Breakdown of string perturbation theory and implications for locality

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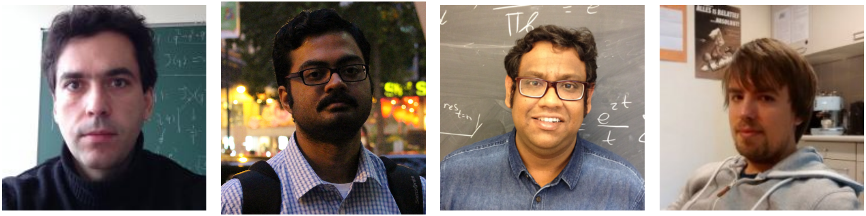


Based on work with Kyriakos Papadodimas, Sudip Ghosh, Souvik Banerjee, Jan-Willem Bryan.

1. S. Ghosh and S. Raju, Loss of locality in gravitational correlators with a large number of insertions, [arXiv: 1706.07424].
2. S. Ghosh and S. Raju, The Breakdown of String Perturbation Theory for Many External Particles, [arXiv:1611.08003].
3. S. Banerjee, J. W. Bryan, K. Papadodimas and S. Raju, A Toy Model of Black Hole Complementarity, [arXiv:1603.02812].

Summary

- Generally accepted that locality breaks down in quantum gravity at the Planck scale

$$[\phi(t, x), \phi(t, x + \vec{\ell}_{\text{pl}})] = ?$$

- In this talk, we will argue that for some observables, nonlocal effects in gravity spread out over macroscopic distances.

Summary

- Observables sensitive to these nonlocal effects are very high point correlators. For example

  $$\langle [\dot{\phi}(t, x_1), \phi(t, x_2) \ldots \phi(t, x_n)] \rangle \neq 0,$$

  even for spacelike $|x_i - x_j| \gg \ell_{\rm pl}$ for sufficiently large n.

- This happens because the algebra of local observables in gravity satisfies polynomial constraints

  $$\phi(t, x_1) \cong \mathcal{P}(\phi(t, x_2), \ldots \phi(t, x_n)).$$

  Similar to black hole complementarity.

- This has significant implications for the information paradox.

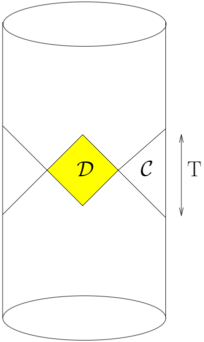
Overview

Empty AdS Complementarity

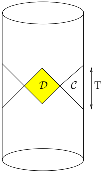
The idea of complementarity $\Leftrightarrow$ high-order polynomial constraints between local operators can be realized precisely in empty anti-de Sitter space.

Empty AdS Complementarity

Operator at center of AdS can be written as complicated polynomial of operators that are uniformly spatially separated!



Implications of Reeh-Schlieder



$$\phi(0) = \sum_{n,m \ll N} c_{nm} |n\rangle\langle m|.$$

Use version of the Reeh-Schlieder theorem to write

$$|n\rangle = X_n[\phi(\mathcal{C})]|0\rangle,$$

where X_n is a simple polynomial.

Vacuum projector

- H is a boundary term in gravity. So

$$P_0 = |0\rangle\langle 0| = \lim_{\alpha \to \infty} e^{-\alpha H},$$

  is an operator in $\mathcal{C}$.

- Can approximate P_0 by a very complicated polynomial in $\mathcal{C}$

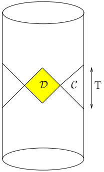
$$P_0 \approx \mathcal{P}[\phi(\mathcal{C})] = \sum_{n=0}^{n_c} \frac{(-\alpha_c H)^n}{n!}.$$

- With

$$\alpha_c = \log N; \quad n_c = N \log(N),$$

  $\mathcal{P}$ is a good approximation to P_0.

Empty AdS Complementarity



Combining the previous formulas we get

$$\boxed{\phi(0) = \sum_{n,m \ll N} c_{nm} X_n[\phi(\mathcal{C})] \mathcal{P}[\phi(\mathcal{C})] X_m^\dagger[\phi(\mathcal{C})]}$$

where X_n are simple polynomials, and $\mathcal{P}$ is a complicated polynomial. Explicitly realizes idea of complementarity and also consistent with approximate locality.

Lessons from AdS

- The root of this nonlocality lies in the Gauss law

$$i[H, \phi(t, x)] = \dot{\phi}(t, x).$$

- For canonically normalized operators, Gauss law leads to $\frac{1}{N}$-suppressed commutators.

$$i[\int h^{00} d^{d-2}\Omega, \phi(t, x)] = \frac{1}{N}\dot{\phi}(t, x).$$

- These $\frac{1}{N}$ effects are enhanced to $O(1)$ effects by the breakdown of $\frac{1}{N}$-perturbation theory.

Flat Space

- Hamiltonian is a boundary term even in flat space.

$$H = M_{\rm pl}^{\frac{d-2}{2}} \int_{\infty} d^{d-2}\Omega n^k (h_{ik,i} - h_{ii,k}).$$

- Leads to nonlocal commutators suppressed by power of $\frac{E}{M_{\rm pl}}$ ← gravitational coupling constant.

  [Donnelly, Giddings, 2016]

- In flat space, the breakdown of gravitational perturbation theory for high-point correlators may signal the loss of locality for such observables.

- The relation between loss of locality and perturbative breakdown in gravity is also natural from the path-integral viewpoint.

Breakdown of String Perturbation Theory

This is the motivation to study the behaviour of string perturbation theory for many external particles.

String perturbation theory breaks down for a large number of external particles, even if the energy per particle stays small.

String Perturbation Theory Limits

- Consider limit where string and Planck scale are widely separated, n is large, and energy-per-particle is small.

$$g_s^2 = \frac{2\pi l_{pl}^{d-2}}{(2\pi\sqrt{\alpha'})^{d-2}} \to 0, \quad n \to \infty,$$

$$\frac{\log(E\sqrt{\alpha'})}{\log(n)} \to -\gamma, \quad 0 < \gamma < \frac{1}{(d-2)}.$$

- String perturbation theory breaks down at least at

$$\frac{\log(g_s)}{\log(n)} = \frac{1}{2}((d-2)\gamma - 1) + O\left(\frac{1}{\log(n)}\right),$$

or

$$\boxed{n \propto g_s^{\frac{-2}{1-(d-2)\gamma}} \Rightarrow n \propto \left(\frac{M_{\text{pl}}}{E}\right)^{d-2}}$$

Unitarity Bounds vs Factorial Growth

- Within perturbation theory, tree amplitudes must satisfy

$$\int d\Pi_{\frac{n}{2}} \left|M^{\text{tr}}(\frac{n}{2} \to \frac{n}{2})\right|^2 \leq 2 \left|M^{\text{tr}}(\frac{n}{2} \to \frac{n}{2})\right|.$$

- Tree-level string tree amplitudes grow as

$$M^{\text{tr}} \propto g_s^{n-2} n!$$

- Factorial growth violates unitarity bounds at

$$n \propto \left(\frac{M_{\text{pl}}}{E}\right)^{d-2}.$$

String Amplitudes using Punctured Spheres

- Usually, tree amplitudes are formulated as integrals over the positions of $(n-3)$ vertex operators on the round sphere.

- Rewrite as integrals over the moduli space of a n-punctured sphere with uniform negative curvature, $R=-1$. eg., in the bosonic string

$$M^{\text{tr}} = g_s^{n-2} \int_{\mathcal{M}_n} d\mu_{\text{WP}} (\det P_1^{\dagger} P_1)^{\frac{1}{2}} (\det \Delta)^{\frac{-d}{2}} \langle V_1 \dots V_n \rangle.$$

[D'Hoker, Giddings, Sonoda, 1987]

$[\alpha' = 2]$

Volume of Moduli Space

- We argue that the main contribution to the amplitude comes from the volume of moduli space. [Caution: some hand-waving involved.]

$$V_{g,n} = \int_{\mathcal{M}_{g,n}} d\mu_{\text{WP}}.$$

- Builds on earlier arguments that $V_{g,0}$ dominates the partition function at genus g.

[Gross, Periwal, D'Hoker, Phong, 1988]

- Volumes of Weil-Petersson moduli spaces have received significant attention in the Mathematical literature.

Volume of Moduli Space

- For any fixed n and large g

$$V_{g,n} \longrightarrow (4\pi^2)^{2g+n-3}(2g+n-3)!\frac{1}{\sqrt{g\pi}}\left(1+\mathrm{O}\left(\frac{1}{g}\right)\right).$$

[Zograf, Mirzakhani, 2008–2013]

- At $n = 0, g \to \infty$, obtain famous (2 g)! growth of string-amplitudes.

[Shenker, 1991]

- At large $g + n$

$$\frac{\log(V_{g,n})}{(n+2g)\log(n+2g)} \to 1.$$

With $g = 0$

$$\boxed{V_{0,n} \propto n!}$$

Numerical Analysis of String Amplitudes

We can verify the $n!$ growth through a numerical analysis of string amplitudes at large n.

Scattering Equations

- At large n, the integral over moduli space localizes to solutions of the scattering equations

$$\sum_{j \neq i} \frac{k_i \cdot k_j}{z_i - z_j} = 0, \forall i.$$

[Gross, Mende, 1987]

[Cachazo, He, Yuan, 2013]

- Exactly $(n-3)!$ solutions $\Rightarrow$ exactly one saddle point per unit volume of moduli space.

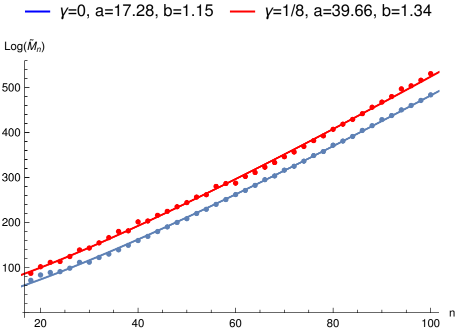
Monte Carlo Numerical Algorithm

- Find random solutions of the scattering equations, and evaluate string-integrand, $I_{\rm str}$, at these solutions. Then use
$$M^{\rm tr} = (n-3)! \langle I_{\rm str} \rangle.$$
[Caution: random sampling may underestimate true answer]

- Procedure works both for the type II superstring and the bosonic string.

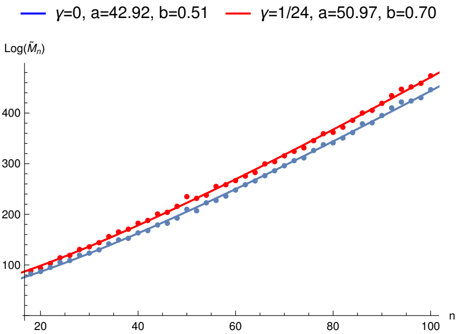
- We examined about 4×10^7 solutions of scattering equations for $n = 4 \ldots 100$. Takes about 8,000 hours of CPU time (Xeon E5@2.5GHz).

Numerical results: Superstring



$$\log(\tilde{M}_n) = \log(M_n^{\text{tr}}) - (n-2)\log(4\pi g_s) + n\log(d-2)$$
$$= a + bn + \log((n-3)!).$$

Numerical results: Bosonic String



Factorial growth is an excellent fit both for the bosonic string and the superstring.

Implications for the Information Paradox

The loss of locality suggested by this perturbative breakdown is precisely sufficient to resolve some versions of the information paradox.

Information Release in Hawking Radiation

- The von Neumann entropy of the emitted black-hole radiation varies with time as follows

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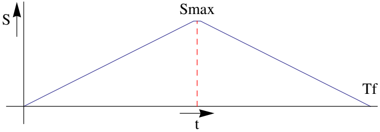
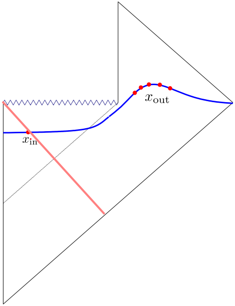


- Information starts to exit the black hole after the Page time.

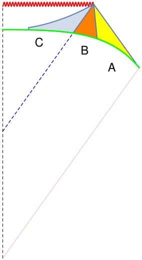
Cloning Paradox

- The black-hole spacetime can then be foliated with nice slices. Information seems both outside and inside: $|\Psi\rangle \to |\Psi\rangle \otimes |\Psi\rangle$?

[Susskind, Thorlacius, Uglum, 93]



Strong Subadditivity Paradox



$$\begin{aligned} C &: r_h - \delta < r < r_h; \\ B &: r_h < r < r_h + \delta; \\ A &: r_h + \delta < r < \infty. \end{aligned}$$

After the Page time, new radiation in *B* is entangled with *A*, and also with C. Violates strong subadditivity of entropy:

$$S_A + S_C \leq S_{AB} + S_{BC}.$$

[Mathur, AMPS, 2009–12]

Measurement Protocols

- It is difficult to extract information or measure the von Neumann entropy of Hawking radiation due to the following theorem:

$$\int \langle \Psi | A | \Psi \rangle d\mu_{\psi} = \mathrm{Tr}(\rho A),$$

$$\int \left(\langle \Psi | A | \Psi \rangle - \mathrm{Tr}(\rho A) \right)^2 d\mu_{\psi} = \frac{\left(\mathrm{Tr}(\rho A^2) - (\mathrm{Tr} \rho A)^2 \right)}{e^S + 1},$$

  with $\rho = e^{-S} \mathbf{I}$. But $S_{\rho} = S$ while $S_{|\Psi\rangle} = 0$.

- This theorem can be beaten by measuring e^S observables.

- Need some order S-point correlators of Hawking radiation to detect cloning/strong subadditivity violation.

Perturbative Breakdown in Information Extraction

- Corresponds to S-matrix elements with S-insertions and typical momentum of order T

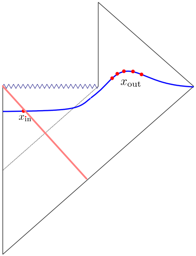
- But perturbation theory breaks down for

$$\frac{(2-d)\log\frac{T}{M_{\text{pl}}}}{\log(n)} = 1 + \text{O}\left(\frac{1}{\log(n)}\right).$$

Reached precisely at n = S.

- So S-point correlators receive non-perturbative corrections. Also possibly sensitive to nonlocal effects.

Black Hole Complementarity



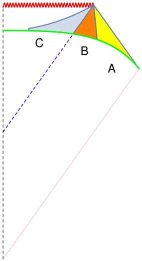
- Our proposal to avoid cloning is to identify operators outside and inside through

$$\phi(x_{in}) \cong P(\phi(x_2), \phi(x_3), \dots \phi(x_S)).$$

Version of complementarity.

- Locality is lost, but no cloning.

Complementarity and Strong Subadditivity



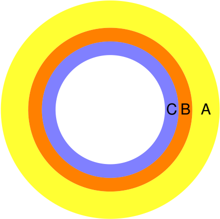
If $$\phi(x_C) \cong P(\phi(x_{A_1}), \phi(x_{A_2}), \ldots \phi(x_{A_S})),$$

the algebra of the theory does not factorize.

$$\mathcal{A} \neq \mathcal{A}_A \otimes \mathcal{A}_B \otimes \mathcal{A}_C \otimes \bar{\mathcal{A}}.$$

So, strong subadditivity inapplicable.

Toy Model of the Information Paradox



Divide a constant time slice of global AdS, into three regions, A,B,C.

- In empty AdS: $\phi(x_C) = \mathcal{P}(\phi(x_{A_1}), \phi(x_{A_2}), \ldots \phi(x_{A_N}))$.
- Information in C is also in A — cloning?
- B is entangled with both C and A — violation of monogamy of entanglement?
- In empty AdS, clear that paradoxes are resolved through a subtle loss of locality.

Summary

- In empty AdS, we can explicitly rewrite a local operator as a complicated polynomial of other spacelike-separated local operators: $\phi(0) = \sum_{n,m} c_{nm} X_n \mathcal{P}_0 X_m^\dagger$.
- Relies on the Gauss law and breakdown of $\frac{1}{N}$ perturbation theory for correlators with $\mathrm{O}\left(N\right)$ insertions.
- Motivates study of limits on flat-space string perturbation theory.
- With small E, large n, provided evidence that string perturbation theory breaks down at least at $n \propto \left(\frac{M_{\rm pl}}{E}\right)^{d-2}$.
- Limit is precisely met for $n = S$, and $E = T$ of a black hole.
- Suggests an elegant resolution to the cloning/strong subadditivity paradoxes: subtle loss of locality for $\mathrm{O}\left(S\right)$-point correlators.