Exploring the black hole interior

The surprising simplicity of near-AdS_2

Juan Maldacena

Based on: - Gao, Jafferis and Wall
- JM, Douglas Stanford and Zhenbin Yang.
- Ioanna Kourkoulou and JM.

Strings 2017
Israel

SYK model

Nearly AdS_2 gravity

Emergent reparametrization symmetry which is spontaneously and explicitly broken

Dominate many aspects of the IR dynamics

$$S = -C \int du \{ f(u), u \}$$

Schwarzian action

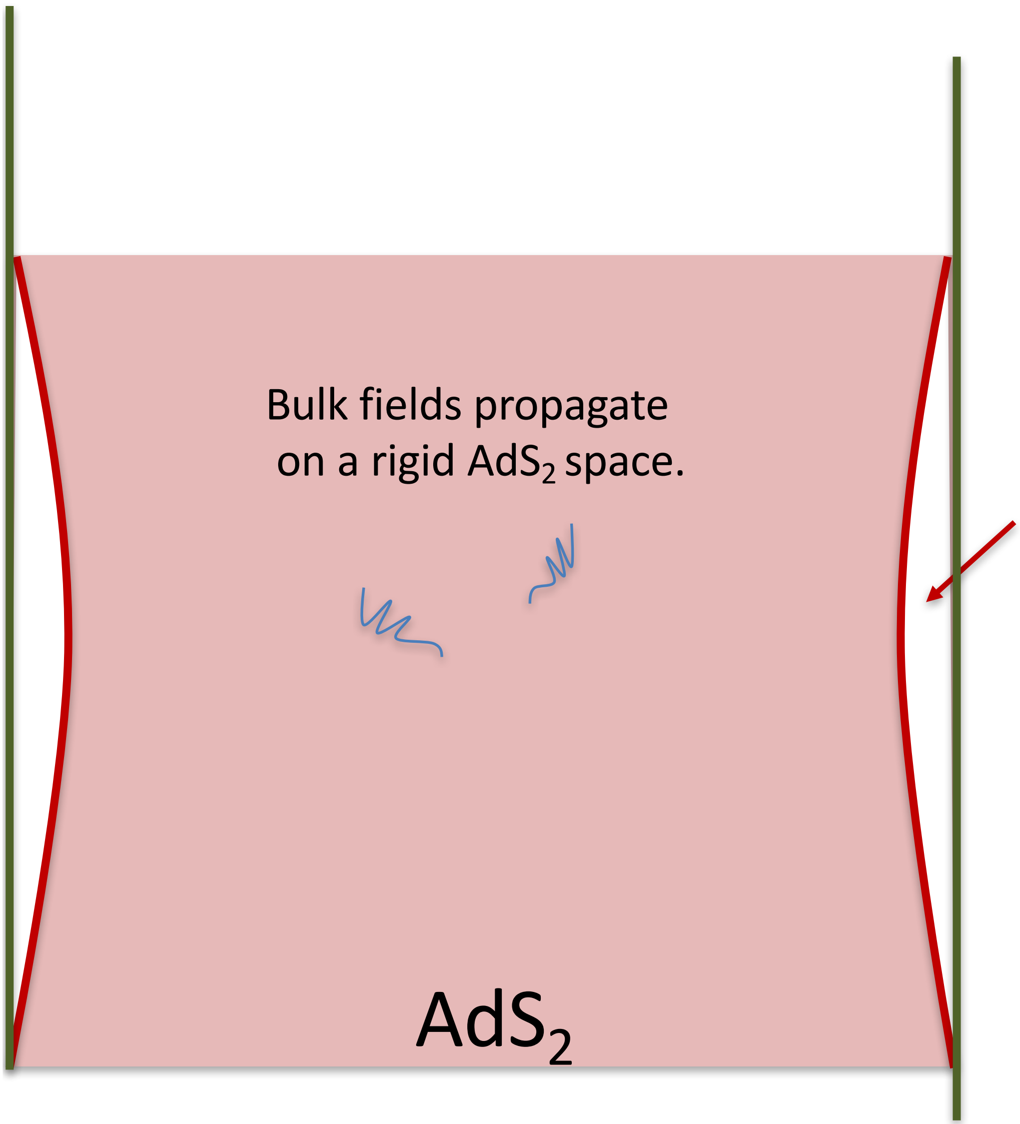
- Low temperature entropy
- Gravitational backreaction
- Chaos exponent
- Wormhole traversability (location of horizon)

Motion of the boundary in AdS_2

Very simple gravitational dynamics in $NAdS_2$

The Schwarzian → motion of the boundary

Nearly AdS_2 Dynamics



Boundary moves in a rigid AdS_2 space, following local dynamical laws.
(like a massive particle in an electric field)

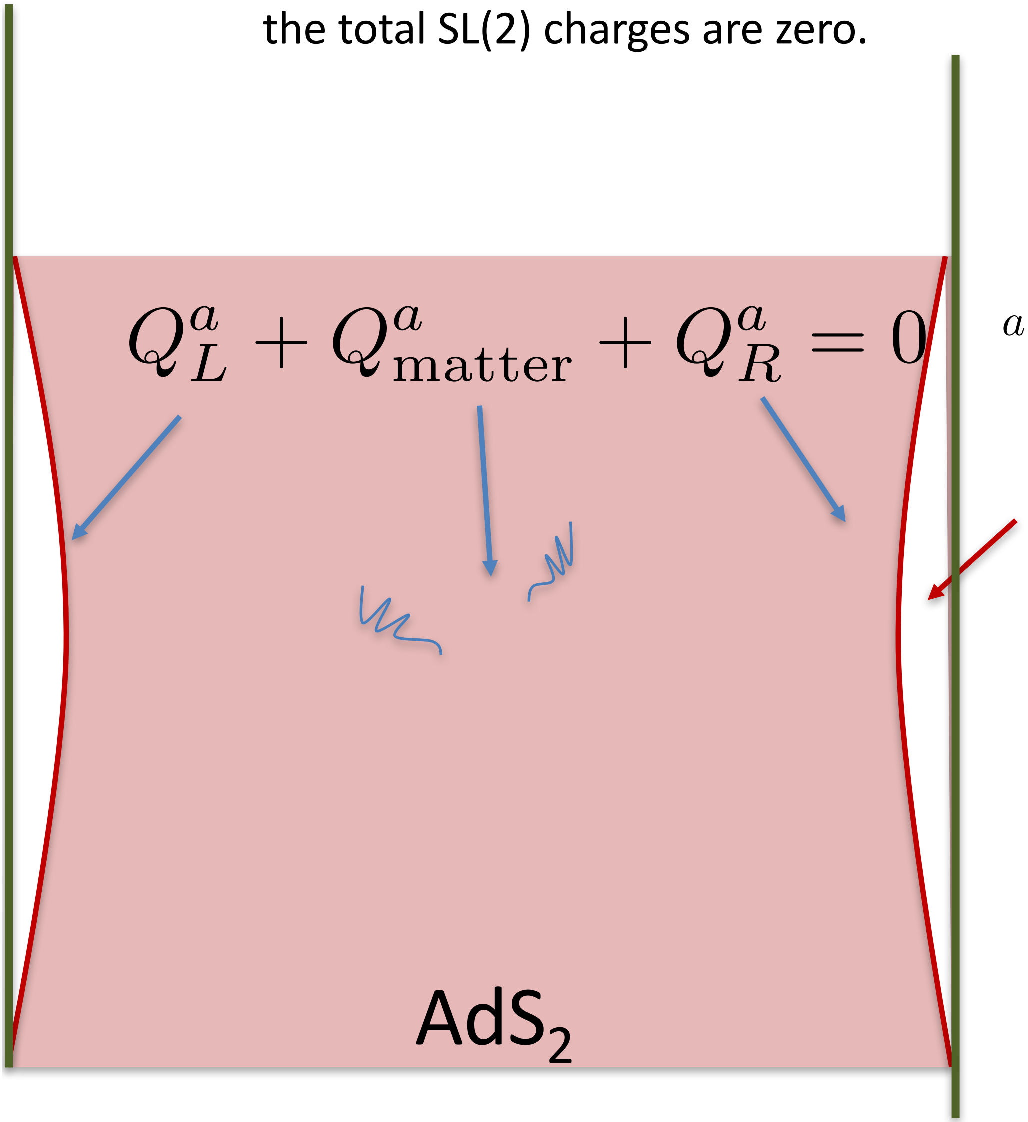
UV particle or UV brane as in a Randall-Sundrum model

Encodes gravitational effects

Constraints

Linked by the constraints that
the total SL(2) charges are zero.

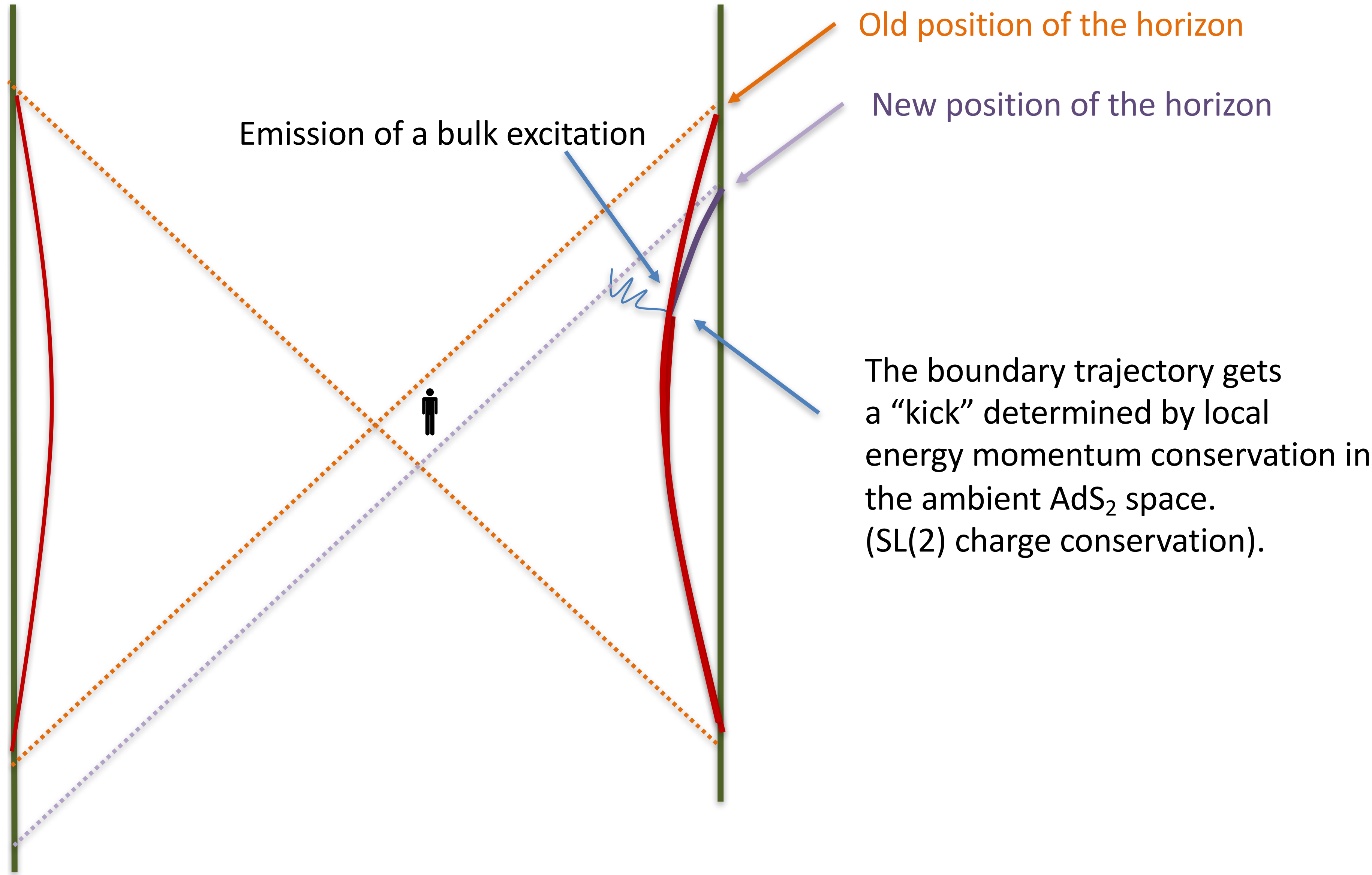
$$Q^a_L + Q^a_{\rm matter} + Q^a_R = 0 \qquad a = 1,2,3$$



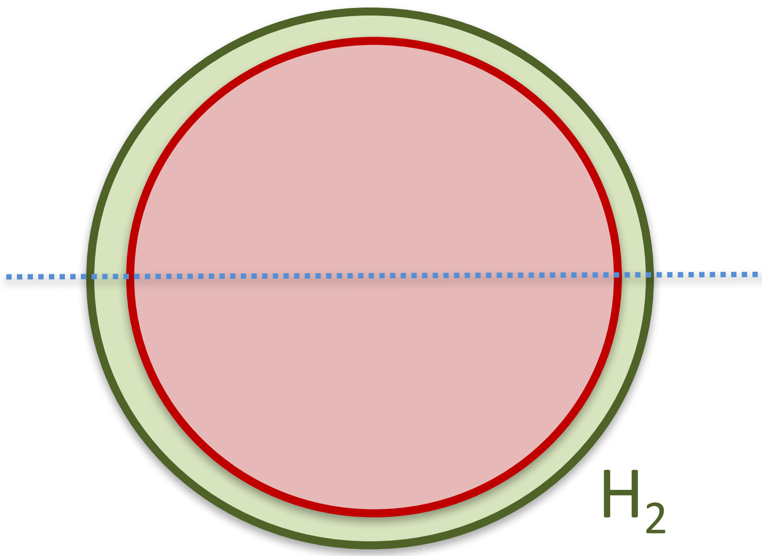
Example

The horizon moves out when you drop in a particle

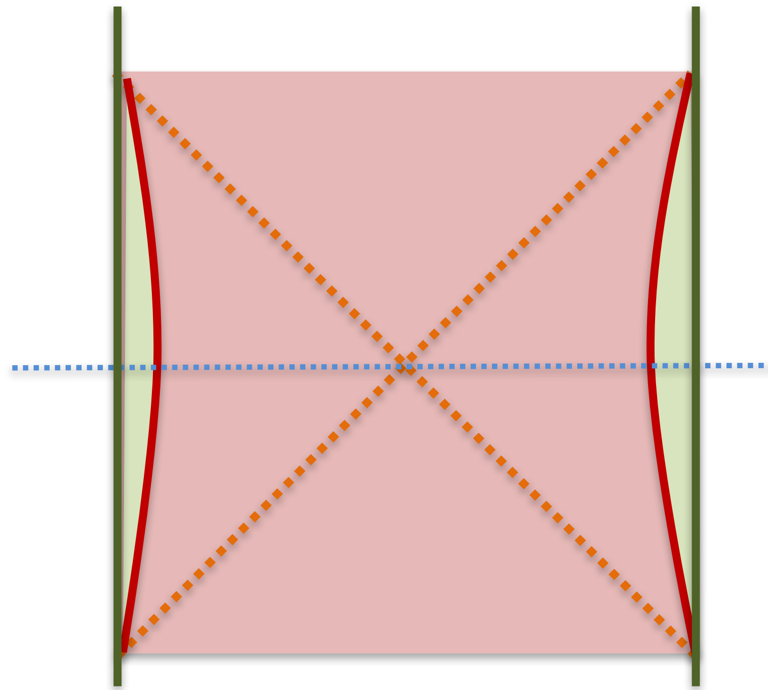
The horizon moves out when you drop in a particle



Euclidean & Lorentzian pictures



Euclidean black hole

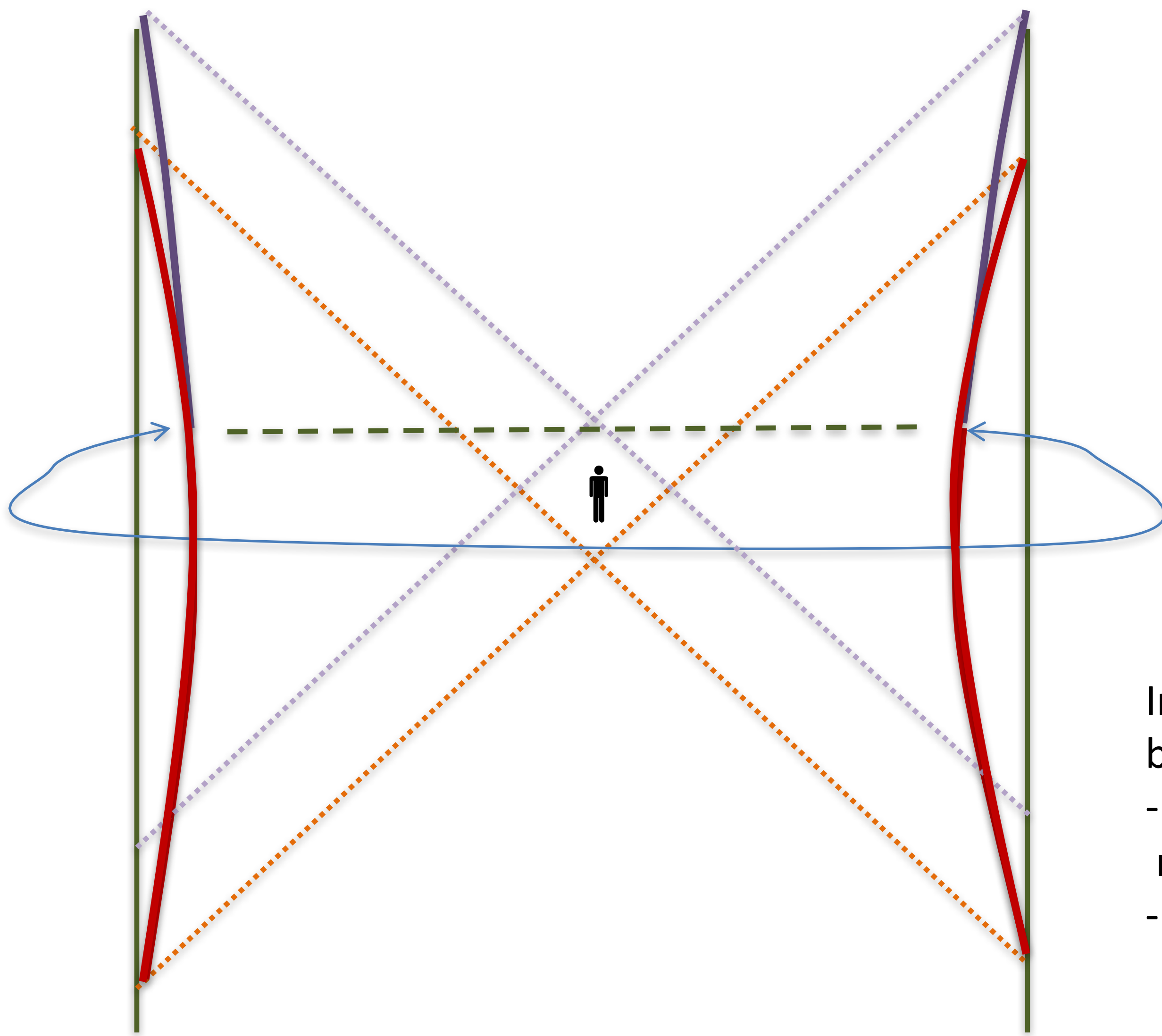


Rindler AdS_2 coordinates, wormhole

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

Interaction between the two boundaries

Gao Jafferis Wall



Insert in the evolution operator

$$e^{ig\phi_L(t_L)\phi_R(t_R)}$$

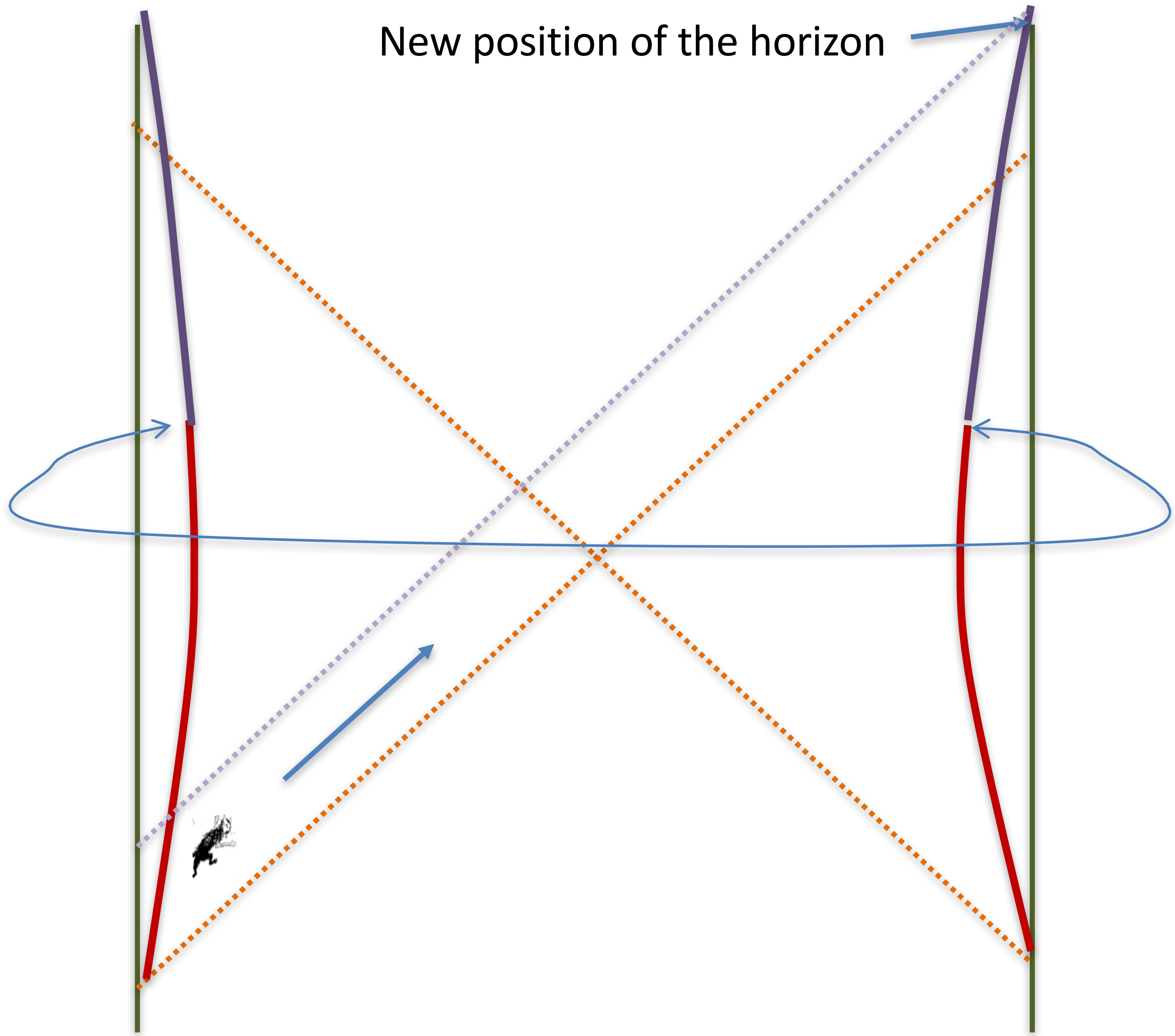
approximate

$$e^{ig\langle\phi_L(t_L)\phi_R(t_R)\rangle}$$

Impulsive force between the two boundaries.

- Can be attractive for the right sign of g .
- Kicks the trajectories <u>inwards.</u>

Interaction makes the wormhole traversable

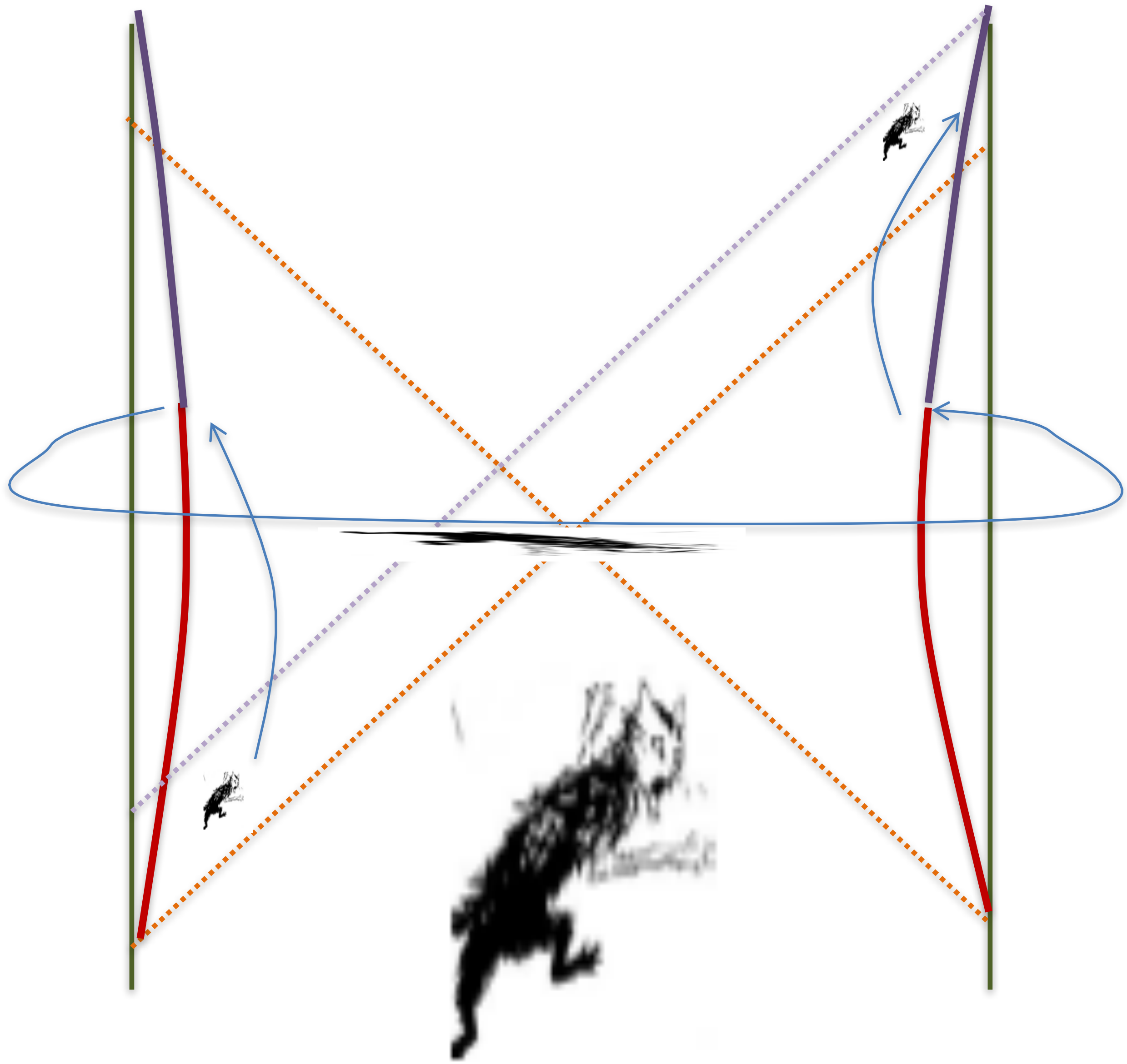


We can now send a signal from the left to the right.

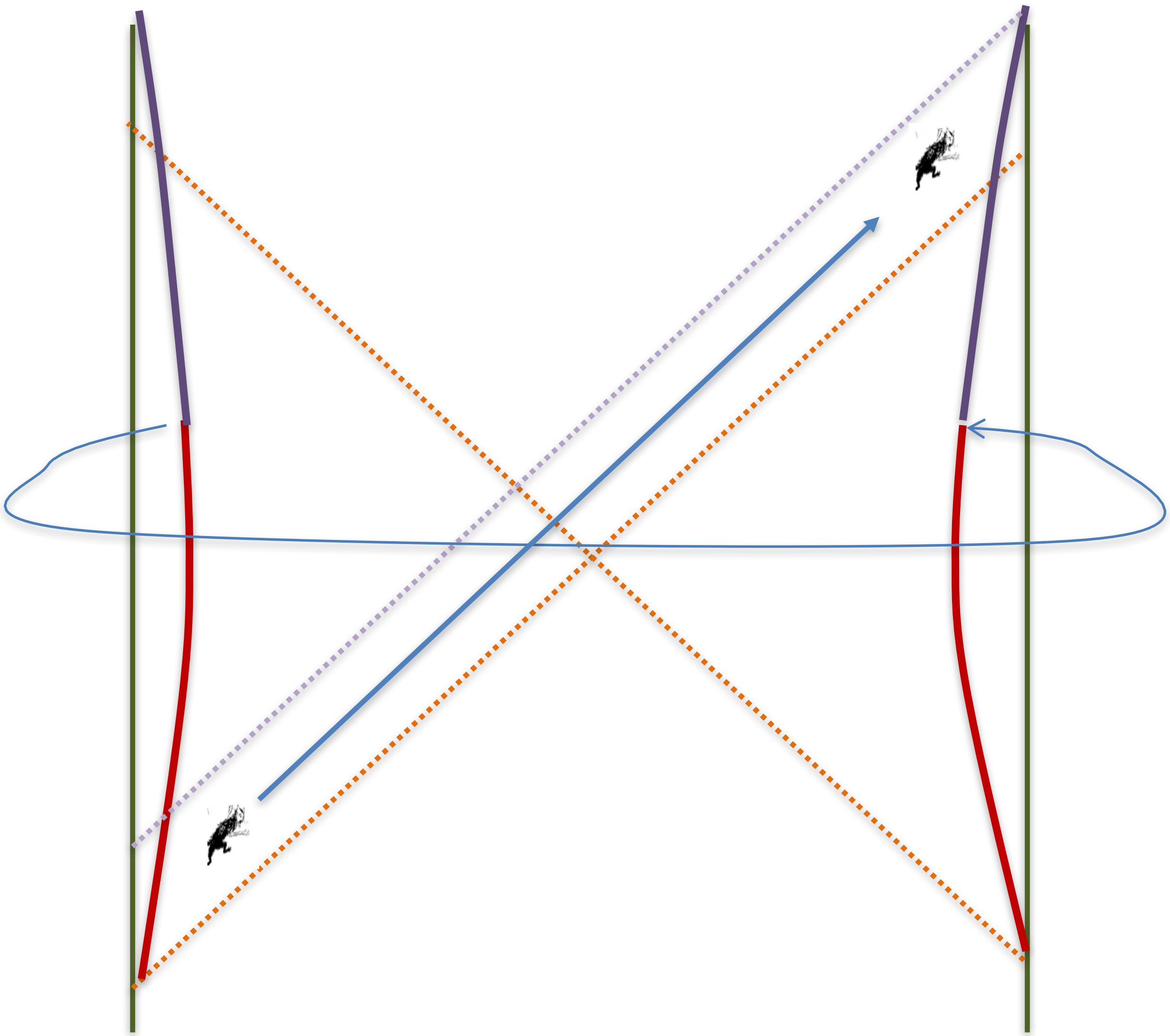
The wormhole has been rendered traversable.

No contradiction because we have a non-local interaction between the two boundaries.

What it is NOT!



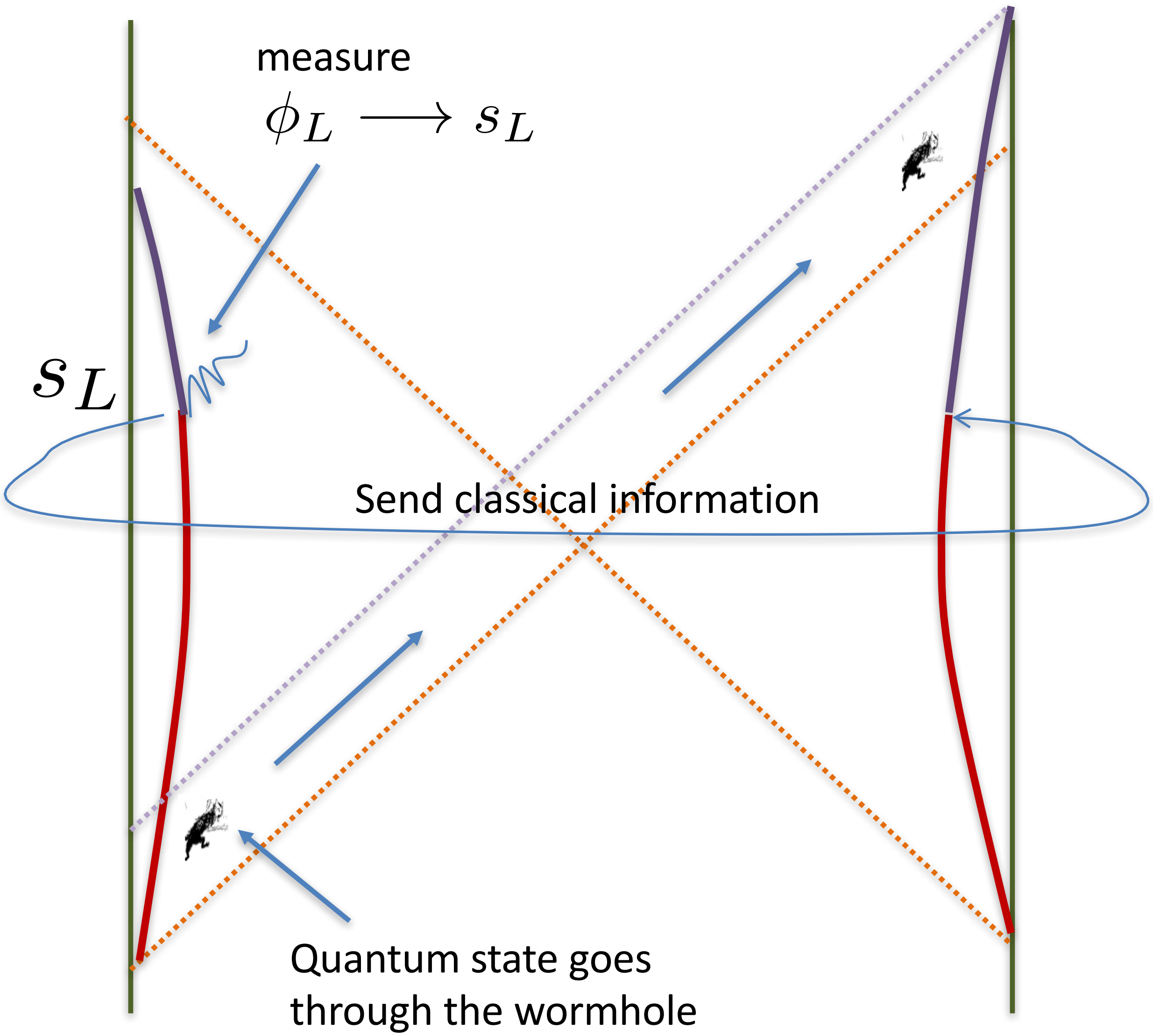
The cat feels fine going through the wormhole



No animals were harmed during this experiment

Simpler protocol

Doing a measurement → Teleportation



Act on the right with

$$e^{i g s_L \phi_R(t_R)}$$

From the point of view of the right, we get the same, whether we measure or not.

Minor differences with usual teleportation discussion:

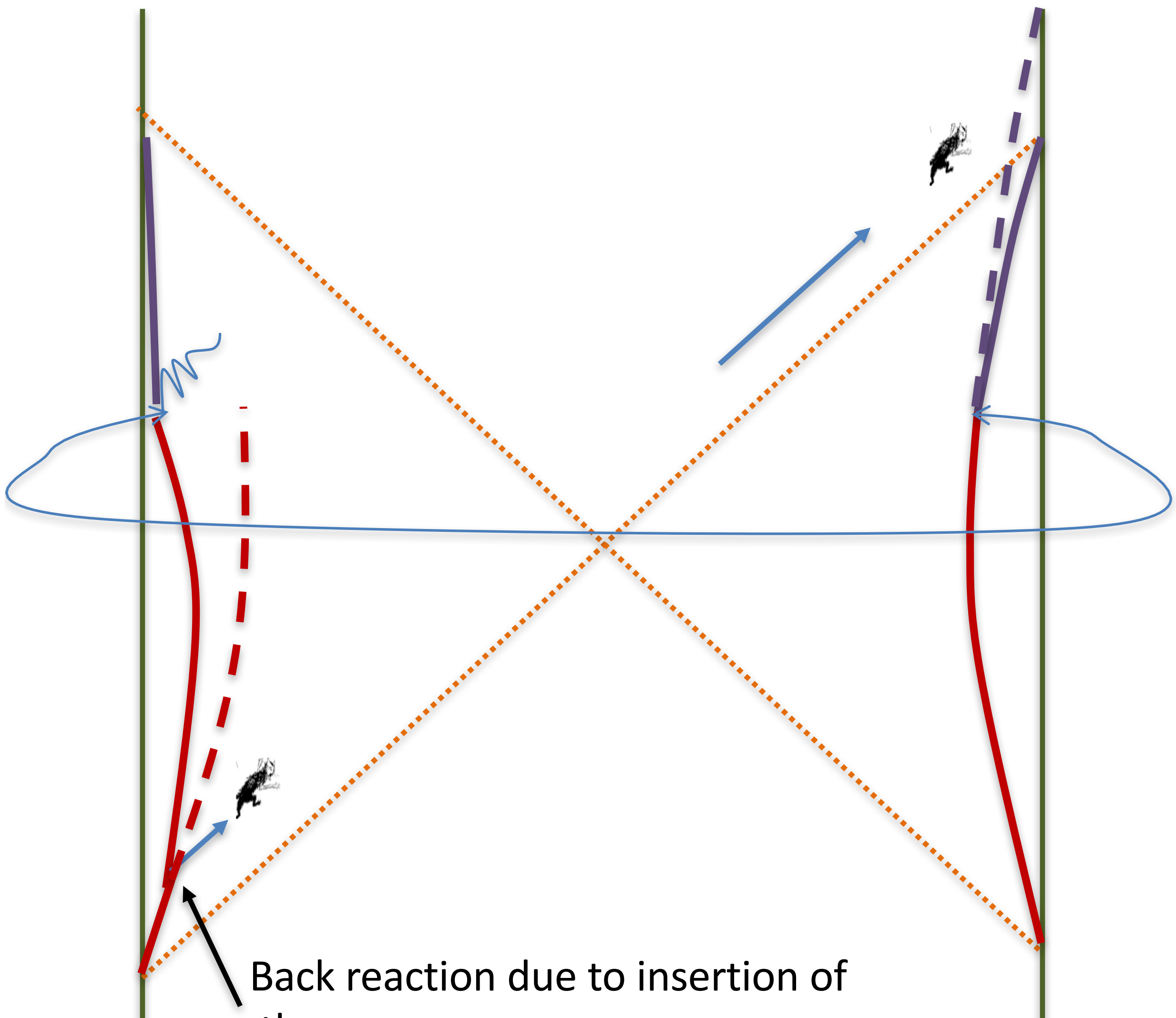
- We do not need to measure the left state completely.

- Encoding and decoding are done via standard Hamiltonian evolution.

As usual:
There is a bound on the information we can send:

$$N_{\text{classical bits}} \geq 2N_{quits}$$

Origin of the bound in gravity



The insertion of the message also gives a small kick to the trajectory.

Moves the insertion points of the non-local operator away from each other

$\langle \phi_L(t_L)\phi_R(t_R)\rangle$ becomes smaller

Attractive force weakens → no opening of the wormhole.

Precise formula for the 2pt function

$$C = \langle e^{-igV} \chi_R(t) e^{igV} \chi_L(-t) \rangle \ , \qquad V = \phi_L(0)\phi_R(0)$$

- Fourier transform the signals we want to send.
- Correlator of V is evaluated on a background with momentum p → p dependent SL(2) transformation on V .
- Effect is amplified by boosts, or chaos $G_N e^t \sim \frac{1}{N} e^t$
- Get an extra phase from $\langle V \rangle$

$$C \sim \int dp (p)^{2\Delta - 1} e^{ip} e^{-ig} e^{i \frac{g}{(1 + pG_N e^t)^{2\Delta}}}$$

Includes gravitational back reaction

Amount of information we can send is roughly g $\qquad \langle V \rangle = 1 \ , \quad \langle \phi_L^2(0) \rangle \sim 1$

Simplified limit

$$C = \langle e^{-igV} \chi_R(t) e^{igV} \chi_L(-t) \rangle \ , \qquad V = \phi_L(0)\phi_R(0)$$

$$C \sim \int dp (p)^{2\Delta - 1} e^{ip} e^{-ig} e^{i \frac{g}{(1+G_N p e^t)^{2\Delta}}}$$

$$C \sim \int dp (p)^{2\Delta - 1} e^{ip} e^{-i(g 2\Delta G_N e^t) p}$$

Just a simple ``translation'' (one of the operations of SL(2)).

The signal does not ``feel'' anything ! Composite objects are simply translated whole !

Shock waves in two dimensions are not felt. (In higher dimensions there are tidal forces).

Quantum mechanical model

The SYK model

Sachdev Ye Kitaev
Georges, Parcollet

$$H = \sum_{i_1, \cdots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

Low energies.

$$S[f] \propto -\frac{N}{J} \int \{f, \tau\} d\tau$$

Same as the action for the
UV boundary
in the gravity description.

$$H = \sum_{i_1,\cdots,i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

Morally like the AdS_2 metric

$$G(t,t') = \langle \psi^i \psi^i(t') \rangle \to \frac{1}{|t-t'|^{2\Delta}}$$

Low energies.

$$S[f] \propto -\frac{N}{J} \int \{f,\tau\} d\tau$$

$$G(t,t')_f = \left[\frac{f'(t) f'(t')}{(f(t)-f(t'))^2} \right]^{\Delta}$$

Same as the action for the UV boundary in the gravity description.

Dependence of Correlators on the boundary position

- We got the same action for the boundary degree of freedom (Schwarzian).
- All that we said about the motion of the UV boundary $NAdS_2$ bulk, also holds for SYK.
- Traversability in SYK !
- Same formula !

Convenient operators

Define the ``spin" operators:

$$S_1 = i\psi^1\psi^2 \ , \quad S_2 = i\psi^3\psi^4 \ , \cdots$$

$$S_k^2 = 1$$

"spin operators", +1, -1 eigenvalues

$$V = \sum S_{kL}(0) S_{kR}(0)$$

Interaction between the boundaries

Pure states in SYK

Measure all spins $S_k \to s_k$ → get the joint eigenstate $|B_s\rangle$

This set generates the full Hilbert space

$$|\Psi\rangle = e^{-H\beta/2}|B_s\rangle$$

Project on to lower energy states

``Diagonal'' correlators on this state

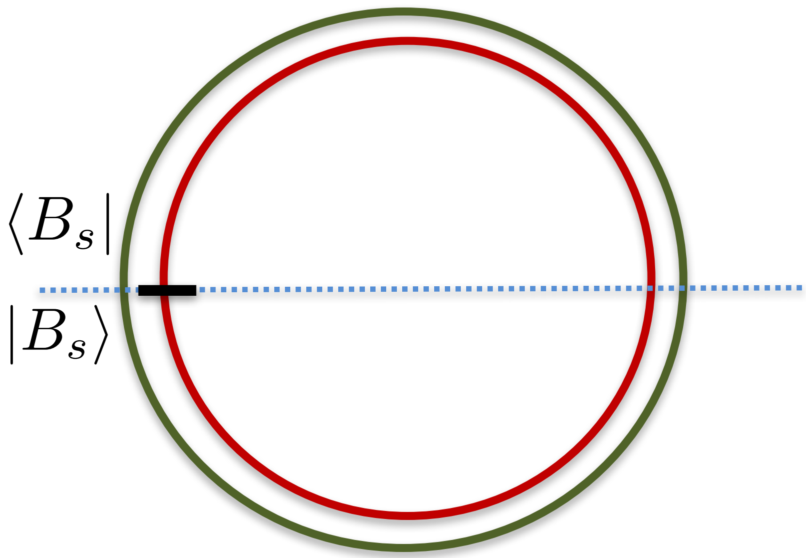
$$\langle\Psi|\psi^i(t)\psi^i(t')|\Psi\rangle = Tr[e^{-\beta H}\psi^i(t)\psi^i(t')] + o(1/N^{q-1})$$

↑ Same as in the thermal state

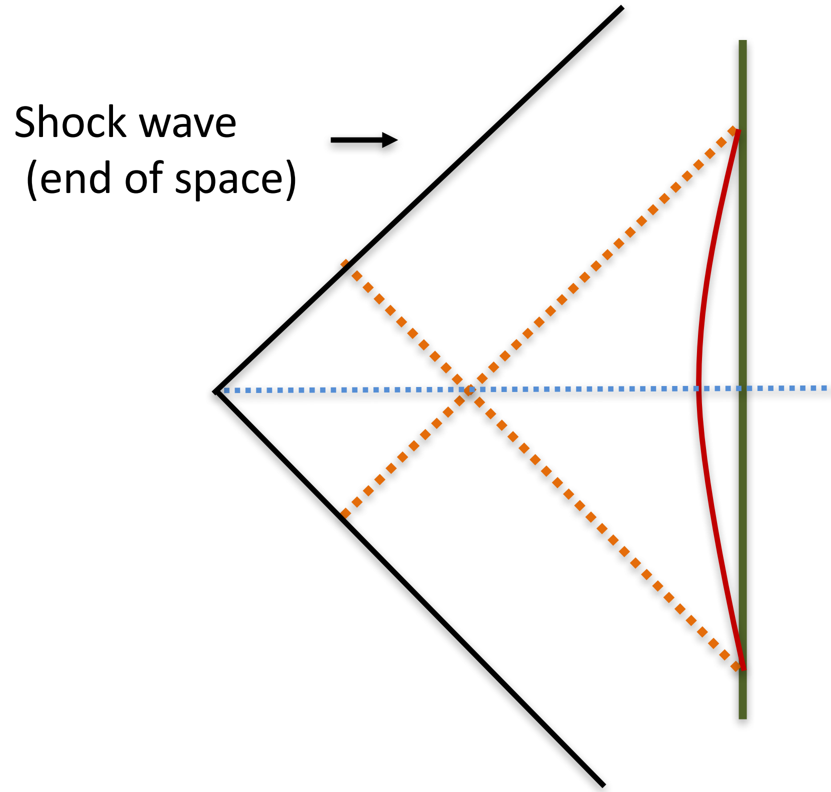
Off diagonal correlators:

$$\langle\Psi|i\psi^1(t)\psi^2(t)|\Psi\rangle = \langle\Psi|S_1(t)|\Psi\rangle = s_1 G_\beta(t)^2 + o(1/N)$$

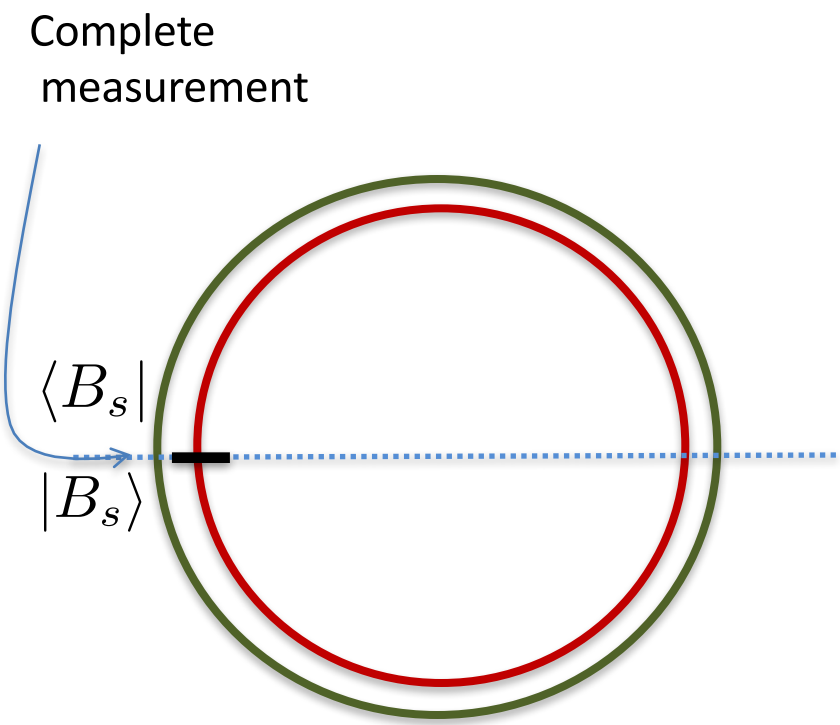
↑ Given in terms of thermal two point function



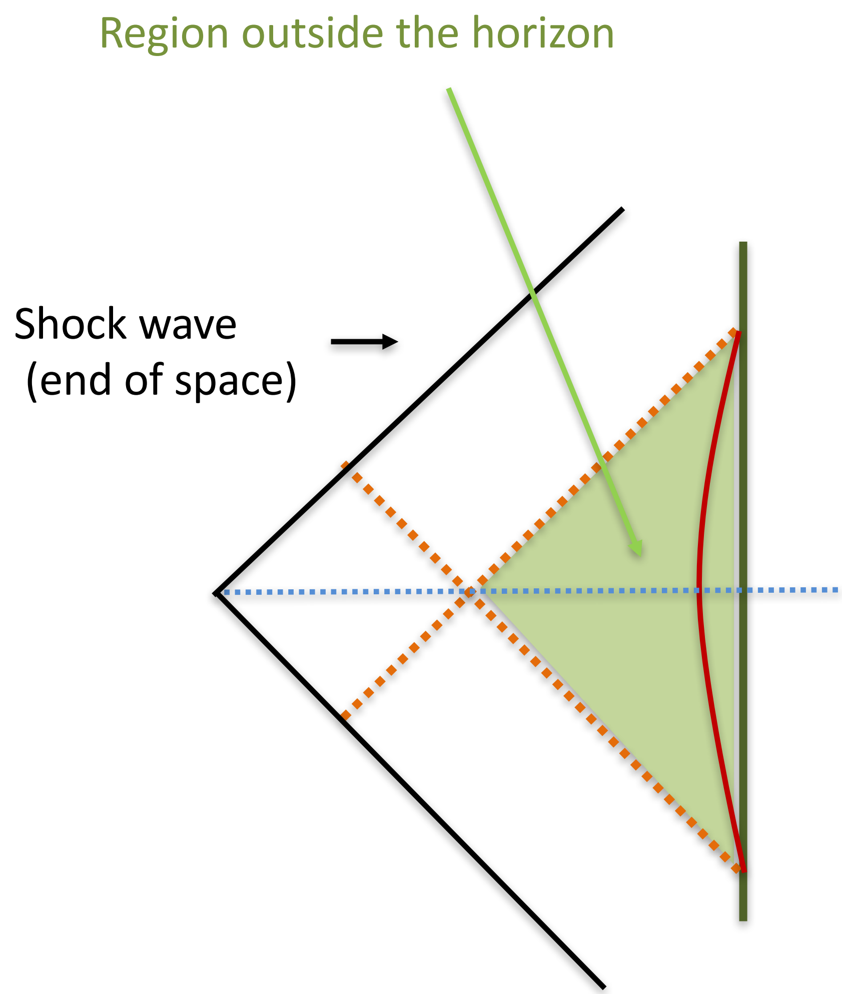
Euclidean configuration



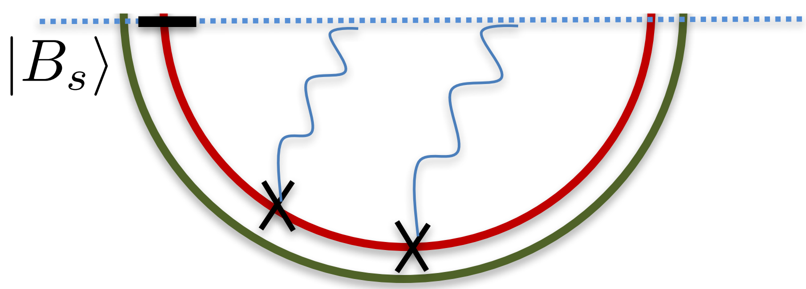
$$|\Psi\rangle = e^{-H\beta/2}|B_s\rangle$$



Euclidean configuration

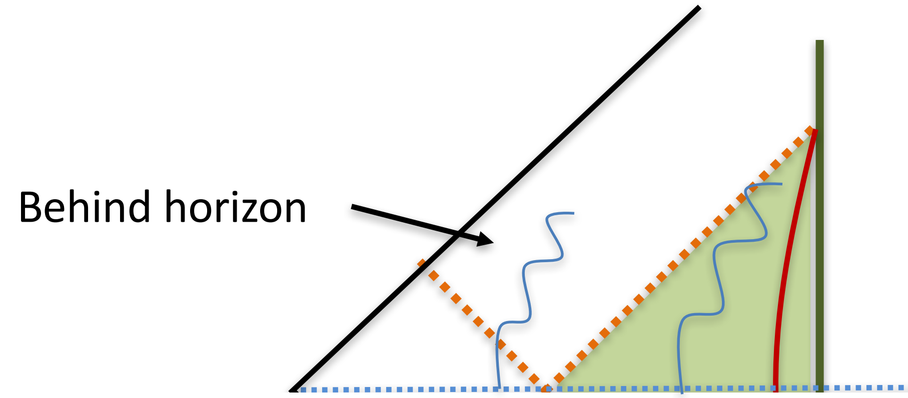


$$|\Psi\rangle = e^{-H\beta/2}|B_s\rangle$$



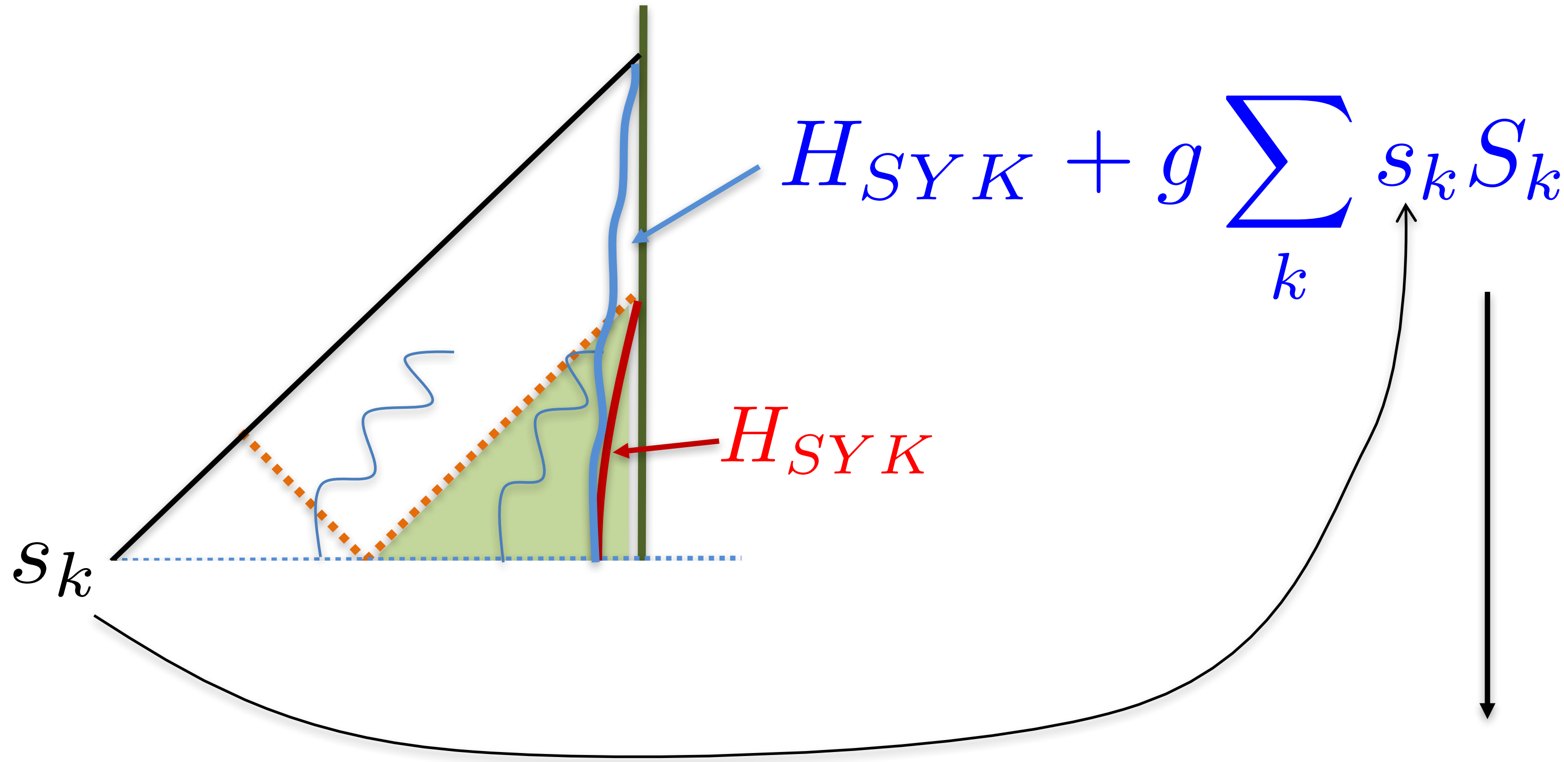
Adding bulk particles

Euclidean preparation



Lorentzian state

How can we see behind the horizon ?



Lorentzian state

See the whole region

$$\langle e^{-ig\int dt\, is_k S_k} \rangle \sim e^{-i\int dt g \langle B_s | is_k S_k | B_s \rangle} \to e^{iS_{\rm extra}(f)}$$

$$S_{\rm extra}[f] = g \int dt (f')^{2\Delta}$$

$$S = S_{\rm Schwarzian} + S_{\rm extra}$$

Gives this modified evolution.

Extra term → new force pushing the boundary particle inwards

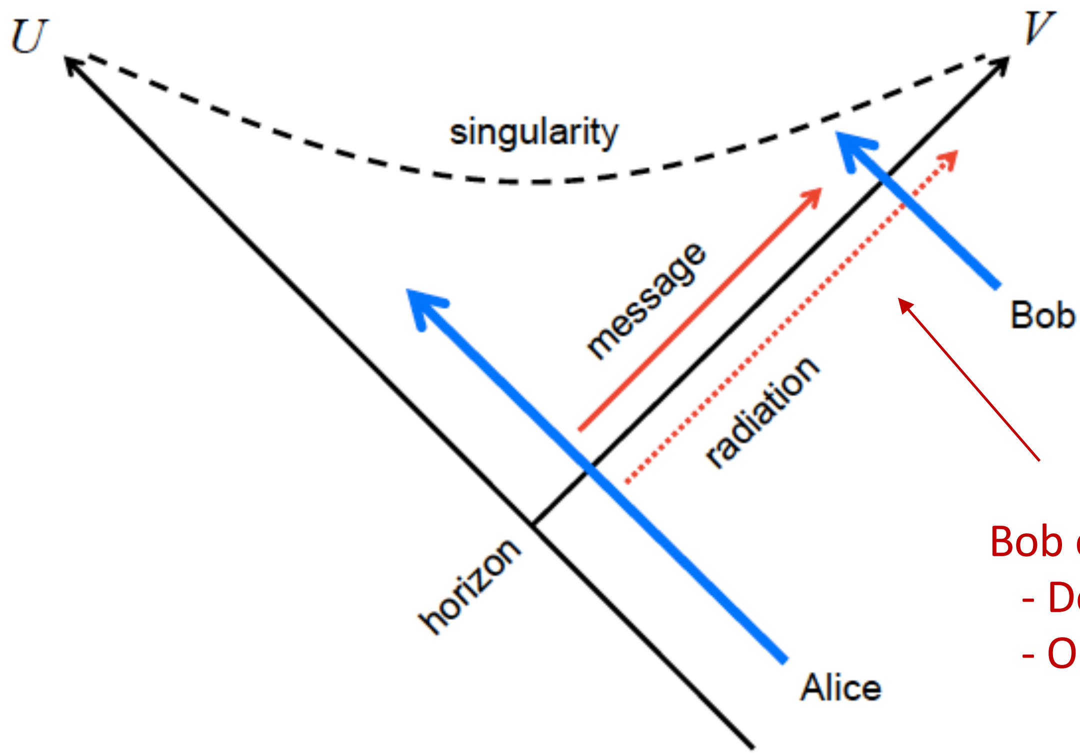
Relation to the black hole cloning paradox

Relation to the black hole cloning paradox

Susskind-Thorlacius
Hayden-Preskill

- Alice has an old black hole.
- Bob has a quantum computer entangled with Alice’s black hole.
- Alice sends in a M-qubit message. And waits for it to scramble in
- Bob only needs a few more than M qubits (2M bits) of Hawking radiation from Alice’s hole to decode the message.

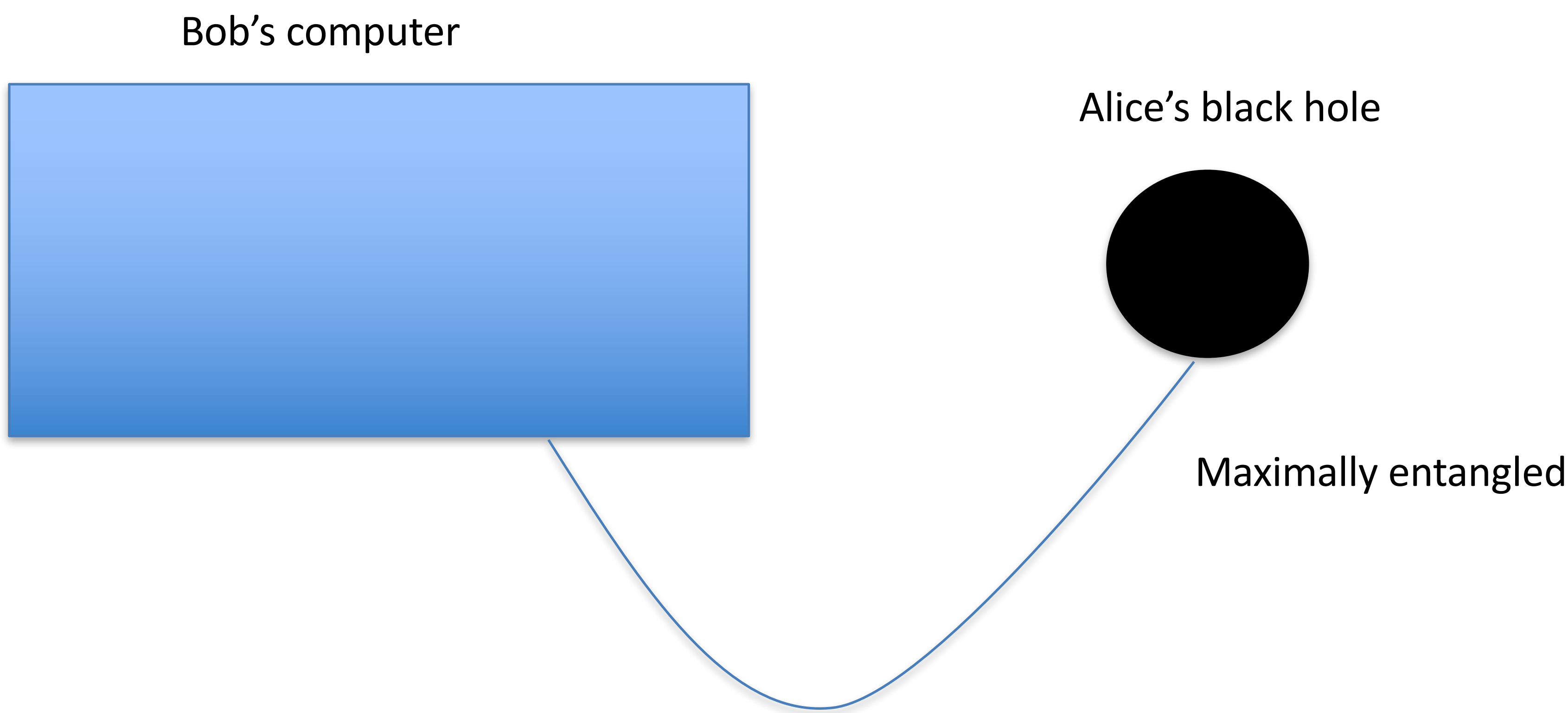
Cloning puzzle



Bob could see two copies:
- Decoded one
- One Alice sent

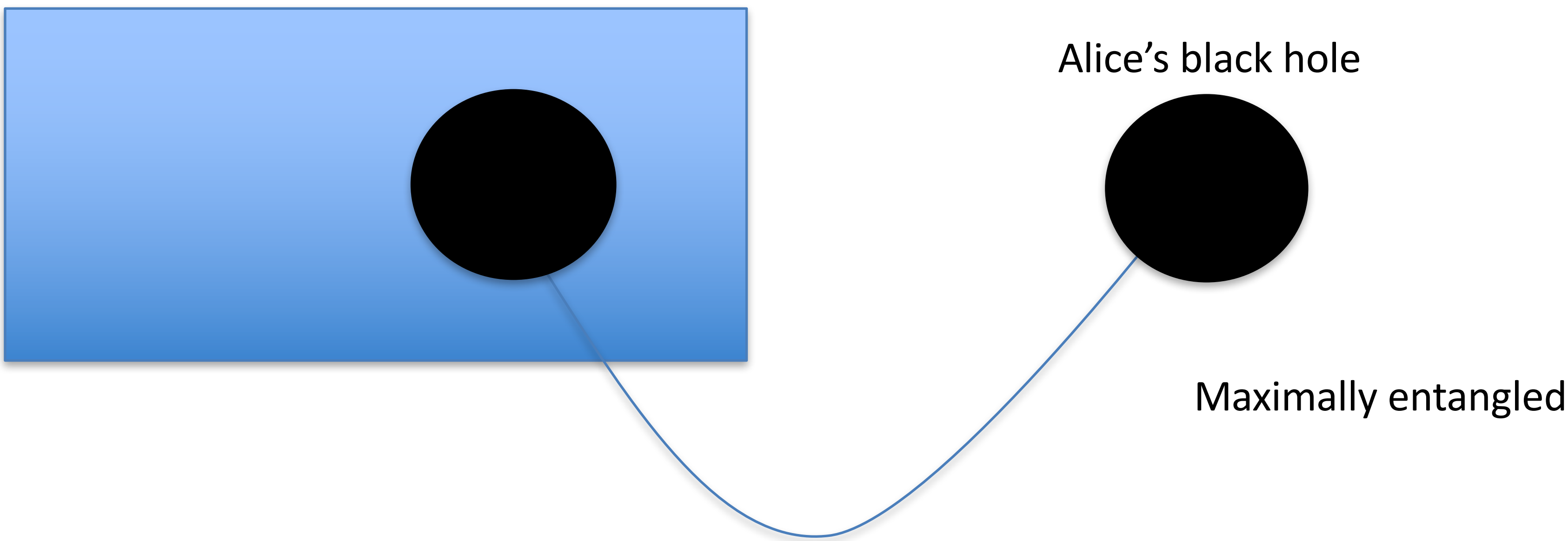
Figure from the paper of Preskill and Hayden.

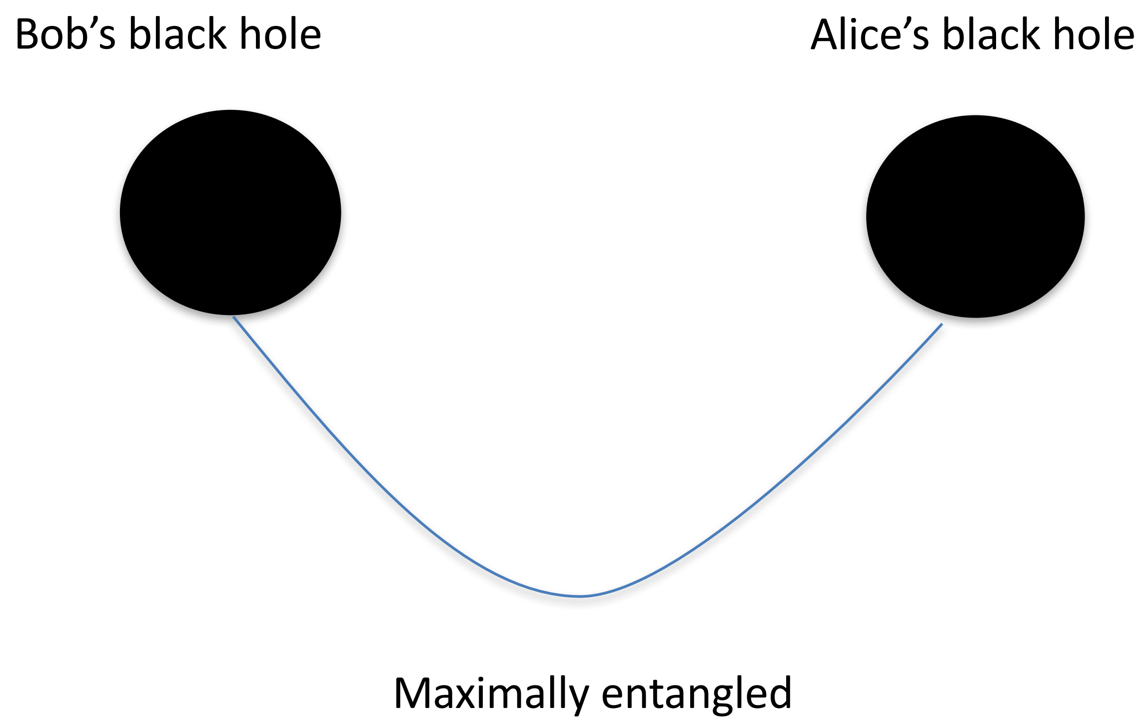
Analysis


Bob's computer
Alice's black hole
Maximally entangled

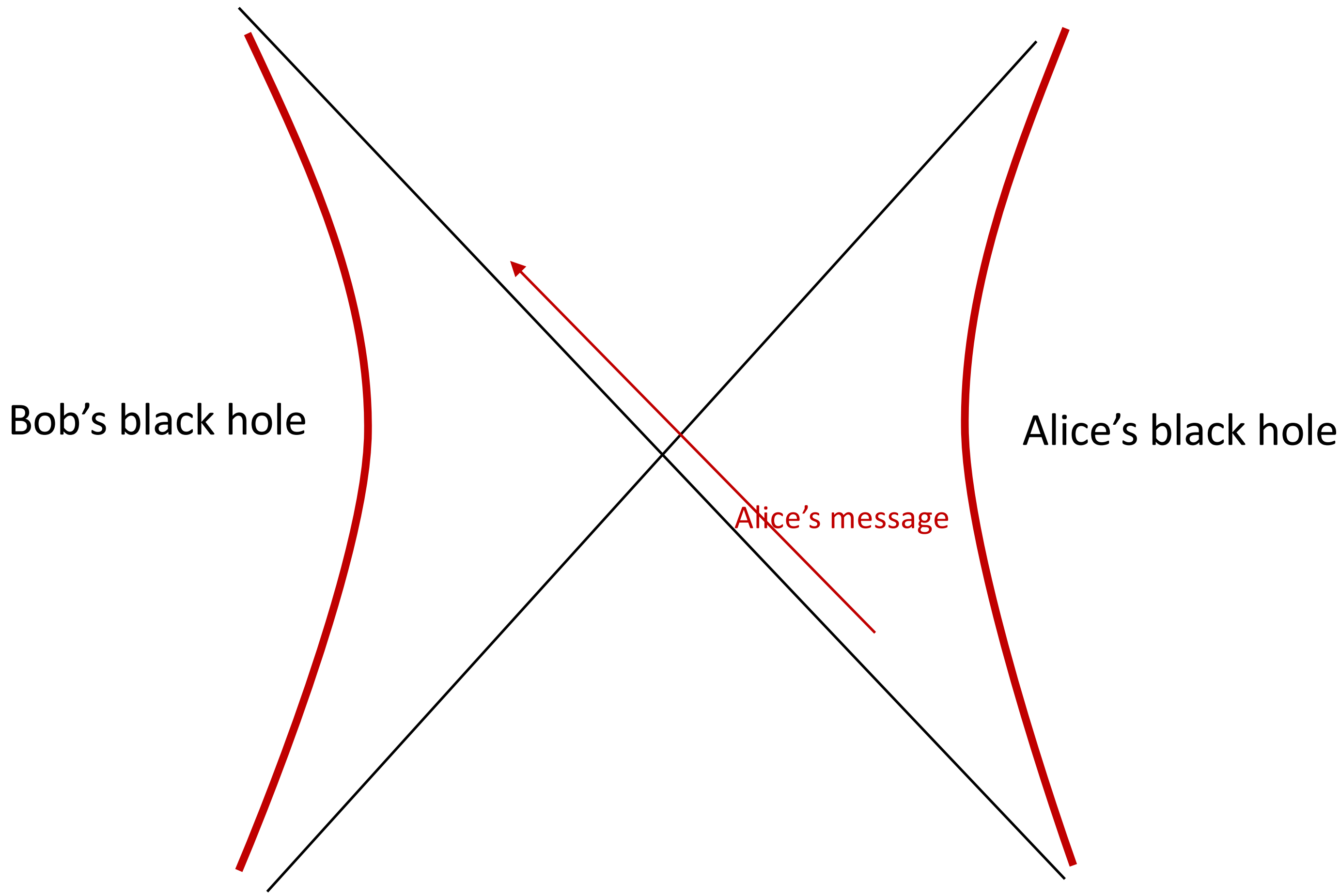
Bob → produces a second black hole, maximally entangled with the first.

(This is hard to do Harlow Hayden)

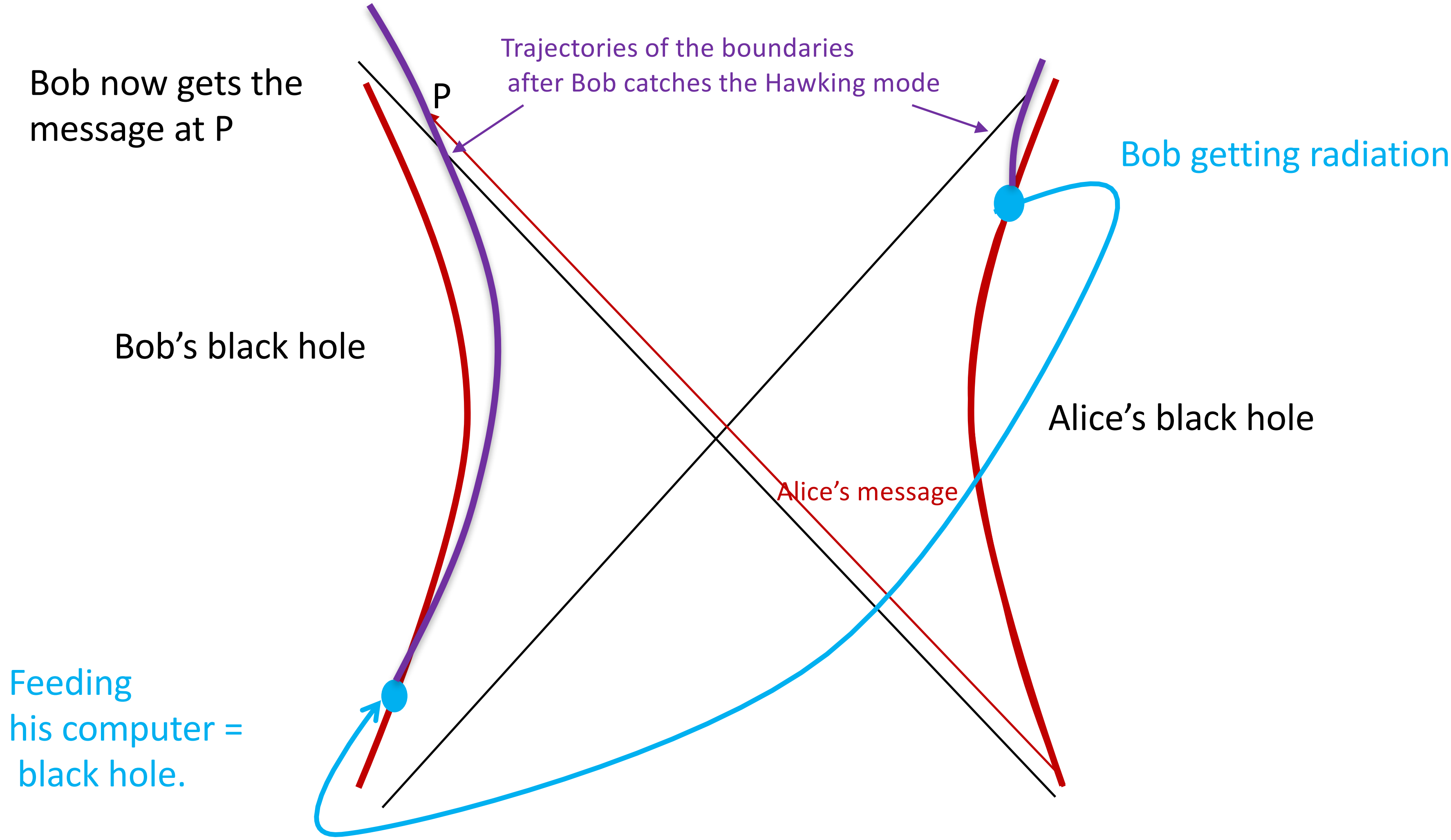



Bob's black hole
Alice's black hole
Maximally entangled

Say they are nearly AdS_2 black holes…

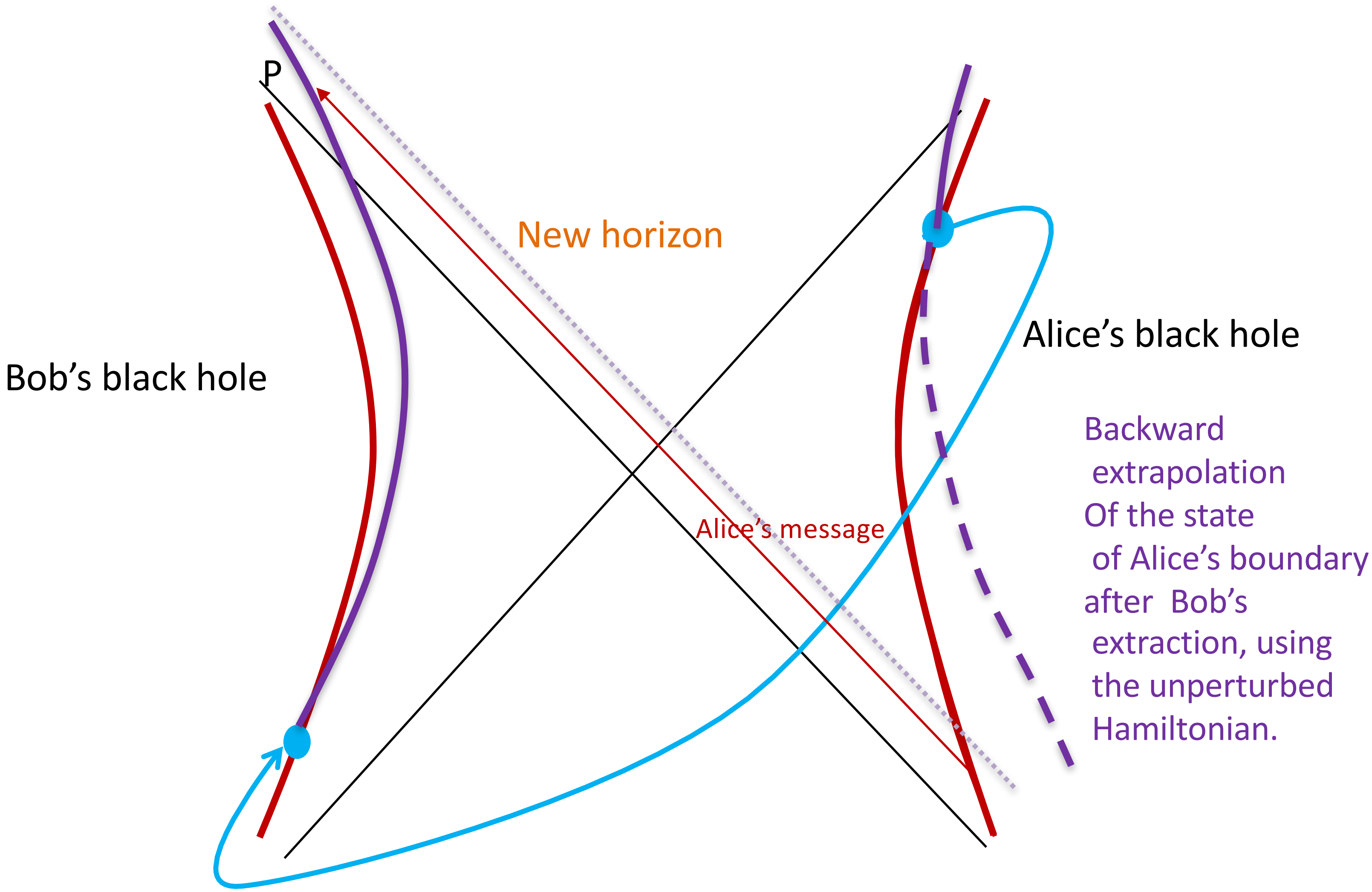

Bob's black hole
Alice's black hole
Alice's message

Bob gets some radiation and feeds it to his computer (black hole).

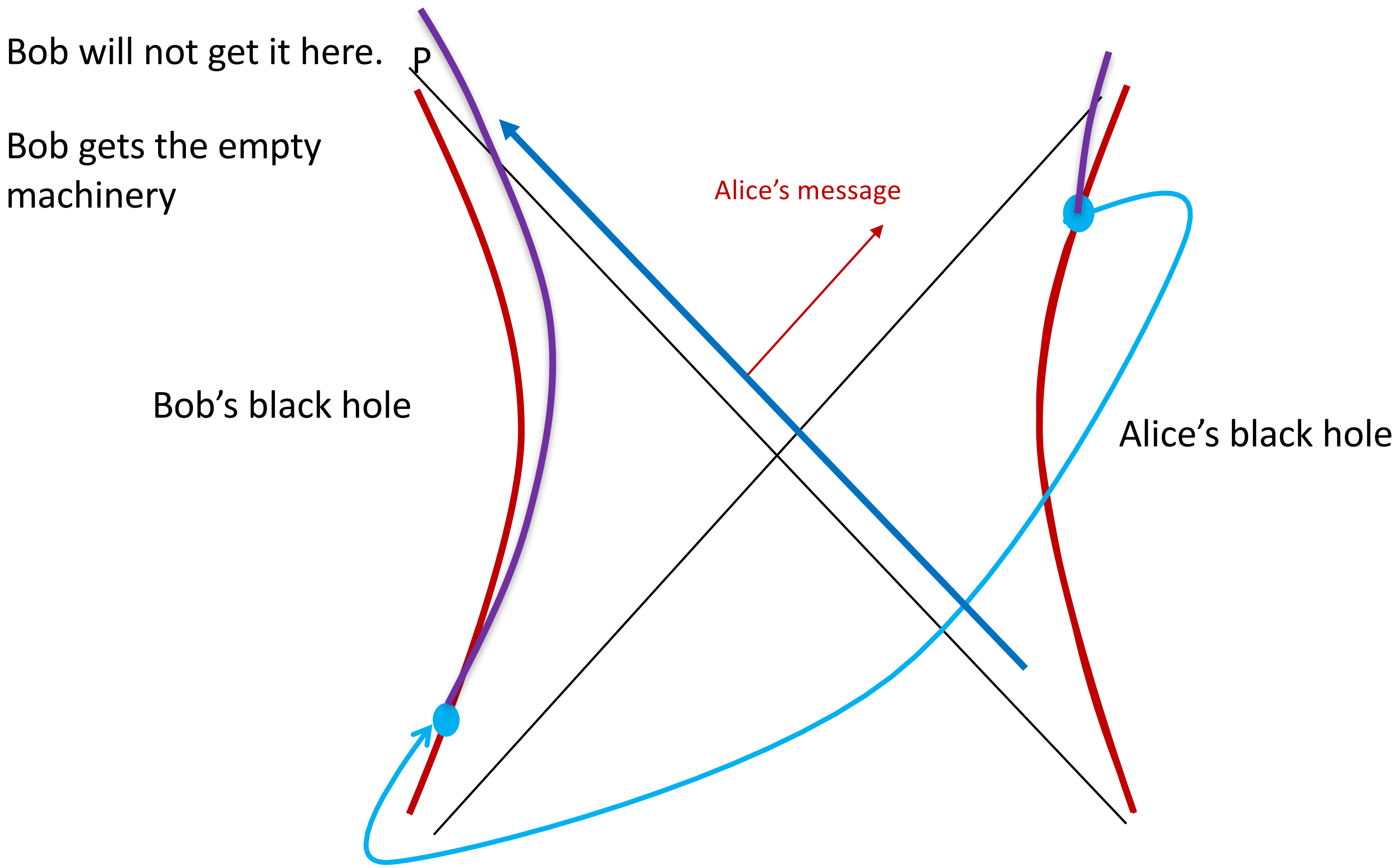


Only one copy of the message in the bulk !

The message switched sides !

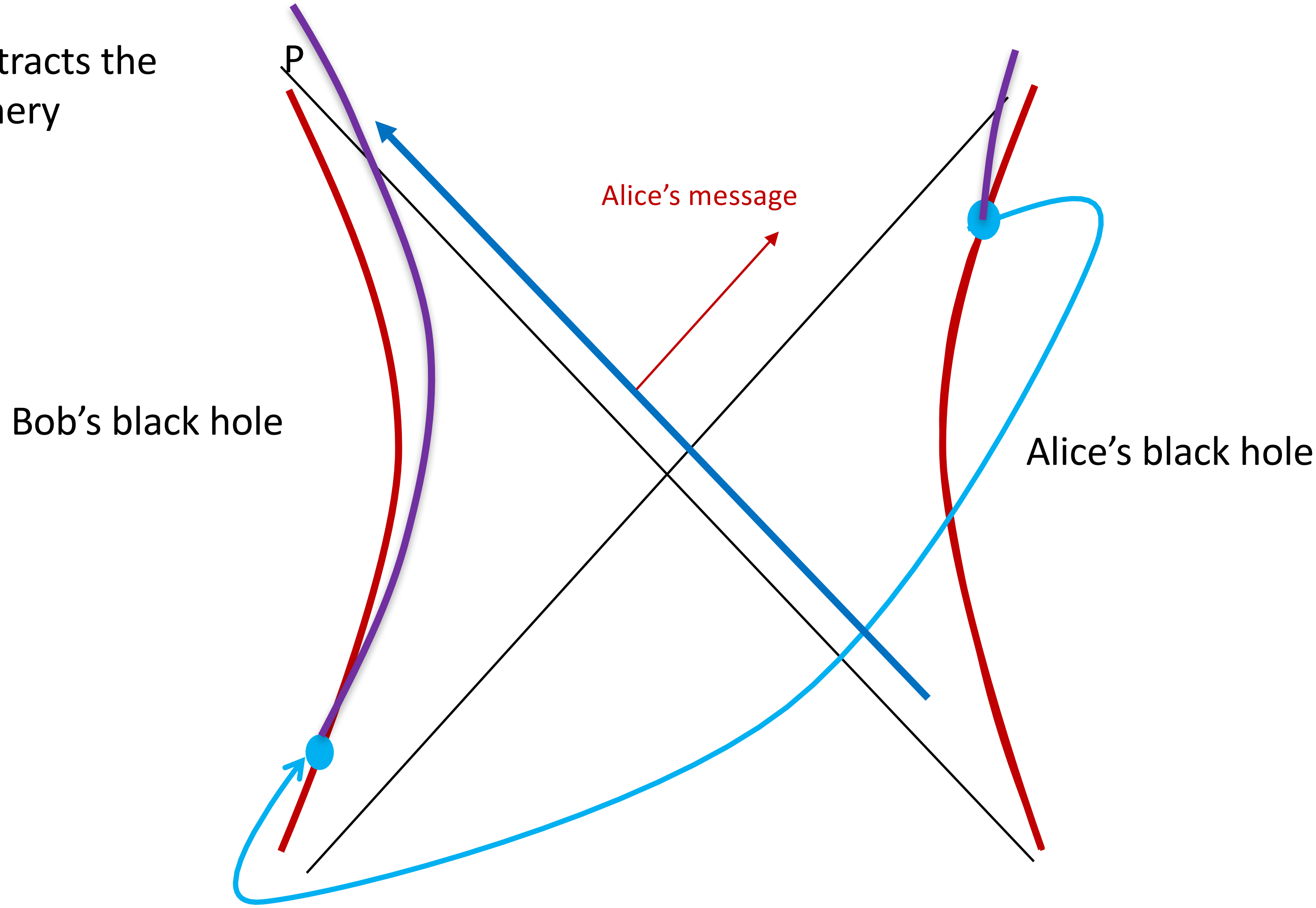


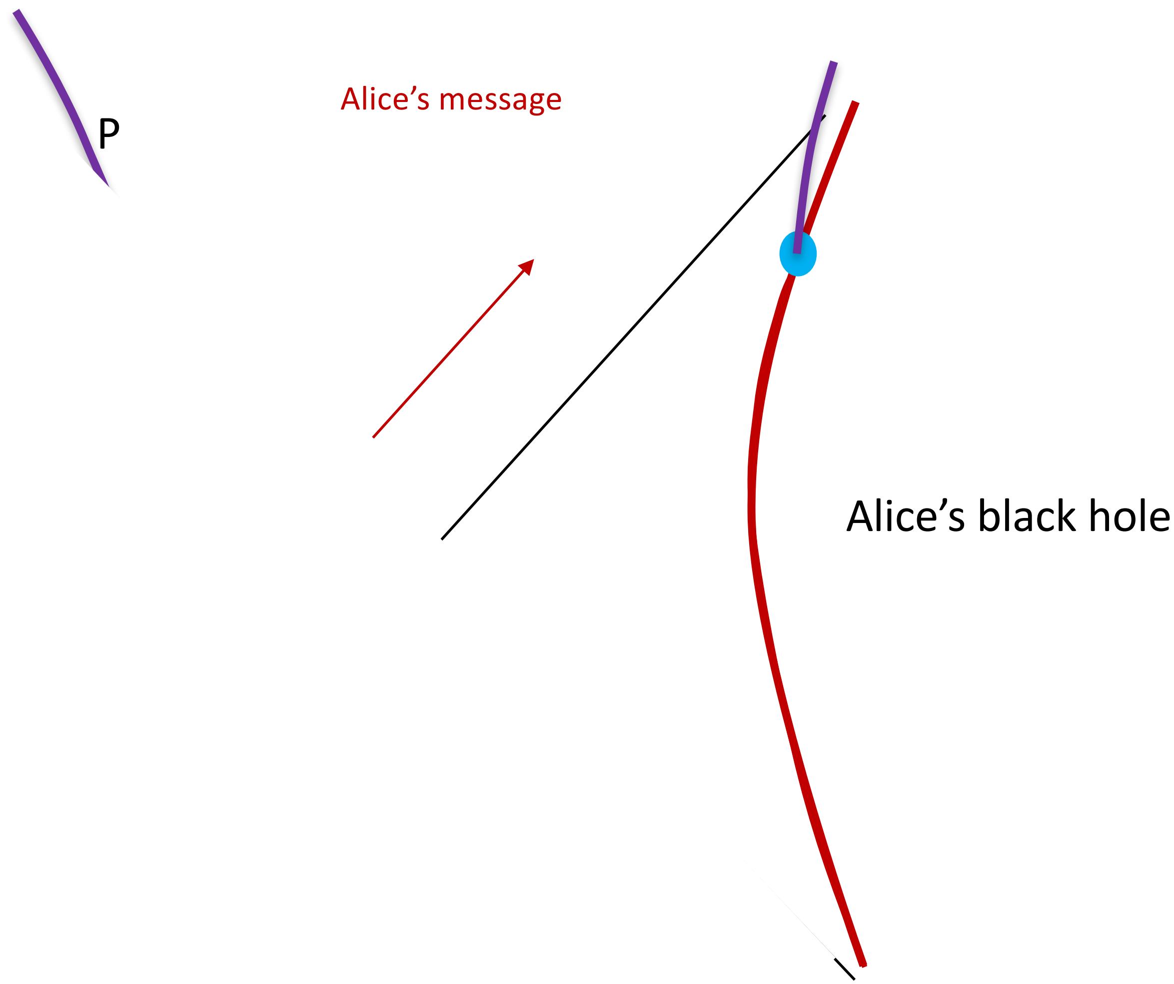
More like the HP figure…



More like the HP figure…

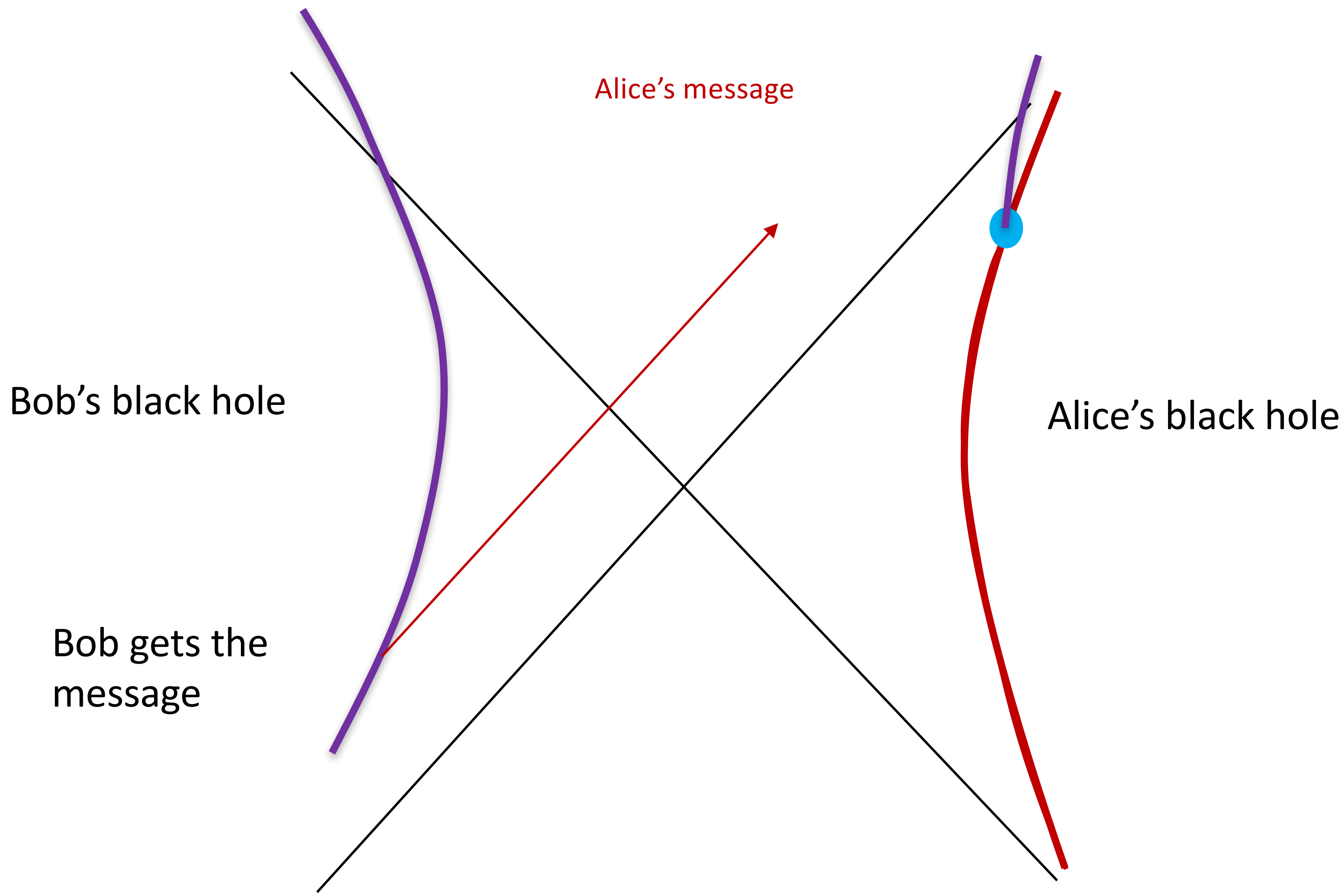
Bob extracts the machinery





P
Alice's message
Alice's black hole

Bob evolves backwards in time



- The message is never duplicated in the bulk picture.
- The process of extracting the message puts it out of reach from Alice.
- No need to invoke unkown new transplanckian physics to solve the no-cloning problem.
- All understandable from standard rules of gravity on the wormhole geometry.
- Assumes ER=EPR.

Conclusions

- Simple picture for the gravitational dynamics of nearly-AdS_2.
- Traversability has a simple origin.
- Nothing special is felt by the traveler.
- Traversability and teleportation.
- Traversability in SYK has the same description.
- Consistent with information transfer bounds.
- We constructed a full set of pure states which appear to have smooth horizons. And generate the Hilbert space.
- We showed how to look at the whole region behind these horizons.

Extra Slides

Nearly AdS_2

Keep the leading effects that perturb away from AdS_2

Jackiw Teitelboim
Almheiri Polchinski

$$\phi_0 \int d^2x \sqrt{g} R + \int d^2x \sqrt{g}\, \phi (R+2)$$

Ground state entropy

Comes from the area of the additional dimensions, if we are getting this from 4 d gravity for a near extremal black hole. General action for any situation with an AdS_2 region.

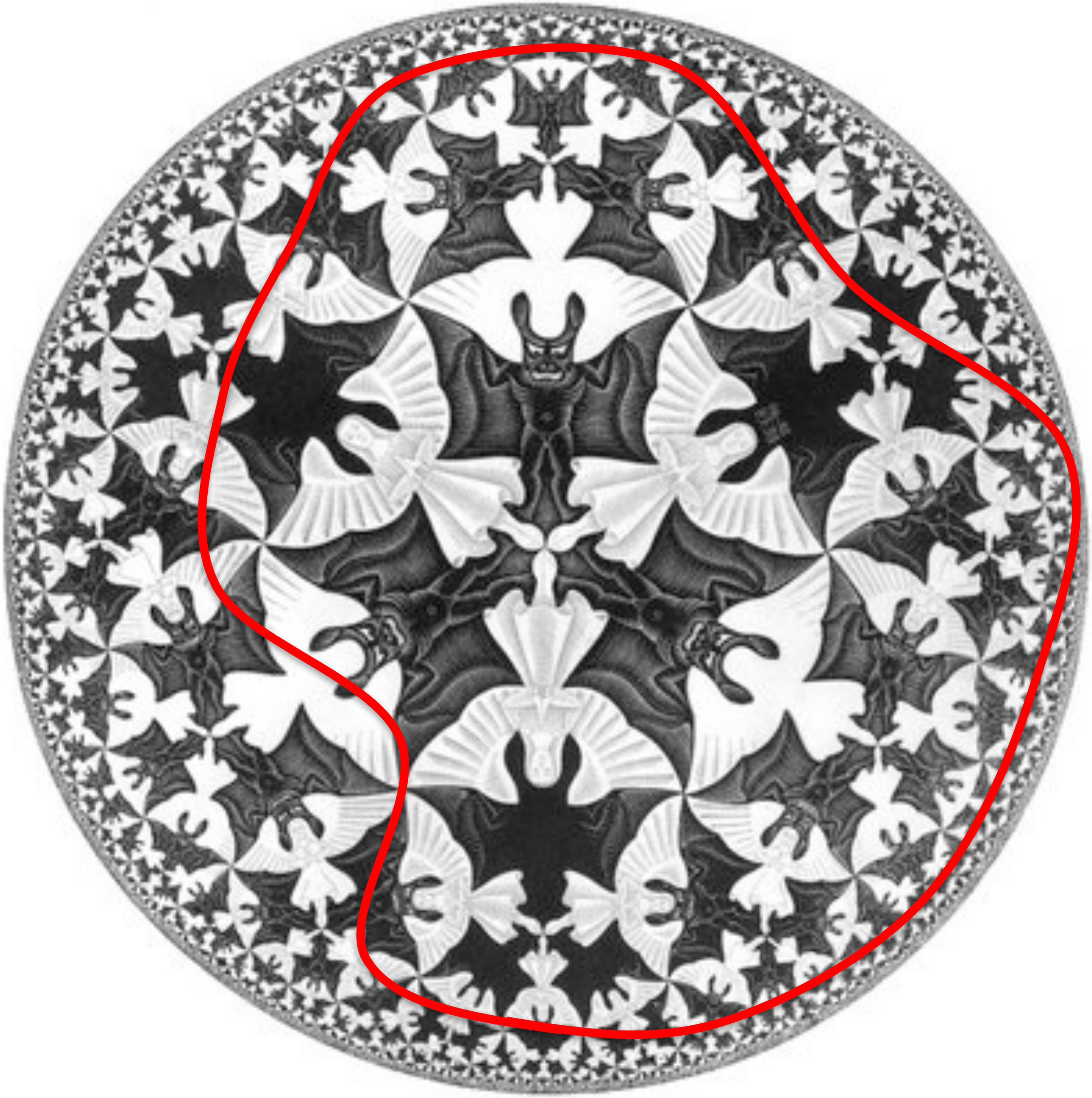
$$\int_{\text{Bulk}} \sqrt{g}\phi(R+2) + \phi_b \int_{\text{Bdy}} K + S_{\text{matter}}[g,\chi]$$

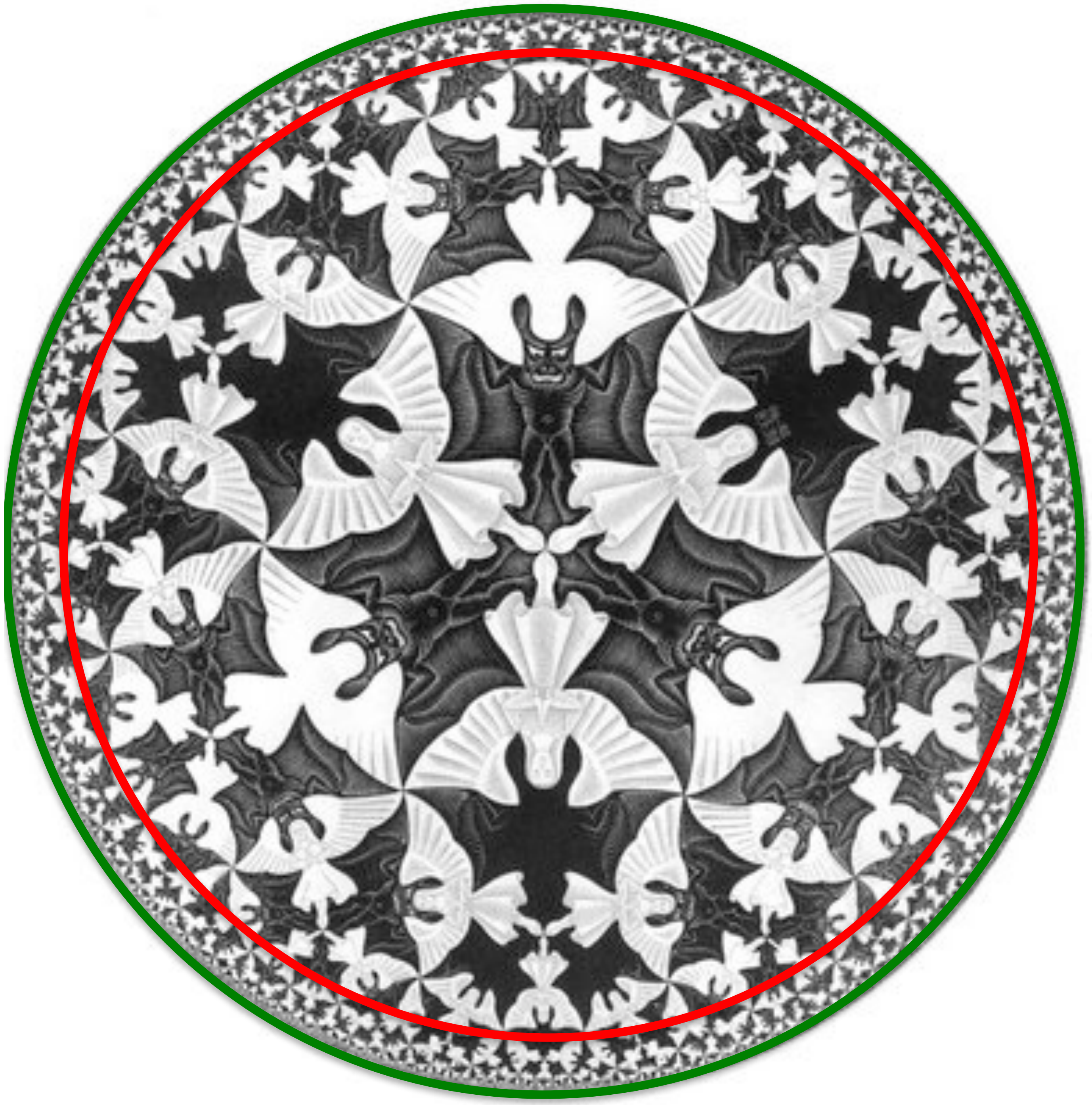
Equation of motion for ϕ → metric is AdS_2 . Rigid geometry !

Only dynamical information → location of the boundary.

$$\phi_b \int_{\text{Bdy}} K$$

Local action on the boundary



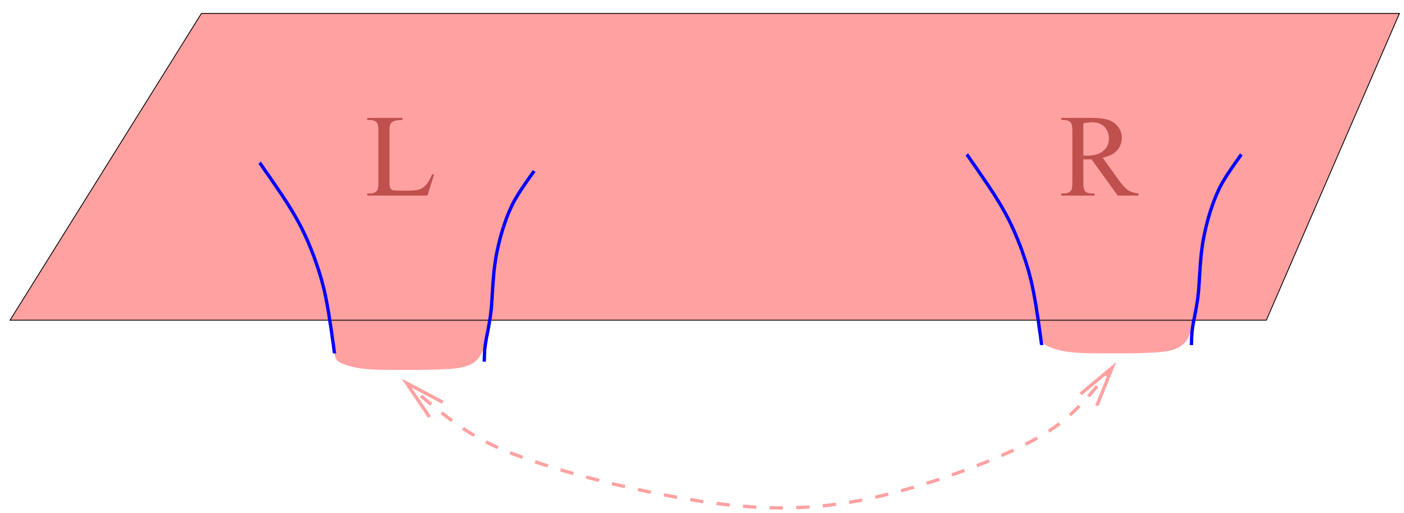


One solution

ADM mass → size

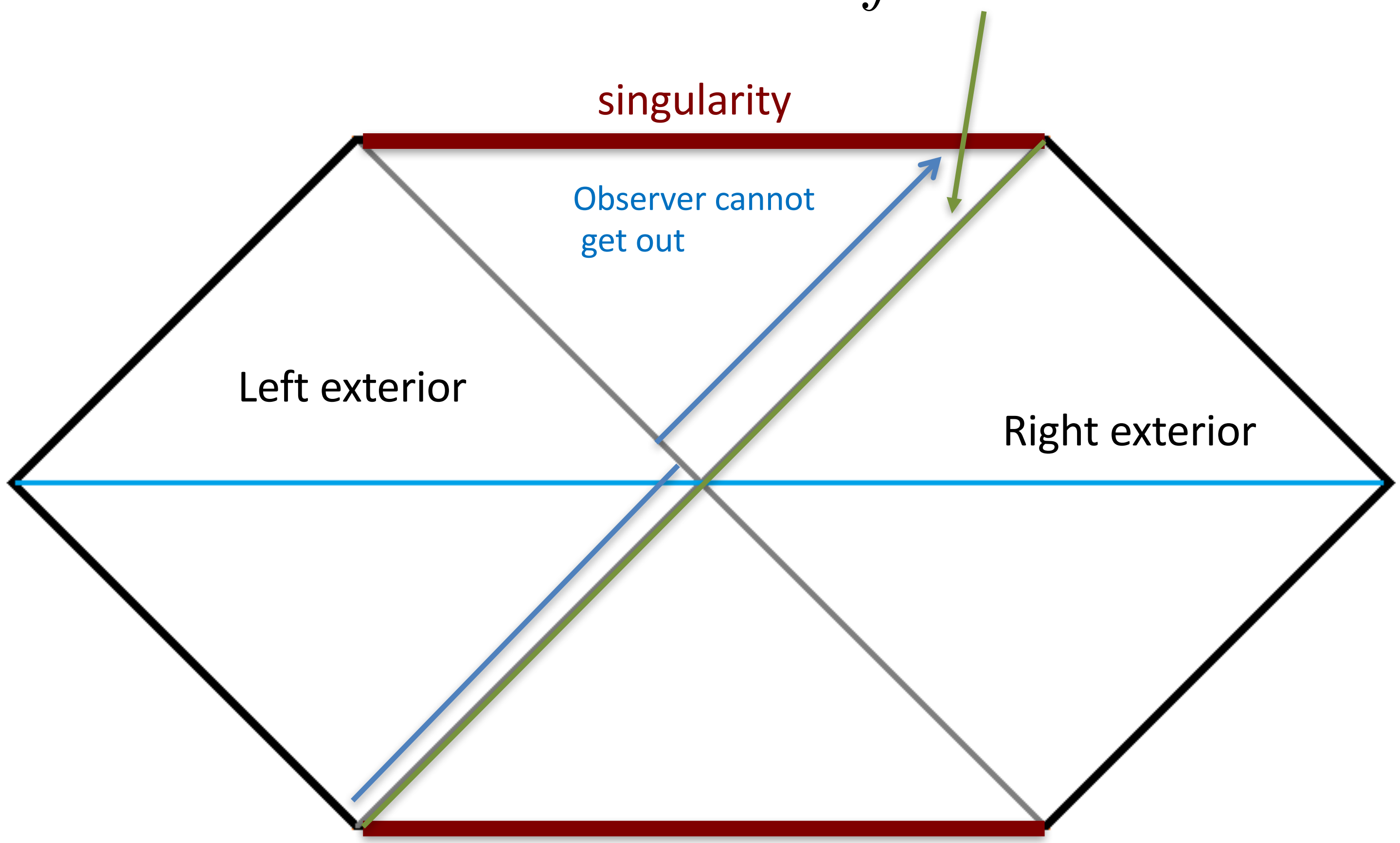
Rest of solutions
Related by AdS_2
isometries

- General relativity has wormhole-like solutions.
- Simplest version is the maximally extended Schwarzschild solution.



These are not traversable. Even quantum mechanically → Integrated null energy condition

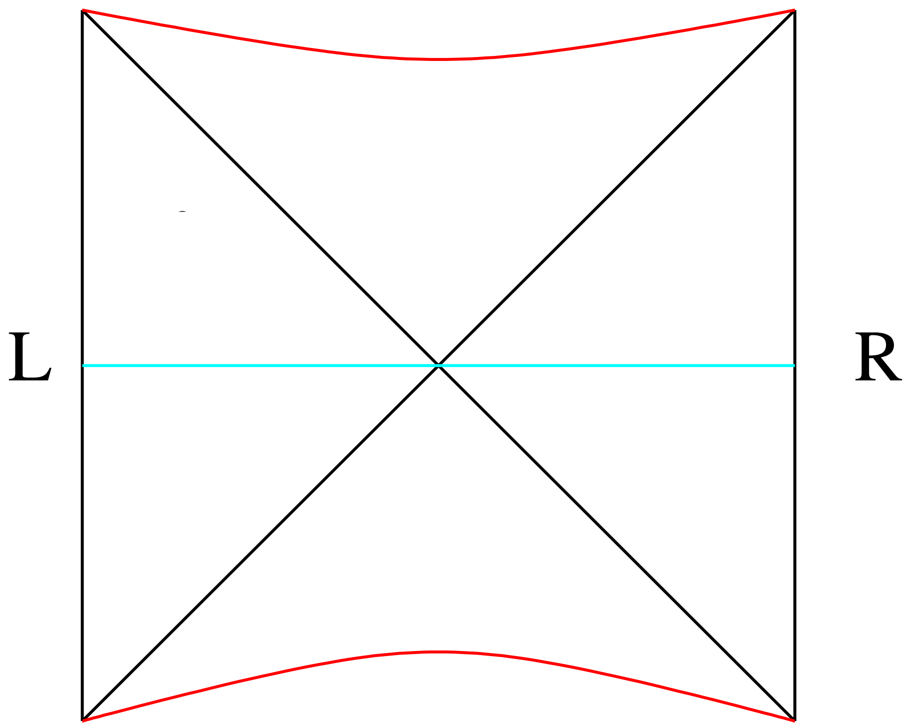
$$\int dx^{+} T_{++} \geq 0$$



- This is good, otherwise general relativity would lead to violations of the principle on which it is based: a maximum propagation speed for signals.

- We will talk about some special situations where it makes sense to talk about traversable wormholes. These do not violate any of the above principles. But they tell us interesting things about black holes.

Kruskal-Schwarzschild-AdS black hole



Geometric connection from entangled

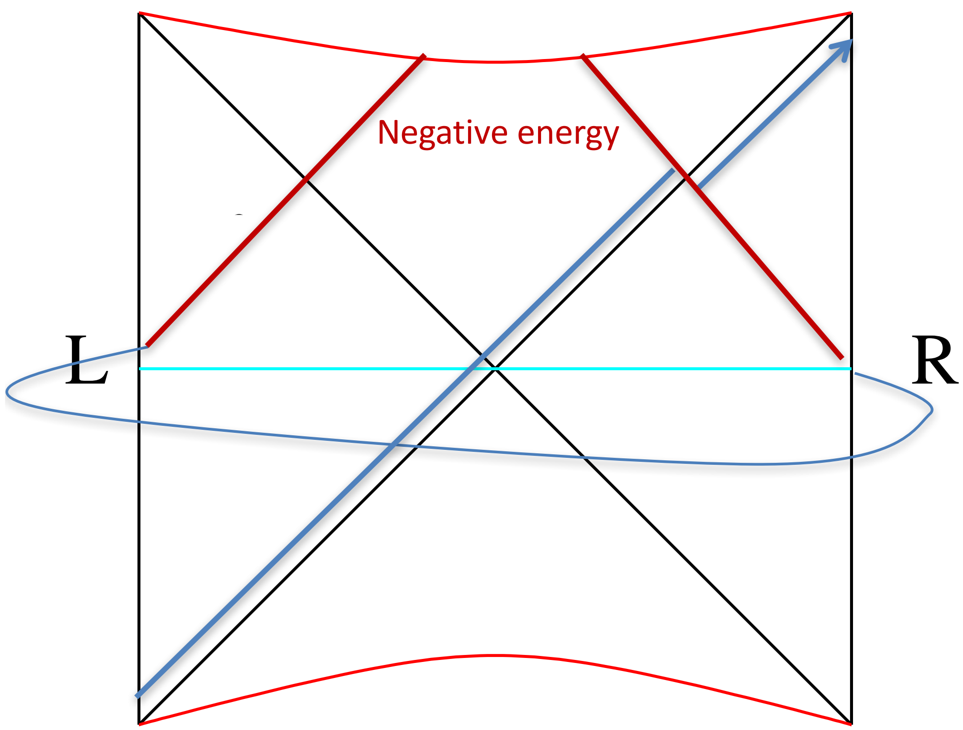
Entangled state in two non-interacting quantum systems.

Israel
JM

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

Traversable wormholes

Gao, Jafferis, Wall



Couple the two field theories.
Direct interaction between fields near each boundary.

$$S_{int} = g\phi_L(0)\phi_R(0)$$

Can create negative energy in the interior →

Gravitational scattering pushes the particle through.

- We will study this phenomenon in more detail for the particular case of nearly-AdS_2 , where the effect is particularly simple
- We will show that the SYK quantum mechanical theory displays the same phenomenon.
- We will show that we can use this to analyze some aspects about cloning of quantum information in black holes.

In Classical mechanics ?

- Two classical systems in the analog of the TFD (same positions and opposite momenta)
- Two cups of water (classical) in the TFD.

- Tap on the left one at some early time, -t .
- At t=0 we let them touch each other and transfer vibrations
- At time t on the right we feel the bump on the right cup.

Similar effect in classical mechanics

- Two classical systems in the analog of the TFD (same positions and opposite momenta)
- At some early time, -t, we perturb x_{2L} on the left side.
- At t=0 we couple $x_{1L}x_{1R}$ (is another coordinate).

$$S_{int} = g(x_{1L}(0) - x_{1R}(0))^2$$

- At time t on the right we measure p_{2R} and find it displaced in a manner correlated with the initial displacement of x_{2L}

Change in x_{1L}
From changing x_{2L}

Change in p_{2R} due to change in p_{1R}

$$\frac{\partial x_{1L}(0)}{\partial x_{2L}(-t)} \frac{\partial p_{2R}(t)}{\partial p_{1R}(0)} \sim \{x_{1L}(0)p_{1L}(-t)\}\{p_{2R}(t)x_{1R}(0)\} \sim (\cdots)^2$$

$$S_{int} = g(x_{1L}(0) - x_{1R}(0))^2$$

Interaction → kick to p_{1R} .

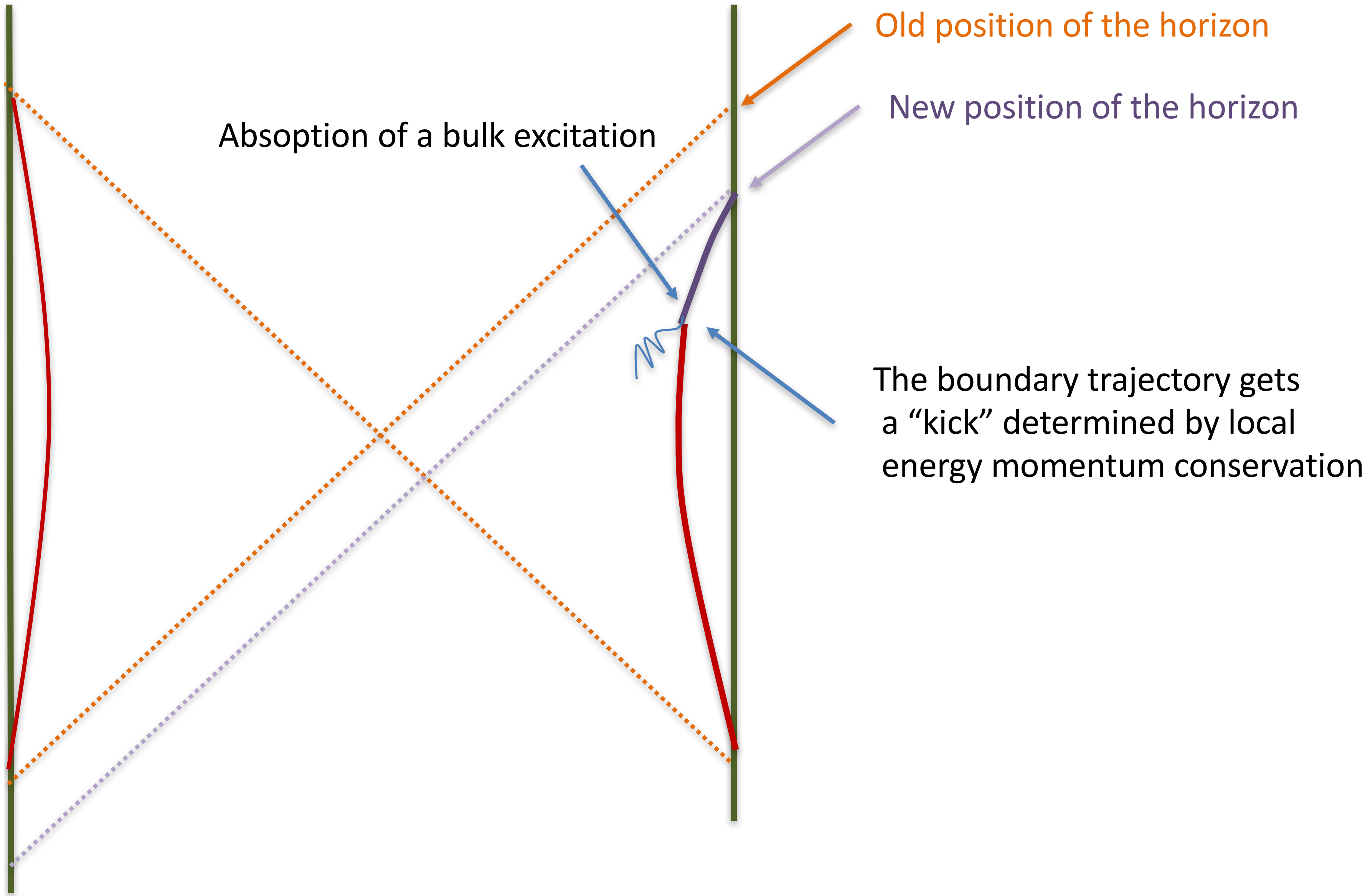
Are equal

Change in p_{2R} is correlated with change of x_{2L}

$$\delta p_{2R}(t) \propto e^{\lambda_L t} \delta x_{2L}(-t)$$

Chaos fueled growth

Dynamics



Old position of the horizon
New position of the horizon
Absoption of a bulk excitation
The boundary trajectory gets
a "kick" determined by local
energy momentum conservation.

The SYK model

N Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$

Sachdev Ye Kitaev
Georges, Parcollet

$$H = \sum_{i_1, \cdots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

Random couplings, gaussian distribution. $\langle J^2_{i_1 i_2 i_3 i_4} \rangle = J^2/N^3$

To leading order → treat J_{ijkl} as an additional field

J = dimensionful coupling. We will be interested in the strong coupling region

$$1 \ll \beta J, \ \tau J \ll N$$

Define new variable

$$G(\tau,\tau') = \frac{1}{N}\sum_i \langle \psi_i(\tau)\psi_i(\tau')\rangle$$

Integrate out fermions and get an action in terms of a new field G

$$S = N f[G(\tau,\tau')]$$

Is analogous to the full bulk gravity + matter action.

There is a particular G that minimizes the action. It is SL(2) invariant. (analogous to the vacuum AdS geometry)

Set of low action fluctuations of this solution. Parametrized by a function of a single variable. Reparametrization mode.

$$G_f = (f'(\tau)f'(\tau'))^{\Delta} G(f(\tau), f(\tau'))$$

Low energy action:

$$S = \frac{N}{J}\int \{f,\tau\} d\tau$$

Same as the action for the UV boundary in the gravity description.

$$H = \sum_{i_1, \cdots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

$$\hat{G}(t,t') = \frac{1}{N} \sum_i \psi^i(t) \psi^i(t')$$

$$\langle \hat{G}(t,t') \rangle = G_c(t,t') \propto \frac{1}{|t-t'|^{2\Delta}}$$

Low energies.

$$\int \mathcal{D}\hat{G} e^{-S[\hat{G}]} \to \int \mathcal{D}f e^{-S[f]}$$

$$S[f] \propto -\frac{N}{J} \int \{f, \tau\} d\tau$$

Same as the action for the UV boundary in the gravity description.

$$\int \mathcal{D}\hat{G} \hat{G}(t,t') e^{-S[\hat{G}]} \to \int \mathcal{D}f e^{-S[f]} (f'(t) f'(t'))^{\Delta} G_c(f(t), f(t'))$$

Same precise formula for the 2pt function

$$C = \langle e^{-igV} \chi_R(t) e^{igV} \chi_L(-t) \rangle \ , \qquad V = g \phi_L(0) \phi_R(0)$$

$\downarrow$

$$C = \langle e^{-igV} \psi_{j\,R}(t) e^{igV} \psi_{j\,L}(-t) \rangle \ , \qquad V = g \frac{1}{K} \sum_{j=1}^{K} \psi_L(0) \psi_R(0)$$

$$C \sim \int dp (p)^{2\Delta - 1} e^{ip} e^{-ig} e^{i \frac{g}{(1 + p e^t)^{2\Delta}}}$$

Amount of information we can send is roughly g

Before transfer: Alice has the message but Bob does not

After transfer: Bob has it but Alice does not !