Black holes and random matrices

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Strings 2017

A question

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- The stakes are high here. Many approaches to understanding the bulk–
  - TFD/Eternal black hole
  - Ryu-Takayanagi
  - Geometry from entanglement
  - Tensor networks
  - ER = EPR
  - Code subspaces
  - ...

  suggest that any complete bulk description of quantum gravity must be able to describe these states.

A diagnostic

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- The "spectral form factor"

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- $Z(\beta)^2 \to Z(2\beta)$. $(= L = e^S$ for $\beta = 0)$
- $e^{2S} \to e^S$, an exponential change. How does this occur?

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  - $O(N)$ singlets
  - nonlocal
  - Nonperturbatively well defined (two replicas)

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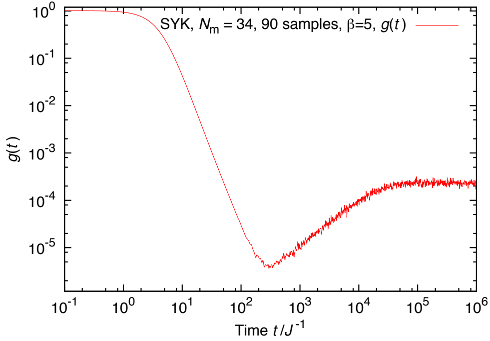
- Finite dimensional Hilbert space, $D = L = 2^{N/2}$, amenable to numerics
- Guidance about what to look for

$ZZ^*(t)$ in SYK

[Jordan Cotler, Guy Gur-Ari, Masanori Hanada, Joe Polchinski, Phil Saad, Stephen Shenker, Douglas Stanford, Alex Streicher, Masaki Tezuka]
([CGHPSSSST])

See also
[Garcia-Garcia–Verbaarschot]

Meaning



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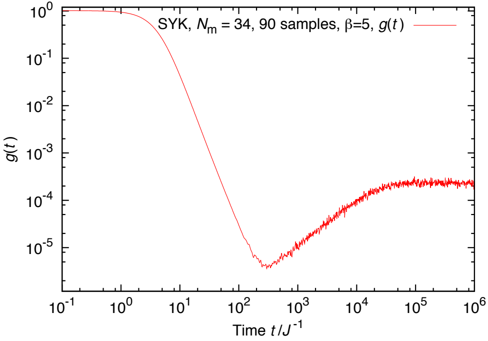

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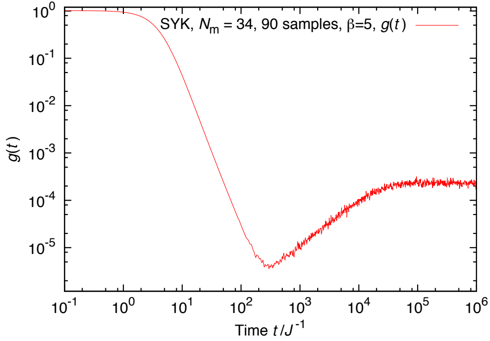

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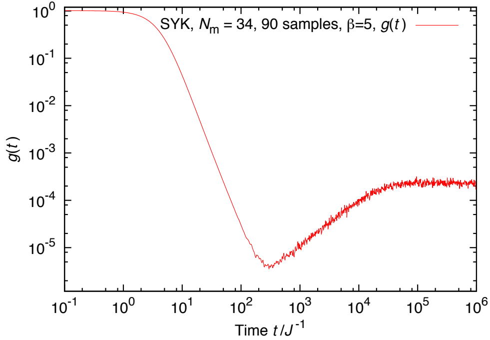

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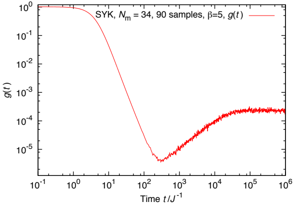
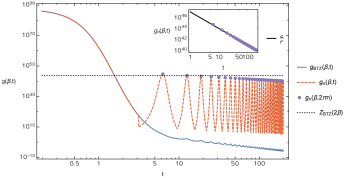
Slope, contd.





Slope is determined by semiclassical quantum gravity–nonuniversal

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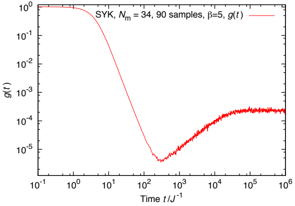

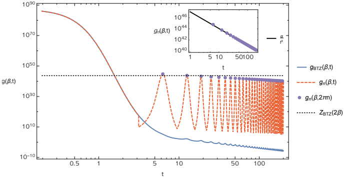

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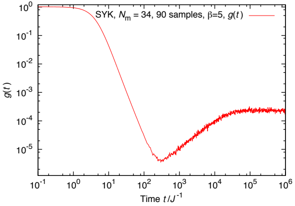

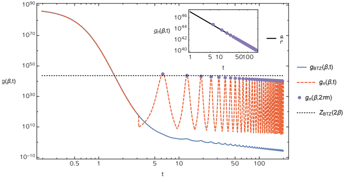

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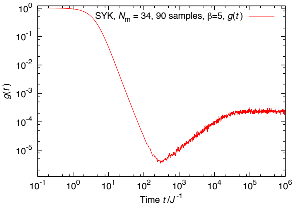
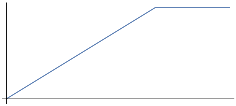
In BTZ summing over modular transforms of blocks gives oscillating slope with power law envelope: nonperturbatively small oscillations in the density of states. [Dyer–Gur-Ari]

The Ramp and Plateau





The Ramp and Plateau are signatures of Random Matrix Statistics, believed to be universal in quantum chaotic systems

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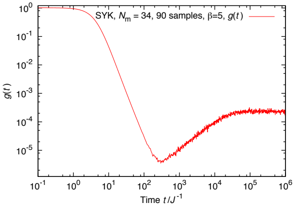

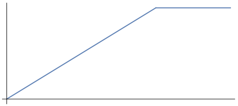

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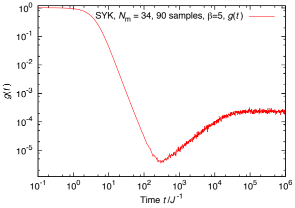

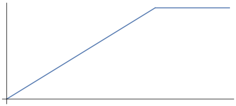

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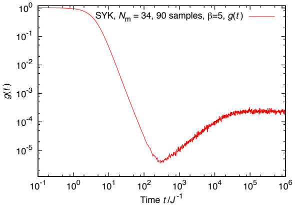

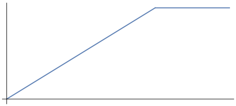

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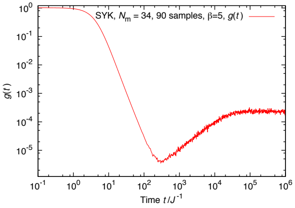

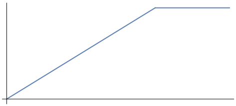

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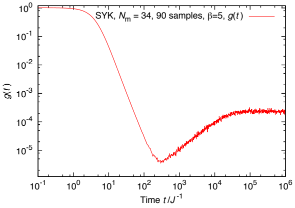

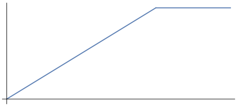

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Some evidence for this in melonic models [Witten; Gurau; Carrozza-Tanasa; Klebanov-Tarnopolsky; Krishnan-Kumar-Sanyal]

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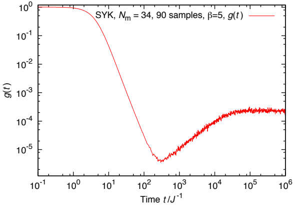
- $q = 2$ SYK in progress [Saad, SS]

Onset of RMT behavior

[Gharibyan-Hanada-SS-Tezuka, in progress]



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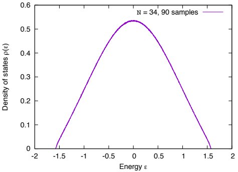

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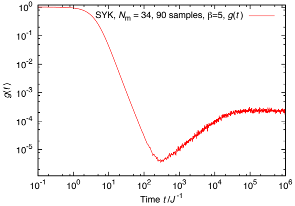

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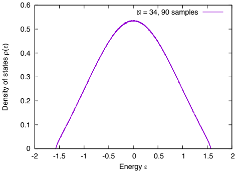

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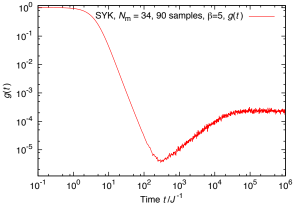

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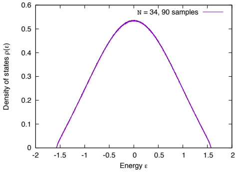

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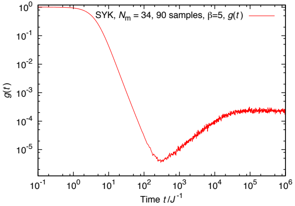

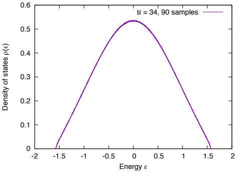

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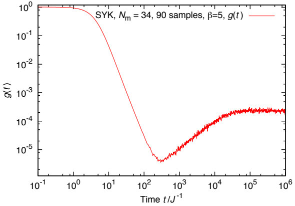

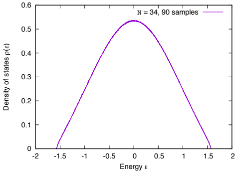

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Follow the ramp below the slope: use Gaussian filter [Stanford]

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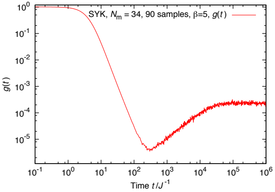
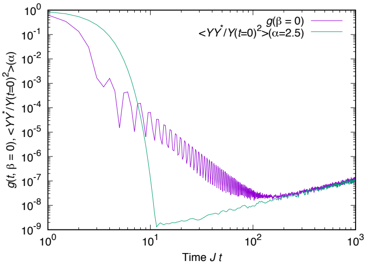
YY^*



Dip time $t_d \sim 200$, $N = 34$



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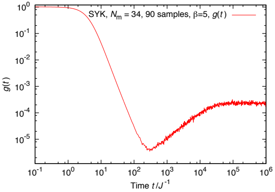

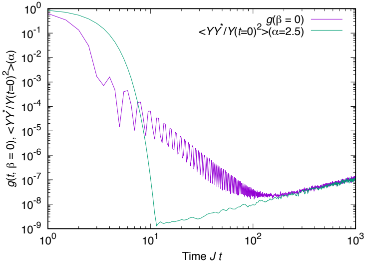

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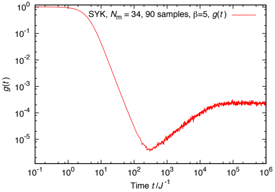

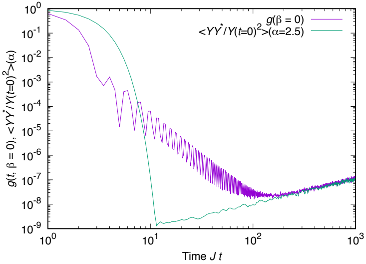

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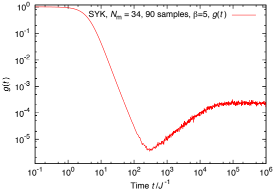

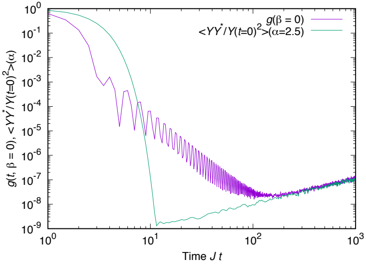

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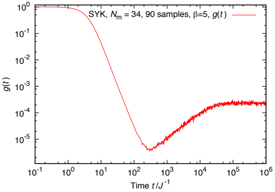

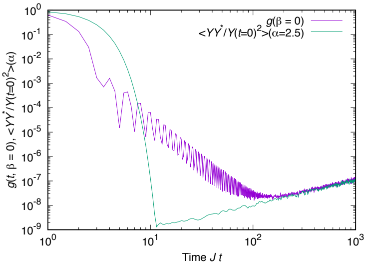

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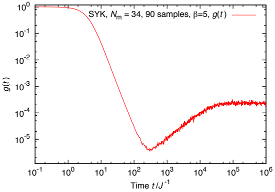

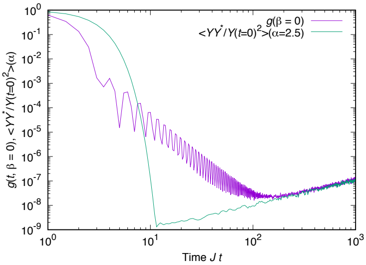

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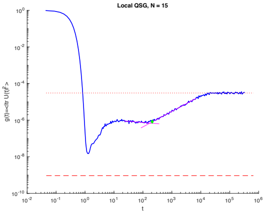

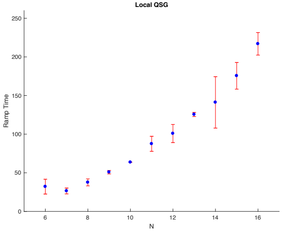

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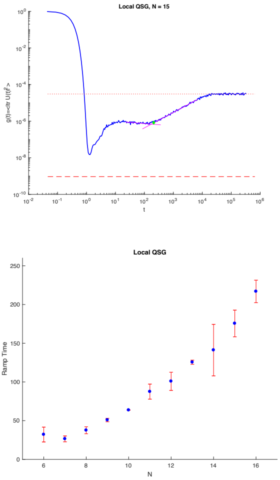

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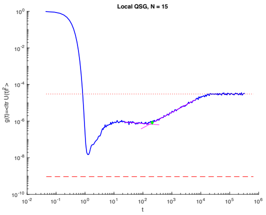

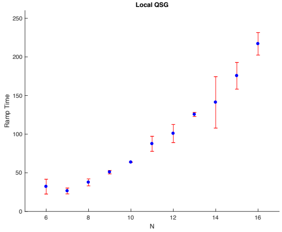

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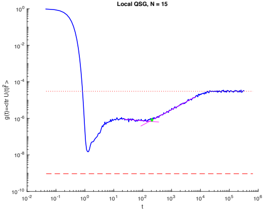

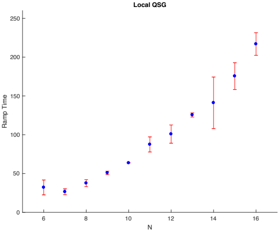

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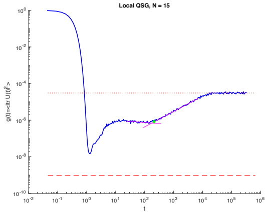

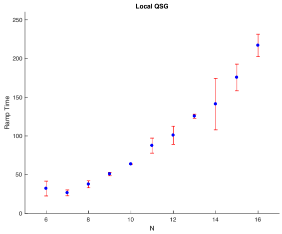

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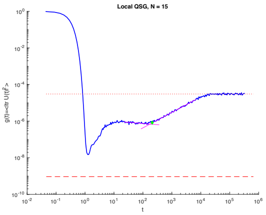

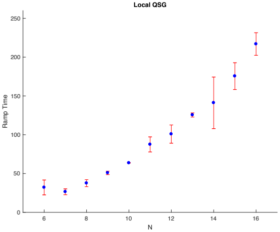

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Maybe not scrambling...

Brownian circuits

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- Can analyze dynamics including scrambling analytically [Oliveira-Dahlsten-Plenio; Lashkari-Stanford-Hastings-Osborne-Hayden; Harrow-Low; ...]

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- Time to randomize last qubit $\sim n$, scrambling time [Nahum-Vijay-Haah; Keyserlingk-Rakovszky-Pollmann-Sondhi]

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- Correlation functions of very complicated operators [Roberts-Yoshida; Cotler-Hunter-Jones-Liu-Yoshida]

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- So these phenomena would appear in small black holes as well, although as an exponentially subleading effect
- We need to know what they mean in quantum gravity!