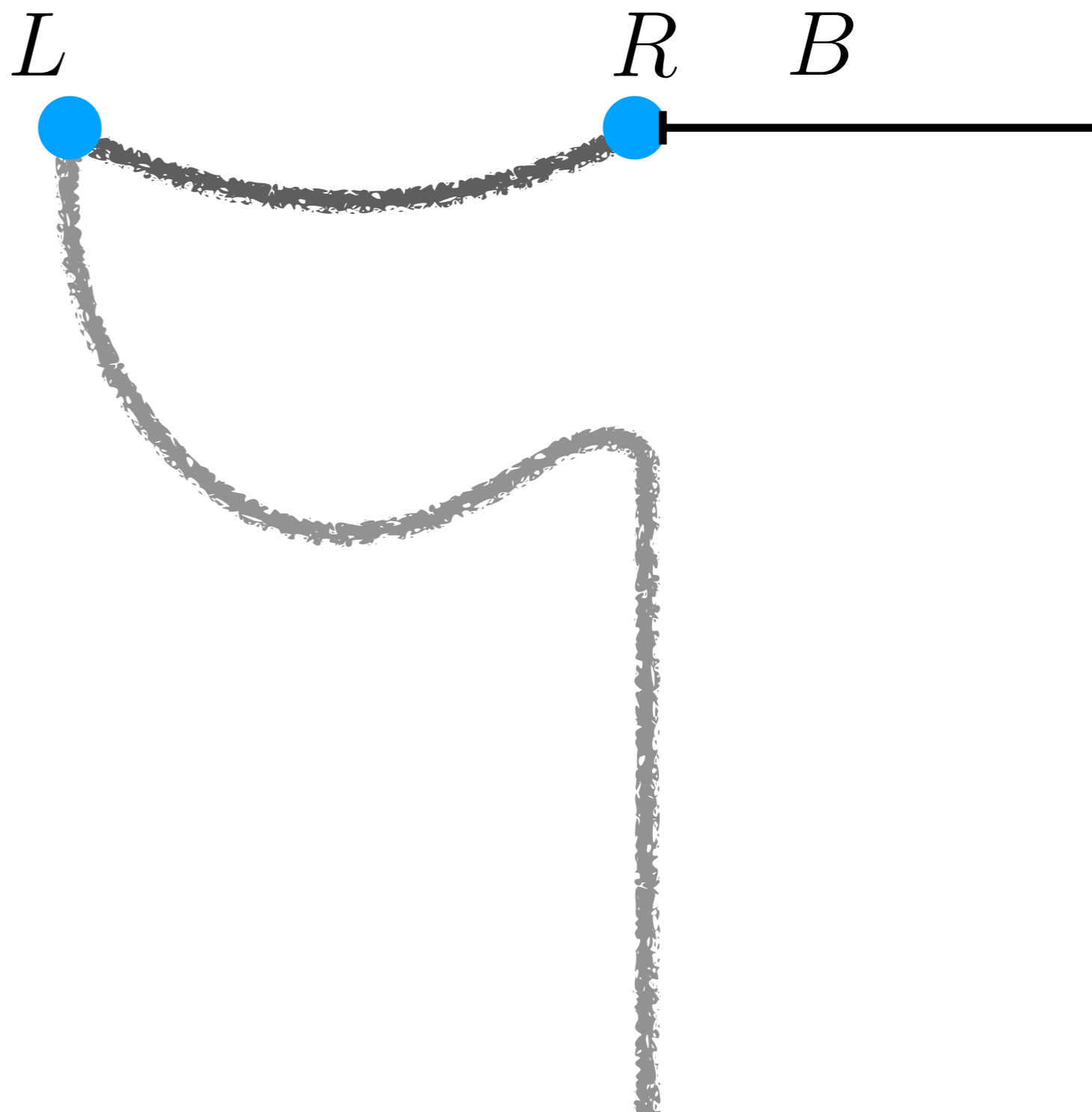
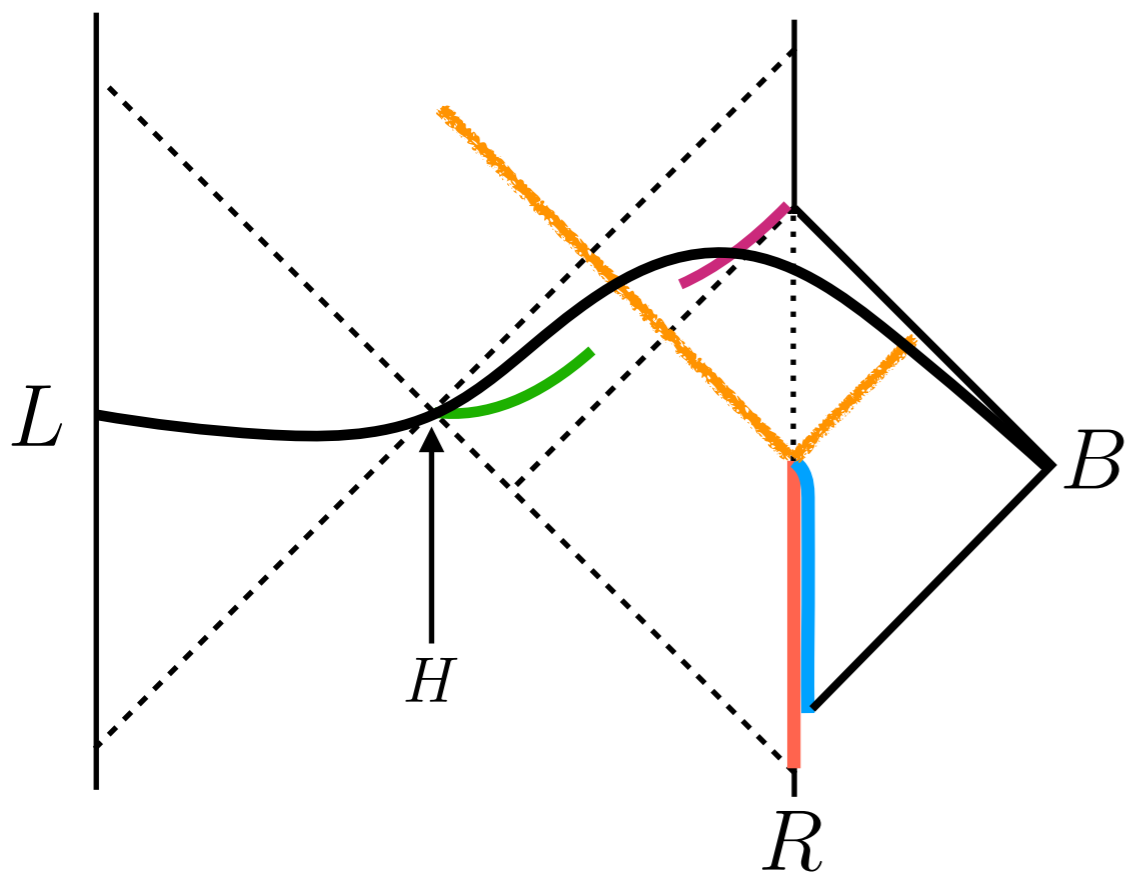
# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



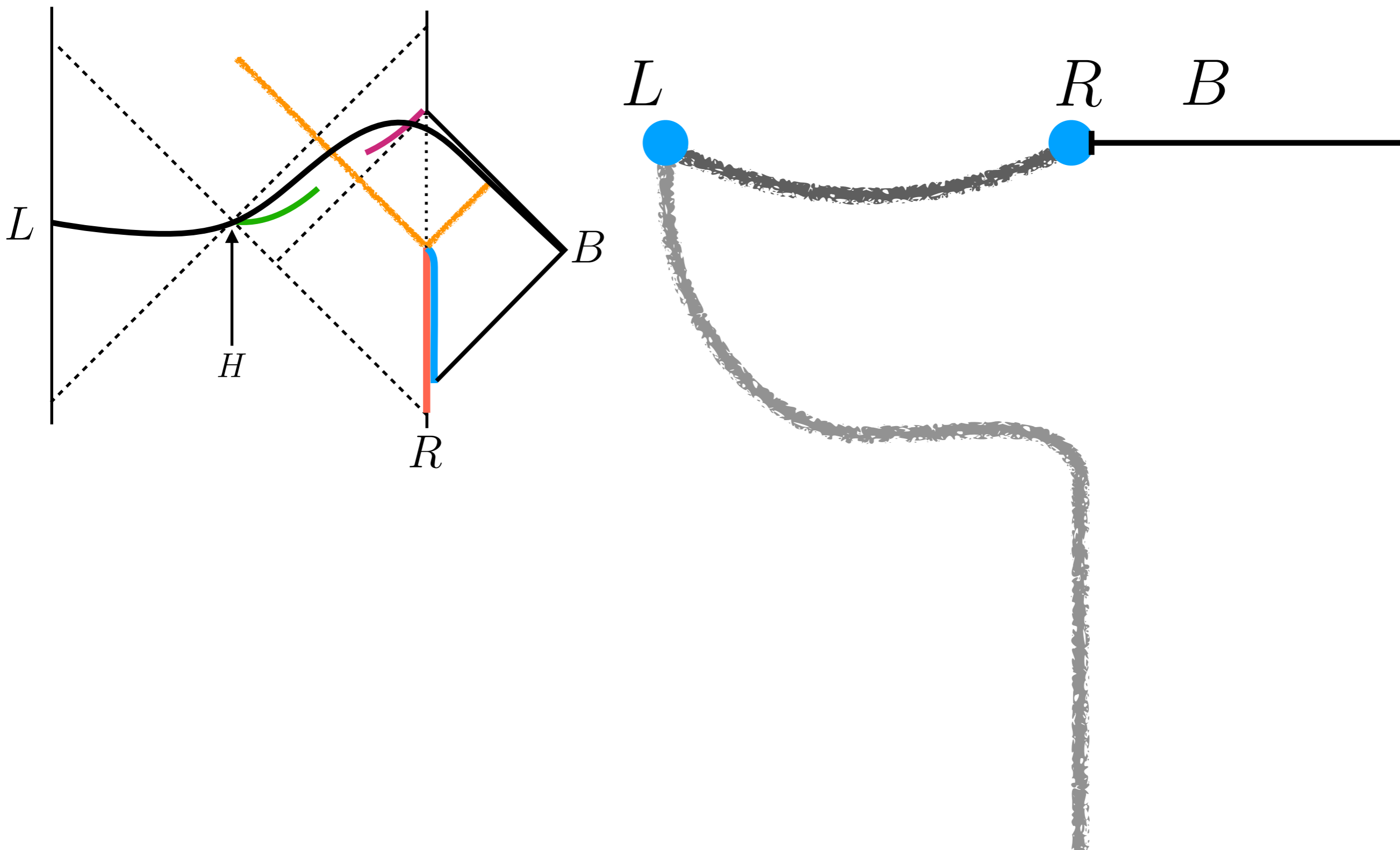
# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



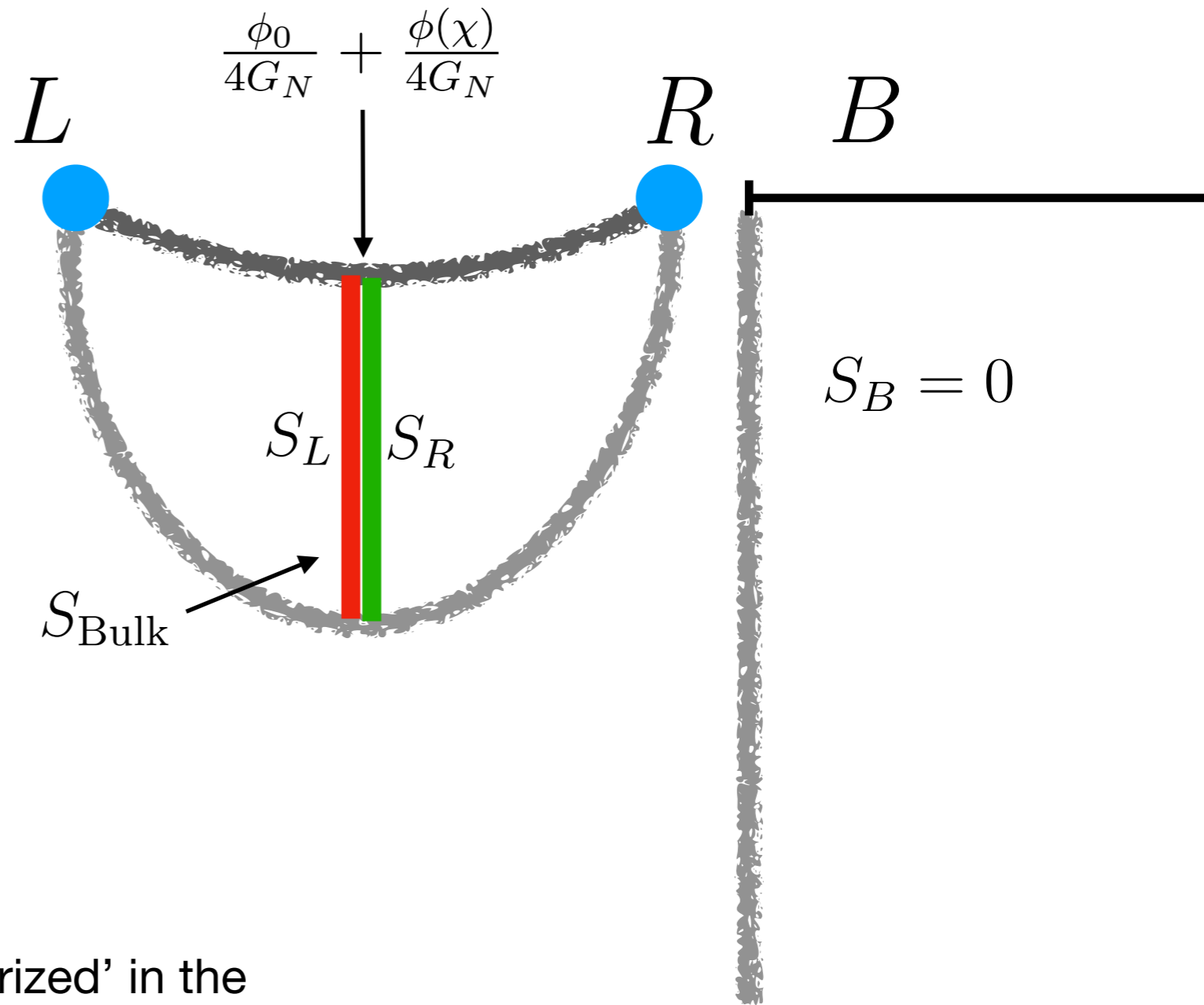
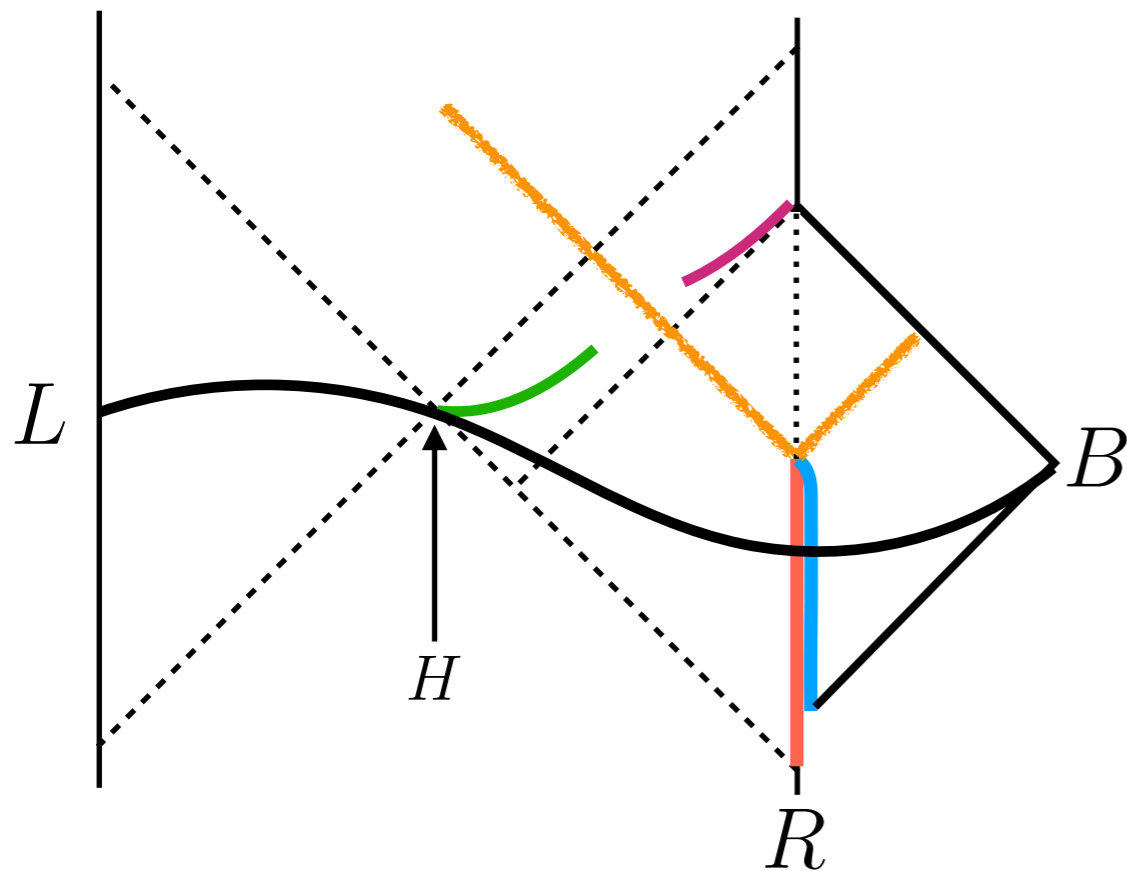
# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



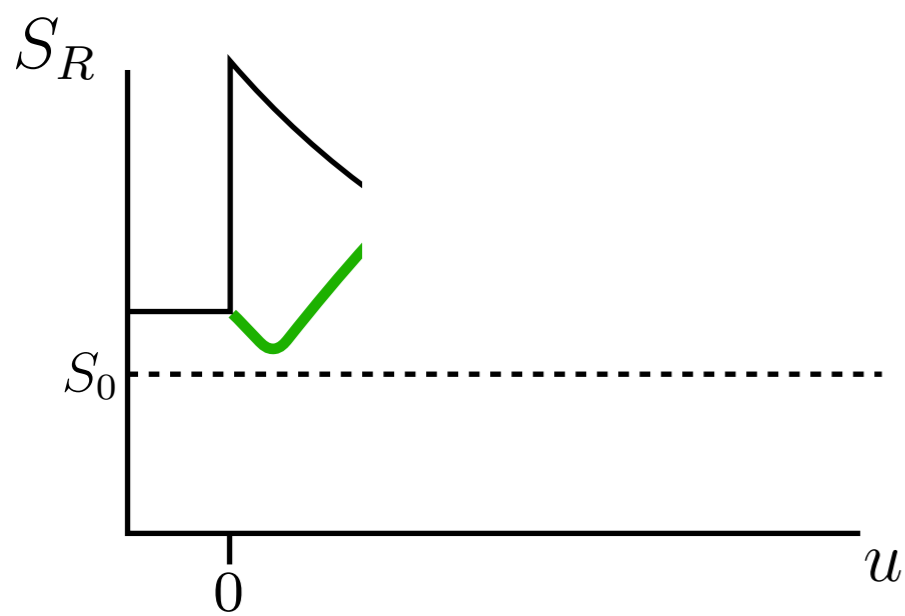
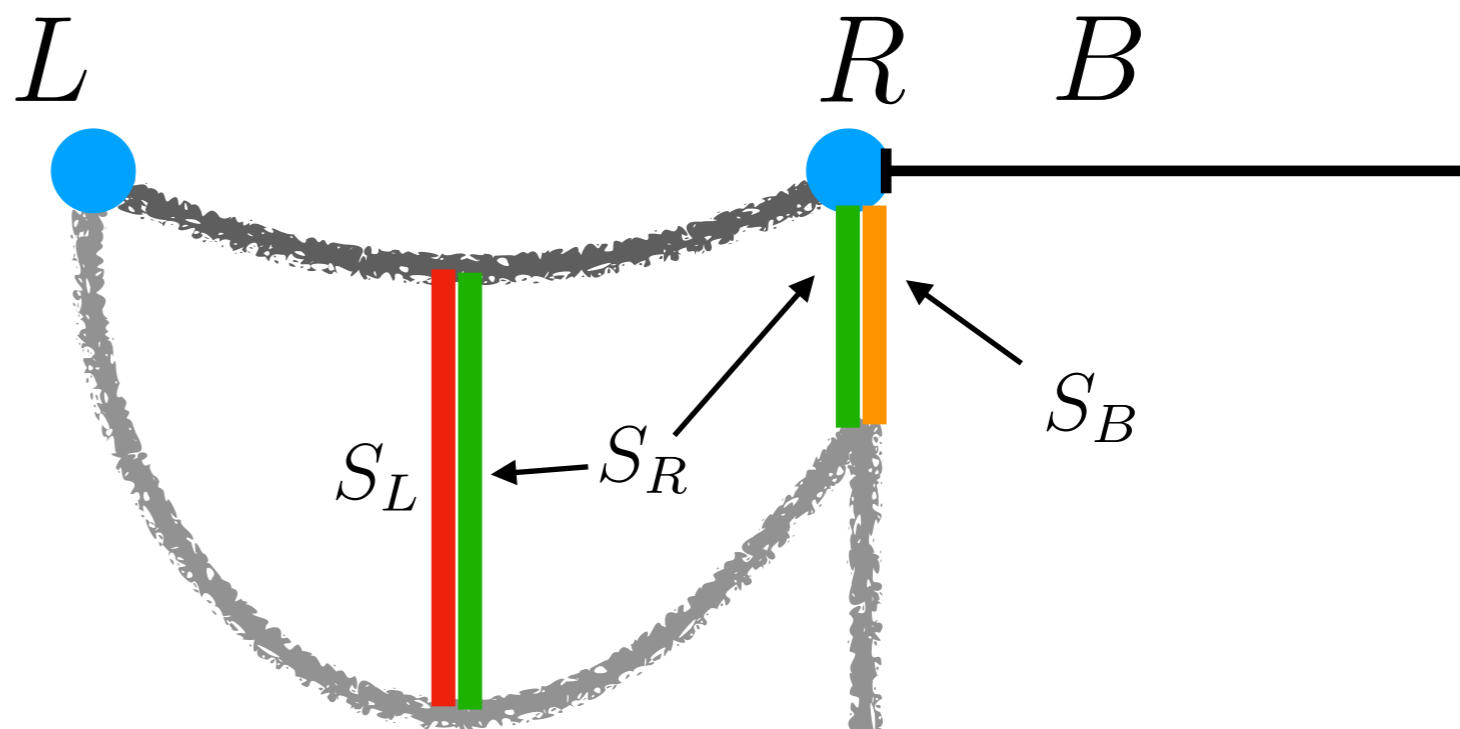
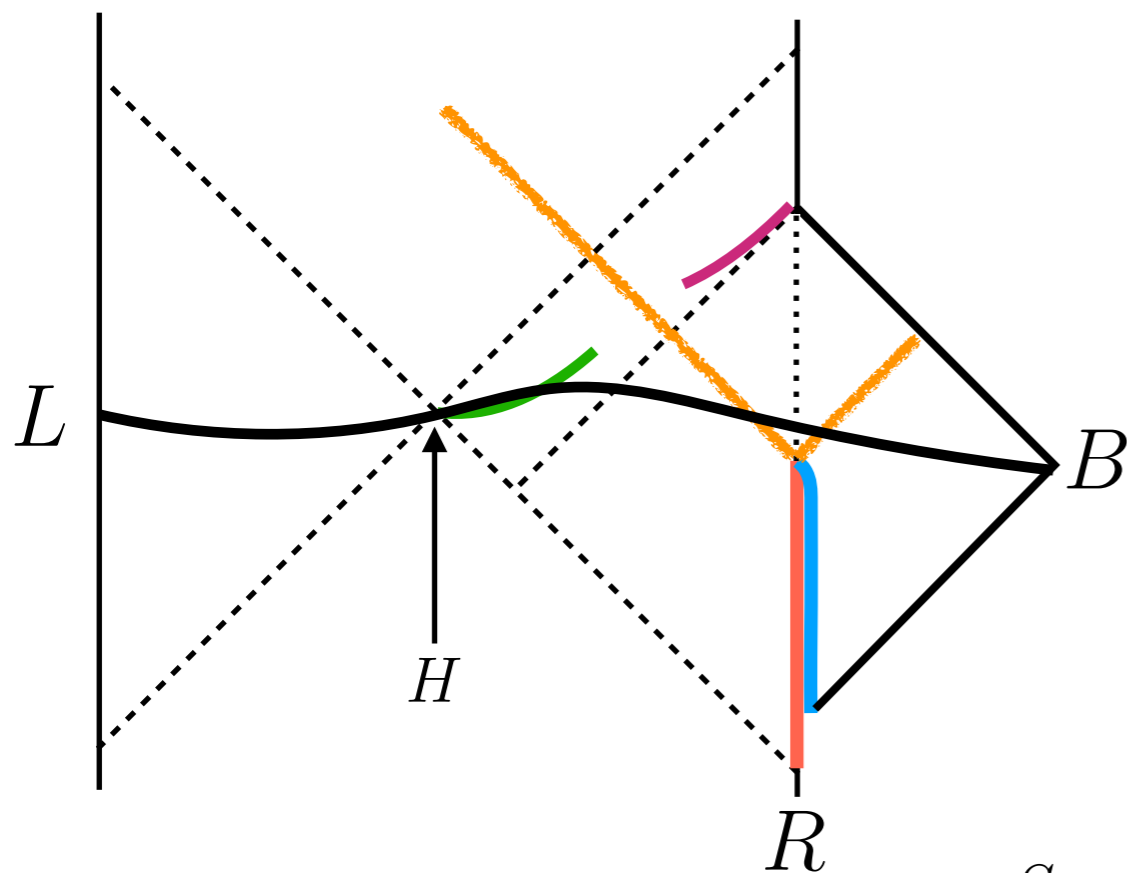
$$S_{gen} = \frac{\phi_0}{4G_N} + \frac{\phi(\chi)}{4G_N} + S_{Bulk}$$

The  $S_{Bulk}$  contribution is 'geometrized' in the new bulk. Apply regular RT.



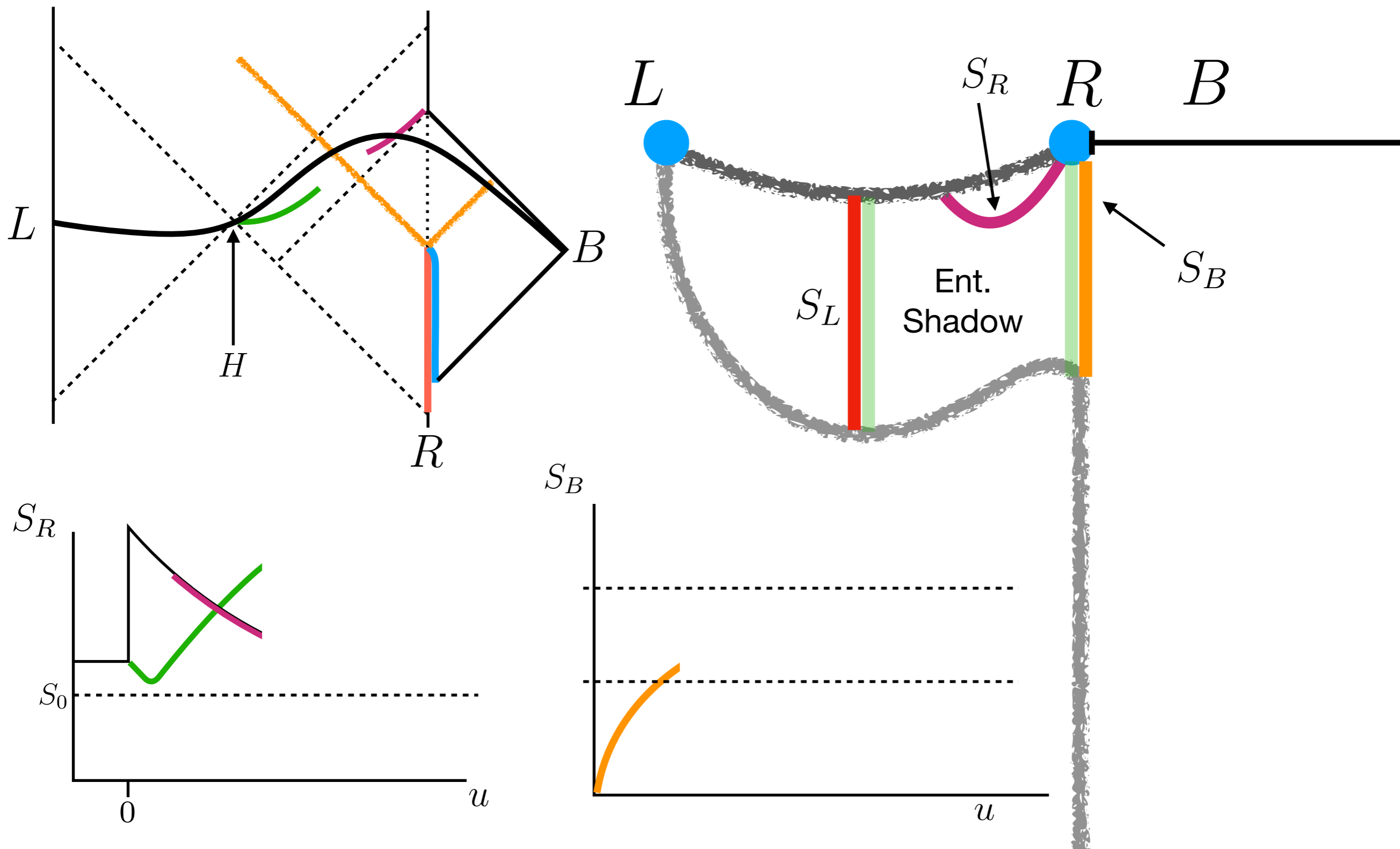
# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



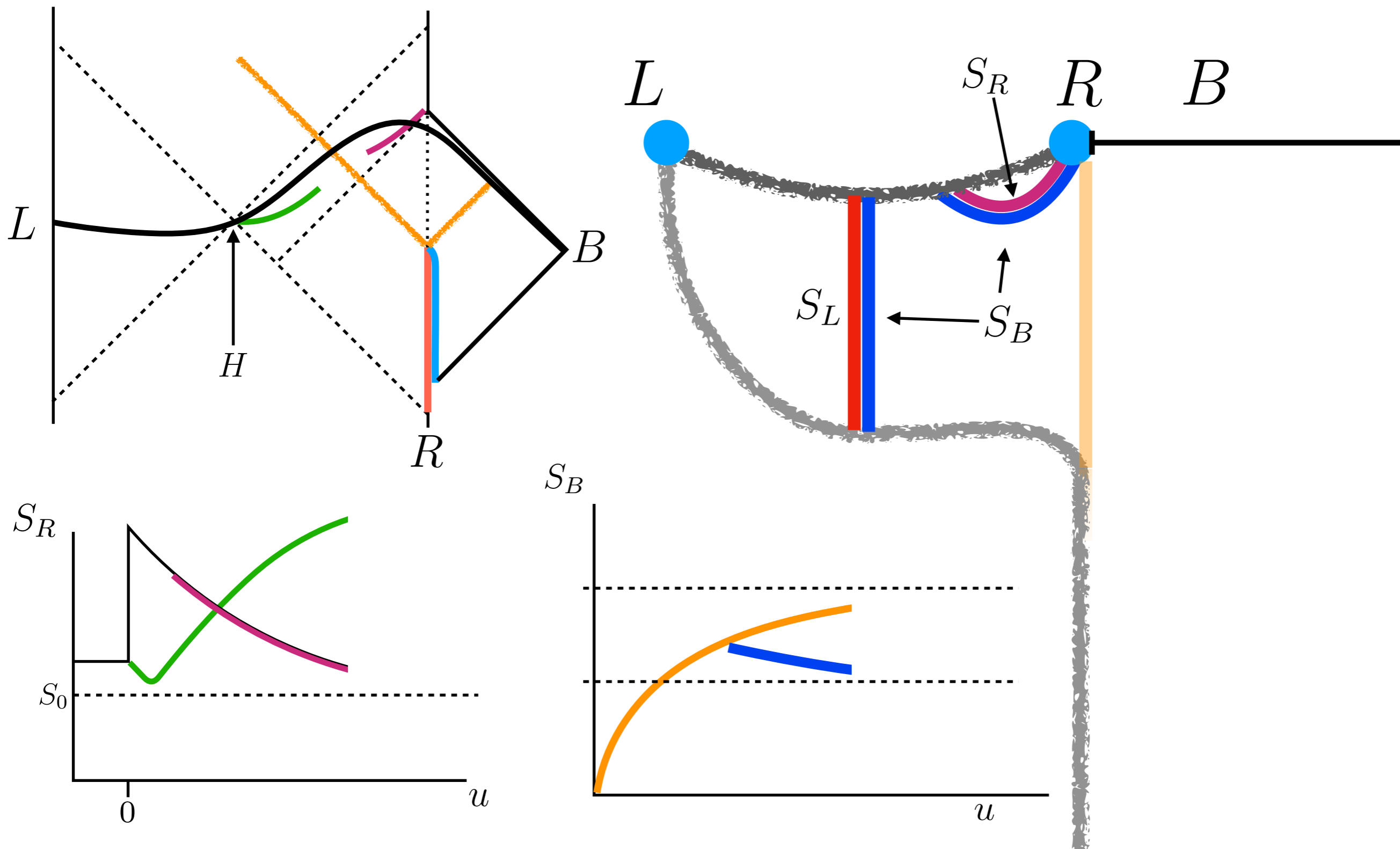
# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



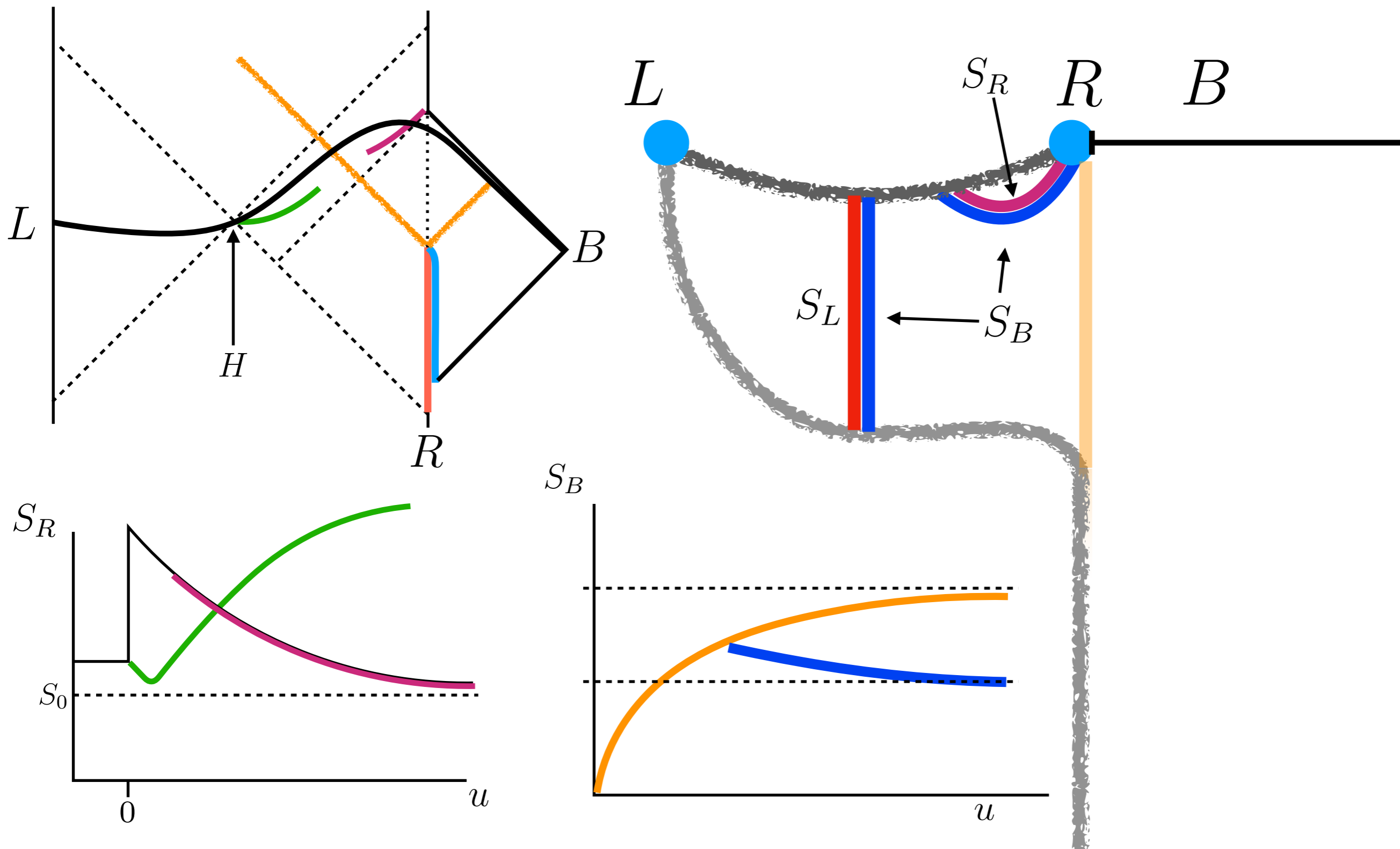
# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



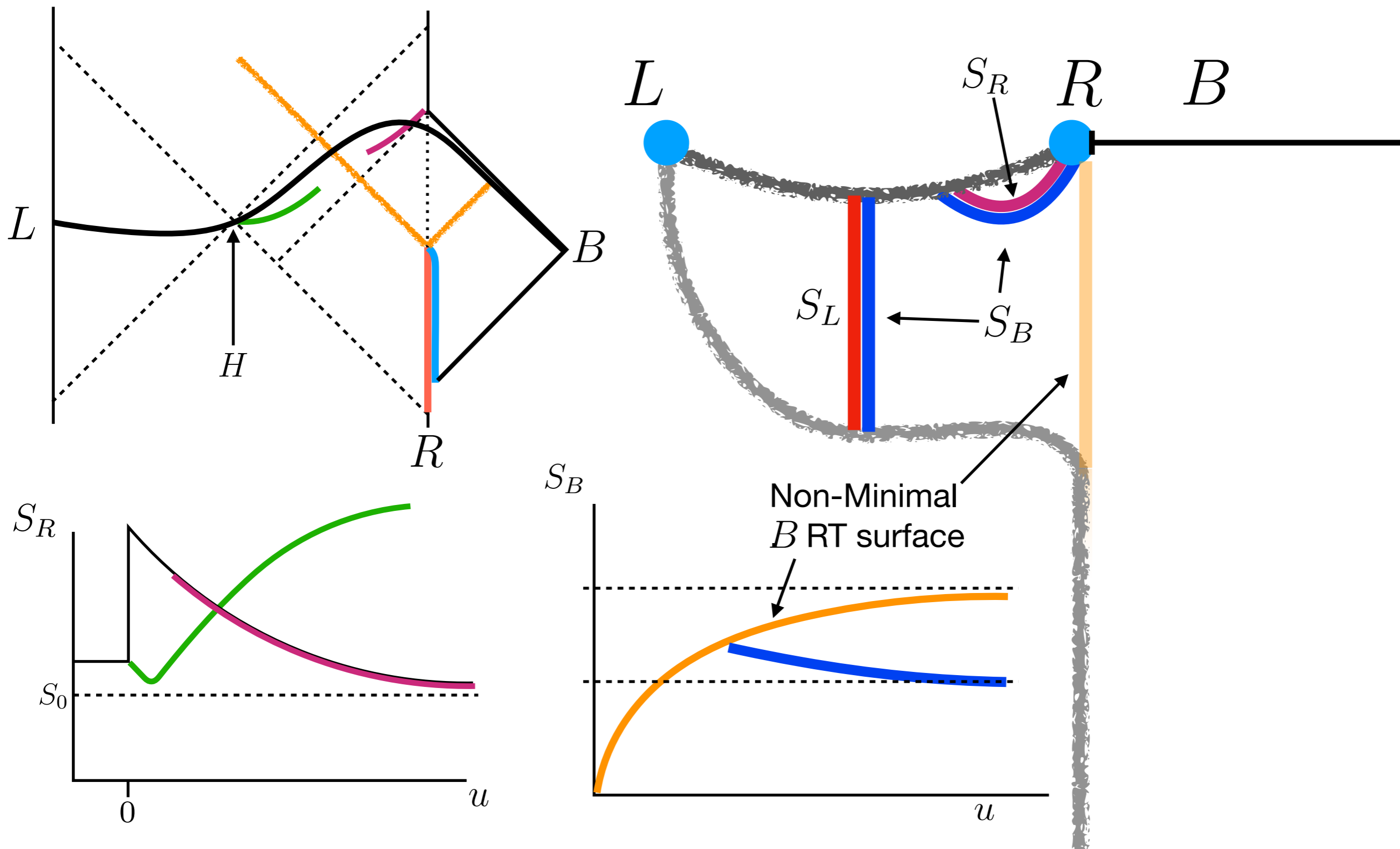
# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



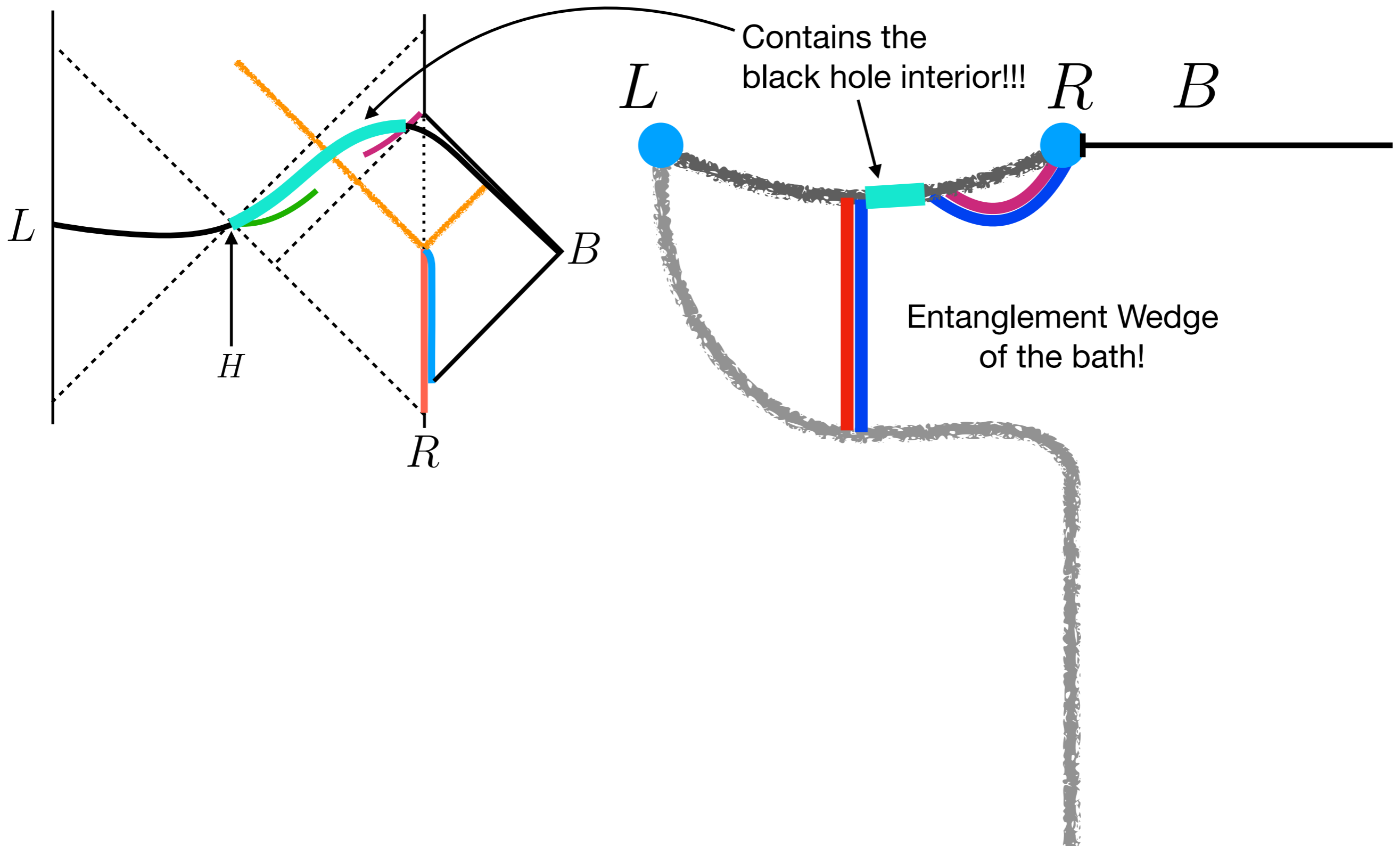
# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



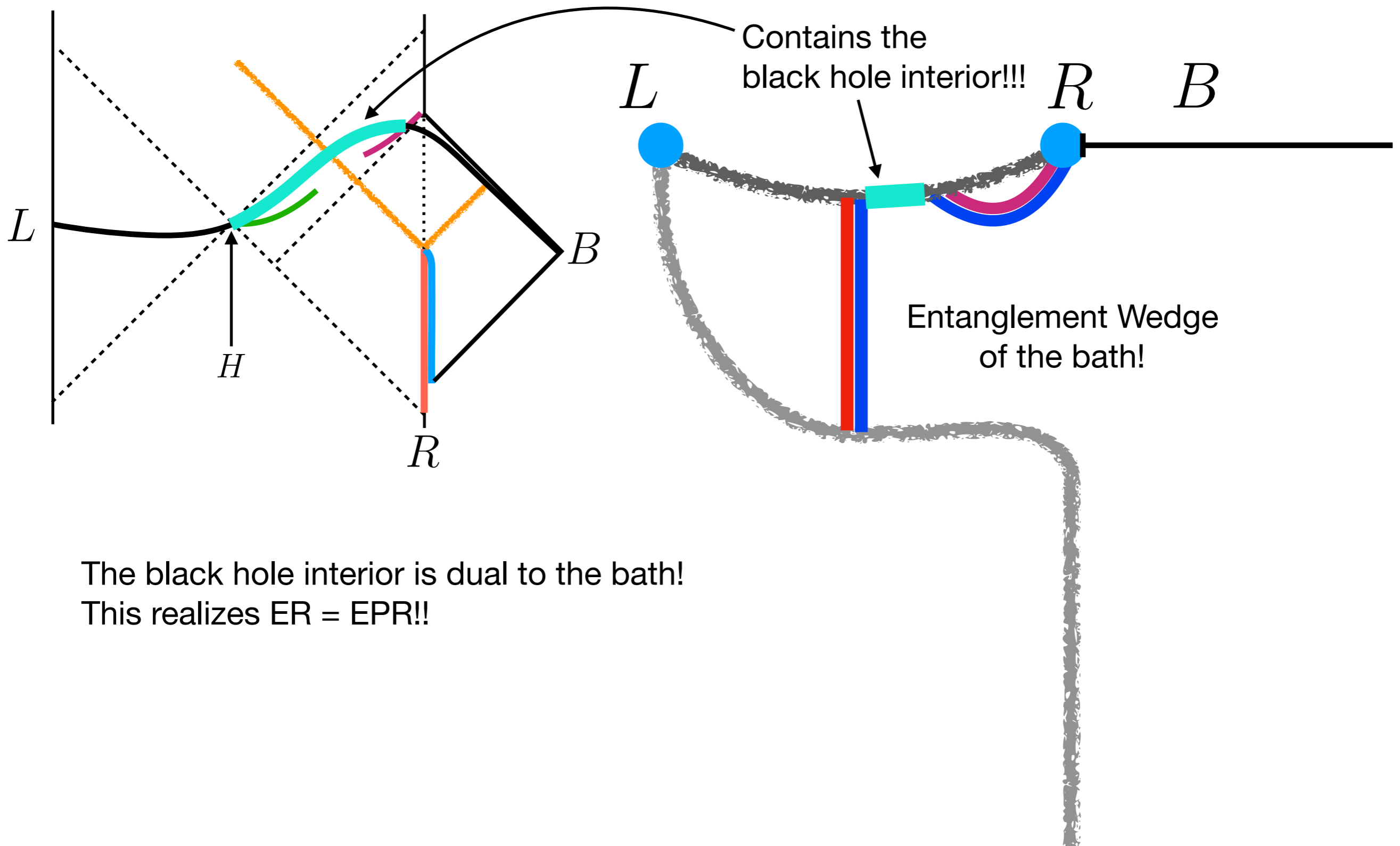
# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



# What if the bulk/bath BCFT was holographic?

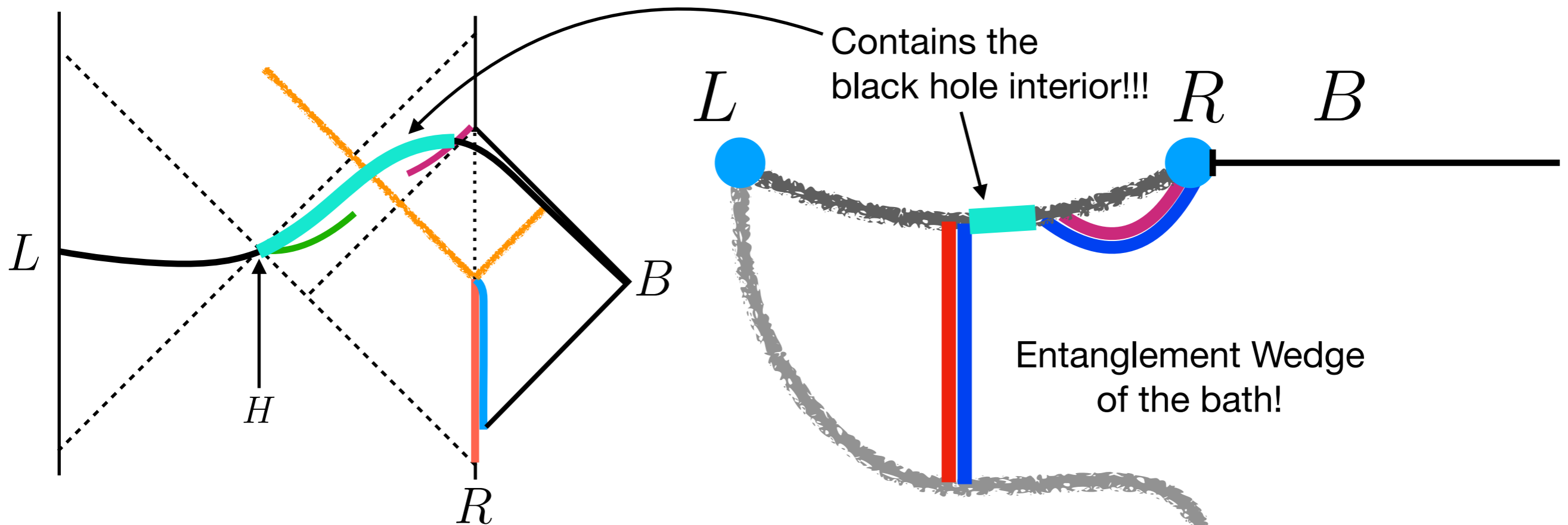
Entropy is given by the RT surface in 2+1d bulk



The black hole interior is dual to the bath!  
This realizes ER = EPR!!

# What if the bulk/bath BCFT was holographic?

Entropy is given by the RT surface in 2+1d bulk



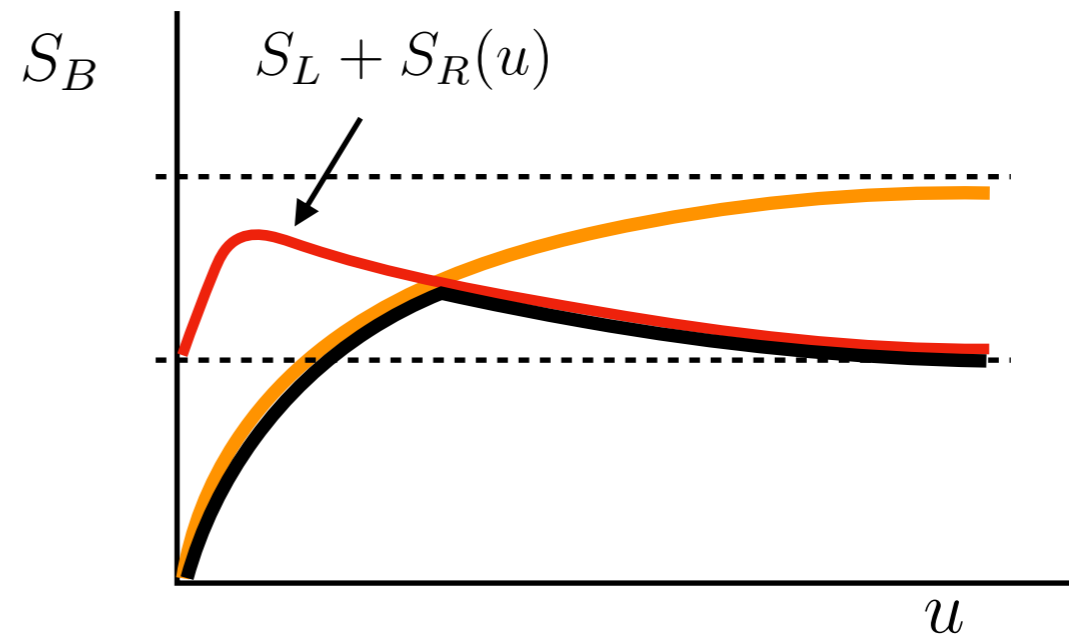
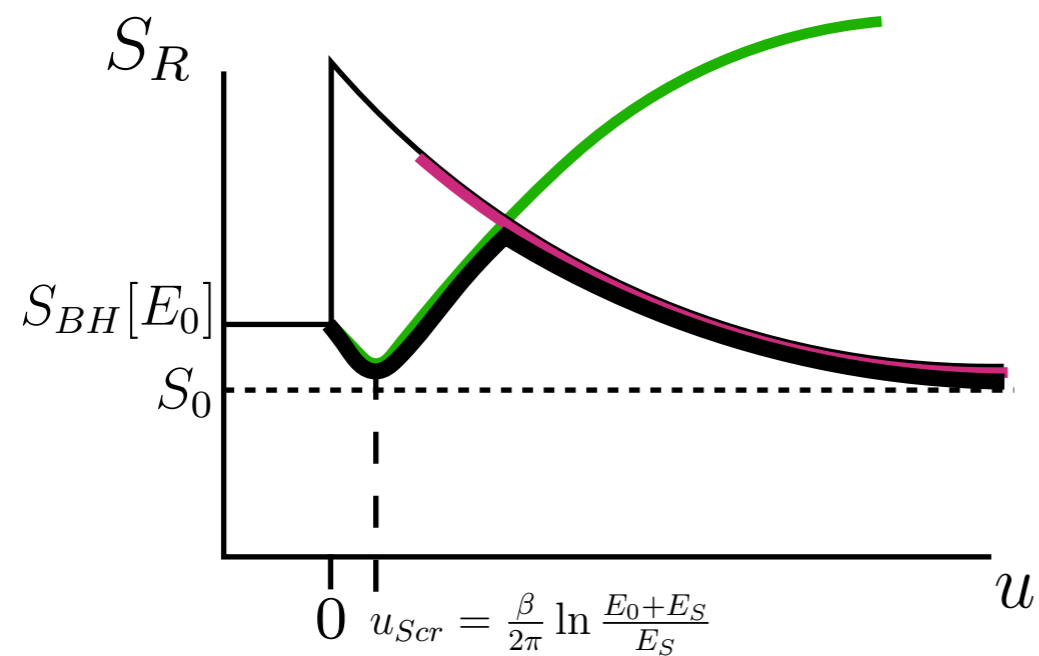
The black hole interior is dual to the bath!  
This realizes  $ER = EPR$ !!

But instead of the 'octopus' spacetime we have a nice "single filet of salmon" - S. Shenker



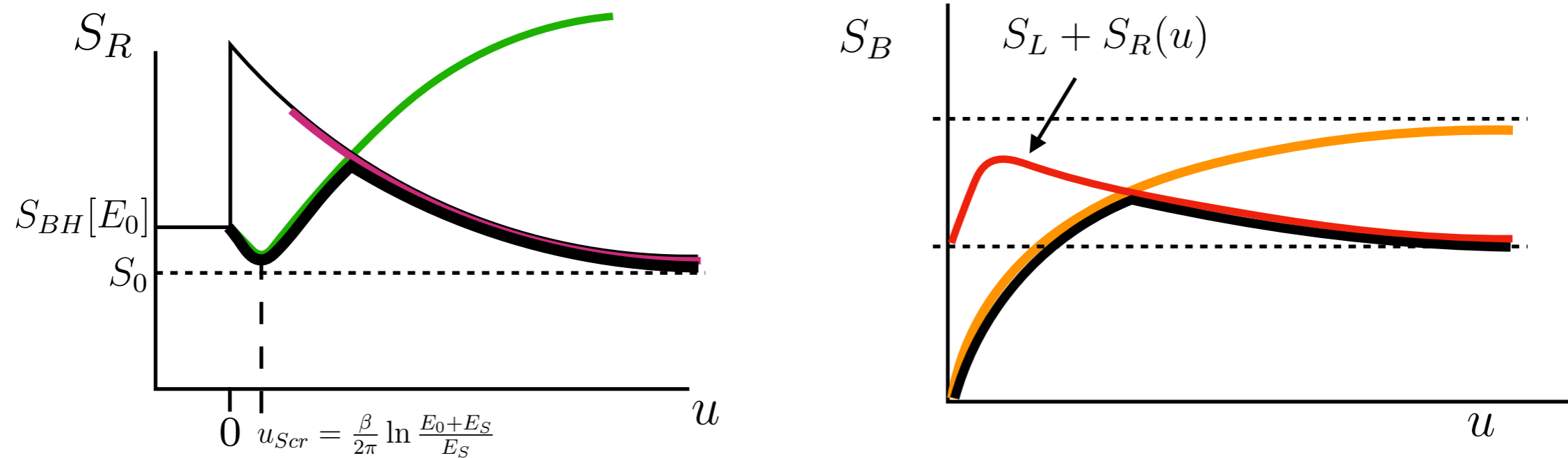
# Punch Line

We obtained the **Page curve** for both the radiation and black hole in the context of evaporating a black hole in 1+1 dimensions coupled to holographic matter.



# Punch Line

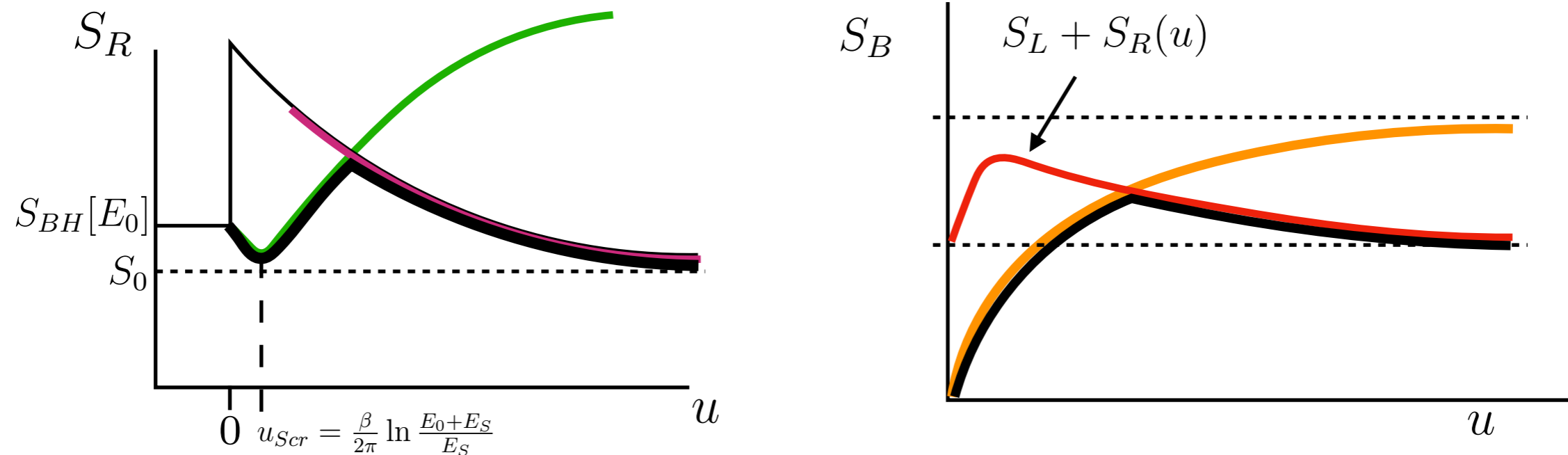
We obtained the **Page curve** for both the radiation and black hole in the context of evaporating a black hole in 1+1 dimensions coupled to holographic matter.



Hawking's 'mistake': He didn't know about the Minimality condition of RT formula!

# Punch Line

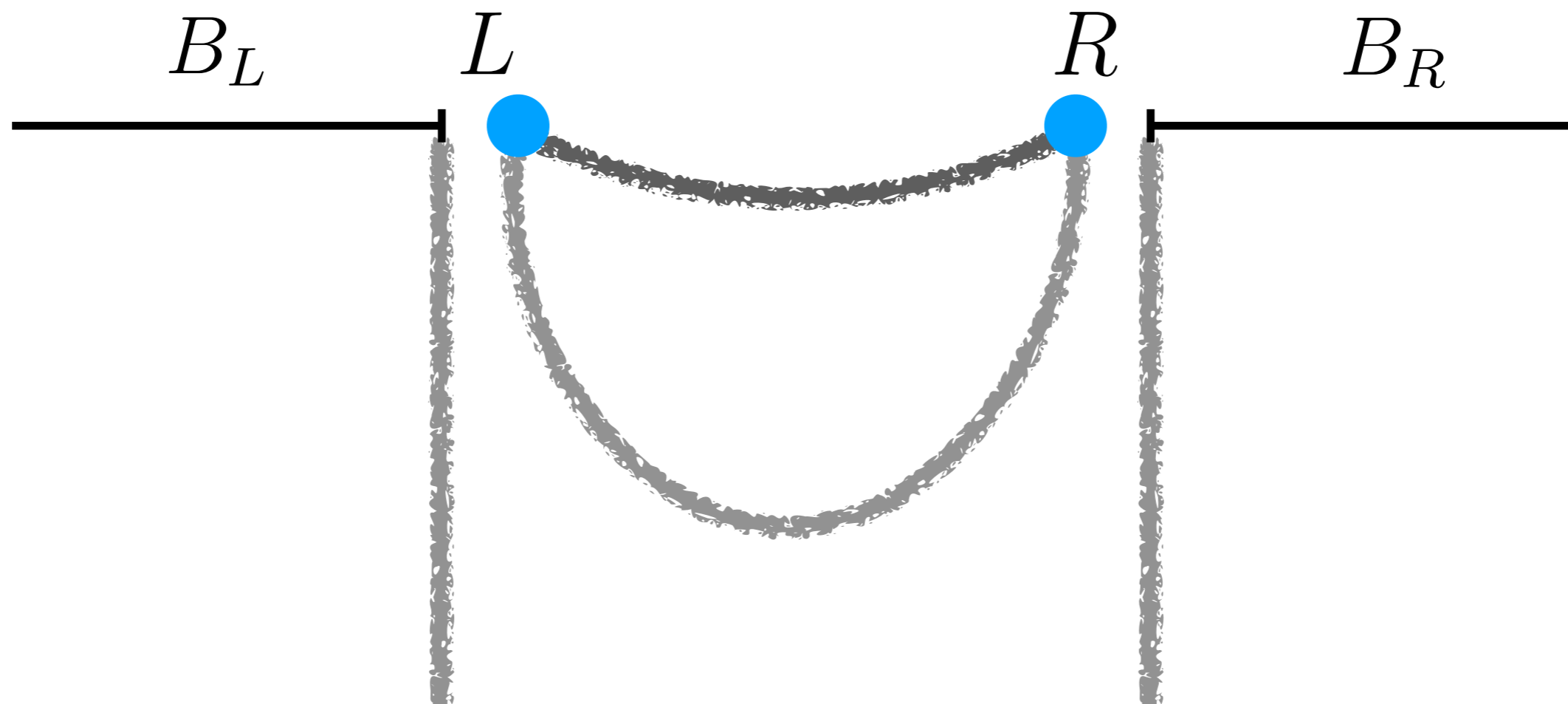
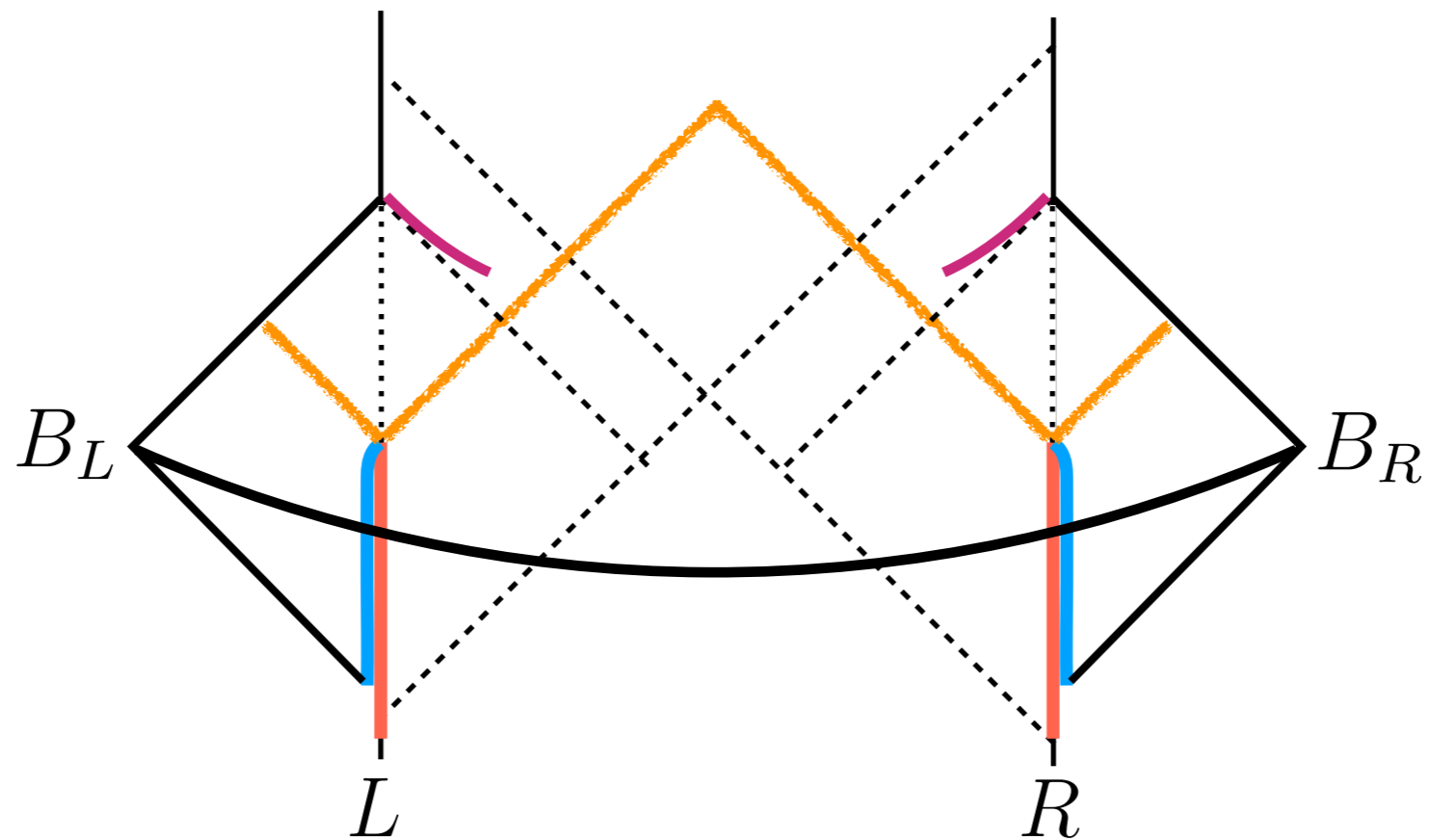
We obtained the **Page curve** for both the radiation and black hole in the context of evaporating a black hole in 1+1 dimensions coupled to holographic matter.

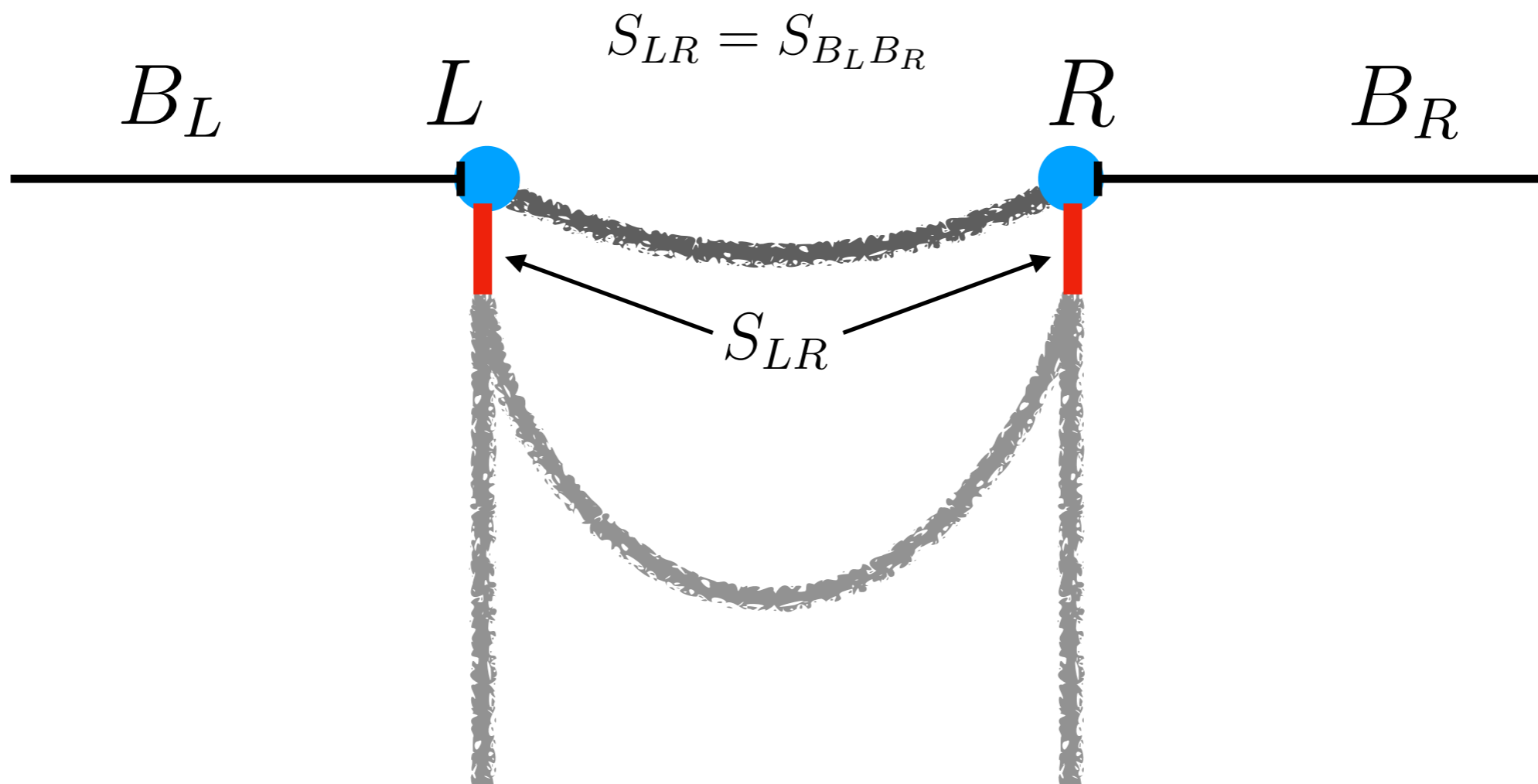
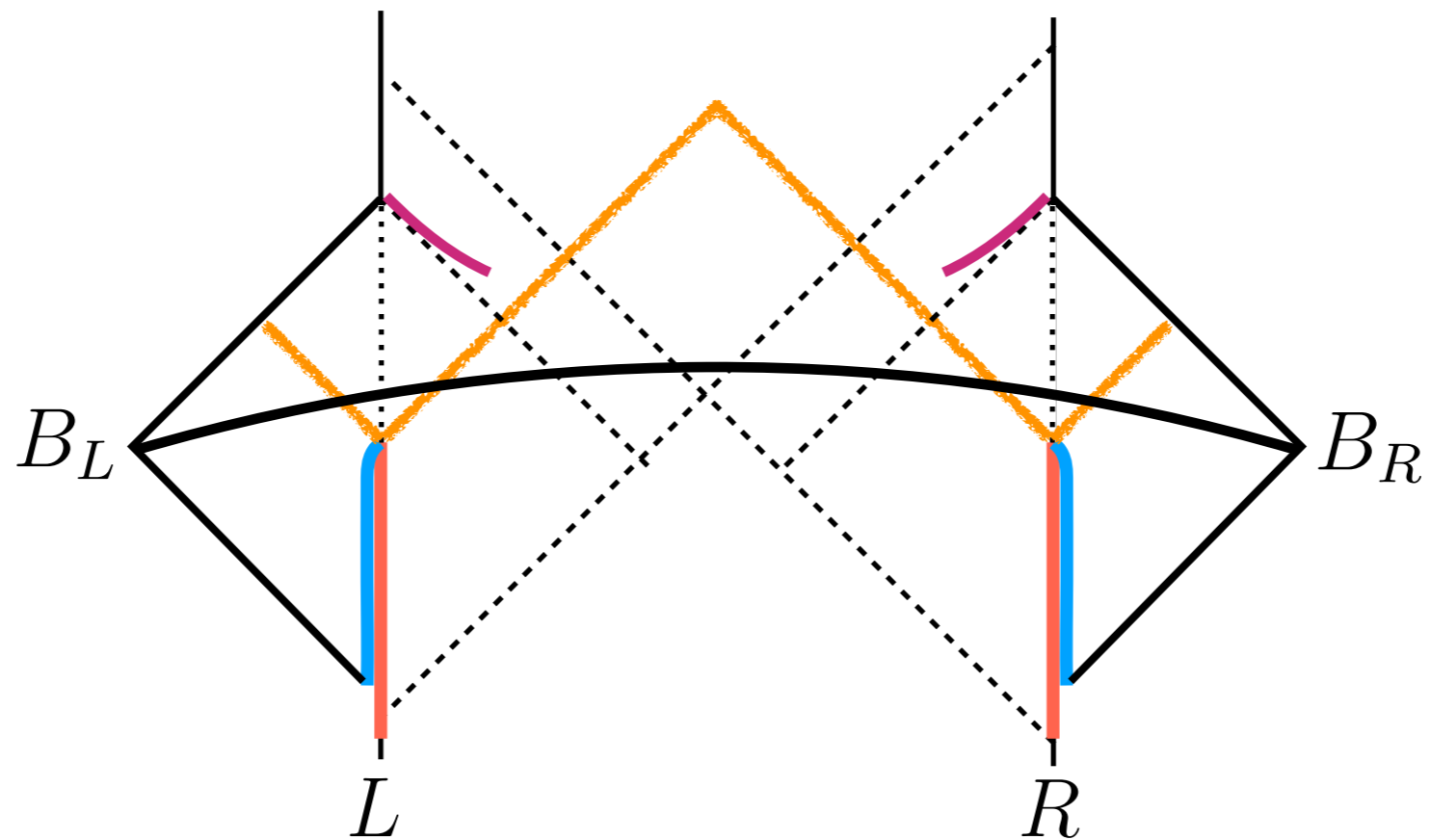


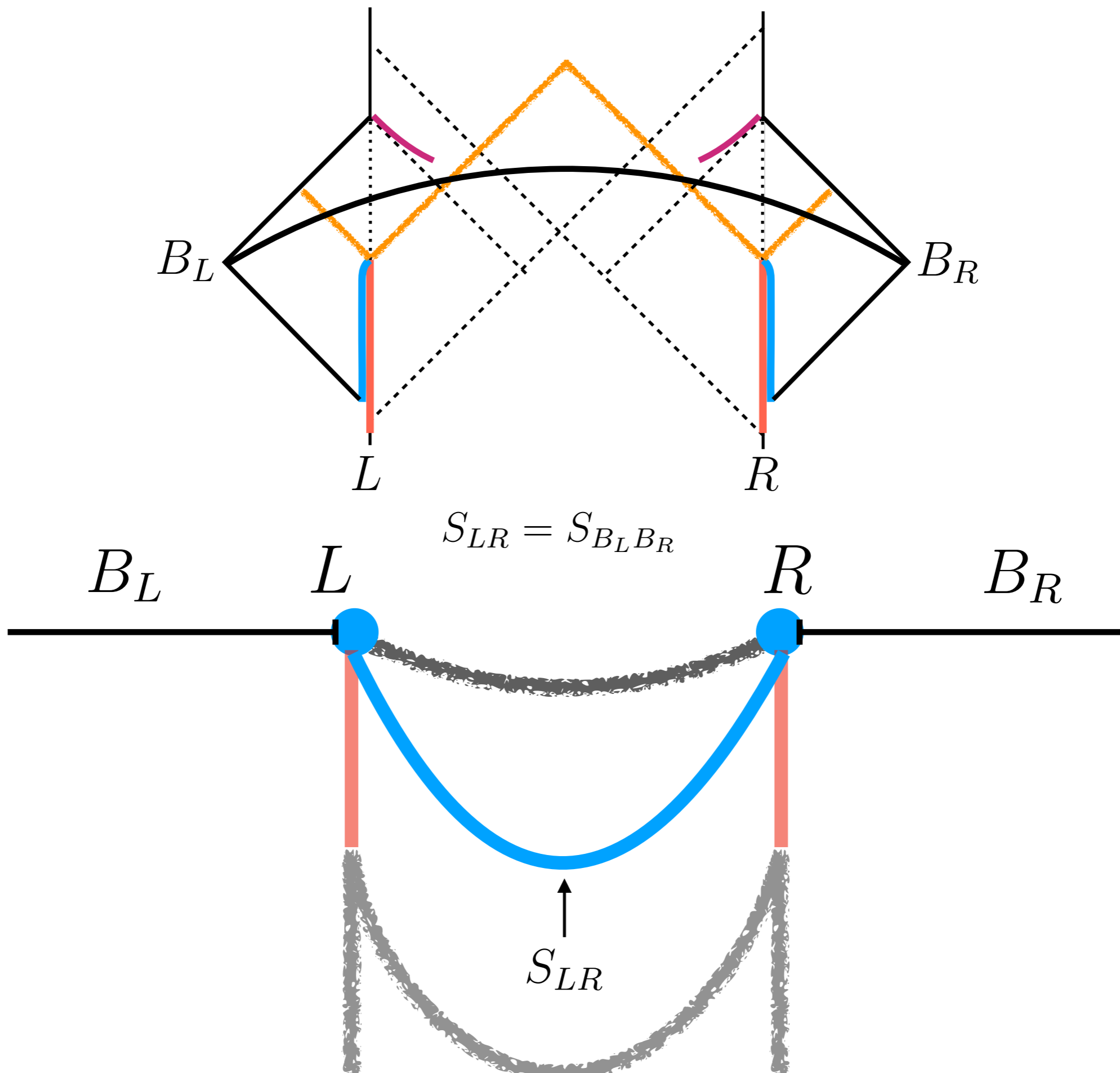
Hawking's 'mistake': He didn't know about the Minimality condition of RT formula!

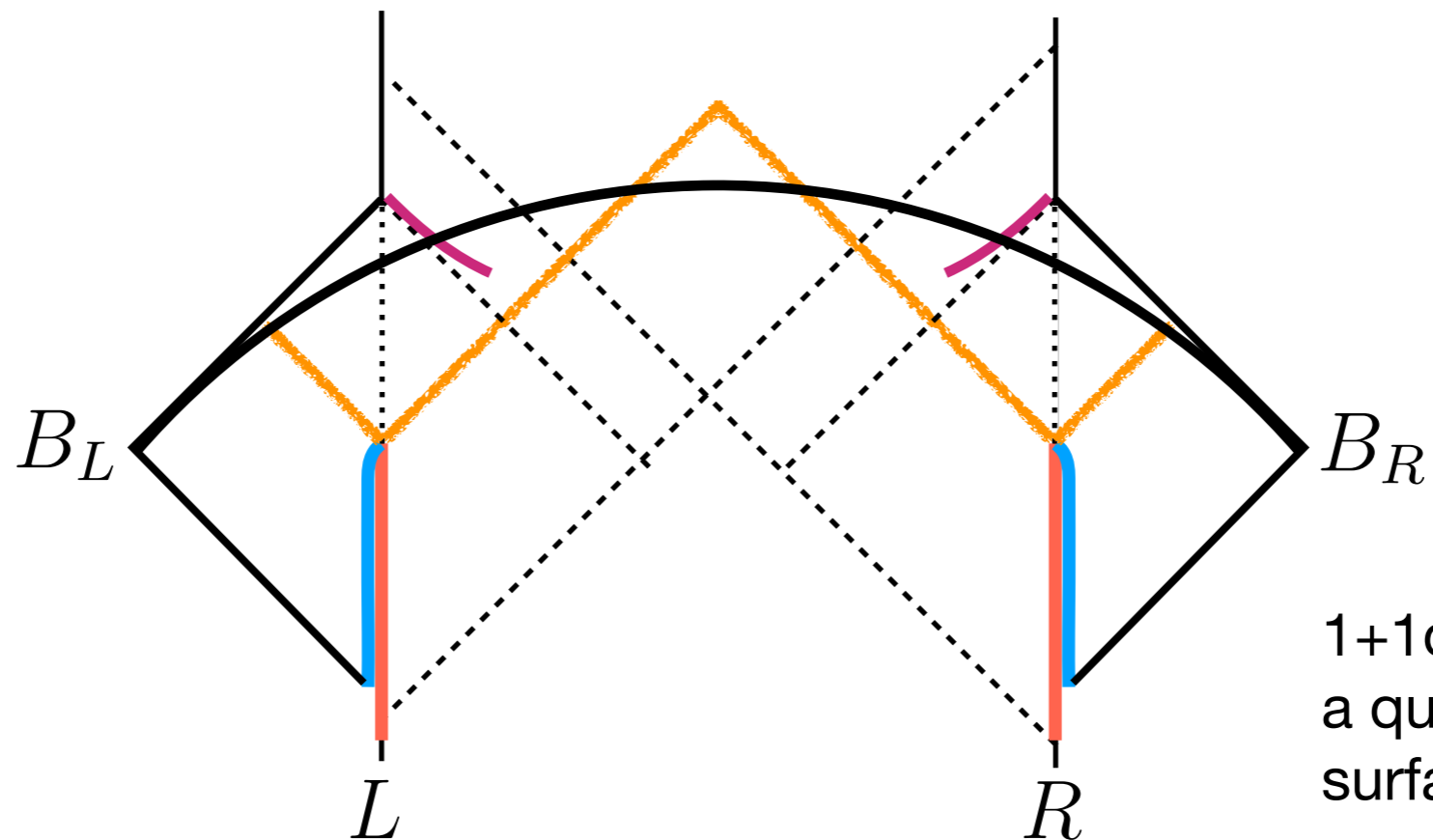
Somebody should ask me about what happens to pure states...



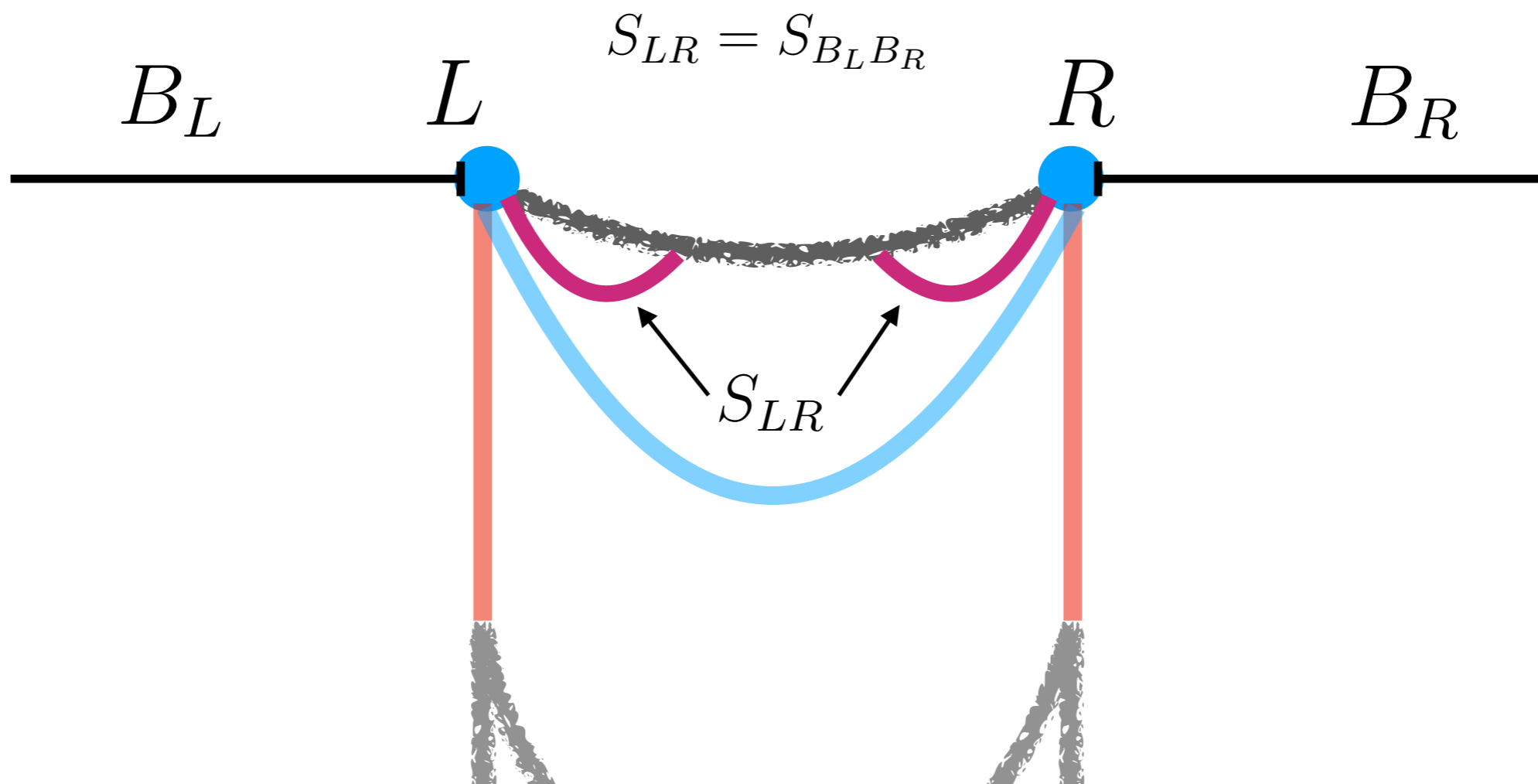




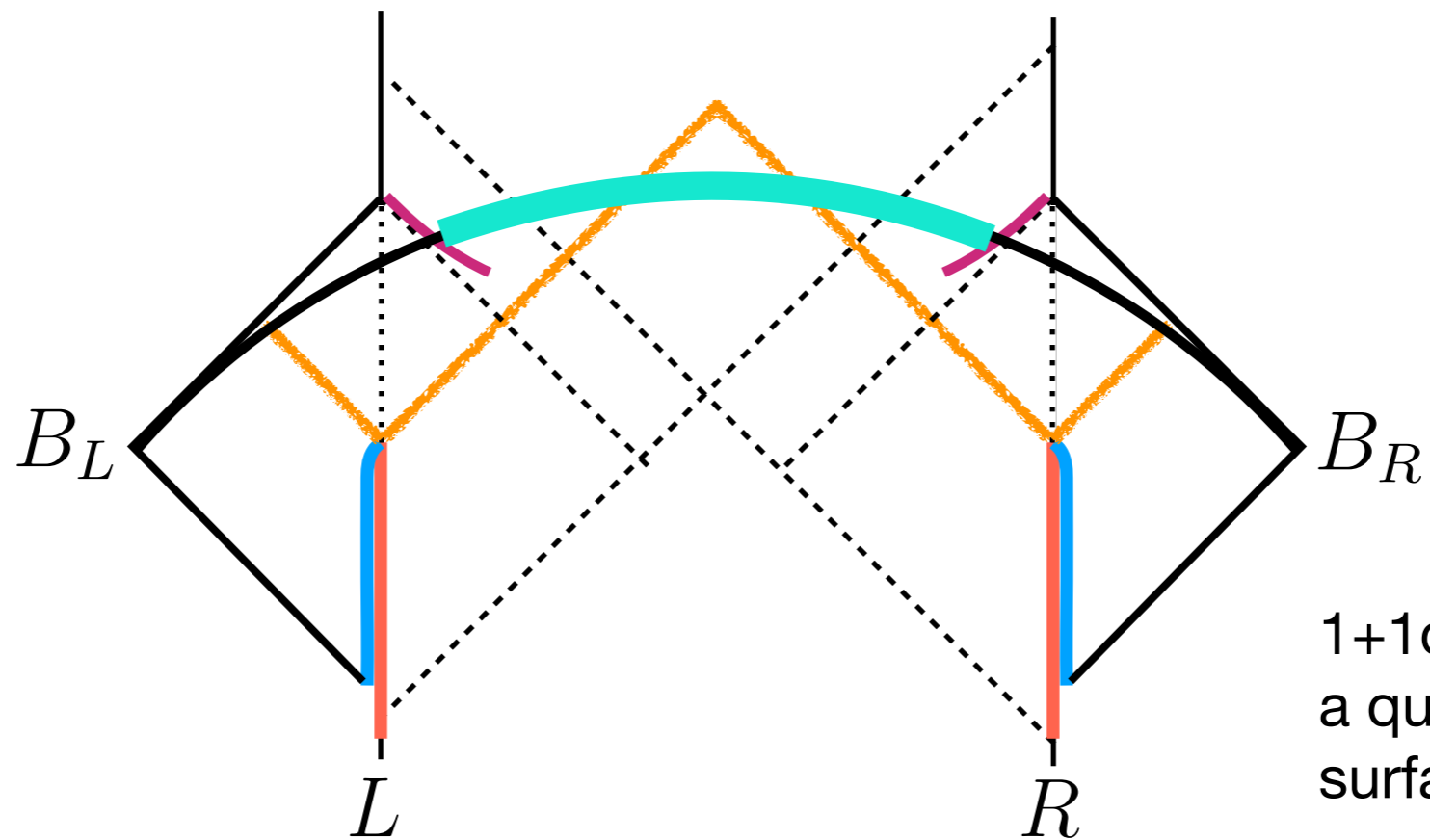




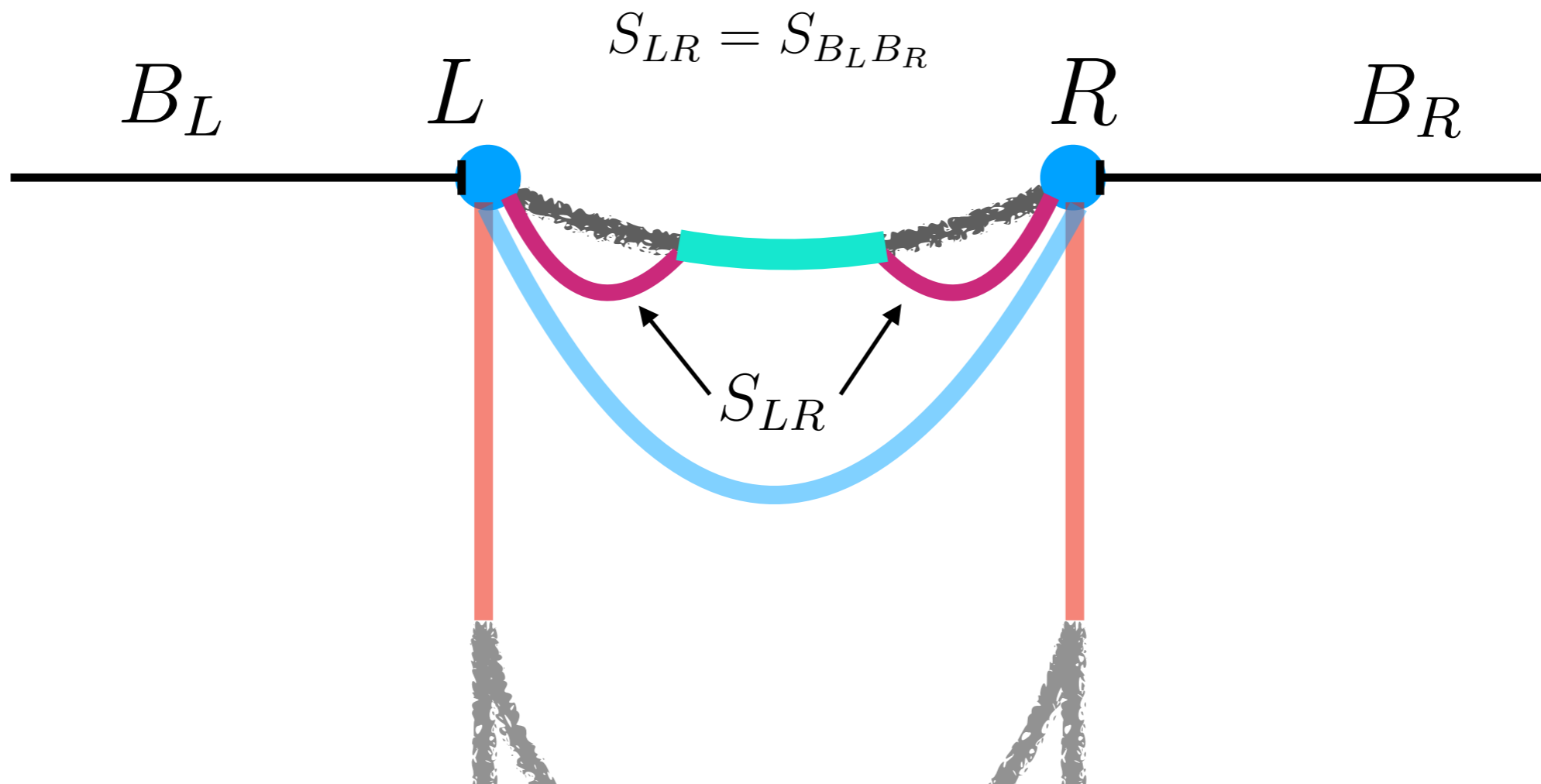
1+1d spacetime develops a quantum extremal surface at the Page time.







1+1d spacetime develops a quantum extremal surface at the Page time.



$$S_{LR} = S_{B_L B_R}$$

$S_{LR}$









