Comments on the Hartle Hawking wavefunction of the Universe

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Based on work with:

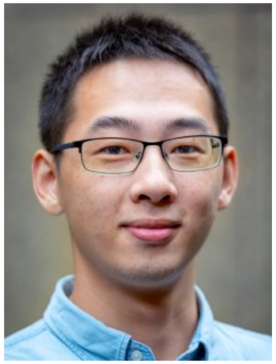
Joaquin Turiaci

Zhenbin Yang

arXiv:1904.01911

Related to paper by
Cotler, Jensen, Maloney:

arXiv:1905.03780



Yiming Chen

Henry Lin

To appear…

The H-H wavefunction of the Universe

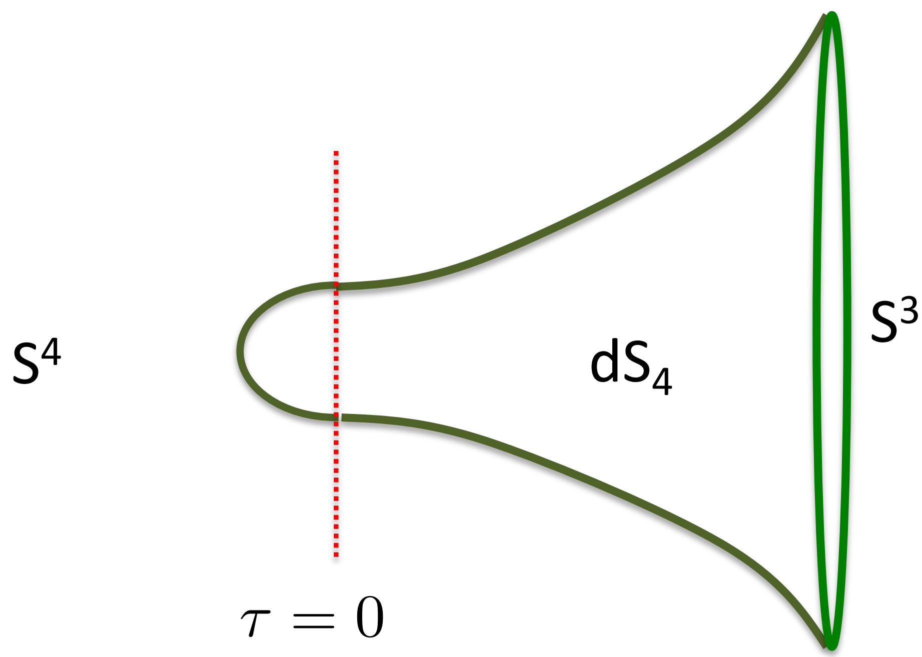
- The small G_N, or semiclassical, limit gives a WKB like approximation to the wavefunction of the universe:

$$\Psi_U \sim e^{iS}\Psi_{\rm Sch} + \cdots$$

``positive frequency part"

- There are many possible solutions.
- HH proposed a particular choice.

- Lorentzian geometries with a deformation into Euclidean signature and with no boundary
- Inspired by the iε prescription.



Hartle-Hawking

$$ds^2 = -d\tau^2 + \sinh^2 \tau d\Omega_3^2$$

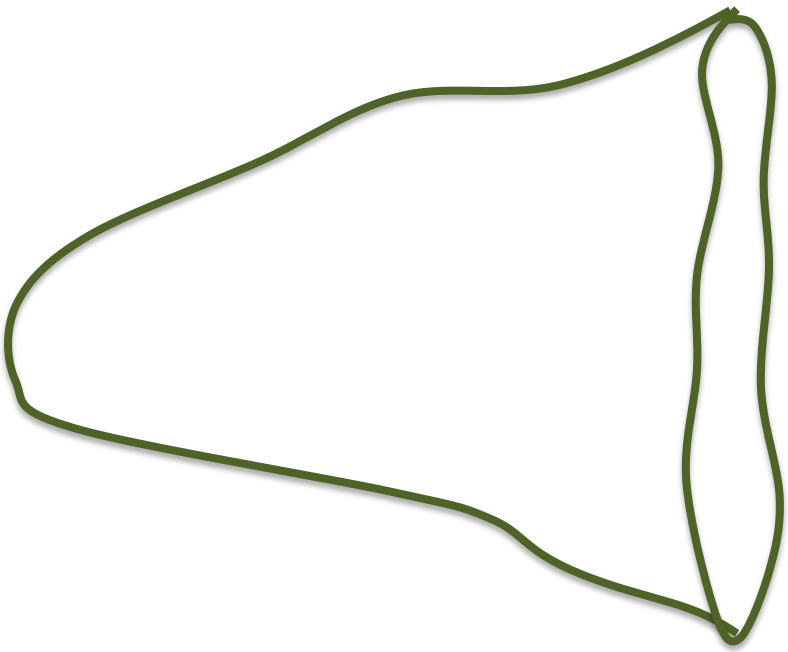
$$|\Psi|^2 = |e^{iS}|^2 = e^{S_{dS}}$$

- The most conservative claim is that this computes the ``positive frequency'' part of the asymptotic wavefunction.

- No claim about other pieces.

- This is what we claim in EAdS, in the standard (Euclidean) AdS/CFT correspondence.

Gives the right fluctuations

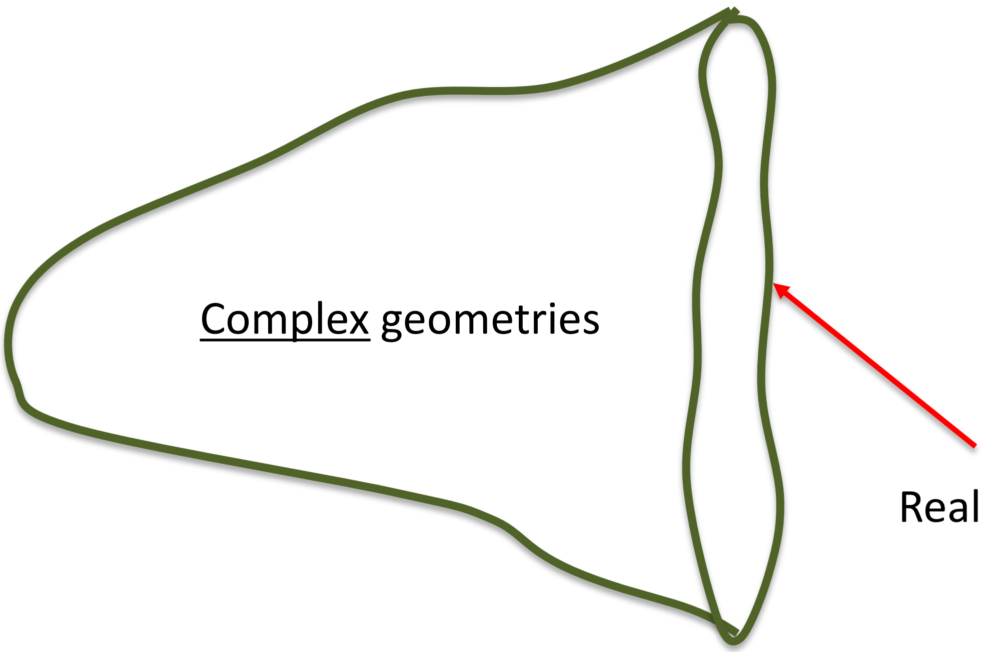


Halliwell Hawking, 1984

Same as the standard de Sitter vacuum fluctuations.

Gives the right fluctuations

Halliwell Hawking, 1984



Real boundary conditions: $g_b = 1 + h$

It is essential that they are complex!

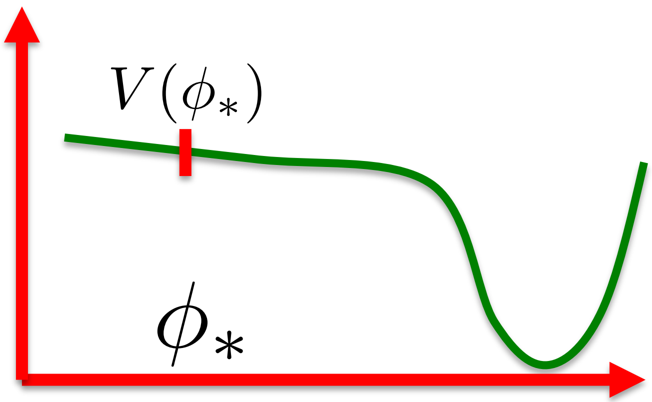
$$|\Psi|^2 = |e^{iS}|^2 \propto e^{-h^2}$$

It has one phenomenological problem

Problem with the HH wavefunction ⚠

- In the context of inflation.
- Calculate the probability that we have a positively curved universe.

$$|\Psi|^2 \sim \exp\left[\frac{M_p^4}{V(\phi_*)}\right]$$



ϕ_*

Value of the inflaton at the time that the length scale of the sphere crosses the horizon during inflation.

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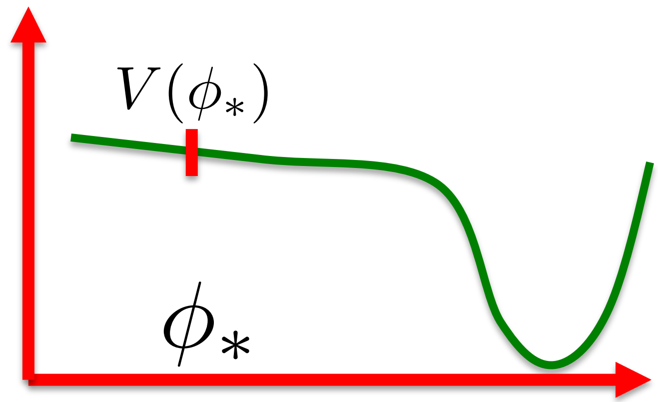


ϕ_*

Value of the inflaton at the time that the length scale of the sphere crosses the horizon during inflation.

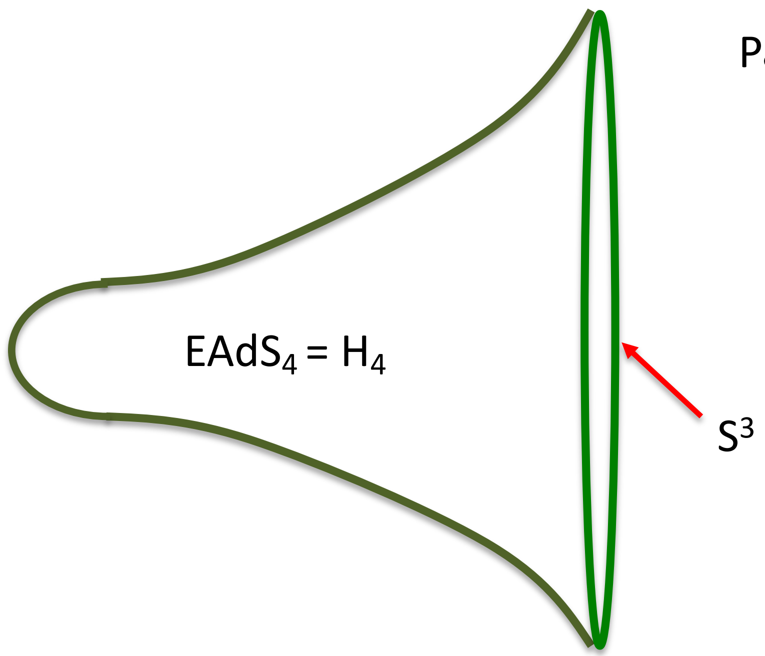
Minimum radius compatible with current curvature, Ω_k, bounds

$$|\Psi|^2 \propto \left(\frac{R_m}{R}\right)^{\frac{1}{A_s}} \sim \left(\frac{R_m}{R}\right)^{7\,10^8}$$

Something is wrong…

Something is right…

This is how we compute EAdS partition functions!



Part of standard AdS/CFT correspondence

Gubser, Klebanov, Polyakov, Witten

Excellent agreements with S^3 partition functions.

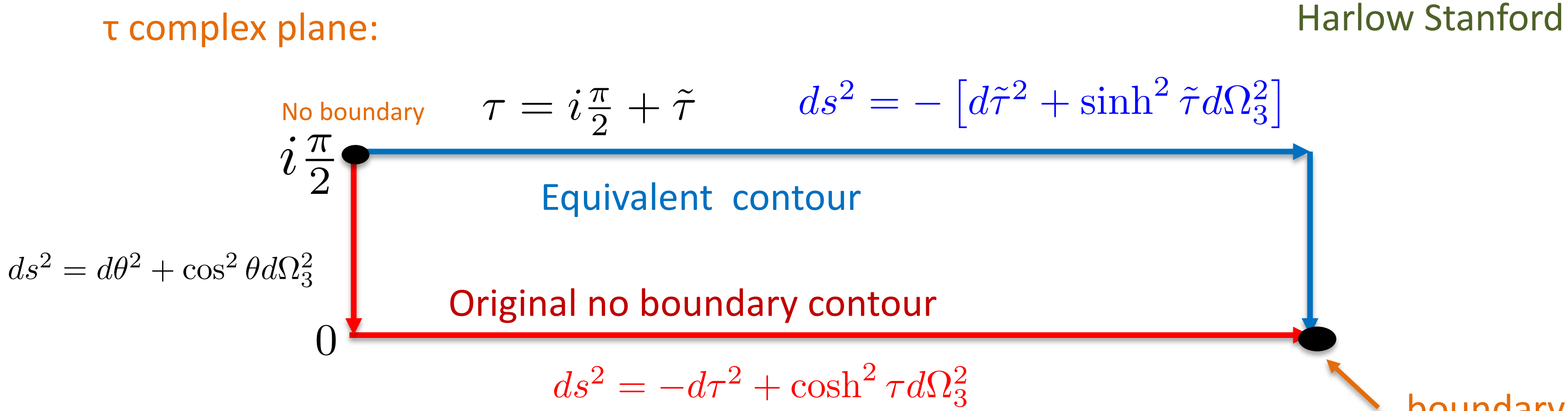
Pestun, Kapustin, Willet, Yaakov
Drukker, Marino, Putrov, Jafferis,
Klebanov, Pufu, Safdi + many others

$$ds^2 = d\rho^2 + \sinh^2 \rho d\Omega_3^2$$

The two problems are closely related

- The de-Sitter problem is related to a “minus” EAdS problem.

JM
Harlow Stanford



- It is a particular analytic continuation.
- We are <u>not</u> modifying the original Lagrangian
- It is a solution with positive cosmological constant.

A comment on the relation to the RG

De Boer, verlinde[2]

….

Heemskerk, Polchinski

….

Euclidean AdS and partition functions

$$\Psi_{\mathrm{U}}[g] = \exp\left[c_0 \int \sqrt{g} + c_2 \int \sqrt{g} R + \cdots\right] \Psi_{\mathrm{ren}}(\hat{g})$$

Counterterms

Function of ratios of metric components.

Euclidean AdS and partition functions

$$\Psi_{\rm U}[g] = \exp\left[c_0 \int \sqrt{g} + c_2 \int \sqrt{g} R + \cdots\right] \Psi_{\rm ren}(\hat{g})$$

Define another solution of the WdW equation:

Argument of wavefunction

$$\Psi_{UV}^{\hat{g}_0}[g] = \exp\left[-c_0 \int \sqrt{g} - c_2 \int \sqrt{g} R + \cdots\right] \delta(\hat{g} - \hat{g}_0)$$

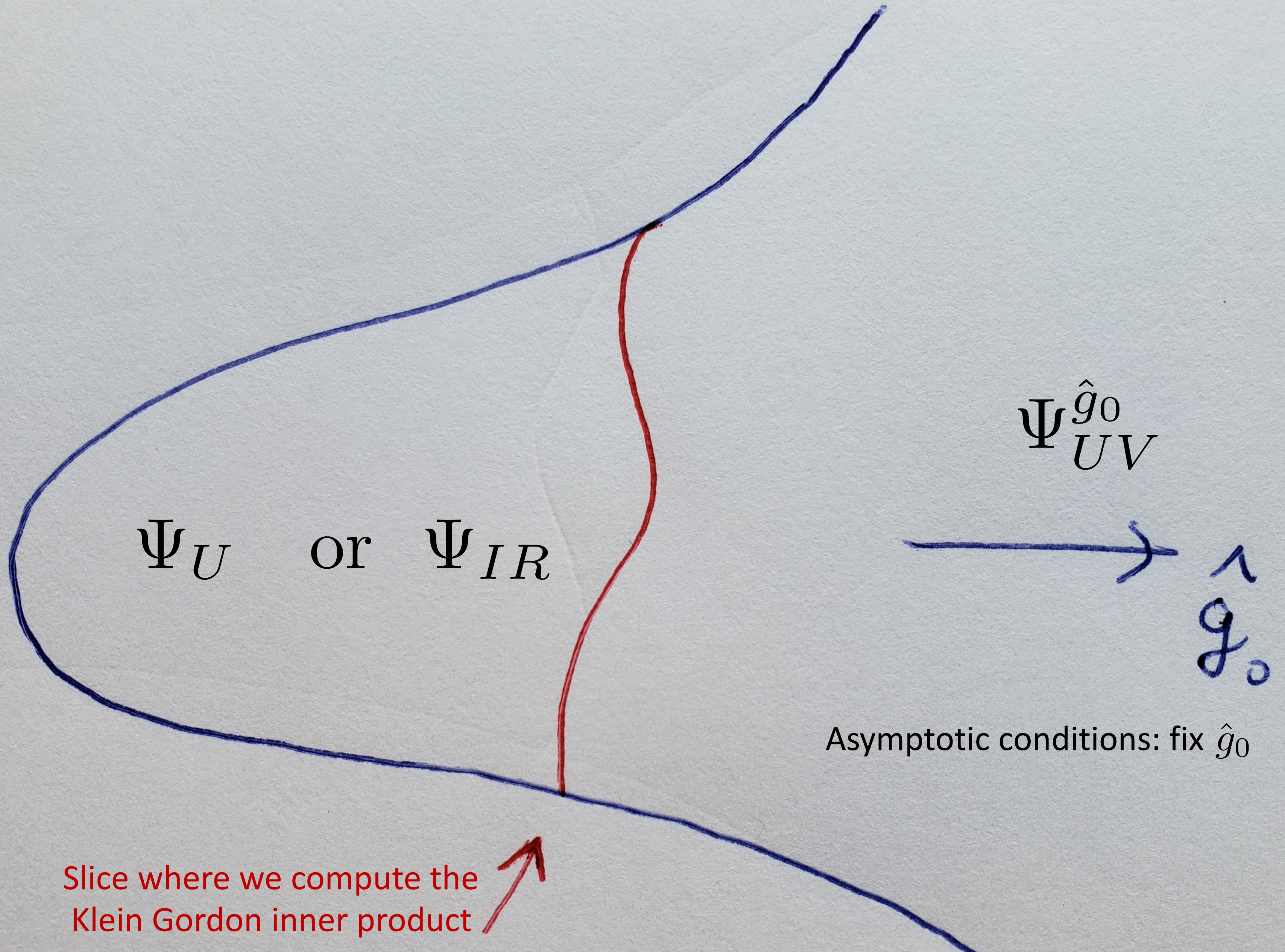
Contains UV Renormalization Group trajectory. 1st order eqn.

Choice of wavefunction

$$Z_{\rm ren}^{\rm CFT}[\hat{g}_0] = \langle \Psi_U, \Psi_{UV}^{\hat{g}_0} \rangle = \langle \Psi_{IR}, \Psi_{UV}^{\hat{g}_0} \rangle \propto \Psi_{\rm ren}(\hat{g}_0)$$

``Klein Gordon'' inner product. Evaluated on a "slice"

Improvement = Now both pieces obey the WdW equation.



Answer is independent of the slice = Renormalization group invariance

End of EAdS and RG comment

- It is worth studying Hartle – Hawking wavefunctions.
- Maybe once we understand them well enough the phenomenological problem will go away.
- Is a kind of “information problem” in cosmology

We will focus on nearly-dS_2 gravity

- Simple de Sitter theory
- Arises as some limit of certain four dimensional computations.

Nearly dS_2 Gravity

$$iS = i\phi_0 \left[\int d^2x \sqrt{g} R - 2 \int_{Bdy} K \right] + i \int d^2x \sqrt{g} \phi (R-2) - i2 \int_{Bdy} \phi_b K$$

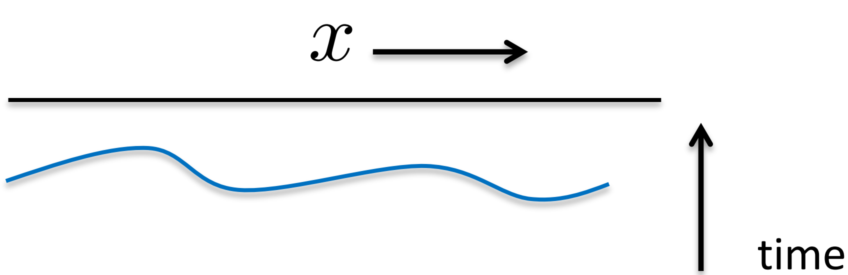
Contributes only to entropy

JT action

metric is fixed to dS_2

Action = $$iS = -i2\phi_b \int K = -i2\phi_b \int du + i2\phi_b \int du \{x(u), u\}$$

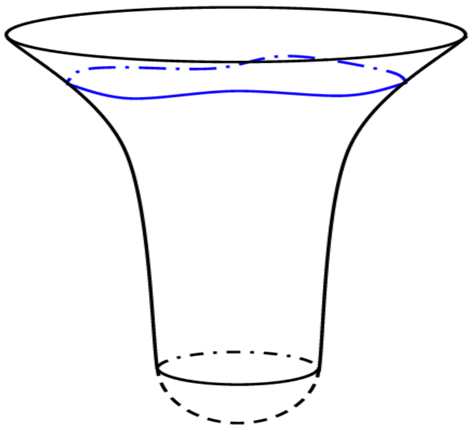
Schwarzian



$$iS = -i2\phi_b \int K = -i2\phi_b \int du + i2\phi_b \int du\{x(u), u\}$$

Length of boundary

Space reparametrization mode



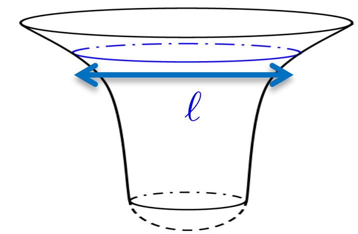
- A Schwarzian mode.
- Off shell mode that we should integrate out.
- Two independent Schwarzian modes: one for the "bra'' and one for the ''ket''.
- Allows us to various quantum gravity corrections.

Two cases

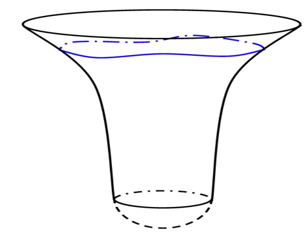
- The wavefunction in JT theory
- Quantum corrections to matter correlators (in JT + matter).

The vacuum HH wavefunction

Classical: $\Psi_{cl} \sim \exp\left(-i2\ell\phi_b + 2\pi\phi_0 + i\frac{4\pi^2\phi_b}{\ell}\right)$



Superspace = 2d = $\phi_b, \ell \longrightarrow \frac{\ell}{\phi_b}$



Quantum: Integrate over fluctuations

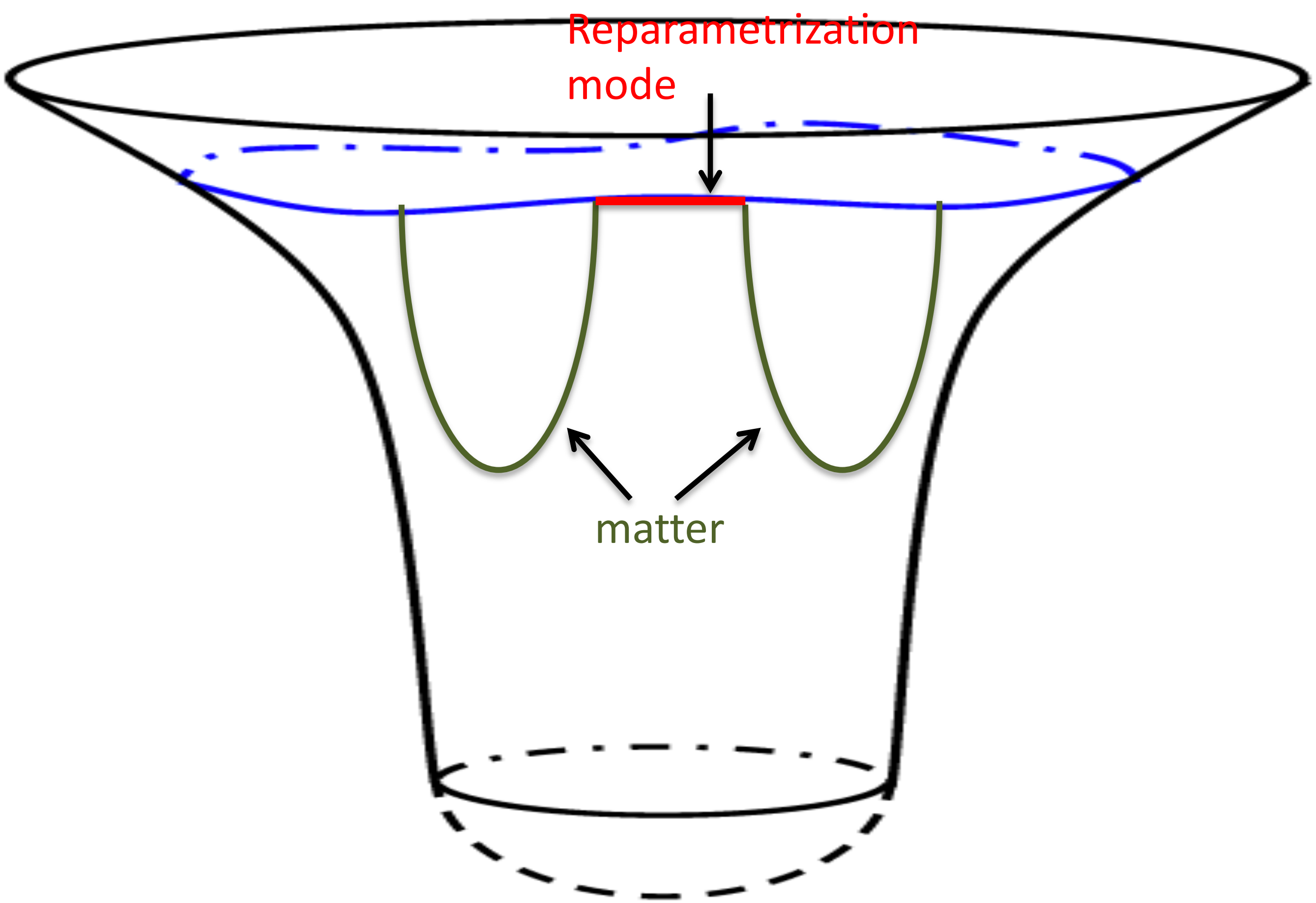
Bagrets, Altland, Kamenev
Stanford Witten
Kitaev Suh
Yang

$$\Psi \sim \frac{1}{\ell^{3/2}}\Psi_{cl}$$

$$|\Psi|^2 \propto \frac{1}{\ell^3}e^{2S_0} \;, \qquad 2S_0 = S_{dS_2}$$

Two separate disks

Gravitational corrections to cosmological correlators



Gravitational corrections to cosmological correlators

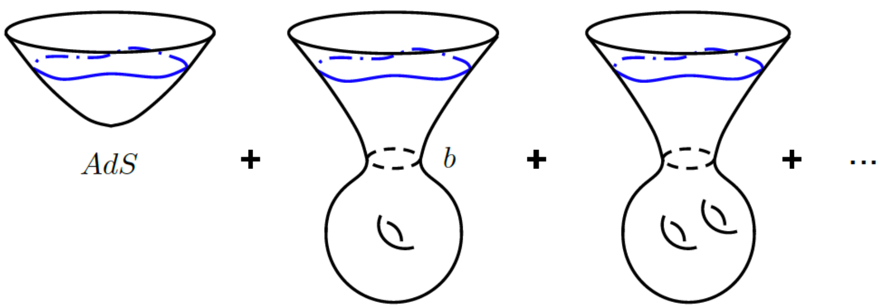
J.M. Stanford Yang

- Similar to N-AdS_2 computations.
- An i in the action for the Schwarzians.
- Two Schwarzians.
- Found the leading corrections.
- Much simpler than gravitational corrections in 4d !

Contributions from other geometries ?

Contributions of other geometries ?

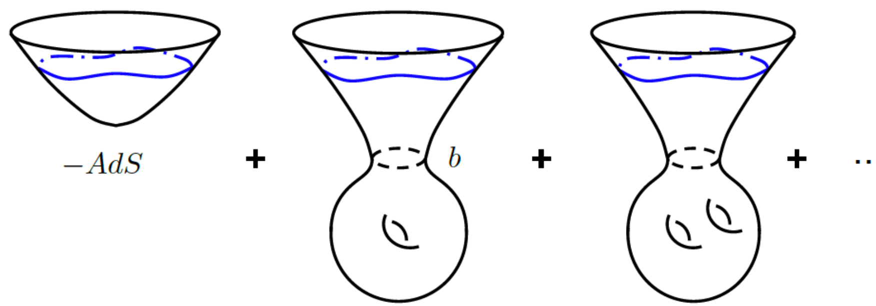
Saad, Shenker and Stanford understood this for N-AdS_2 gravity.



Sum over other surfaces with constant <u>negative</u> curvature.

Sum over geometries in N-dS_2

- Need positive curvature geometries.
- Naively only S^2
- But we also have ``minus'' all the previous ones.



Is an identical problem. Only that we have an extra i for the wiggles at infinity.

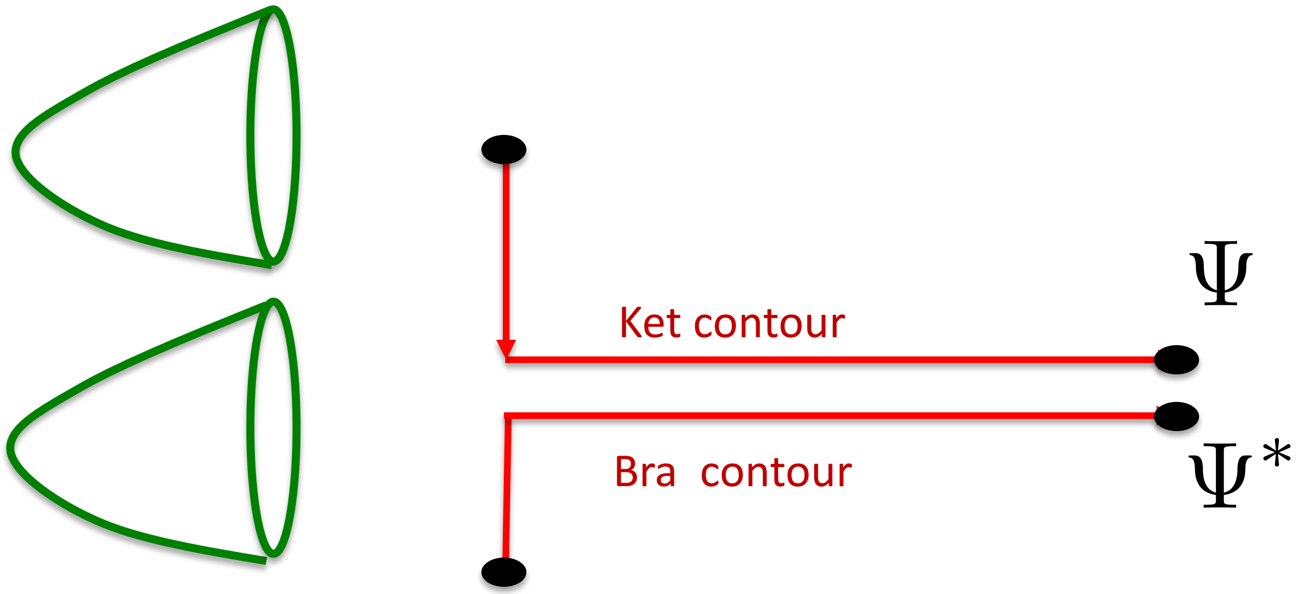
$$\Psi \propto Z_{SSS}(\beta, S_0)|_{\beta \to -i\ell}$$

SSS=Saad, Shenker, Stanford

Also given by a matrix integral!

Pure state or density matrix?

Pure state



Pure state or density matrix?

Density Matrix

Page

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Barvinsky, Deayet, Kamenshchik

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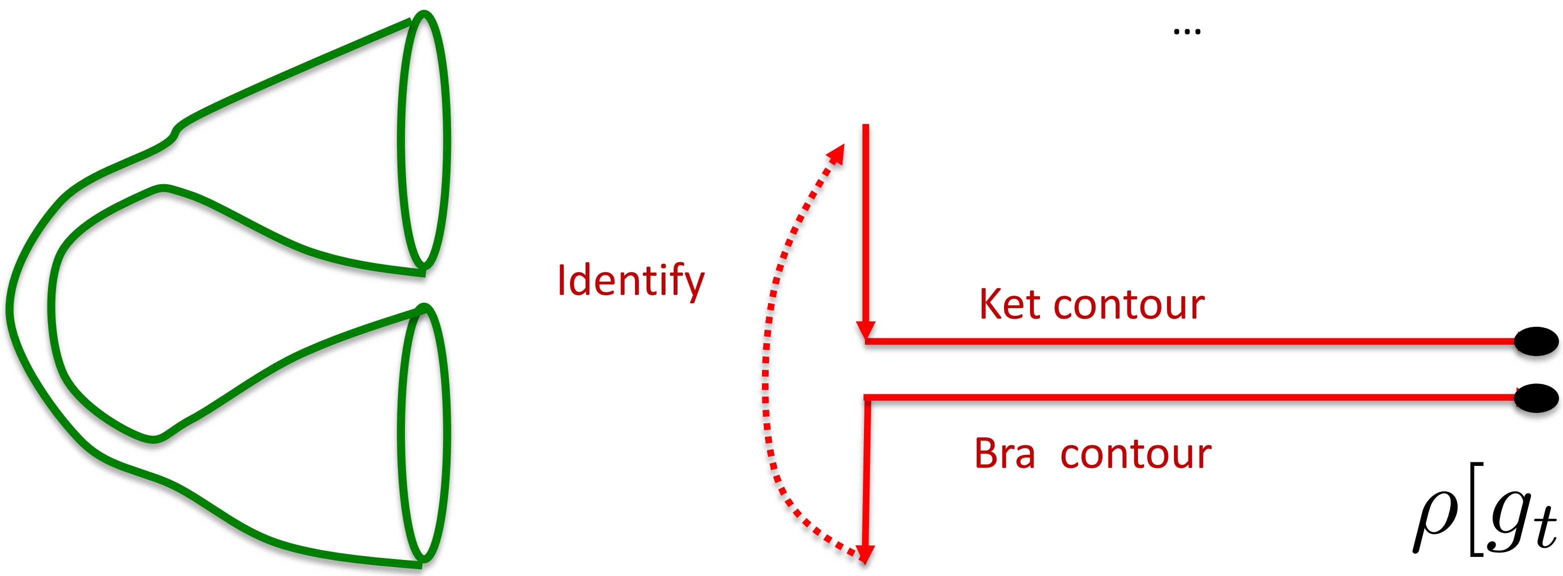


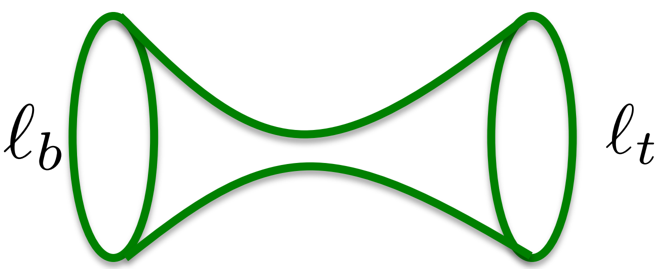
$$\rho[g_t, g_b]$$

Bra-ket reunification

Ordinary thermal states in QFT are produced by such a contour.

In $NAdS_2$ gravity

- We also have the ``double cone'' geometry considered by Saad, Stanford and Shenker.



We could interpret it as the HH wavefunction for two entangled universes.

Or, as the contribution to the density matrix of a single universe.

$$\rho[\ell_b, \ell_t] \propto Z_{SSS}[\beta_1, \beta_2]|_{\beta_1 \to -i\ell_t, \beta_2 \to i\ell_b} \propto i \frac{\sqrt{\ell_t \ell_b}}{\ell_t - \ell_b}$$

It is formally hermitian

It diverges when the two lengths are equal (along the diagonal)

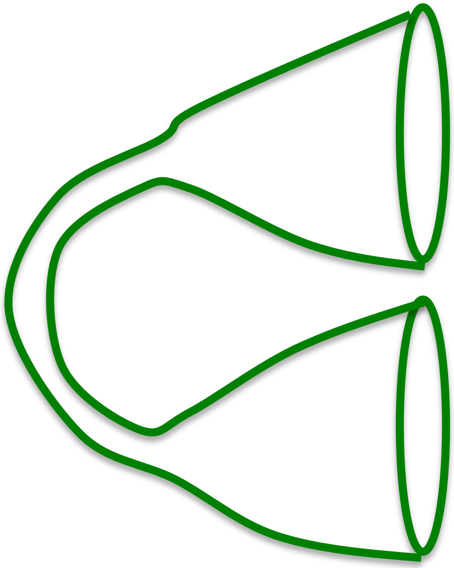
Small correction relative to the two disks, since it does not have the de-Sitter entropy contribution

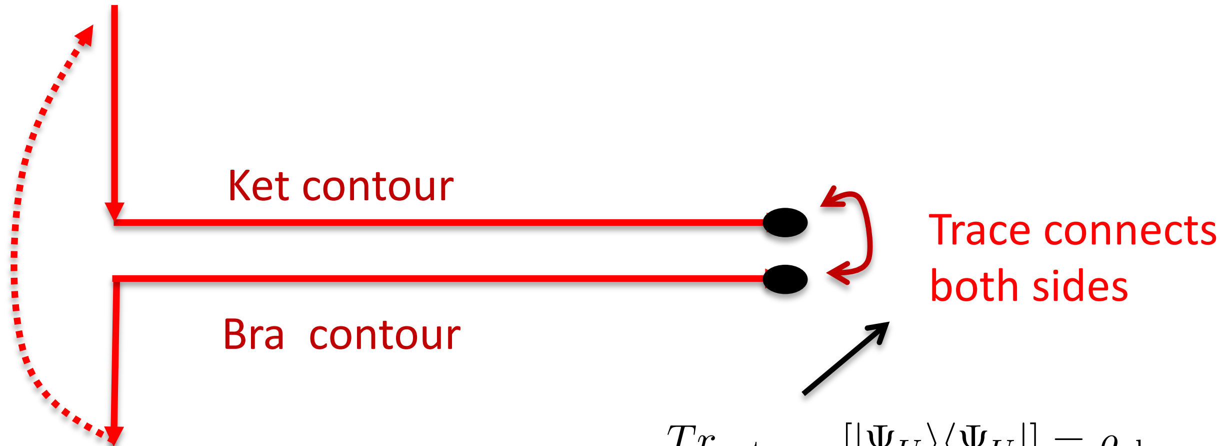
More general theories of gravity

- The standard static patch solution of dS can be viewed in this way → thermal equilibrium
- We searched for <u>time dependent</u> solutions of this type but we did not find one that we liked.
- Unreliable regimes. ($N_{fields} \sim S_{dS}$) Barvinsky, Deayet, Kamenshchik
- Unstable solutions. (unstable Euclidean wormholes)
- Bad wavefunctions. (wavefunctions unsuppressed in some directions)

Motivation/Speculation

Cosmological wormholes?





$$Tr_{\text{not seen}}[|\Psi_U\rangle\langle\Psi_U|] = \rho_{\text{observed}}$$

In cosmology there are regions we do not see!

dS/CFT + coupling → wormhole (``double trace'' deformations) (related to Bousso, Maloney, Strominger & Gao, Jafferis, Wall, JM. Qi , Saad, Shenker, Stanford)

Could there be a way to directly compute the observed density matrix ?

Conclusions

- We reviewed the HH wavefunction.
- Slightly improved picture for the RG flow in usual AdS/CFT.
- Introduced Nearly-dS_2 gravity.
- Computed perturbative quantum gravity effects in Nearly-dS_2
- We mentioned that the sum over topologies is the same as the one considered by Saad, Shenker, Stanford.
- We discussed why it seems natural to expect ``cosmological wormholes''. But we owe you a good example… (other than the dS static patch)

The End