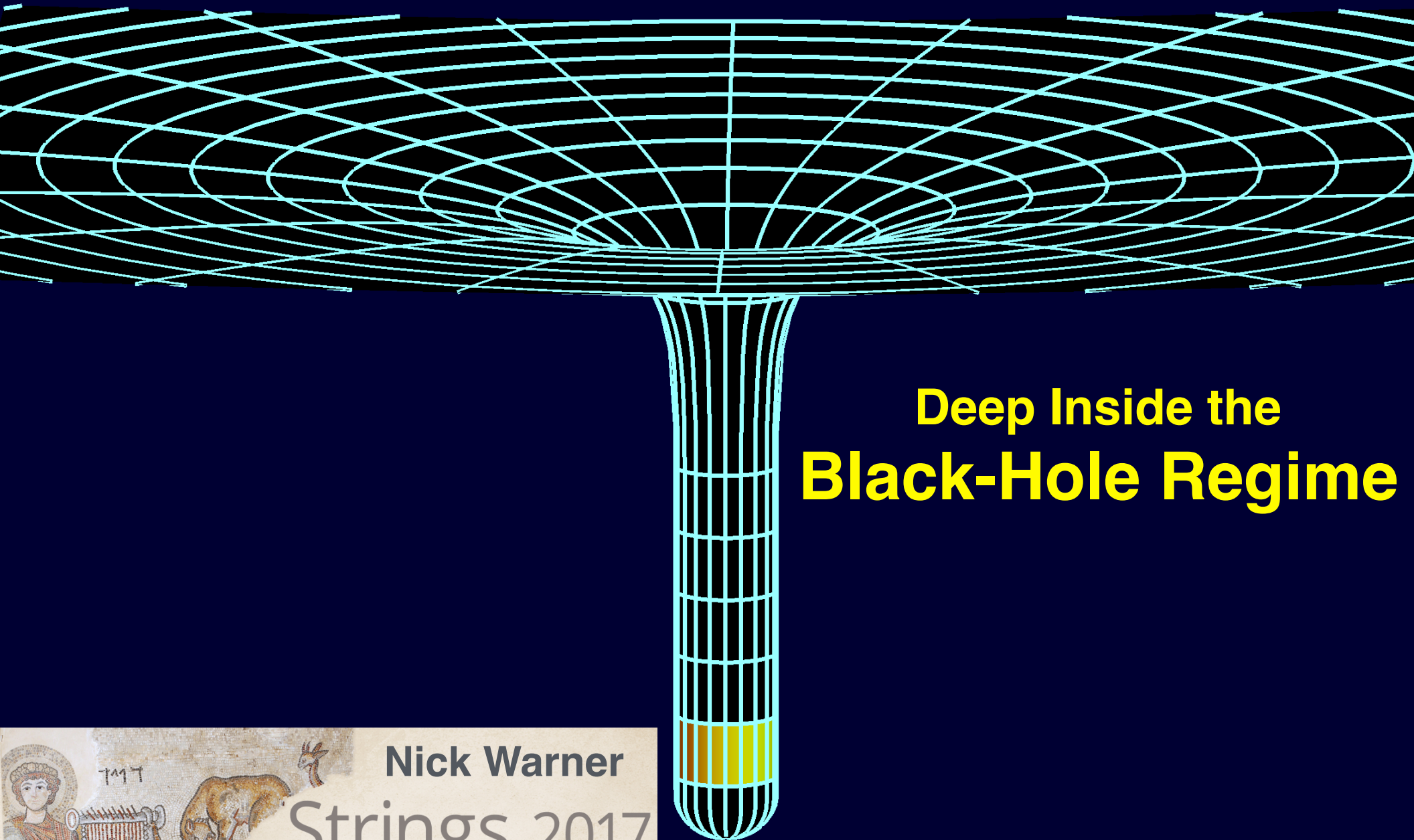


# Microstate Geometries



**Deep Inside the  
Black-Hole Regime**



Research supported supported in part by  
DOE grant DE- SC0011687

# Outline

- Microstate Geometries D1-D5-P Holography
- Some families of D1-D5-P states
- Building the holographic duals: Microstate geometries with  $AdS_2/BTZ$  throats
- The MSW string
- Holographic duals of some MSW states

Based on Collaborations with:

**I. Bena, S. Giusto, E. Martinec, R. Russo, M. Shigemori, D. Turton.**

**arXiv:1607.03908, arXiv:1703.10171, arXiv:1708.XXXXX**

# Microstate Geometry Program

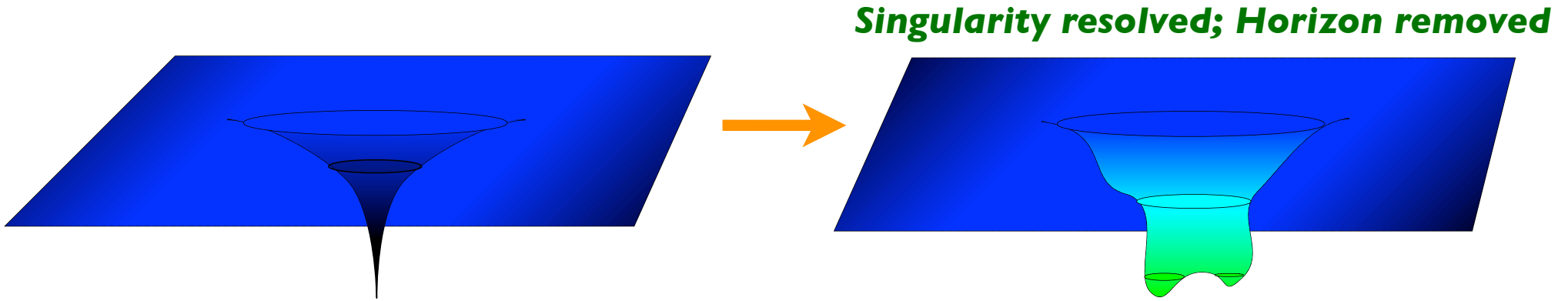
*Microstate Geometry*  $\equiv$  **Smooth, horizonless solutions** to the **bosonic** sector of **supergravity** with the same asymptotic structure as a given black hole/ring

**Singularity resolved; Horizon removed**



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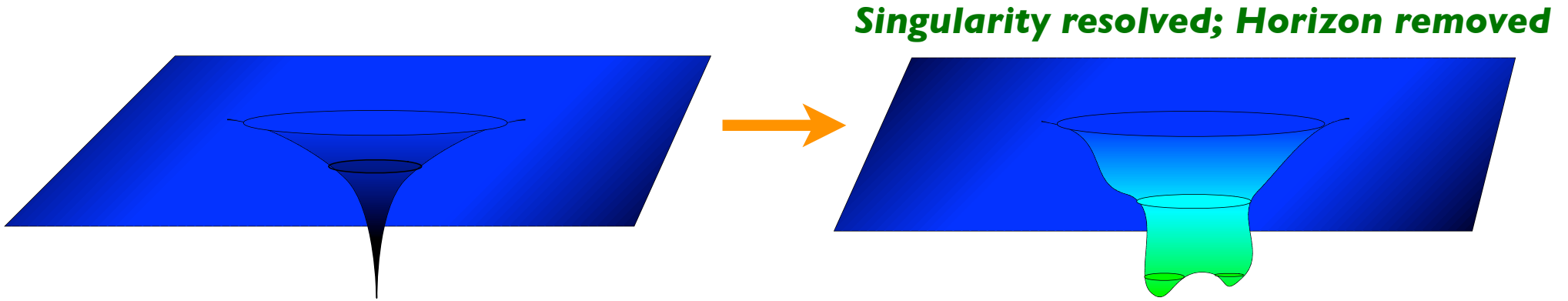


**Supergravity** because we seek stringy resolutions at the horizon scale

- ▶ **Very long-range effects**  $\Rightarrow$  Massless limit of strings ...

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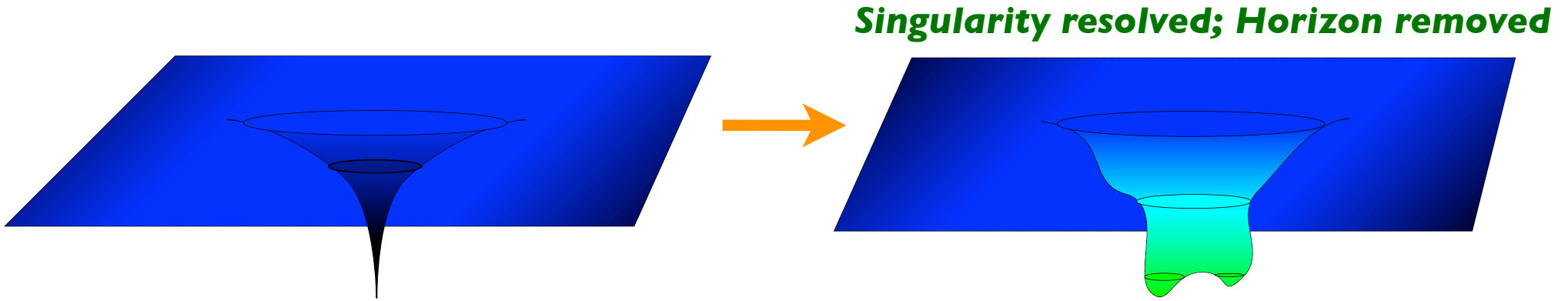
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▶ **Very long-range effects**  $\Rightarrow$  Massless limit of strings ...

**What is the form of generic, BPS, time-independent horizonless, smooth solutions in supergravity?**

**What CFT states do they describe?**

## **Primary Motivation for Microstate Geometries**

Resolving the black-hole information problem seems to require microstate structure to be encoded and supported at the horizon scale

### **Microstate Geometries**

- ***The only (known) mechanism that can support structure at the horizon scale***
- ***Supergravity captures the universal, macroscopic features of microstate structure***
- Semi-classical analysis: ***To what extent can supergravity encode microstate structure?***

# Black-Hole Microstates and CFT's



## Black-Hole Microstates and CFT's

- **D1-D5 CFT:** A (4,4) supersymmetric CFT with  $c = 6 N_1 N_5$

$\frac{1}{4}$  BPS states = (R,R)-ground states

$\frac{1}{8}$  BPS states = ( $\underbrace{\text{any left-moving state}}_{N_P}$ , R ground state)

*Strominger-Vafa state counting for BPS black hole in five dimensions:*

$$S = 2\pi \sqrt{N_1 N_5 N_P}$$

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- **MSW String:** A (0,4) supersymmetric CFT *(Maldacena-Strominger-Witten)*

M5 brane wrapping a divisor in a  $\text{CY}_3$ . Dual class,  $P \in H^2(\text{CY}_3, \mathbb{Z})$

MSW string CFT lives on remaining (1+1) dimensions of M5 brane

Central charge  $c = 6 D$ ,  $D = \frac{1}{6} \int_{\text{CY}_3} P^3$

*State counting for BPS black hole in four dimensions:*  $S = 2\pi \sqrt{D N_P}$

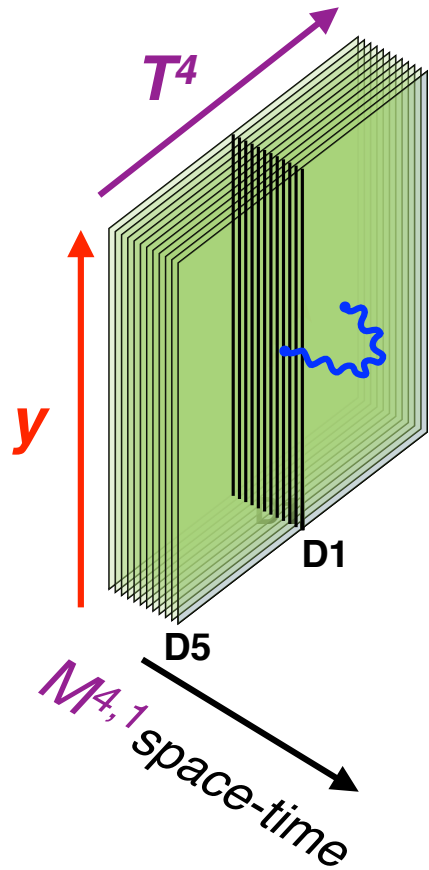
## One Focus of the Microstate Geometry Program

Describe the strongly coupled gravity duals of these CFT states.

To what extent can these CFT states be captured in supergravity?

⇒ Universal gravity dual of both D1-D5 and MSW.

# The D1-D5 CFT



Open D1-D5 superstrings moving in  $T^4$   
with  $N \equiv N_1 N_5$  Chan-Paton labels:  $(T^4)^N/S_N$

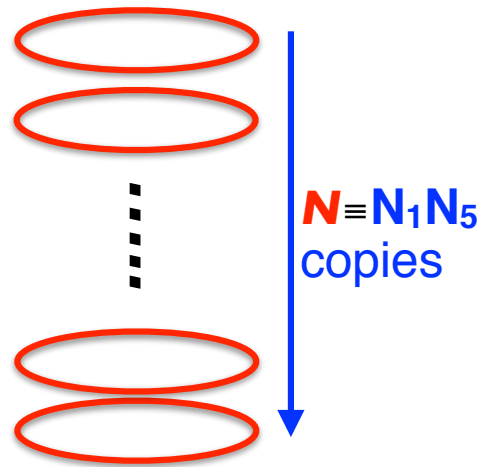
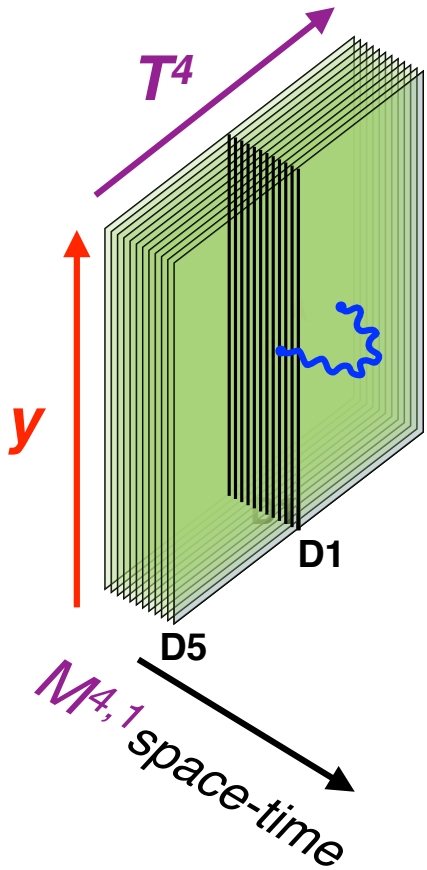
$\Rightarrow$  CFT on common D1-D5 direction,  $(t,y) \Leftrightarrow (u,v)$   
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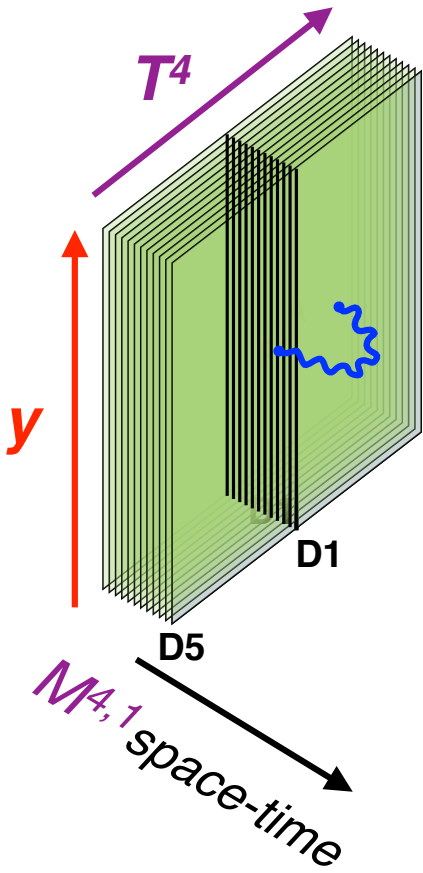
Maximally spinning ( $1/4$  BPS) RR-ground state:



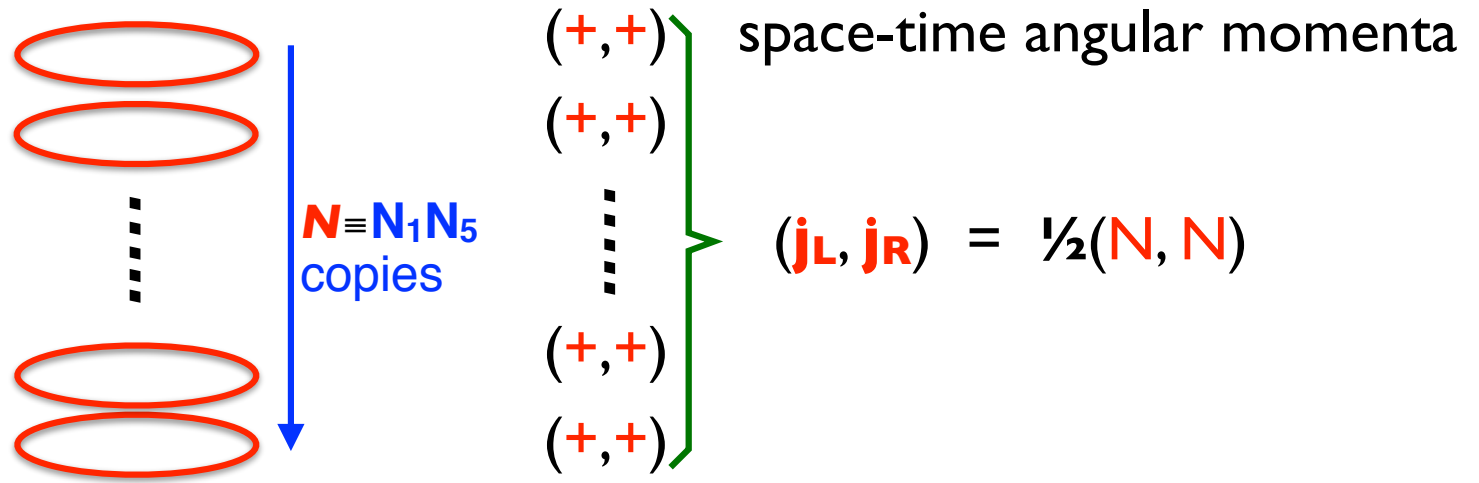
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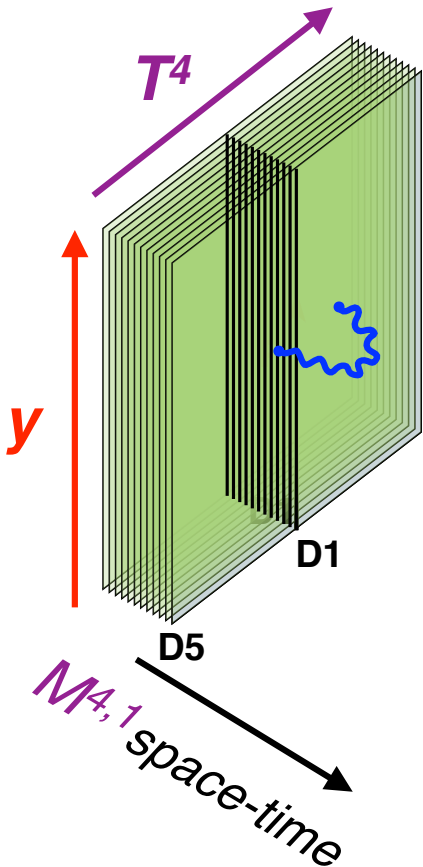
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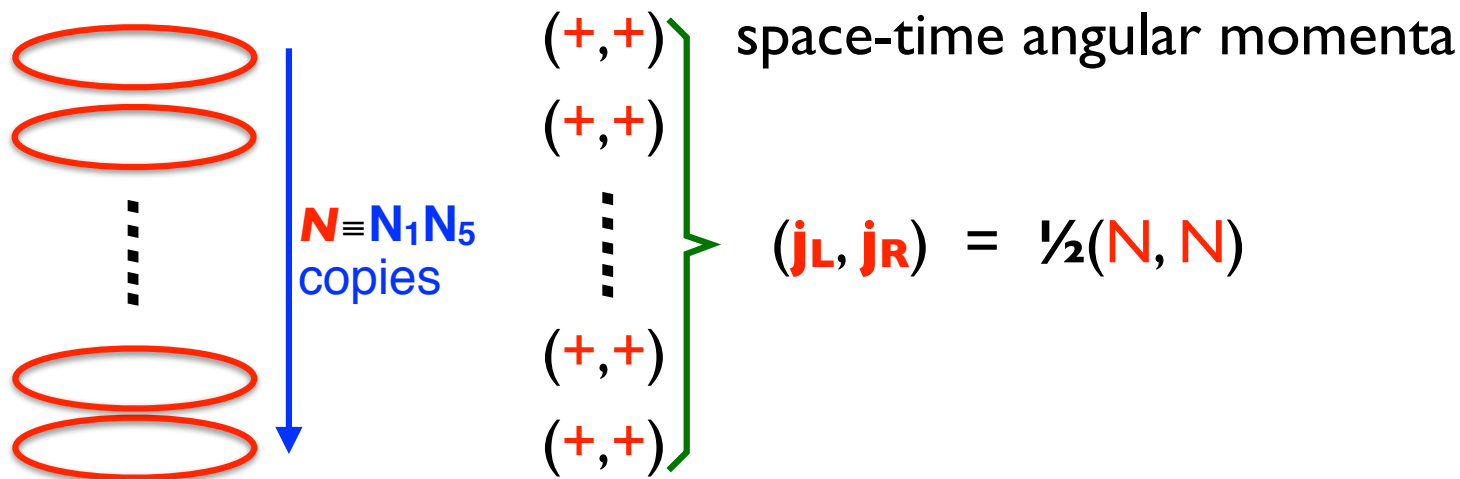
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Maximally spinning ( $1/4$  BPS) RR-ground state:



Holographic dual: Maximally spinning supertube in  $R^{4,1}$

Supertube profile spins out into  $M^{4,1}$  space-time

$$(g_1(v), g_2(v), g_3(v), g_4(v)) \in \mathbb{R}^4$$

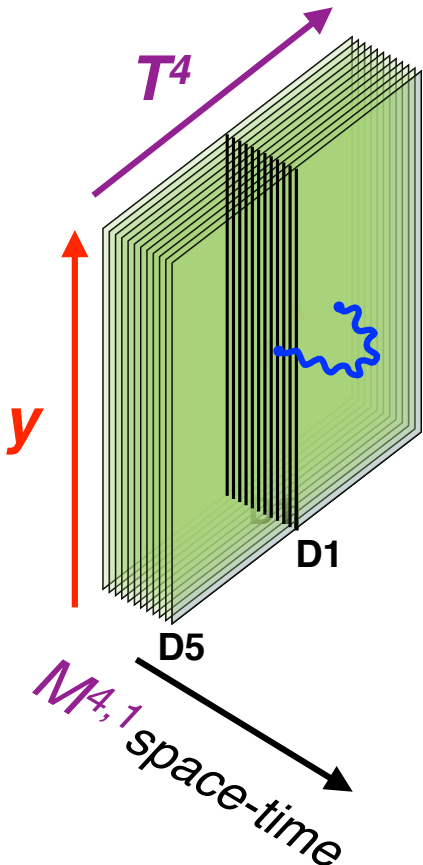
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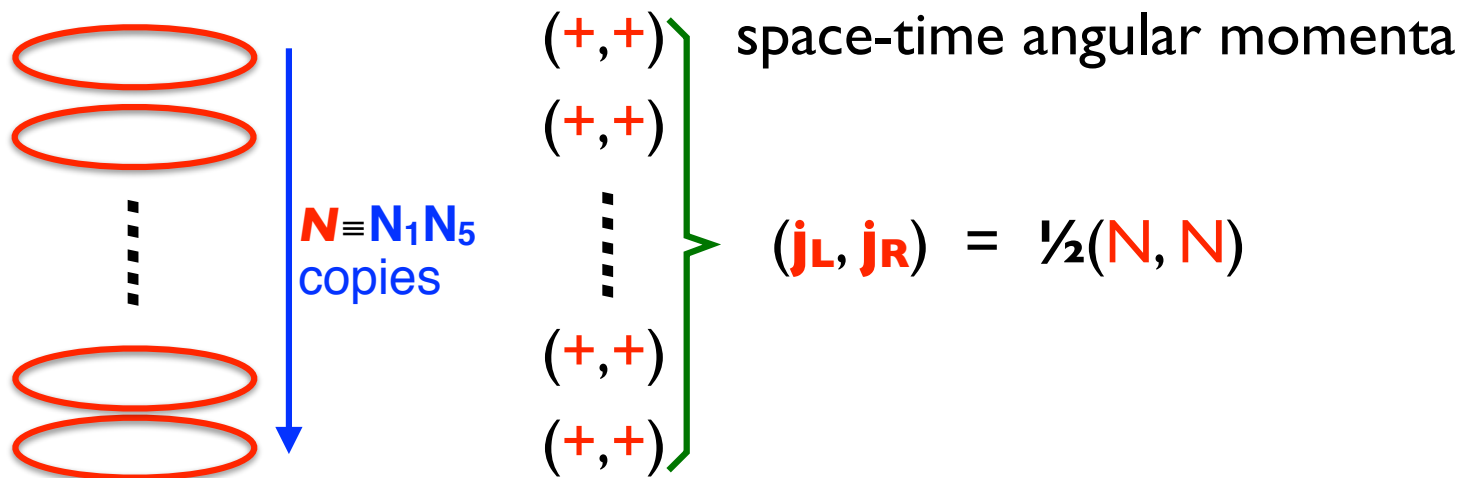
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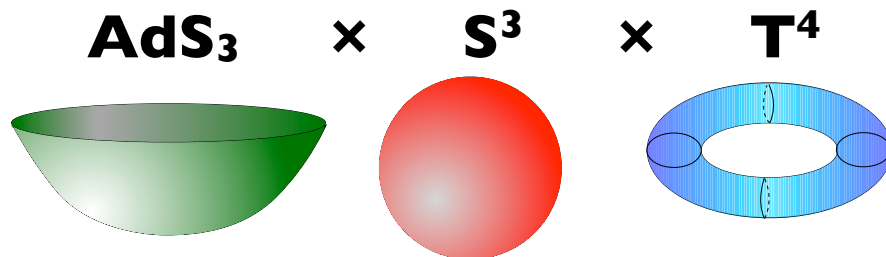
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$$Q_1 Q_5 = R^2 a^2$$

$$\left. \begin{aligned} (g_1(v), g_2(v), g_3(v), g_4(v)) &\in \mathbb{R}^4 \\ g_1(v) + ig_2(v) &= a e^{2\pi i v/R} \\ g_3(v) = g_4(v) &= 0 \end{aligned} \right\}$$

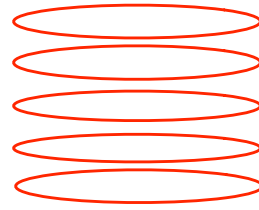
$\xrightarrow{\text{back-react}}$





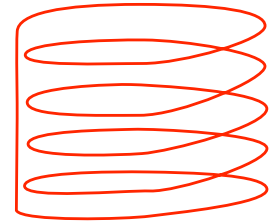
# More general $\frac{1}{4}$ BPS profiles

Orbifold CFT:  $k$  twisted sector



$k$  loops

$$|+\frac{1}{2}, +\frac{1}{2}\rangle^k$$



Length  $k$  loop

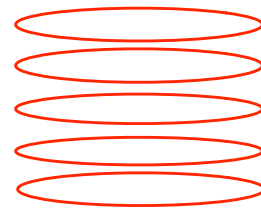
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Orbifold CFT:  $k$  twisted sector

*Act with fermion zero modes*

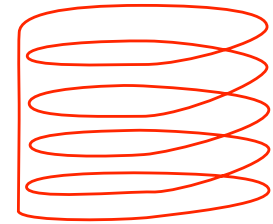


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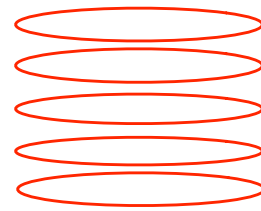
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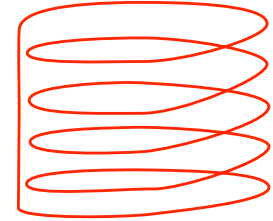


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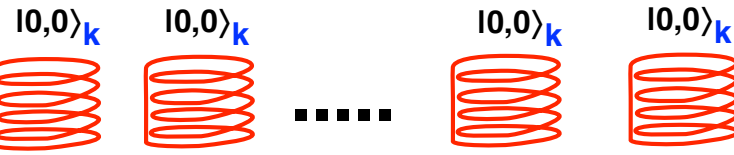


## More general class of D1-D5 ground state

$|+\frac{1}{2}, +\frac{1}{2}\rangle$   $|+\frac{1}{2}, +\frac{1}{2}\rangle$       $\dots$       $|+\frac{1}{2}, +\frac{1}{2}\rangle$   $|+\frac{1}{2}, +\frac{1}{2}\rangle$



$\sim a^2$  copies

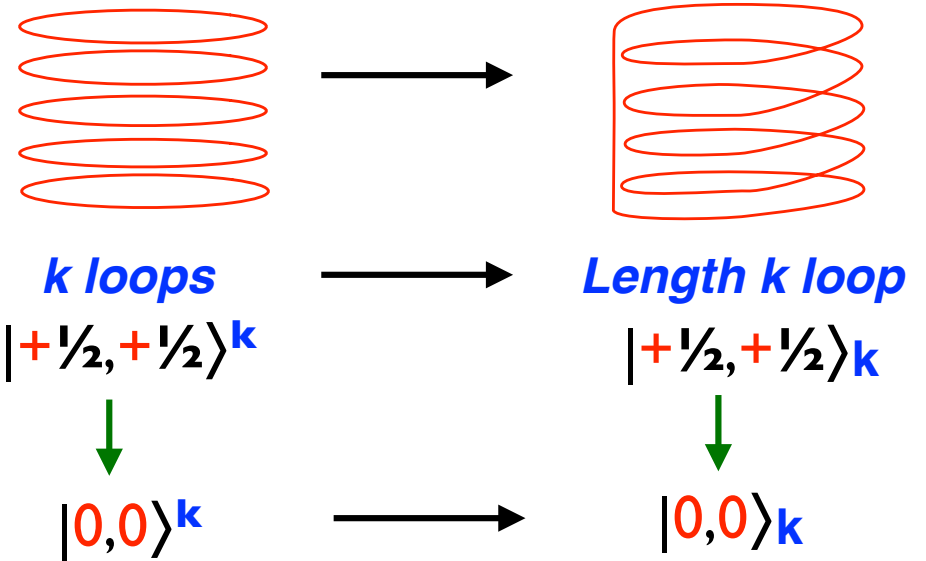


$\sim b^2$  copies

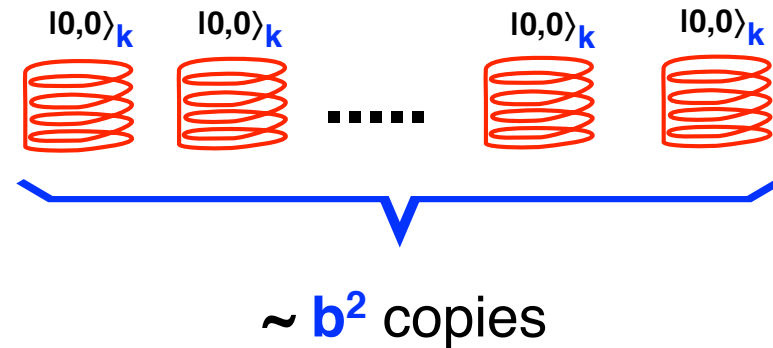
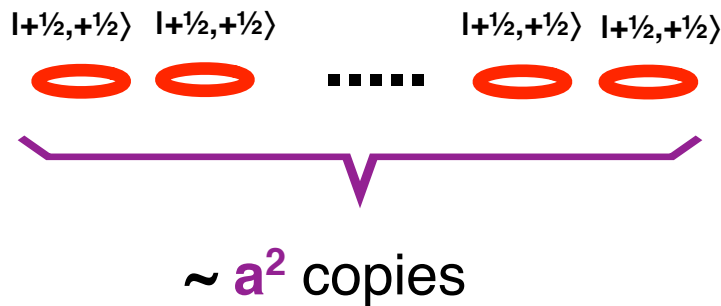
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## More general class of D1-D5 ground state



## Holographic dual supertube profile

$$g_1(v) + ig_2(v) = \mathbf{a} e^{2\pi i v/R}$$

$$“g_5(v)” = \mathbf{b} \sin(2\pi \mathbf{k} v/R)$$

Partitioning of charges:  $Q_1 Q_5 = R^2 (\mathbf{a}^2 + \mathbf{b}^2)$

# Families $\frac{1}{8}$ BPS states in the D1-D5-P system

Generic  $\frac{1}{8}$  BPS state: Add general left-moving excitations

Momentum charge,  $Q_P = L_{0,left}$        $S = 2\pi \sqrt{Q_1 Q_5 Q_P}$  (Strominger-Vafa)

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Very special families of momentum excitations: “Supergraviton gas”

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## Quantum numbers

$$\text{Define } \mathcal{N} = \frac{N_1 N_5}{a^2 + b^2}$$

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**D1-D5  $|+\frac{1}{2}, +\frac{1}{2}\rangle$  residue**

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*D1-D5  $|+\frac{1}{2}, +\frac{1}{2}\rangle$  residue*

$$Q_1 Q_5 = R^2 (\mathbf{a}^2 + \mathbf{b}^2)$$

## Special forms:

Adding pure momentum:  $m = 0$ .

Vanishing angular momentum:  $m = 0, \mathbf{a} \rightarrow \mathbf{0}$ .

## The “Supergraviton gas”

We know the supergravity duals of arbitrary superpositions of states of the form:

$$\left( \left| +\frac{1}{2}, +\frac{1}{2} \right\rangle_1 \right)^{N_0} \otimes \left[ \bigotimes_{k_i, m_i, n_i} \left( \frac{1}{m_i! n_i!} (J_{-1}^+)^{m_i} (L_{-1} - J_{-1}^3)^{n_i} |00\rangle_{k_i} \right)^{N_{k_i, m_i, n_i}} \right]$$

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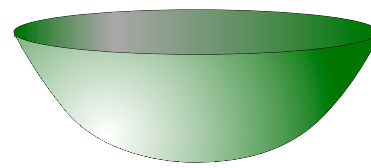
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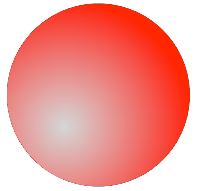
## Holographic duals

Add momentum and angular momentum excitations to D1-D5 profiles:

$$g_1(v) + ig_2(v) = \mathbf{a} e^{2\pi i v/R} \quad “g_5(v)” = \mathbf{b} \sin(2\pi \mathbf{k} v/R)$$



AdS<sub>3</sub> (u, **v**, r)



S<sup>3</sup> (θ, **ψ**, **φ**)

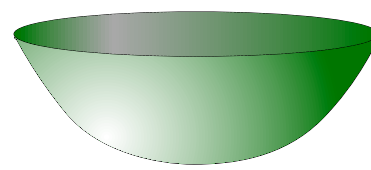
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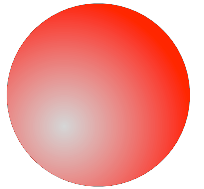
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Add momentum and angular momentum excitations to D1-D5 profiles:



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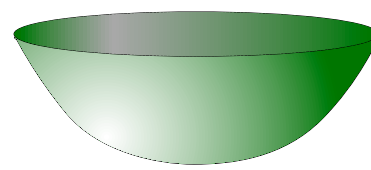
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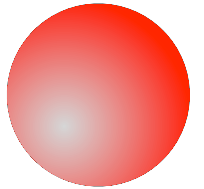
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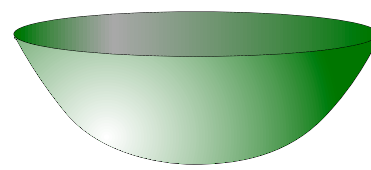
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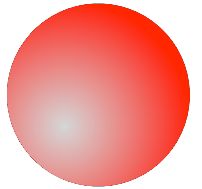
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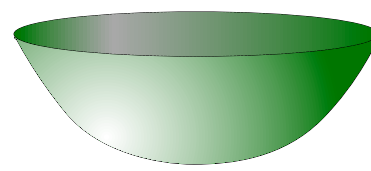
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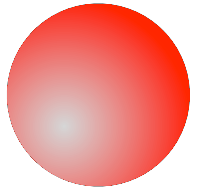
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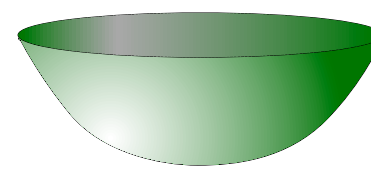
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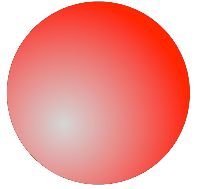
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# Building the Fluctuating BPS Microstate Geometries

IIB Supergravity on  $T^4$ : Supergravity + two (anti-self-dual) tensor multiplets in six-dimensions

Six-dimensional metric ansatz:

*(Gutowski, Martelli and Reall)*

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}} (dv + \beta)(du + \omega - \frac{1}{2} Z_3 (dv + \beta)) + \sqrt{\mathcal{P}} V^{-1} (d\psi + A)^2 + \sqrt{\mathcal{P}} V d\vec{y} \cdot d\vec{y}$$

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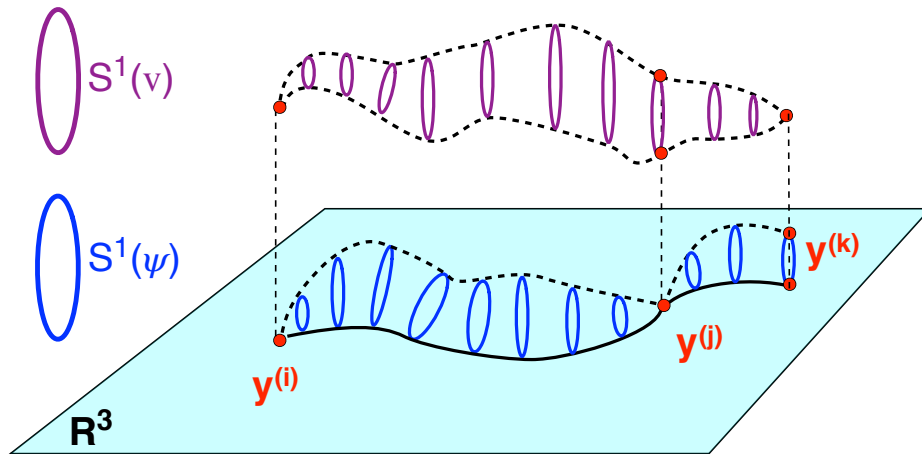
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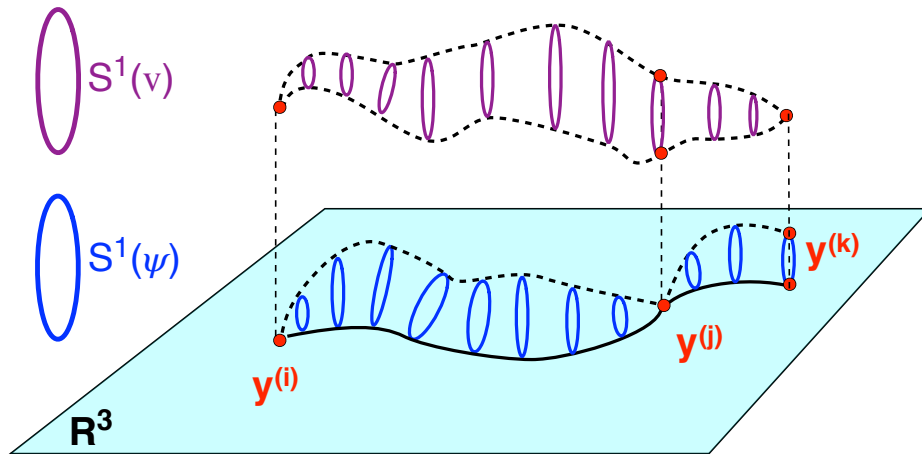
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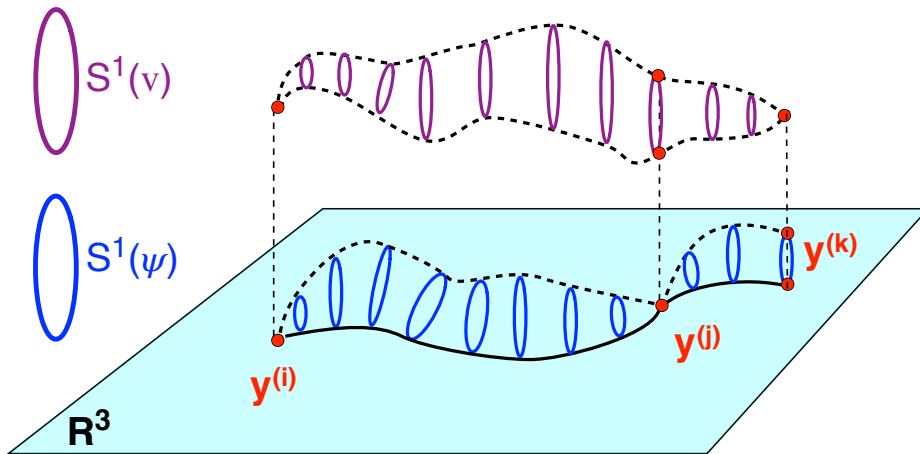
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Maxwell Fields

$$G^{(a)} = d \left[ -\frac{1}{2} \frac{\eta^{ab} \mathbf{Z}_b}{\mathcal{P}} (du + \omega) \wedge (dv + \beta) \right] + \frac{1}{2} \eta^{ab} *_4 D \mathbf{Z}_b + \frac{1}{2} (dv + \beta) \wedge \Theta^{(a)}$$

$$\mathcal{P} \equiv \frac{1}{2} \eta^{ab} \mathbf{Z}_a \mathbf{Z}_b \equiv \mathbf{Z}_1 \mathbf{Z}_2 - \frac{1}{2} \mathbf{Z}_4^2$$

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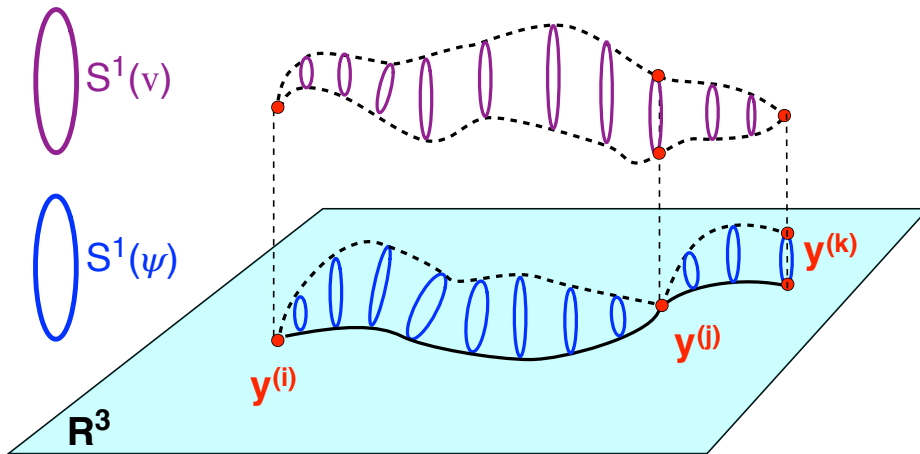
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# The BPS Equations

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Layer 1: Conditions on Maxwell Fields    A homogeneous **linear** system

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$$\text{where } D\Phi \equiv d_{(4)}\Phi - \beta \wedge \partial_v \Phi$$

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*Interesting families of particular solutions known. General solution not known.*

## Linear system of gravitational BPS equations:

**Critical to constructing the holographic duals of a generic superpositions of the states on multiple, independent strands:**

$$\left( \left| +\frac{1}{2}, +\frac{1}{2} \right\rangle_1 \right)^{N_0} \otimes \left[ \bigotimes_{k_i, m_i, n_i} \left( \frac{1}{m_i! n_i!} (J_{-1}^+)^{m_i} (L_{-1} - J_{-1}^3)^{n_i} |00\rangle_{k_i} \right)^{N_{k_i, m_i, n_i}} \right]$$

# A Family of Microstate Geometries deep in the Black-Hole Regime

Add pure momentum states

$$N_0 + N_{1,0,n} = \mathbf{N}_1 \mathbf{N}_5$$

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↓

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⏟

*D1-D5 residue*  
*All angular momentum*

↓

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⏟

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Geometry:

← Flat Space

Flat Space →

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$$\rho_*^2 \sim \frac{Q_P}{Q_1 Q_5}$$

**Scale of  $S^1$  stabilizes at  $\rho_* \ell_{\text{AdS}} R$**

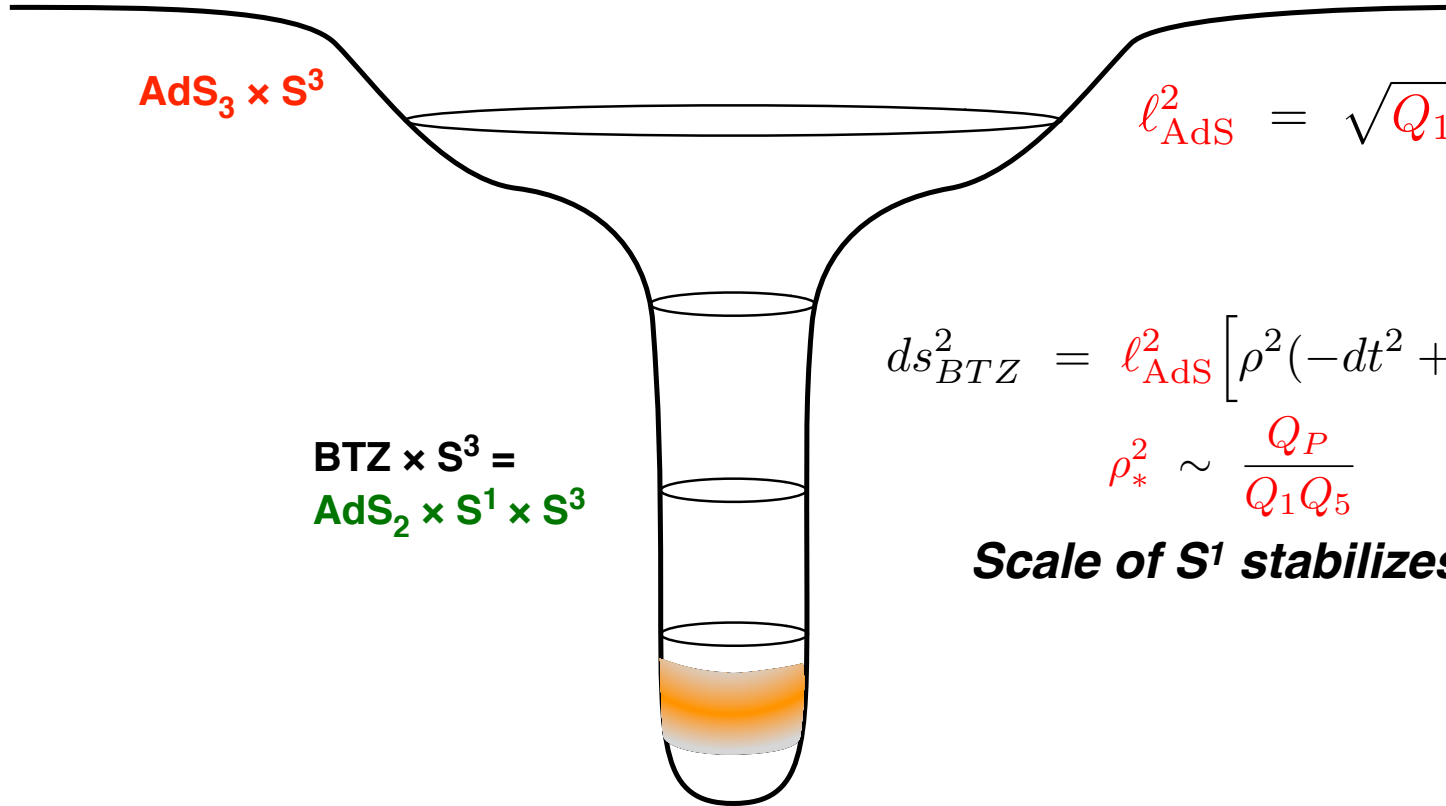
$$\left( \left| +\frac{1}{2}, +\frac{1}{2} \right\rangle_1 \right)^{N_0} \otimes \left( \frac{1}{n!} (L_{-1} - J_{-1}^3)^n |00\rangle_1 \right)^{N_{1,0,n}}$$

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All angular momentum

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*P excitations*  
Angular momentum  $\equiv 0$



# A Family of Microstate Geometries deep in the Black-Hole Regime

Add pure momentum states

$$N_0 + N_{1,0,n} = \mathbf{N}_1 \mathbf{N}_5$$

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As  $a \rightarrow 0$ :  $j_L = j_R \rightarrow 0$   
Depth of  $\text{AdS}_2$  throat  $\rightarrow \infty$

## **Several significant results**

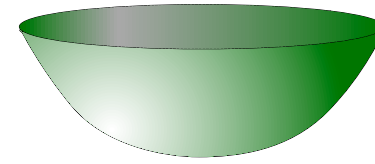
## Several significant results

- **First deep, scaling microstate geometry in Black-Hole regime with  $j_L = j_R \rightarrow 0$**
- **Deep, scaling microstate geometry that goes to BTZ**
- **Deep, scaling  $\Rightarrow$  Arbitrarily large red-shifts**  
**Microstate Geometry  $\Rightarrow$  Smooth cap-off**
- **Momentum excitations localize at the bottom of the BTZ throat**
- **Holographic dictionary in  $AdS_3$  for deep  $AdS_2/BTZ$  throat**
- **Geometry dual to states counted by Strominger-Vafa**

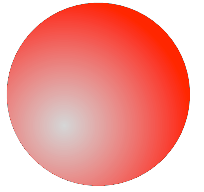
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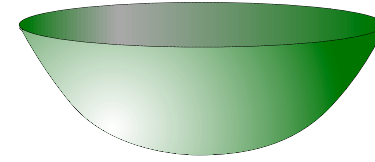
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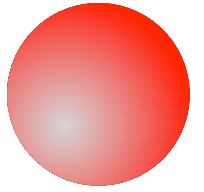
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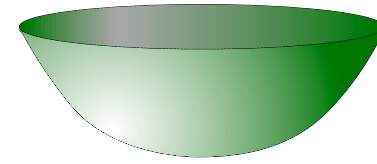
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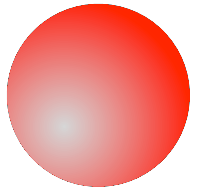
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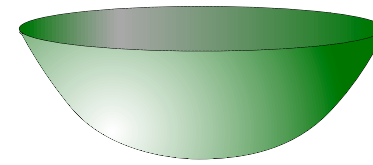
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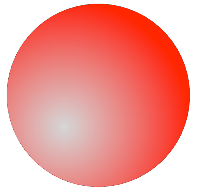
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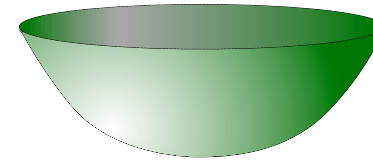
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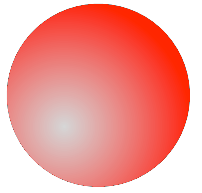
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Before doing this: first enrich the family of solutions

It is relatively easy to generalize the entire IIB construction to include a KKM dipole charge,  $\kappa$ , to the D1-D5 system

# Some T-dualities

Starting configuration

| IIB | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|---|---|---|---|---|---|---|---|---|---|
| D1  | ↑ | * | * | * | * | ↑ | ↔ | ↔ | ↔ | ↔ |
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**T-dualize 3 times to IIA:**



| IIA | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|---|---|---|---|---|---|---|---|---|---|
| D4  | ↑ | * | * | * | ↑ | ↑ | ↔ | ↔ | ↑ | ↑ |
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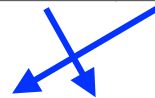
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**Uplift to M theory**



| M  | 0 | 1 | 2 | 3 | 5 | 4 | 10 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|----|---|---|---|---|
| M5 | ↑ | * | * | * | ↑ | ↑ | ↑  | ↔ | ↔ | ↑ | ↑ |
| M5 | ↑ | * | * | * | ↑ | ↑ | ↑  | ↑ | ↑ | ↔ | ↔ |
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## M-theory background

D1-D5-KKM solution  $\rightarrow$  M5-M5-M5 charges:  $(\mathbf{Q}_1, \mathbf{Q}_5, \mathbf{\kappa})$   
+ dipolar/dissolved M2-M2-M2 charges

Dualities + compactification on  $\psi$  lattice:

D1-D5-KKM (4,4) supersymmetry  $\rightarrow$  M5-M5-M5 (0,4) supersymmetry

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Add momentum along common circle (5) ... untouched in duality

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|-----|---|---|---|---|---|---|---|---|---|---|
| D1  | ↑ | * | * | * | * | ↑ | ↔ | ↔ | ↔ | ↔ |
| D5  | ↑ | * | * | * | * | ↑ | ↑ | ↑ | ↑ | ↑ |
| KKM | ↑ | * | * | * | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| P   | ↑ |   |   |   |   | ↑ |   |   |   |   |

| M  | 0 | 1 | 2 | 3 | 5 | 4 | 10 | 6 | 7 | 8 | 9 |
|----|---|---|---|---|---|---|----|---|---|---|---|
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→ Momentum excitations of MSW string wrapping (5) direction ..

## MSW string vs M5 on $T^6$ (or $K3 \times T^2$ )

- ▶ **MSW:** Single M brane wrapped on very ample divisor of  $CY_3$
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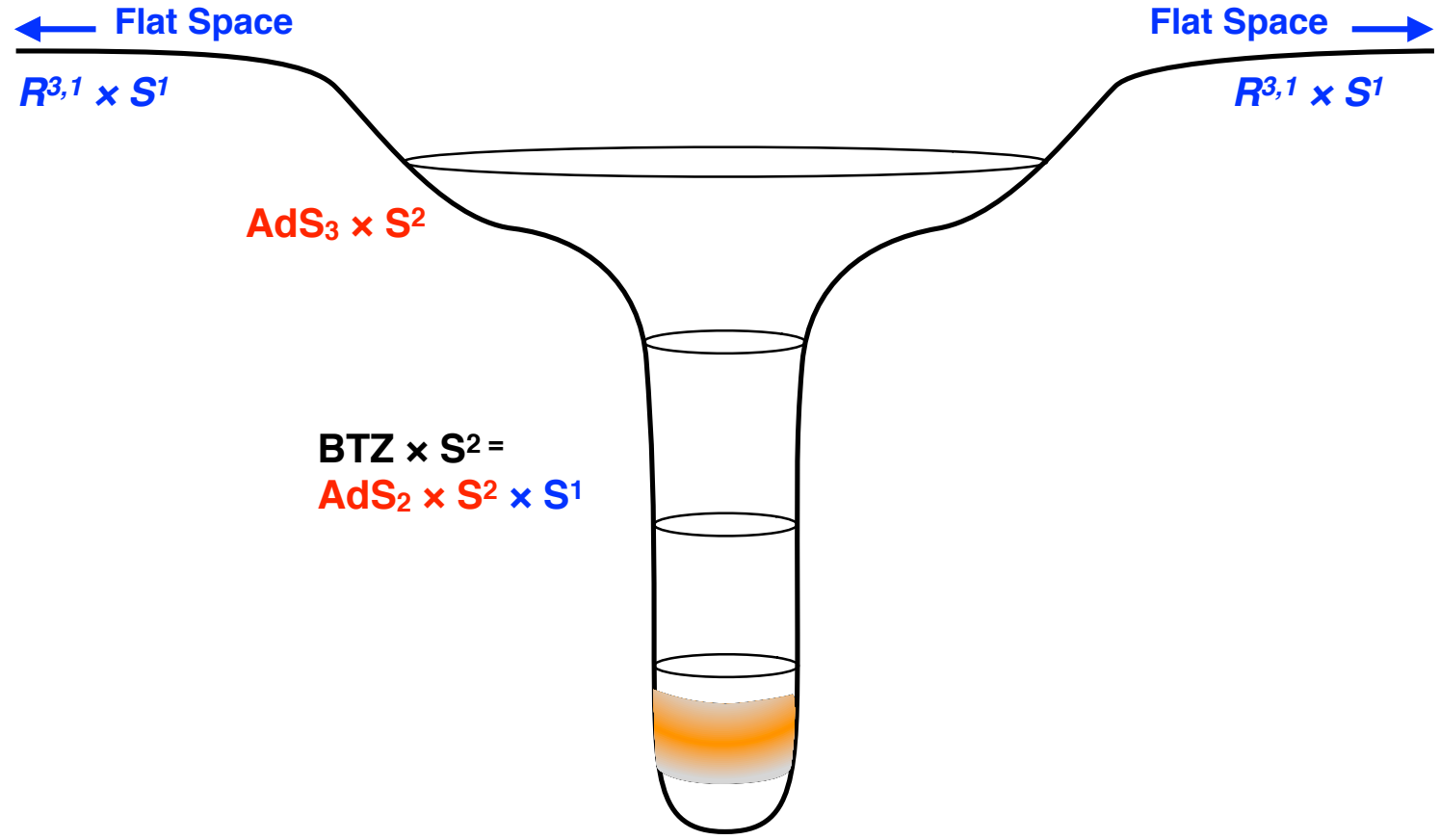
- ▶ **MSW:** Single M brane wrapped on very ample divisor of  $CY_3$
- ▶ **Here:** Multiple, disjoint M branes  $T^4$ 's in  $T^6$
- Non-trivial fluctuations require turning deforming Kahler moduli of the tori, “bending” disjoint M5's into one another ...

### Universality of the five-dimensional solution:

- We have reduced to five-dimensions and so our solution is valid for any Calabi-Yau compactification with the same set of M5-brane charges

# Fluctuating Microstate Geometries for MSW Strings

Previous picture compactified on Hopf fiber of  $S^3$ .



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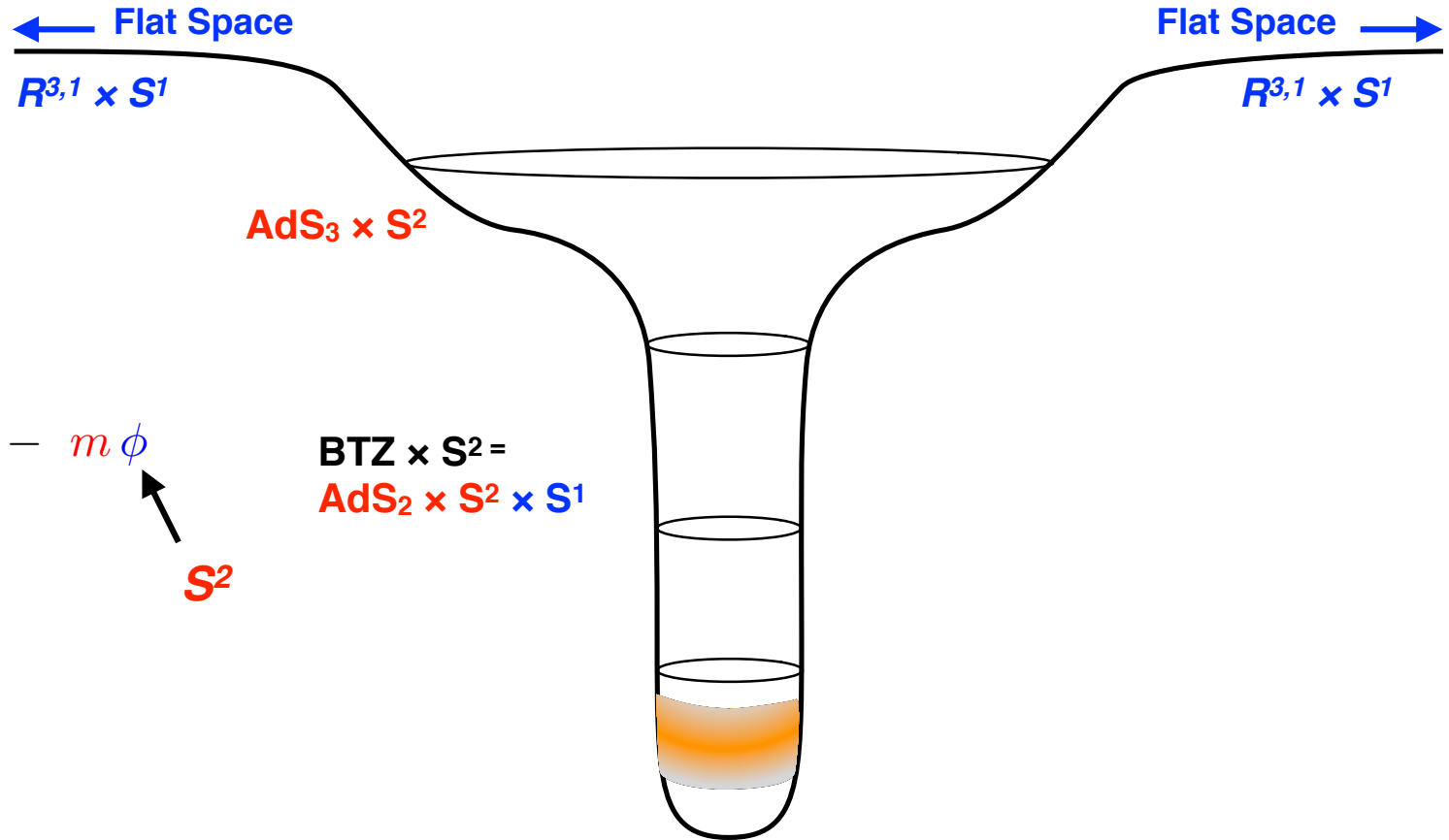
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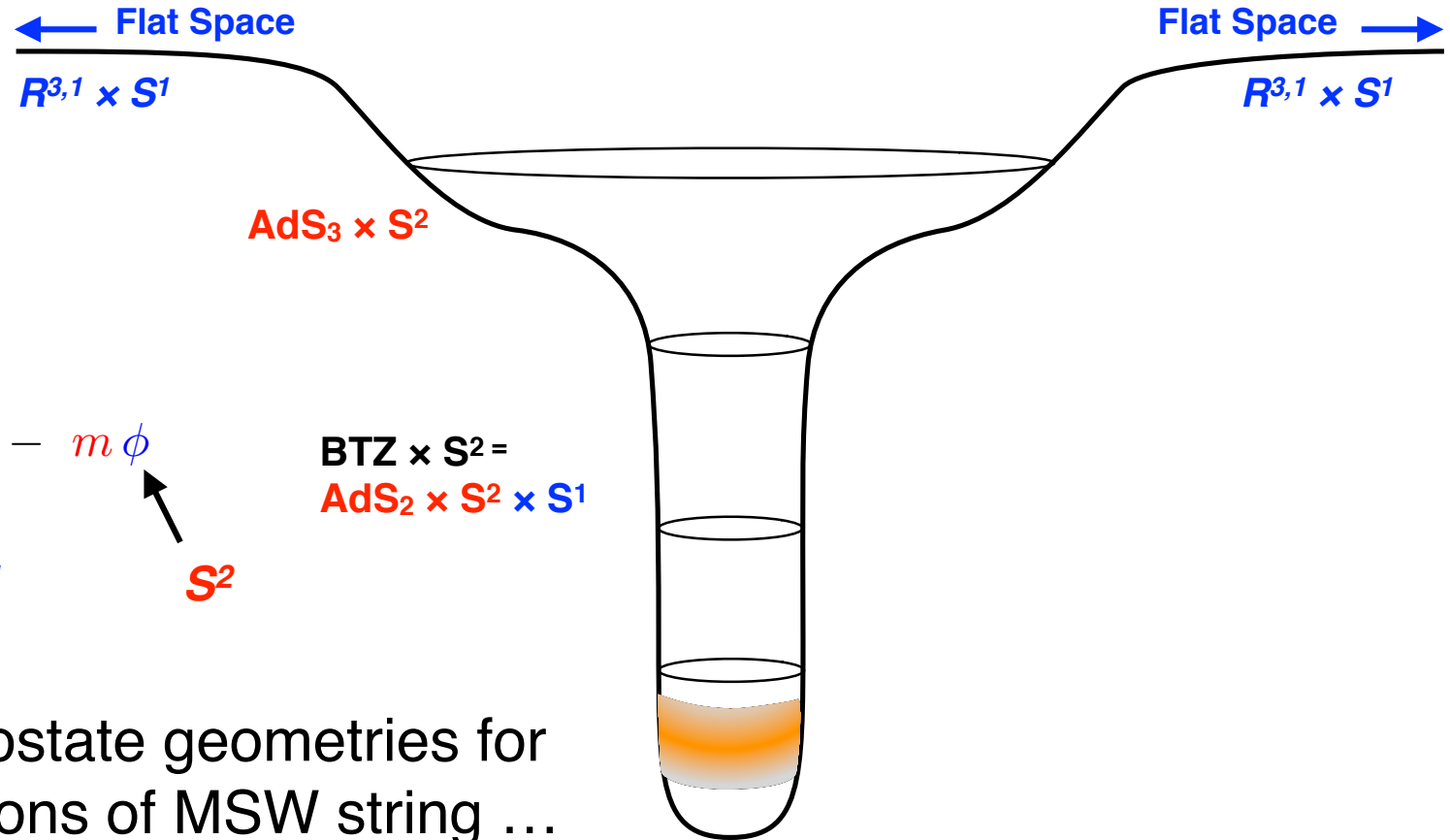
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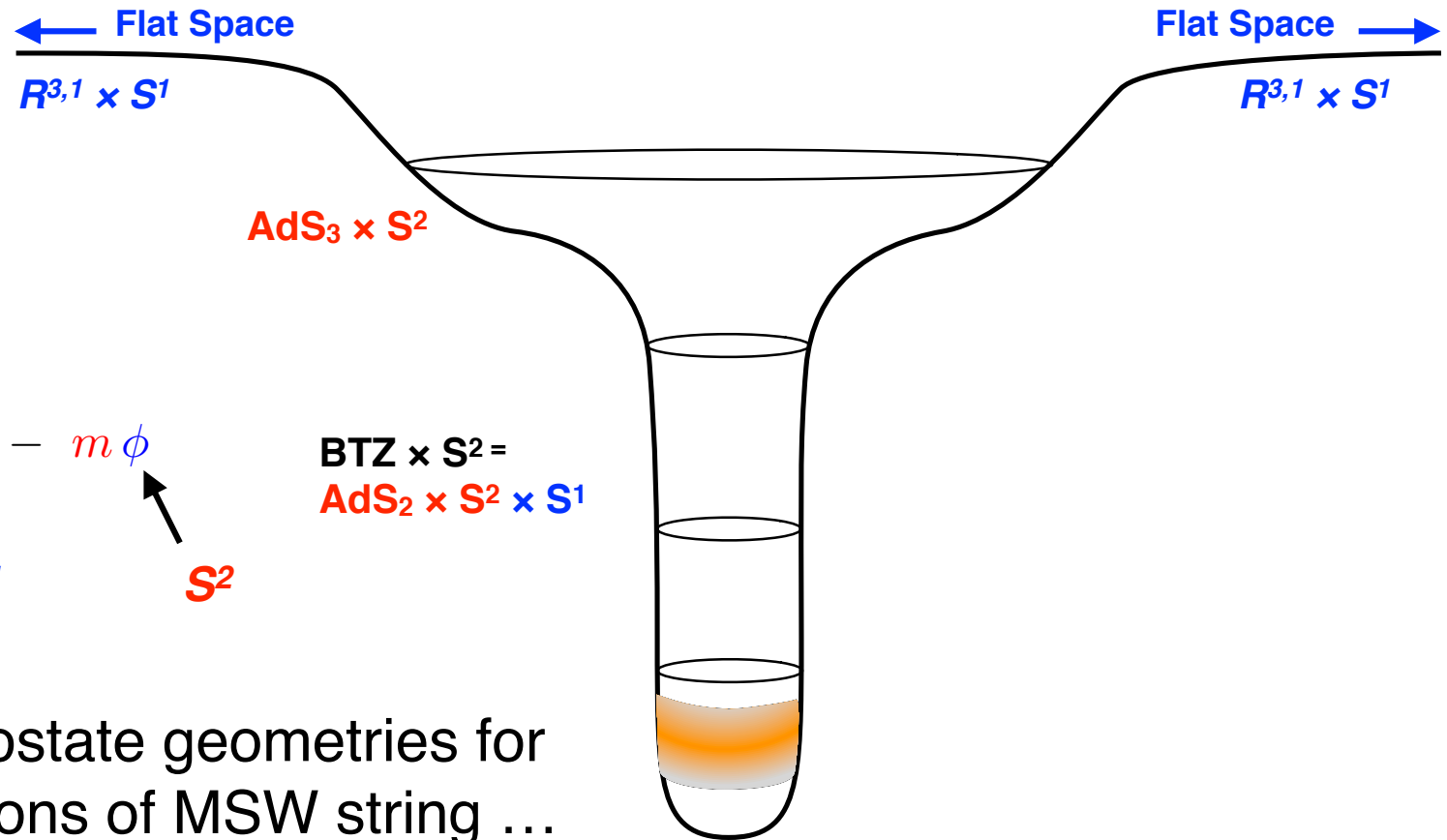
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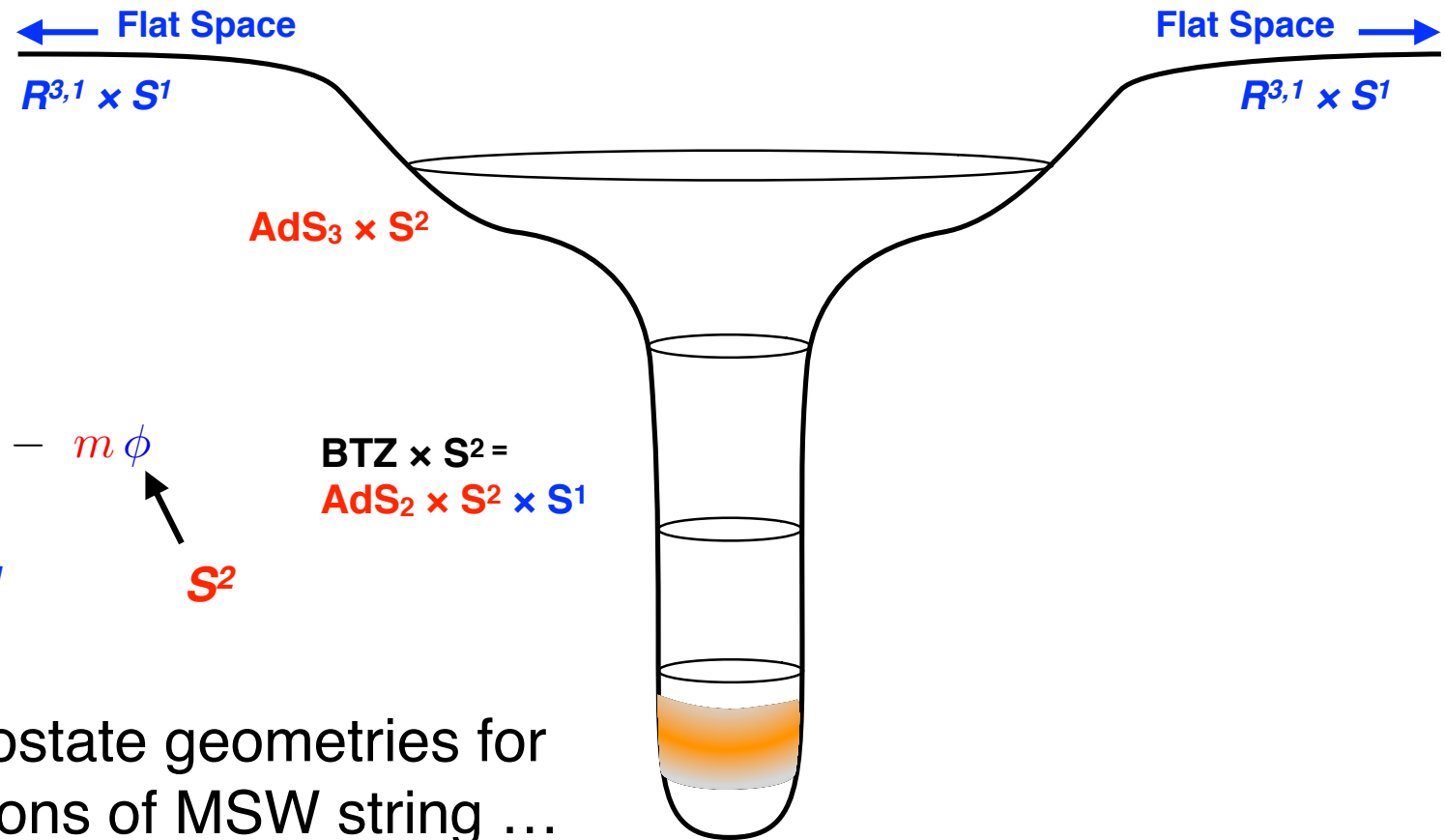
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**Here:** Precise, fully back-reacted, capped-off BTZ  $\times$   $S^2$  realization of the deconstructed configurations ...

..... related to D1-D5-P microstate structure



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## Open issues

- ▶ Twisted sector excitations. Relation to multi-centered geometries?
- ▶ Holography/CFT states of MSW string dual to new microstate geometries
- ▶ Probe the IR physics/large-t correlators of these new geometries