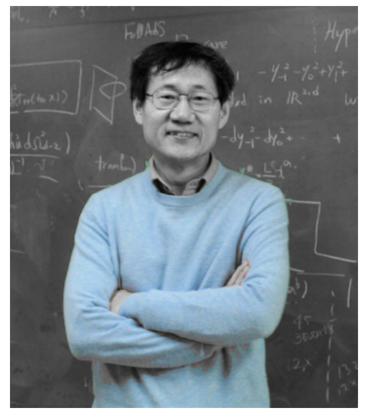
Based on [JY, 1906.08815]
[Y. Qi, S. Sin, JY, 1906.00996]
[P. Narayan, JY, 1903.08761]


String 2019 - Brussels, Belgium

Chaos in Higher Spin Gravity

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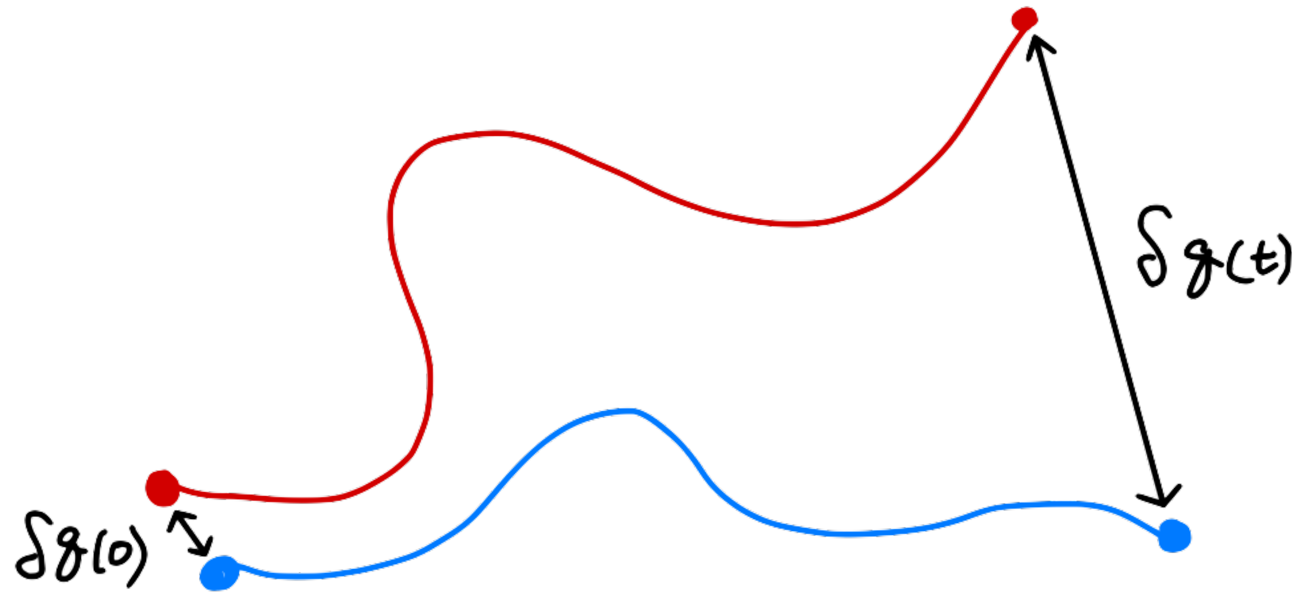
Brussels, Belgium
July, 9, 2019

imagine the impossible
KIAS KOREA INSTITUTE FOR ADVANCED STUDY

Introduction: Chaos and OTOC

❖ Butterfly Effect is the sensitivity of the system to the initial condition.

$$\frac{\delta q(t)}{\delta q(0)} \sim e^{\lambda t}$$



❖ In quantum system, the exponential growth can be captured by the out-of-time-ordered correlation (OTOC)

$$\langle V(t)W(0)V(t)W(0)\rangle \sim 1 - \frac{1}{c}e^{\lambda_L t}$$

λ_L: Lyapunov exponent

❖ **Bound on Chaos** [Maldacena, Shenker, Stanford, 1503.01409]

$$\lambda_L \leqq \frac{2\pi}{\beta}$$

- saturated by black hole, SYK(-like) models: Maximal Chaos $\lambda_L = \frac{2\pi}{\beta}$

Schwarzian Action

❖ The **Schwarzian action** is responsible for the saturation of the bound on chaos in SYK-like models.

$$S = \int d\tau \left(-\frac{c}{12}\mathrm{Sch}[\phi(\tau),\tau] - \frac{c}{24}[\phi'(\tau)]^2 \right) \qquad \mathrm{Sch}[\phi(\tau),\tau] \equiv \frac{\phi'''}{\phi'} - \frac{3}{2}\left(\frac{\phi''}{\phi'}\right)^2$$

❖ The Schwarzian action appear as

- ❖ Low Energy Effective action of maximally chaotic models (e.g. SYK-like models)
- ❖ Boundary action for 2D dilaton gravity on the nearly-AdS_2
- ❖ Coadjoint orbit $\mathrm{Diff}(S^1)/SL(2)$

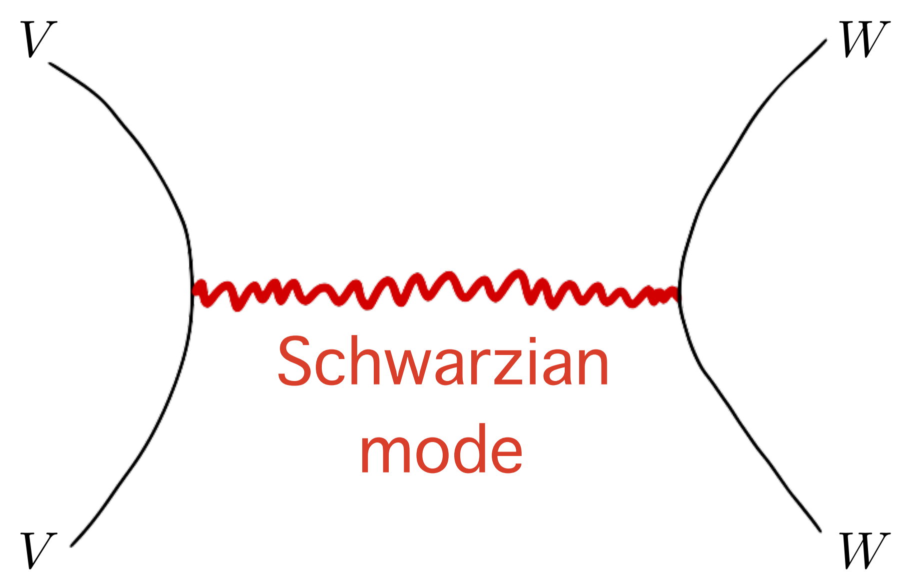
GOAL

- ❖ The contributions of Schwarzian mode to OTOCs
  - assume that the contribution of the Schwarzian mode dominates
- ❖ Higher spin generalization of Schwarzian action at finite temperature
- ❖ The contribution of the higher spin Schwarzian mode to OTOC

The Contribution of Schwarzian to OTOC

❖ Schematically, the contribution of Schwarzian mode to OTOC

$$\langle V(t)W(0)V(t)W(0)\rangle \supset$$



cf. [F, Haehl, M. Rozali]

❖ **Propagator** of Schwarzian mode: In large c expansion around the saddle point, the quadratic action gives the (free) propagator of the Schwarzian mode

❖ "**Coupling**" to matter two point function: We expand two point functions "dressed" by Schwarzian mode (or, conformal transformed two point function) w.r.t. infinitesimal Schwarzian mode

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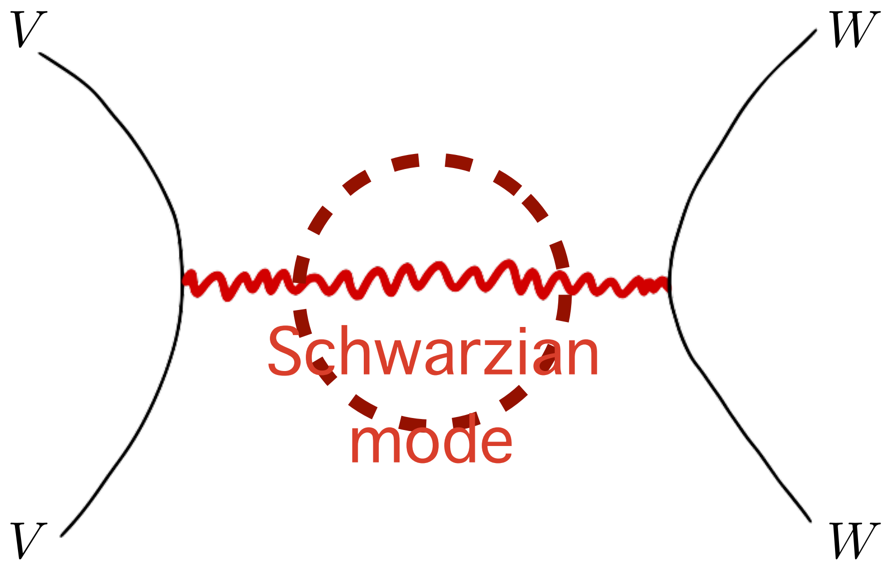

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$$S = \int d\tau \left(-\frac{c}{12}\mathrm{Sch}[\phi(\tau),\tau] - \frac{c}{24}[\phi'(\tau)]^2\right) \xrightarrow{\phi(\tau) = \tau + \frac{1}{\sqrt{c}}\epsilon(\tau)} S = cS^{(0)} + S^{(2)} + \frac{1}{\sqrt{c}}S^{(3)} + \frac{1}{c}S^{(4)}\cdots$$

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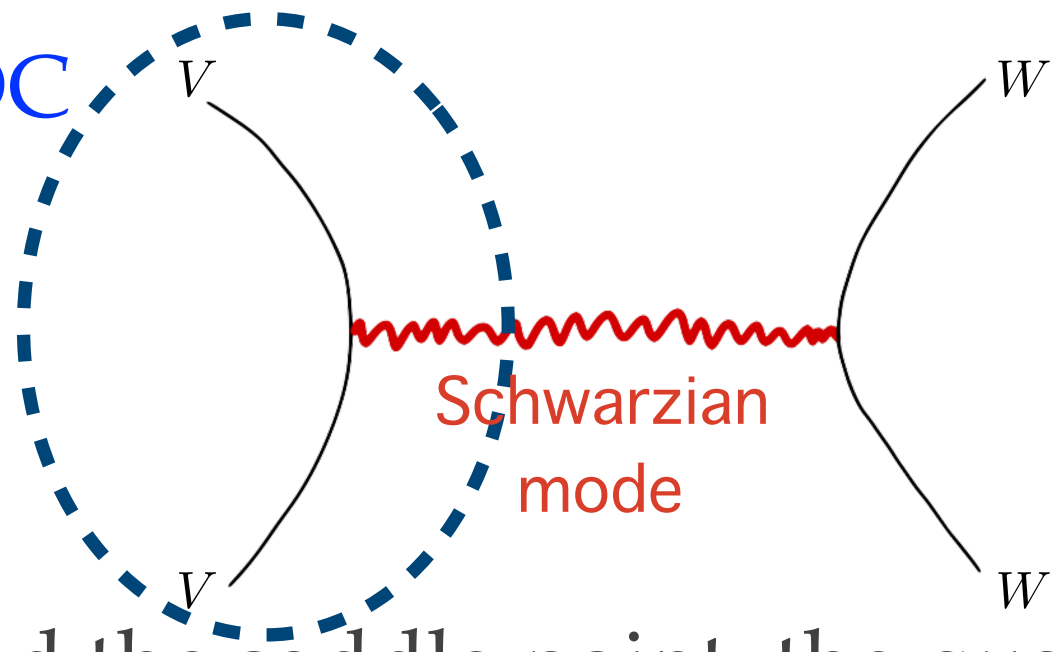

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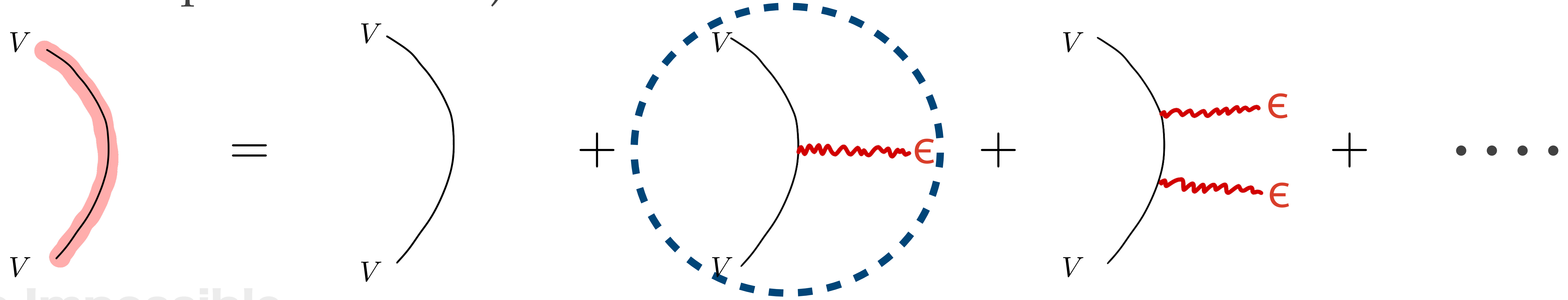

Coadjoint Orbit

- General coadjoint orbit of reparametrization [E. Witten, 1988], …, [D. Stanford, E. Witten, 1703.04612]:

$$S = \int d\tau \left(-\frac{c}{12}\mathrm{Sch}\left[\phi(\tau), \tau\right] + b_0 [\phi'(\tau)]^2 \right)$$

  - Diff(S1)/SL(2): $b_0 = -\frac{c}{24}$
  - Diff(S1)/U(1): $b_0 \neq -\frac{cn^2}{24} \quad (n \in \mathbb{Z})$

- In the semi-classical analysis (large c limit), the stability condition of the quadratic action requires

$$S^{(2)} = \frac{\pi}{12} \sum_{|n|>n_0} n^2 \left(n^2 + \frac{24}{c} b_0 \right) \epsilon_{-n}\epsilon_n \longrightarrow -\frac{c}{24} \leqq b_0$$

: **Stability Condition** [E. Witten, 1988]

- Propagator of Schwarzian mode (from quadratic action)

- Lyapunov exponent:

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: **Stability Condition** [E. Witten, 1988]

❖ Propagator of Schwarzian mode (from quadratic action)

ϵ  ϵ

$$\langle \epsilon_{-n}\epsilon_n \rangle = \frac{\frac{6}{\pi}}{n^2\left(n^2 + \frac{24}{c}b_0\right)}$$

❖ Lyapunov exponent:

$$\lambda_L = \frac{2\pi}{\beta}\sqrt{\frac{24}{c}|b_0|}$$

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  - Diff(S^1)/SL(2): $b_0 = -\frac{c}{24}$ ☞ maximal Lyapunov exponent [Maldacena, Stanford], [Jevicki, Suzuki, JY], [Jensen], [Verlinde, Mertens, Turiaci], [Narayan, JY], [Mandal, Nayak, Wadia], [Kitaev, Suh], [Castro, Papadimitriou], [Poojary], [Jahnke, Kim, JY]…
  - Diff(S^1)/U(1): $b_0 \neq -\frac{cn^2}{24}$ $(n \in \mathbb{Z})$ ☞ reduced Lyapunov exponent [Banerjee, Altman], [Garcia-Garcia, Romero-Bermdez, Tezuka], [Nosaka, Rosa, JY], [Ferrari, Schaposnik Massolo], [Ferrari, Valette], …

- In the semi-classical analysis (large c limit), the stability condition of the quadratic action requires

$$S^{(2)} = \frac{\pi}{12} \sum_{|n|>n_0} n^2 \left(n^2 + \frac{24}{c} b_0 \right) \epsilon_{-n} \epsilon_n \quad \longrightarrow \quad -\frac{c}{24} \leqq b_0$$

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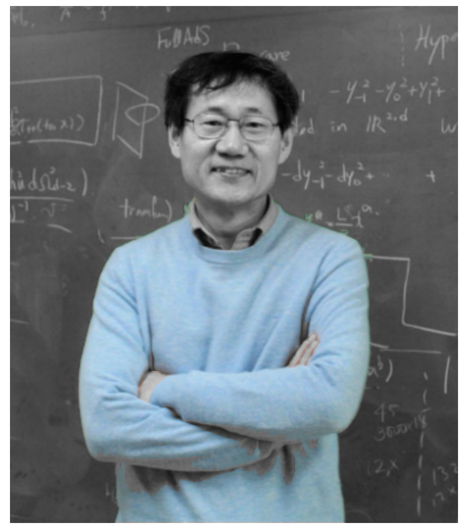


$^1/_c$ Correction to OTOC

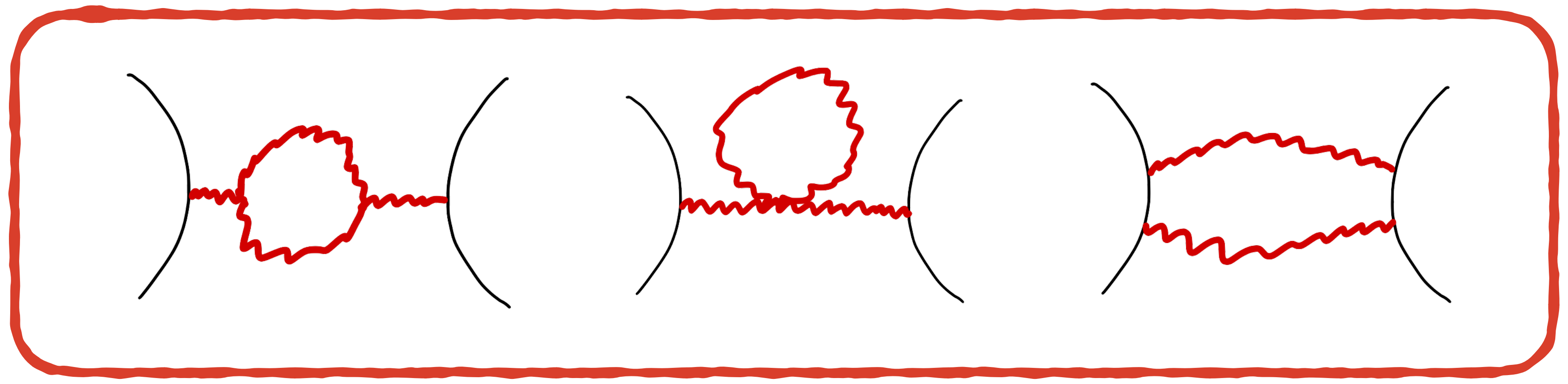
❖ Cubic and quartic vertices from 1/c expansion of Schwarzian action (Diff(S¹)/SL(2))

$$S = cS^{(0)} + S^{(2)} + \frac{1}{\sqrt{c}} S^{(3)} + \frac{1}{c} S^{(4)} \cdots$$

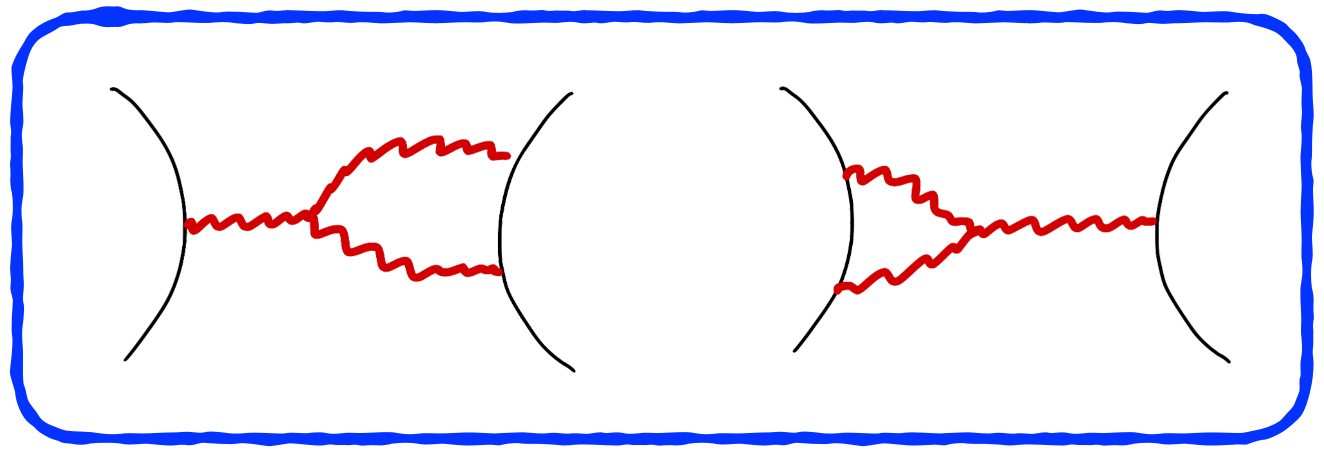
❖ 1/c correction to OTOC

$$\langle \text{OTOC} \rangle \sim 1 - \frac{1}{c} e^{\frac{2\pi}{\beta} t} + \frac{1}{c^2} \mathcal{F}^{(2)}(t) + \mathcal{O}(1/c^3)$$

cf. [J. Kaplan, A.L. Fitzpatrick]



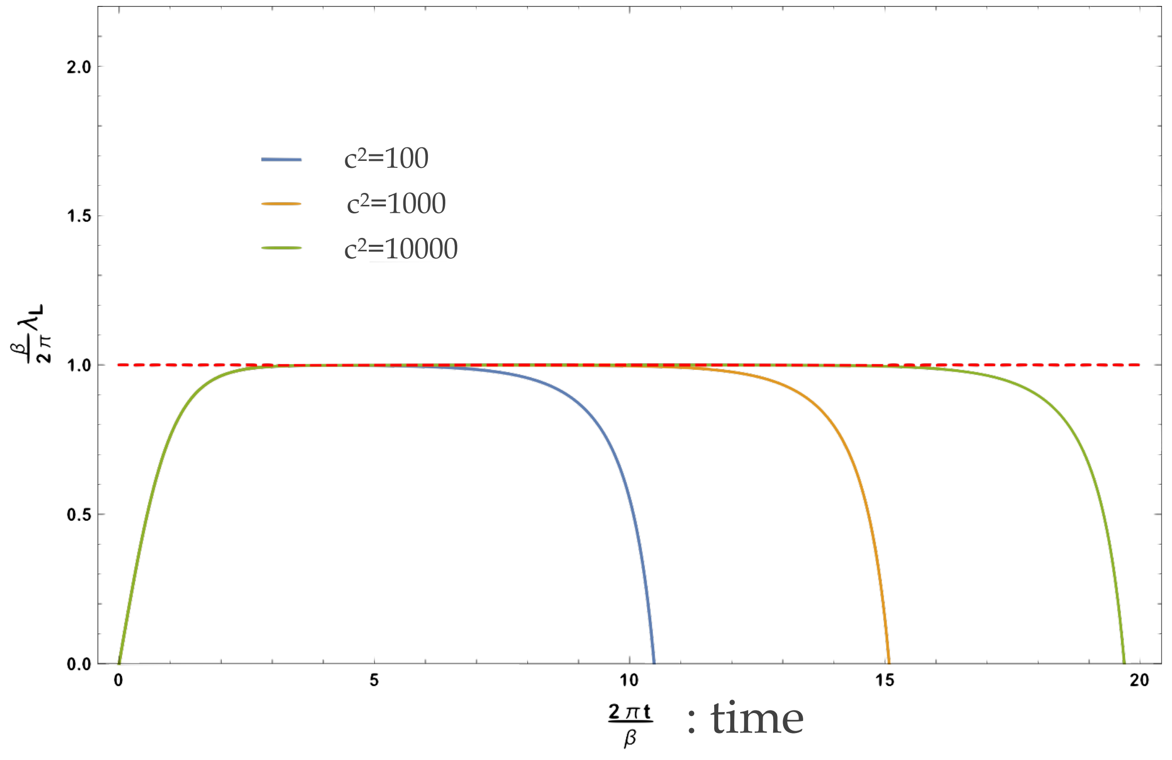
Speed up the exponential growth



Slow down the exponential growth

❖ In total, the contribution of order $1/c^2$ **slows down** the exponential growth of OTOC.

Lyapunov Exponent



❖

Higher Spin Generalization

❖ [E. Perlmutter,1602.08272] evaluated Lyapunov exponent from the semi-classical W_N vacuum block

$$\lambda_L^{(s)} = \frac{2\pi}{\beta}(s-1) \qquad \text{(s: spin)}$$

❖ Question: How can we evaluate the Lyapunov exponent in the higher spin gravity?

- ❖ The higher spin generalization of Schwarzian action was observed long time ago.[Bershadsky, Ooguri, 1989][A. Marshakov, A. Morozov, 1990][W. Li, S. Theisen, 2015][H. Gonzlez, D. Grumiller, J. Salzer, 2018]

$$\tilde{\mathcal{T}}_2 = \left\{ \frac{e_+'''}{e_+'} - \frac{4}{3}\left(\frac{e_+''}{e_+'}\right)^2 - \frac{1}{6}\frac{e_+''}{e_+'}\frac{f_+''}{f_+'} \right\} + e_+ \leftrightarrow f_+ \qquad \tilde{\mathcal{T}}_3 = \left\{ \frac{1}{12}\left[\frac{e_+''''}{e_+'} - 5\frac{e_+'''}{e_+'}\frac{e_+''}{e_+'} + \frac{40}{9}\left(\frac{e_+''}{e_+'}\right)^3 \right] - \frac{1}{6}\frac{e_+'''}{e_+'}\frac{f_+''}{f_+'} + \frac{5}{18}\frac{e_+''}{e_+'}\left(\frac{f_+''}{f_+'}\right)^2 \right\} - e_+ \leftrightarrow f_+$$

❖ Two difficulties in the higher spin generalization: We don't know **the finite W transformation** (for finite temperature).

- ☑ Higher spin Schwarzian action at finite temperature

$$\mathrm{Sch}[\phi(\tau),\tau] \longrightarrow \mathrm{Sch}[\tan\frac{\pi\phi(\tau)}{\beta},\tau] = \mathrm{Sch}\,[\phi(\tau),\tau] + \frac{2\pi^2}{\beta^2}(\phi')^2$$

- ☑ "Coupling" to matter at finite temperature

$$\langle \mathcal{O}(\tau_1)\mathcal{O}(\tau_2)\rangle \sim \frac{1}{\tau_{12}^{2\Delta}} \longrightarrow \langle \mathcal{O}(\tau_1)\mathcal{O}(\tau_2)\rangle \sim \frac{[f'(\tau_1)f'(\tau_2)]^{\Delta}}{[f(\tau_1)-f(\tau_2)]^{2\Delta}}$$

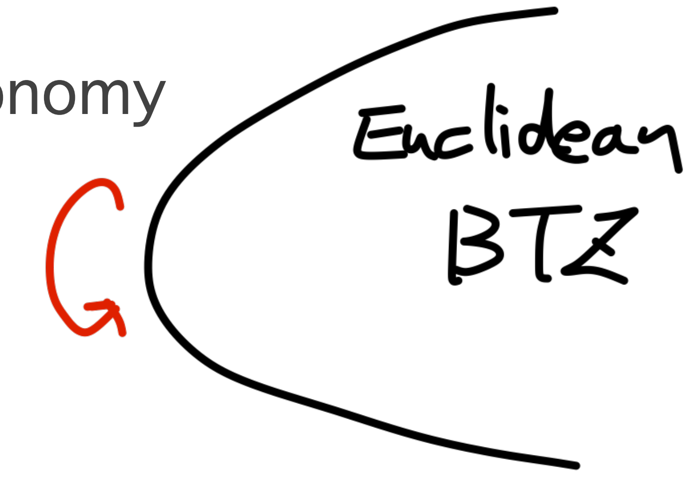
⇒ **Higher spin generalization?**

Finite Temperature Higher Spin Schwarzian

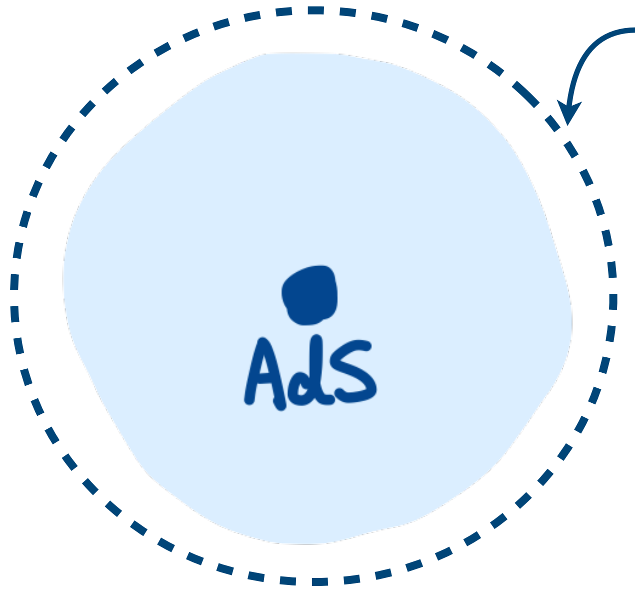
❖ SL(N,C) Chern-Simons gravity with suitable boundary term, boundary condition and smoothness condition
[J. de Boer, J. Jottar, 1302.0816]

$$I_{CS} = \frac{ik_{cs}}{4\pi}\int_{\mathcal{M}} \mathbf{Tr}\,\left[CS(A) - CS(\bar{A})\right] \qquad CS(A) = A\wedge dA + \frac{2}{3}A\wedge A\wedge A$$

→ holonomy

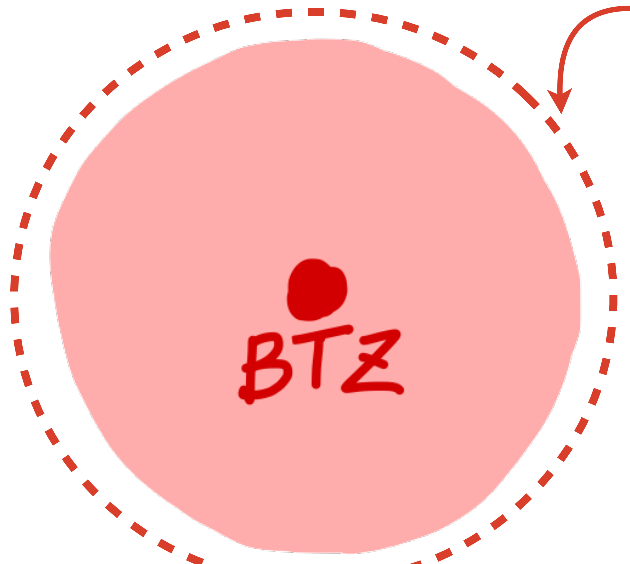


❖ Asymptotic AdS solutions: smooth fluctuation around AdS, BTZ



fixed holonomy

fluctuation around AdS solution



fixed holonomy

fluctuation around BTZ solution

❖ Quadratic boundary action of **asymptotic AdS solutions** fluctuating around BTZ [P. Narayan, JY, 1903.09086].

e.g. SL(3) case:

$$I^{(2)}_{\text{on-shell}} = \frac{4\pi^4 ik_{cs}}{\tau^3}\sum_{n\geq 2} n^2(n^2-1)\zeta^{(2)}_{-n}\zeta^{(2)}_n + \frac{2\pi^6 ik_{cs}}{3\tau^5}\sum_{n\geq 3} n^2(n^2-1)(n^2-4)\zeta^{(3)}_{-n}\zeta^{(3)}_n$$

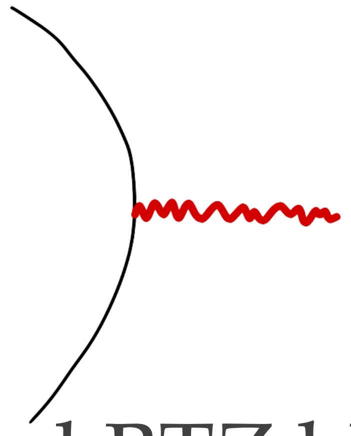
read off propagators →

$$\langle\zeta^{(2)}_{-n}\zeta^{(2)}_n\rangle \sim \frac{1}{n^2(n^2-1)}$$

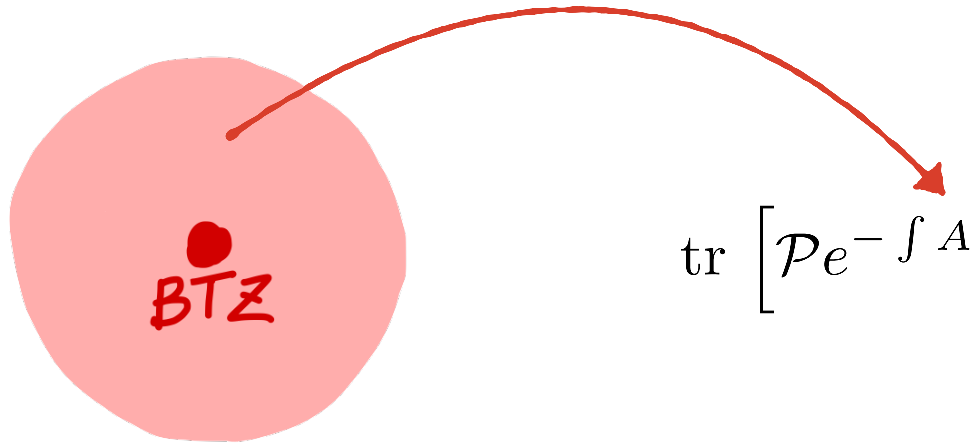
$$\langle\zeta^{(3)}_{-n}\zeta^{(3)}_n\rangle \sim \frac{1}{n^2(n^2-1)(n^2-4)}$$

"Coupling" to Higher Spin Schwarzian

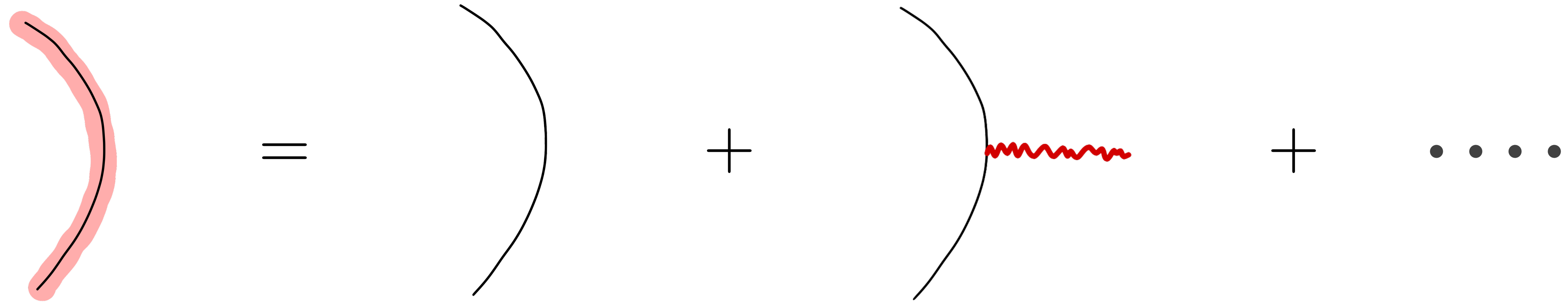
❖ Three methods to find "coupling" between matter two point function and Schwarzian mode:



❖ **Wilson line** dressed by higher spin Schwarzian mode (evaluated for the fluctuation around BTZ black hole)



$$\mathrm{tr}\left[\mathcal{P}e^{-\int A}\, c_0\, \mathcal{P}e^{-\int \bar{A}}\right]$$



❖ **Null relations** in large c limit, Ward identity

$$W^{(s)}_{-n}\phi \sim (L_{-1})^n\phi + \mathcal{O}(1/c) \qquad \delta_\zeta\langle\mathcal{O}(z_1)\mathcal{O}(z_2)\rangle \sim \oint dz\, \zeta(z)\langle W^{(s)}(z)\mathcal{O}(z_1)\mathcal{O}(z_2)\rangle$$

❖ **Recursion relations** among different spin cases

❖ Three independent methods leads to the same result for all spins. e.g.

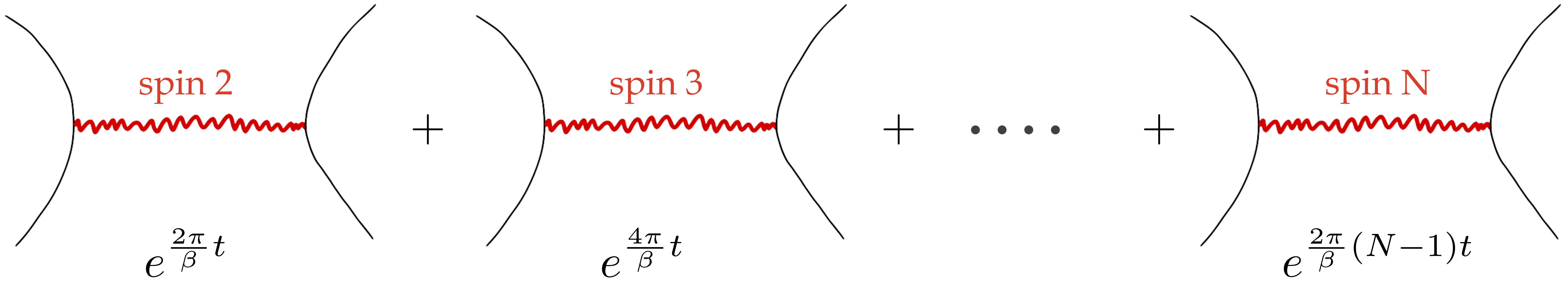
$$f_{2,n} \sim e^{-\frac{2\pi i n\chi}{\tau}}\left[n\cos\frac{2\pi n\sigma}{\tau} - \frac{\sin\frac{2\pi n\sigma}{\tau}}{\tan\frac{2\pi\sigma}{\tau}}\right] \qquad f_{3,n} \sim e^{-\frac{2\pi i n\chi}{\tau}}\left[2n^2\sin\frac{2\pi n\sigma}{\tau} + 6n\frac{\cos\frac{2\pi n\sigma}{\tau}}{\tan\frac{2\pi\sigma}{\tau}} - 2\frac{1+2\cos^2\frac{2\pi\sigma}{\tau}}{\sin^2\frac{2\pi\sigma}{\tau}}\sin\frac{2\pi n\sigma}{\tau}\right]$$

[P. Narayan, JY, 1903.08761]

Lyapunov Exponent in Higher Spin Gravity

❖ In SL(N) Chern-Simons higher spin gravity, the OTOC behaves like

$\langle V(t)W(0)V(t)W(0)\rangle \supset$



❖ Violation of chaos bound: $\lambda_L = \frac{2\pi}{\beta}(N-1)$

☑ Non-unitary of SL(N) Chern-Simons higher spin gravity (the semi-classical limit of Vasiliev hs[λ] higher spin gravity)

Also, see [J. David, S. Khetrapal, S. Kumar, 1707.07166, 1906.0067]

Summary

- ❖ The stability condition of coadjoint orbit leads to the chaos bound.
- ❖ The $1/c$ correction slows down the exponential growth of OTOC.
- ❖ We developed techniques to evaluate the finite temperature higher spin Schwarzian and W transformation of two point function perturbatively.