Black holes, random matrices, baby universes, and D-branes

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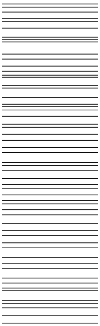
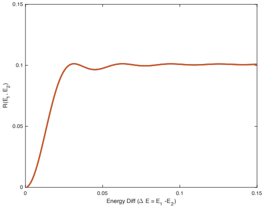
Motivation

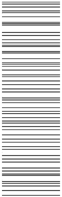
- What is the gravitational explanation for the discreteness of the energy spectrum of quantum black holes?
- Discreteness determines the long time behavior of correlation functions, and of the spectral form factor. An aspect of the black hole information problem [Maldacena]. $\star$



Ensembles of quantum systems

- Simplicity after averaging. Consider an **ensemble** of unitary finite entropy quantum systems (e.g., SYK). Aspects of discreteness still visible.
- Compute $R_2(E, E') = \langle \rho(E)\rho(E') \rangle$ in SYK.

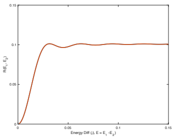




[You-Ludwig-Xu; Garcia-Garcia–Verbaarschot; Cotler-Gur-Ari-Hanada-Polchinski-Saad-Shenker-Stanford-Streicher-Tezuka (CGHPSSSST)]

- Eigenvalue spacing $\sim e^{-S} \sim e^{-N_{SYK}}$.
- A smoking gun for discreteness in an **averaged** quantity.

Random matrix statistics



$$\langle \rho(E)\rho(E') \rangle \sim e^{2S} - \frac{1}{2(\pi(E-E'))^2}(1-\cos(2\pi e^{S}(E-E')))$$

- The "Sine kernel formula" for the eigenvalue correlations in (GUE) random matrix theory [Dyson; Gaudin; Mehta].
- Conjectured to be **universal** in quantum chaotic systems [Wigner; Dyson; Berry; Bohigas-Giannoni-Schmit; ...].
- What is the gravitational explanation for this pattern in SYK? (And then, in more general systems with a gauge/gravity dual?)
- What is the origin of the **doubly exponential** $e^{ie^S} \sim e^{ie^N}$ behavior?

The SYK model and JT gravity

- We will focus on the observable

$$\langle Z(\beta)\rangle = \langle Tr e^{-\beta H_{SYK}}\rangle = \int dE \langle\rho(E)\rangle e^{-\beta E}$$

- At low energies, large β, this is gauge/gravity dual to (2D) Jackiw-Teitelboim (JT) gravity on the disk, (a piece of Euclidean AdS_2), to all orders in $G_N \sim 1/N$

  [Jensen; Maldacena-Stanford-Yang; Engelsoy-Mertens-Verlinde].
- Try to understand eigenvalue statistics in the JT gravity limit. Not a uniform limit – a model of the model.

Jackiw-Teitelboim (JT) gravity

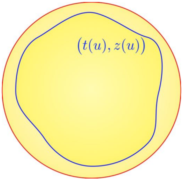
- Jackiw-Teitelboim (JT) gravity (metric $g_{\mu\nu}$ and dilaton ϕ):

$$I = \underbrace{-\frac{S_0}{2\pi}\left[\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}R + \int_{\partial\mathcal{M}}\sqrt{h}K\right]}_{\text{topological term } = S_0\,\chi(\mathcal{M})}$$

$$-\left[\underbrace{\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}\phi(R+2)}_{\text{sets } R=-2} + \underbrace{\phi_0\int_{\partial\mathcal{M}}\sqrt{h}(K-1)}_{\text{gives action for boundary}}\right].$$

- Localizes on $R = -2$ geometries, with fluctuating boundary.
- The ground state entropy S_0 is $\sim \frac{1}{G_N} \sim N$. Surfaces $\mathcal{M}$ with Euler character $\chi(\mathcal{M})$ are weighted by $e^{S_0\chi(\mathcal{M})} \sim e^{N\chi(\mathcal{M})}$. For a surface with g handles and one boundary the weight is $e^{S_0(1-2g)}$.

JT gravity density of states



- $Z(\beta)$, at leading order in e^{S_0}, is described by a geometry with one fluctuating boundary of fixed length β/ϵ. Not a geodesic boundary. $Z(\beta)$ is one loop exact (in $G_N = 1/N$) [Bagrets-Altland-Kamenev; CGHPSSSST; Stanford-Witten].
- Gives (by inverse Laplace transform) the density of states

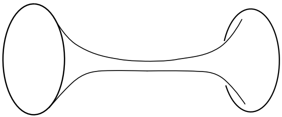
$$\rho_0^{\mathrm{total}}(E) = e^{S_0} \frac{\sinh(2\pi\sqrt{E})}{4\pi^2}.$$

- A disk has Euler character $\chi = 1$ giving the weight $e^{S_0} \sim e^N$.

Cylinder

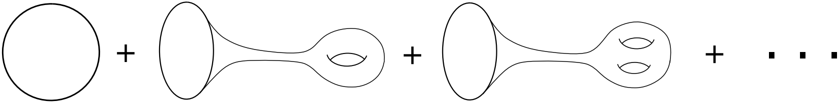
- Study topologies beyond the disk.
- The leading contribution to $\langle Z(\beta_1)Z(\beta_2)\rangle = \langle Tre^{-\beta_1 H}Tre^{-\beta_2 H}\rangle$ comes from two disconnected disks, $\chi = 2$, of order e^{2S_0}.
- The cylinder (euclidean spacetime wormhole) contribution has $\chi = 0$ and is of order one $\sim e^{0 \cdot S_0}$.

  [Maldacena-Qi; Harlow-Jafferis; Saad-SS-Stanford]
- It gives the $1/(E - E')^2$ term in $\langle \rho(E)\rho(E')\rangle$, corresponding to the ramp in the spectral form factor [Saad-SS-Stanford].



Sum over topologies

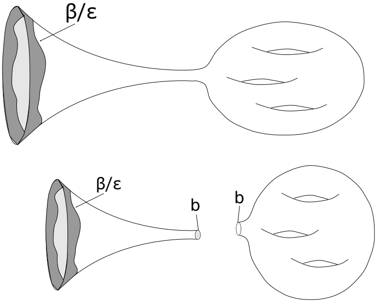
- Here we study contribution of surfaces with any number (g) of handles to $Z(\beta)$ ($\chi = 1 - 2g$).



- $Z(\beta) = \sum_g Z^{(g)}(\beta) \times e^{(1-2g)S_0} = e^{S_0} \sum_g Z^{(g)}(\beta) \times (e^{-S_0})^{2g}$
- Looks like a perturbative string genus expansion, **but here** $g_s = e^{-S_0} \sim e^{-1/G_N} \sim e^{-N}$.
- These are nonperturbative effects in G_N. Joining and splitting of baby JT universes, a "**third quantized**" description.
- (Only hints for higher topologies in SYK, as opposed to JT.)

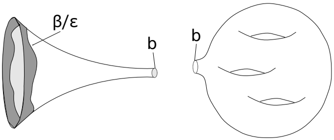
Sum over topologies, contd.

All of these geometries have the same asymptotically AdS_2 region (the trumpet) glued along a **geodesic** boundary of length b to a higher genus Riemann surface.



These are not saddle points of the JT action. The $R = -2$ constraint allows us to do the full path integral.

Doing the higher genus path integral



$$Z^{(g)}(\beta) = \int b db \; Z_{\text{Trumpet}}(\beta, b) \times V_{g,1}(b) \; .$$

- $Z_{\text{Trumpet}}(\beta, b)$ is one loop exact, $= \exp(-b^2/4\beta)/2\sqrt{\pi\beta}$. (use techniques of [Stanford-Witten]). (See also Blommaert-Mertens-Verschelde).
- $V_{g,1}(b)$ is the Weil-Petersson (WP) volume of the moduli space of Riemann surfaces with a single geodesic boundary.

Mirzakhani's recursion and topological recursion

- The WP volumes $V_{g,1}(b)$ can be computed efficiently [Witten; Kontsevich; Mirzakhani].
- Mirzakhani's recursion builds up higher genus surfaces from lower genus ones by sewing geodesic boundaries together and integrating over the intermediate boundary lengths.
- This recursion can be mapped onto matrix model loop equations for resolvents, in the streamlined form of "topological recursion" [Eynard-Orantin]. rank $H \sim e^N$.
- Input for topological recursion is the (double scaled) genus 0 eigenvalue density $\rho_0^{\mathrm{total}}(E)$ ($\Longleftrightarrow$ the spectral curve $y(x)$), and the cylinder diagram (universal).
- To get $V_{g,1}(b)$ use the (disk) JT density ! [Eynard-Orantin]

$$\rho_0^{\mathrm{total}}(E) = \frac{e^{S_0}}{4\pi^2} \sinh(2\pi\sqrt{E})$$

$$y(x) = \frac{\sin(2\pi\sqrt{x})}{4\pi}$$

Putting it together

- $V_{g,1}(b) =$ integral transform of genus g resolvent $R^{(g)}(x)$. $(V_{g,1}(b) = \mathcal{I}[b, R^{(g)}(x)].)$

$$\begin{aligned}
Z_{JT}^{(g)}(\beta) &= \int bdb\ Z_{\mathrm{Trumpet}}(\beta, b) V_{g,1}(b) \\
&= \int bdb \exp(-b^2/4\beta)/2\sqrt{\pi\beta} \times \mathcal{I}[b, R^{(g)}(x)] \\
&= \frac{1}{2\pi i} \int_{\mathcal{C}} dx e^{-\beta x} R^{(g)}(x) \\
&= \int dE \rho^{(g)}(E) e^{-\beta E} \\
&\equiv Z_{MM}^{(g)}(\beta)\ .
\end{aligned}$$

- To all orders in the genus expansion.
- JT gravity is a matrix model, with random matrix eigenvalue statistics. What is the “bulk” interpretation of this? $\star$

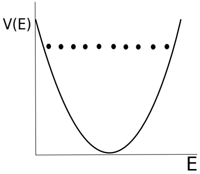
The genus expansion diverges

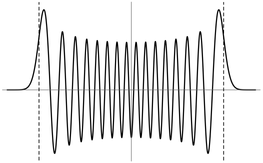
- $Z(\beta) = \sum_g Z^{(g)} e^{(1-2g)S_0}$
  Series of nonperturbative baby universe joinings and splittings.
- $Z^{(g)} \sim (2g)!$. The series diverges.
- Such $\sum_g (2g)!\ g_s^{2g-1}$ behavior is generic in perturbative string theory, indicating the existence of e^{-c/g_s} nonperturbative effects [SS].
- These are due to D-branes – described by arbitrary numbers of disconnected world sheets ending on the brane [Polchinski].
- There should be D-branes in the JT String – here we would have an arbitrary number of disconnected spacetimes.
- Third quantization: $e^{-c/g_s} \to \exp(-c\ e^{S_0}) = \exp(-c\ e^N)$.
  Doubly exponential.

FZZT branes

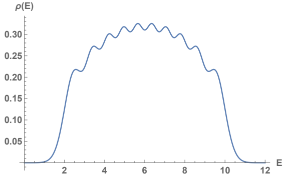
- The matrix integral gives a (non-unique) nonperturbative definition of the JT string. (Contour choice.)
- Nonperturbative effects in matrix models are due to the dynamics of discrete eigenvalues, not smooth densities. Dual description by branes.
- In the early 2000's a detailed understanding was developed of the branes present in the "minimal strings" described by $c \leq 1$ minimal matter coupled to Liouville gravity, and their matrix duals [Fateev, Klebanov, Kutasov, Maldacena, Martinec, McGreevy, Moore, Seiberg, Shih, Teschner, Verlinde, Zamolodchikov, Zamolodchikov, ...].
- Related, parallel work on branes in topological strings and their matrix duals by [Aganagic, Dijkgraaf, Klemm, Marino, Vafa, ...].
- The insertion of an FZZT probe brane is described by $\langle \psi(E) \rangle = e^{-LV(E)/2} \langle det(E - H) \rangle$.
  E describes boundary condition at edge of brane. ($L = rank\ H$.)
- Take this technology over directly.

Determinants





$e^{-LV(E)/2}\langle det(E-H)\rangle$



$\langle \rho(E) \rangle$

- Determinants are sensitive probes of discreteness.
  [standard probe in quantum chaos]
- Oscillations are a leading effect in $\langle det(E-H)\rangle$. They are subleading in $\langle \rho(E)\rangle$.

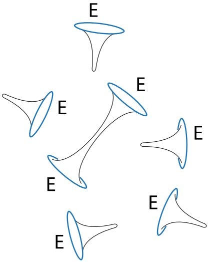
From determinants to D-branes

Use topological/minimal string technology. At leading order, the FZZT brane insertion is determined by exponentiating disks, just as in D-branes [Polchinski].

$$det(E-H) = \exp\left(Tr\log(E-H)\right)$$
$$\langle det(E-H)\rangle = \langle 1 + Tr\log(E-H) + (Tr\log(E-H))^2/2 + \ldots\rangle.$$

At leading order $\langle (Tr\log(E-H))^k\rangle = \langle Tr\log(E-H)\rangle^k$. So

$$\langle\psi(E)\rangle = e^{-LV(E)/2}\langle det(E-H)\rangle \sim e^{-LV(E)/2}\exp(\langle Tr\log(E-H)\rangle)$$
$$= \exp(Disk(E))$$

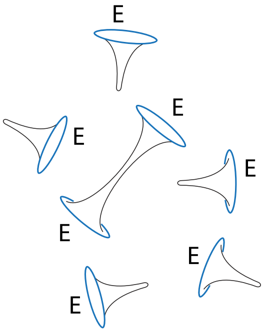


- Many traces.
- Many disconnected spacetimes.
- Like many disconnected world sheets.

From determinants to D-branes, contd.

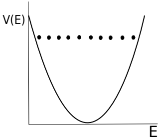
FZZT brane insertion:

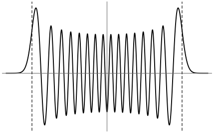
$$\langle \psi(E) \rangle = e^{-LV(E)/2} \langle det(E-H) \rangle \sim e^{-LV(E)/2} \exp(\langle Tr \log(E-H) \rangle)$$
$$= \exp(\mathrm{D}isk(E))$$



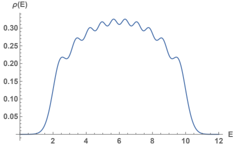
- $Tr \log(E-H) = \int^E dx Tr \frac{1}{x-H}$
- $\mathrm{D}isk(E) = \int^E dx (R^{(0)}(x) - LV'(x)/2)$
  $= \pm i\pi \int^E dx \rho_0^{\mathrm{total}}(x)$
- $\exp(\mathrm{D}isk(E)) \sim \exp(i\pi \int^E dx \rho_0^{\mathrm{total}}(x))$
- $\sim \exp(ie^{S_0}) \sim exp(i/g_s)$.
- Doubly exponential.
- Rapidly oscillating.

Resolvents and densities





$e^{-V(E)/2}\langle det(E-H)\rangle$



$\langle \rho(E)\rangle$

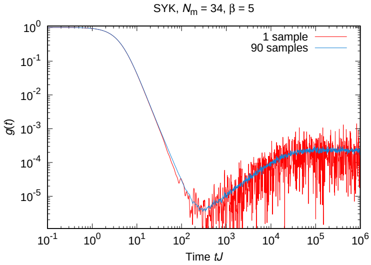
- We can go from determinants to resolvents and densities using the basic identity: [standard technique in quantum chaos]

$$Tr\frac{1}{E-H} = \partial_E \frac{\det(E-H)}{\det(E'-H)}\bigg|_{E'\to E}.$$

- A brane - antibrane "dipole."
- Determine nonperturbative contributions to $\langle \rho(E)\rangle$ from D-branes.
- A similar, more elaborate, analysis involving ratios of four determinants gives a D-brane calculation of the pair correlation function $\langle \rho(E)\rho(E')\rangle$. We recover the Sine kernel result. (See Phil Saad's poster).

Non-averaged systems

- This analysis raises lots of questions. $\star$
- One involves the transition from averaged to non-averaged systems, like SYM. (See also Douglas Stanford's talk.)
  For a non-averaged system the spectral form factor is very erratic [Prange].
- What is the bulk explanation for this erratic behavior? $\star$



$$\langle Z(\beta + it) Z(\beta - it) \rangle$$

Semiclassical quantum chaos

- An analogy: semiclassical chaos in ordinary quantum mechanical systems, like quantized billiards.
- Use the path integral (Gutzwiller trace formula), summing over periodic orbits

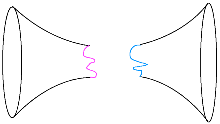
$$Tre^{-iHt/\hbar} \sim \sum_a e^{\frac{i}{\hbar}S_a}$$

- The spectral form factor becomes:

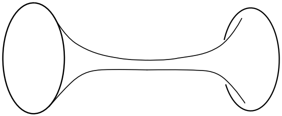
$$Tre^{iHt/\hbar}\ Tr^{-iHt/\hbar} \sim \sum_{ab} e^{\frac{i}{\hbar}S_a} e^{-\frac{i}{\hbar}S_b}$$

Semiclassical quantum chaos, contd.

$$\sum_{ab} e^{\frac{i}{\hbar} S_a} e^{-\frac{i}{\hbar} S_b}$$

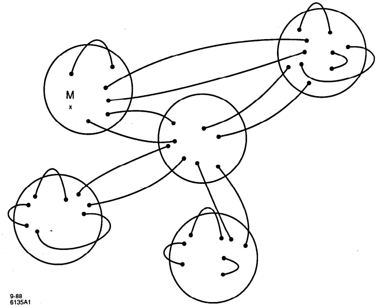


- Long times $t \to$ long orbits $\to$ large phases $\to$ large fluctuations.
- But on averaging (over time, say) in the ramp region the only terms that survive are the ones where $a = b$, up to a time translation. This is Berry's "diagonal approximation" that gives the ramp. The spacetime wormhole connection is produced by averaging – (not a spatial wormhole produced by entanglement) [Maldacena-Maoz].

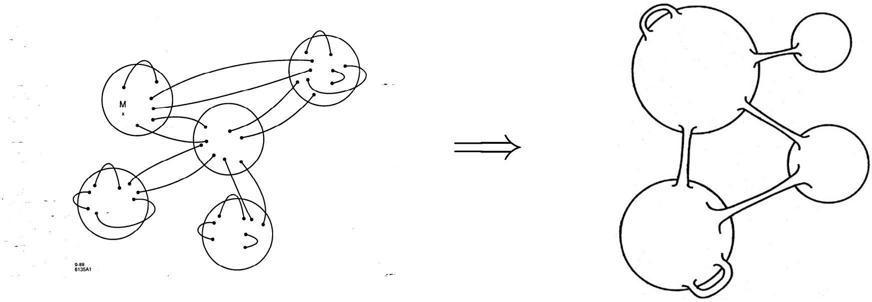


Fluctuating couplings

These ideas are reminiscent of Coleman's ideas about the relation between Euclidean wormholes and fluctuating couplings [see also, Banks, Fischler, Giddings-Strominger, Hawking, Klebanov, Rubakov, Susskind ...].
Consider many baby universes, each with Lagrangians with the same fluctuating couplings (around fixed reference values). Expand and perform Wick contractions: [figure from Klebanov-Susskind, 1988]



Wormholes from fluctuating couplings



"Fatten" tubes by including a number of operators.
The full sum over disconnected spacetimes produces doubly exponential quantities, reminiscent of those discussed above.

A parting thought

- Perhaps we have a choice about the bulk description of spectral statistics:
- an unaveraged description with simple topology but with exceedingly detailed and complicated information about microstates, like the individual orbits in the microscopic phase space and their intricate, rapidly fluctuating phases (fuzzballs ?!);
- **or** an averaged description made up of more familiar geometrical objects. But the price for this simplicity seems to be third quantization, wormholes, and D-branes.

Thank you

Thank You

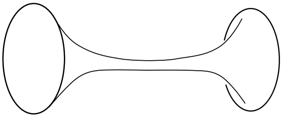
Backup Slides

Eigenvectors/ETH

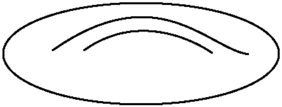
- What about the random matrix behavior of eigenvectors?
- Eigenstate Thermalization Hypothesis (ETH) [Deutsch, Srednicki]
- Probe with correlation functions (two sided, $\beta \to 2\beta$).
- $$\langle O(t)O(0)\rangle = \sum_{mn} |\langle m|O|n\rangle|^2 e^{i(E_m - E_n)t} e^{-\beta(E_m+E_n)}$$
- ETH implies $|\langle m|O|n\rangle|^2 \sim e^{-S}$.
- At late times we expect
$$\langle O(t)O(0)\rangle \to e^{-S}|Z(\beta+it)|^2$$
- e^{-S} times the spectral form factor.

Eigenvectors/ETH, contd.

- The ramp in the spectral form factor, is a two Hilbert space, two boundary, quantity. It is described in JT by the cylinder, $\chi = 0$, of order e^0.



- The correlator is a one Hilbert space, one boundary, quantity. We expect its ramp to be described by [Saad-SS-Stanford, Blommaert-Mertens-Verschelde]



- The "lid" has $\chi = -1$ so it is of order e^{-S}, as expected from ETH.

Asymptotic formulas for WP volumes

- A mathematical application.
- What is the asymptotic behavior of the WP volume $V_{g,1}(b)$ for large genus g and arbitrary values of b/g?
- Based on extensive numerical evidence for $g = 1 \dots 20$, Zograf conjectured a formula for this asymptotic behavior for the region b fixed, $g \to \infty$.

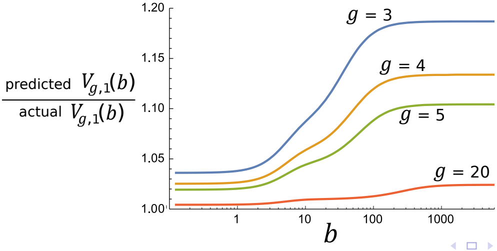
$$V_{g,1}(b) \approx \frac{4(4\pi^2)^{2g-\frac{3}{2}}}{(2\pi)^{3/2}} \Gamma(2g - \tfrac{3}{2}) \frac{\sinh(\frac{b}{2})}{b} \ .$$

- (Mirzakhani, Zograf and Petri later proved most of this directly from the recursion relation.)

Asymptotic formulas for WP volumes

- For many integrals the large order behavior of a perturbation series is determined by nontrivial saddle points of the integral. (Borel resummation, resurgence...)
- The matrix integral has a nontrivial saddle point where one eigenvalue moves away from the large L saddle point distribution. “One eigenvalue instanton” (ZZ brane).

  [Neuberger, David, Ginsparg-Zinn-Justin, Shenker]
- This gives an analytic determination of the asymptotic behavior for all b/g, confirming Zograf’s conjecture in the region for which it applies.



Comments

- Random matrix behavior is universal. Is the D-brane mechanism?
- JT $\to$ SYK. It seems that the extra effects present in SYK do not destabilize the mechanism, except for the expected change in $\rho_0(E)$. (Not true for ZZ brane effects).
- Averaged SYM. The key ingredients, the Disk and the Cylinder, can both be computed in bulk gravity (or string theory). These give the expected answer. The real question is whether there are large corrections to these terms.
  In general it seems that large corrections would be expected primarily in integrable systems. Chaotic ones seem to be the simplest. A sharp argument does not yet exist, though.
- How do we calculate all this in a "two boundary" description?