

THE ENTANGLEMENT ENTROPY OF TYPICAL PURE STATES AND REPLICA WORMHOLES

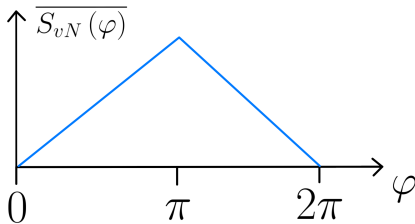
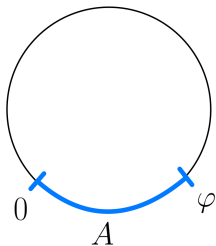
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Based on 2105.15059.

Strings 2021 Gong Show

The goal

- In a 1+1 dimensional QFT on a S^1 , we consider the von Neumann entanglement entropy of an interval $A = [0, \varphi]$ for **typical pure states**.
- Typically expect a “Page curve” in $S_{vN}(\varphi)$.
- We will reproduce it using a holographic calculation of “ $\overline{S_{vN}(\varphi)}$ ”: the entropy averaged over a specific ensemble of high energy pure states with $E \sim \frac{1}{\beta} \gg 1$.

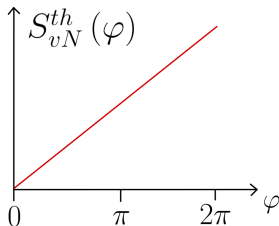
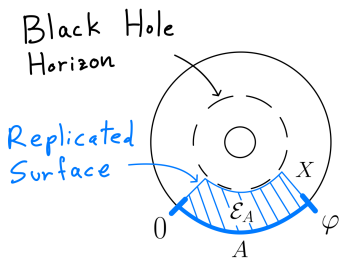


Reminder: thermal entanglement entropy

For $\rho_{th} = \frac{1}{Z} \exp(-\beta H)$, the replica trick reduces to

$$S_n^{th}(\varphi) \sim \text{[Diagram of } n \text{ replicas connected by a blue line with a knot]} \dots \text{[Diagram of } n \text{ replicas connected by a blue line with a knot]}$$

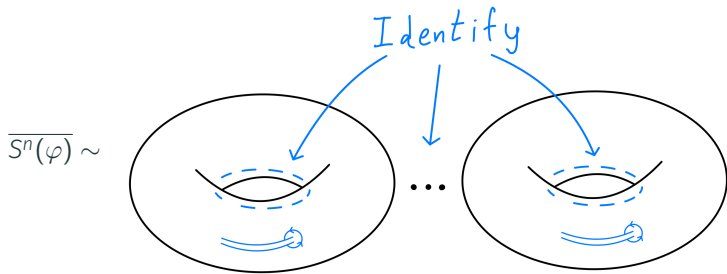
The holographic dual of the replica trick [Lewkowycz, Maldacena] gives [Azeyanagi, Nishioka, Takayanagi]



Typical pure state - CFT side

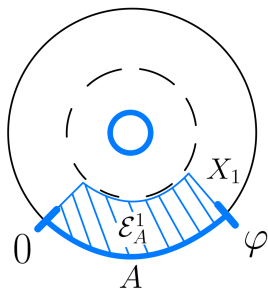
For a specific ensemble of pure states, the averaged entropy is given by a similar path integral in the CFT replica trick.

Singular geometry that further identifies all the replicas together:

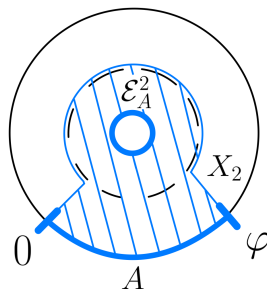


Typical pure state - gravity side

The dual gravitational calculation has two saddles due to the singular geometry:



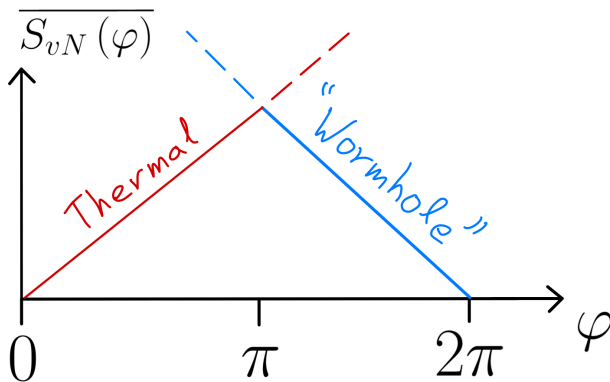
A thermal saddle with increasing entropy.



A “wormhole” saddle with decreasing entropy. Connects the replicas “behind the horizon”.

Reproduce the Page curve in $\overline{S_{vN}(\varphi)}$!

Result - a Page curve



(This is not an evaporation process!)

THANK YOU!
QUESTIONS?