
$|0\rangle$
$|\psi\rangle$
H
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Complexity and Geometry

holographic
duality

Complexity and Geometry

holographic
duality

bulk (AdS)

Complexity and Geometry

holographic
duality

boundary (CFT)

bulk (AdS)

Complexity and Geometry

holographic
duality

boundary (CFT)

bulk (AdS)

Complexity and Geometry

classical property

holographic duality

boundary (CFT)

bulk (AdS)

Complexity and Geometry

classical property gravitational theory

boundary (CFT)

holographic duality

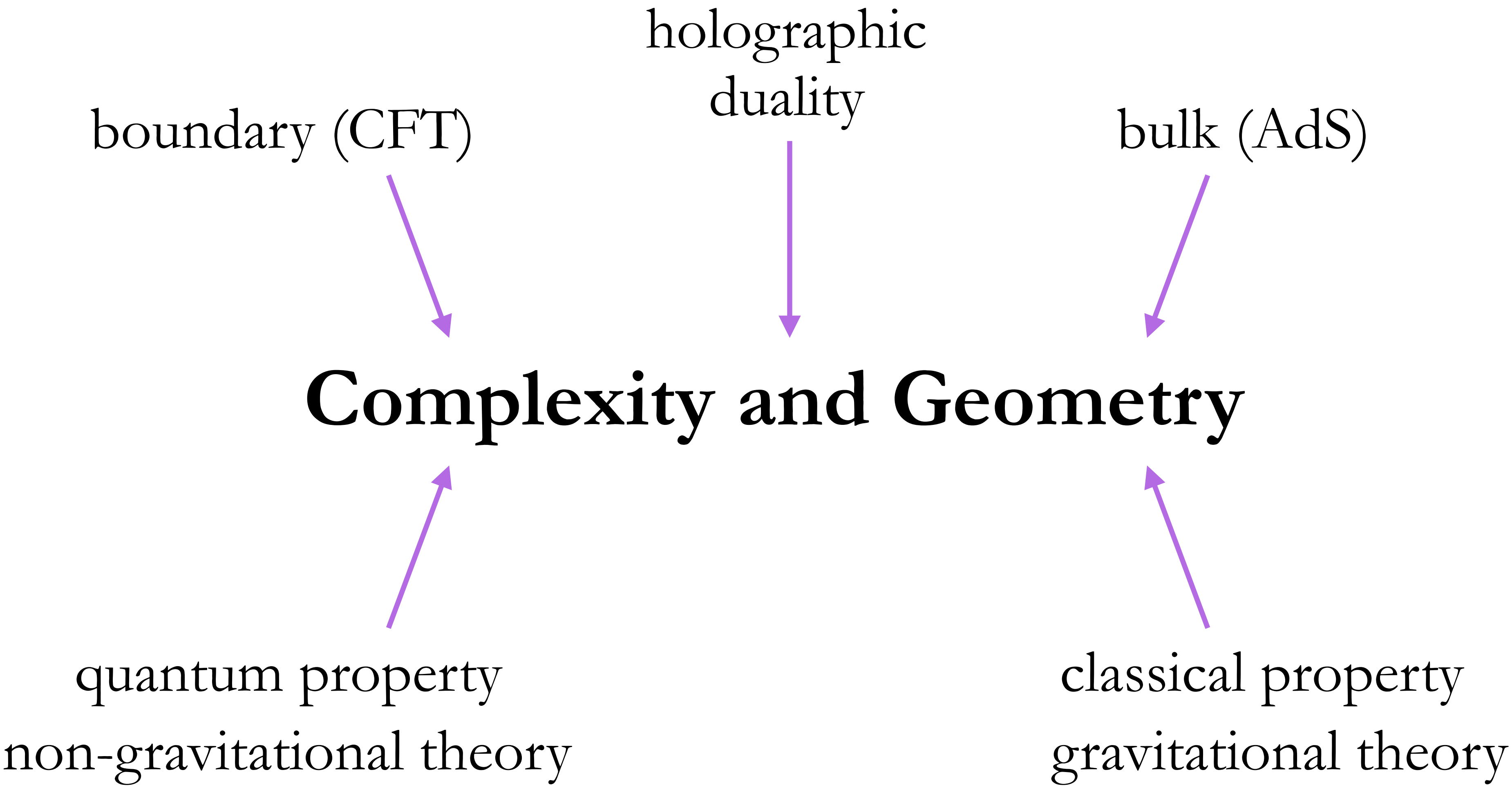
bulk (AdS)

Complexity and Geometry

quantum property

classical property

gravitational theory


holographic
duality
boundary (CFT)
bulk (AdS)
Complexity and Geometry
quantum property
non-gravitational theory
classical property
gravitational theory

holographic
duality

boundary (CFT)

bulk (AdS)

Complexity and Geometry ?

quantum property
non-gravitational theory

classical property
gravitational theory

Complexity and Geometry

classical property
gravitational theory

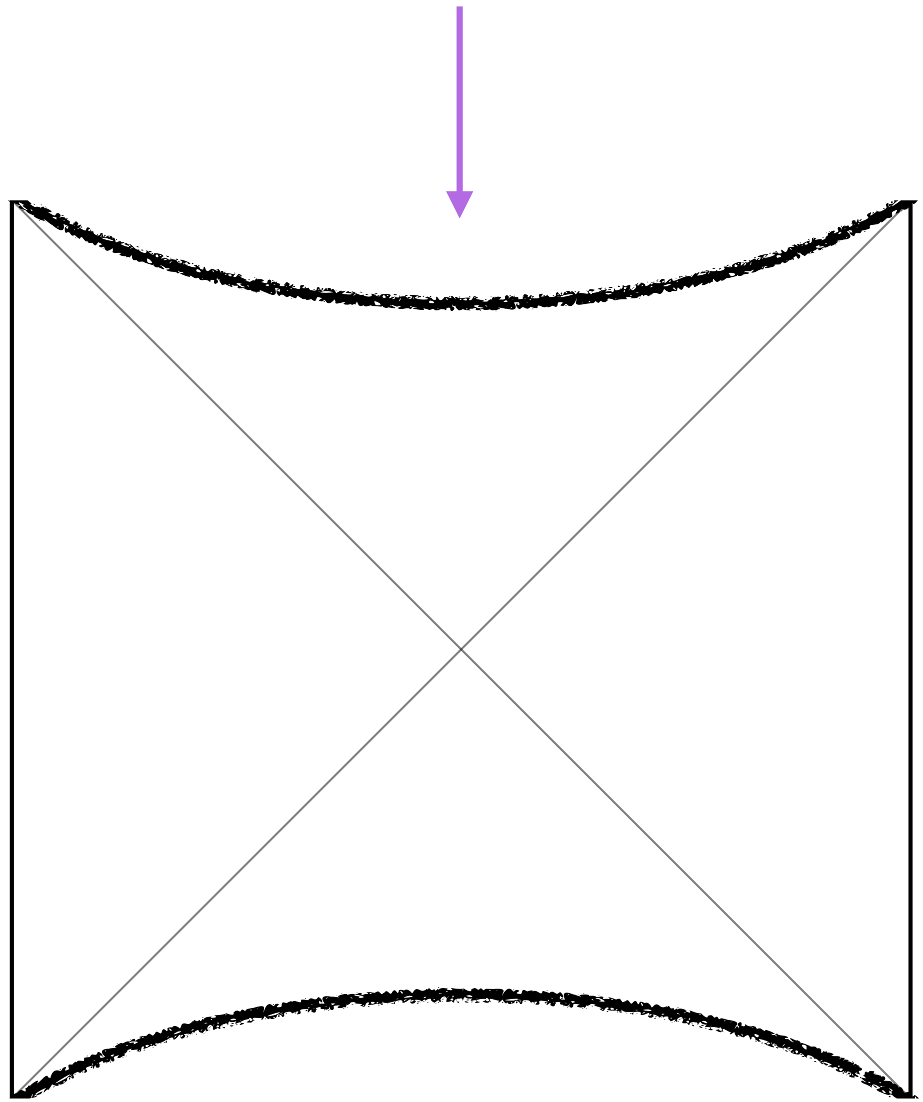
Complexity and Geometry

‘size’ of wormhole
for black hole in AdS

classical property
gravitational theory

‘size’ of wormhole

for black hole in AdS



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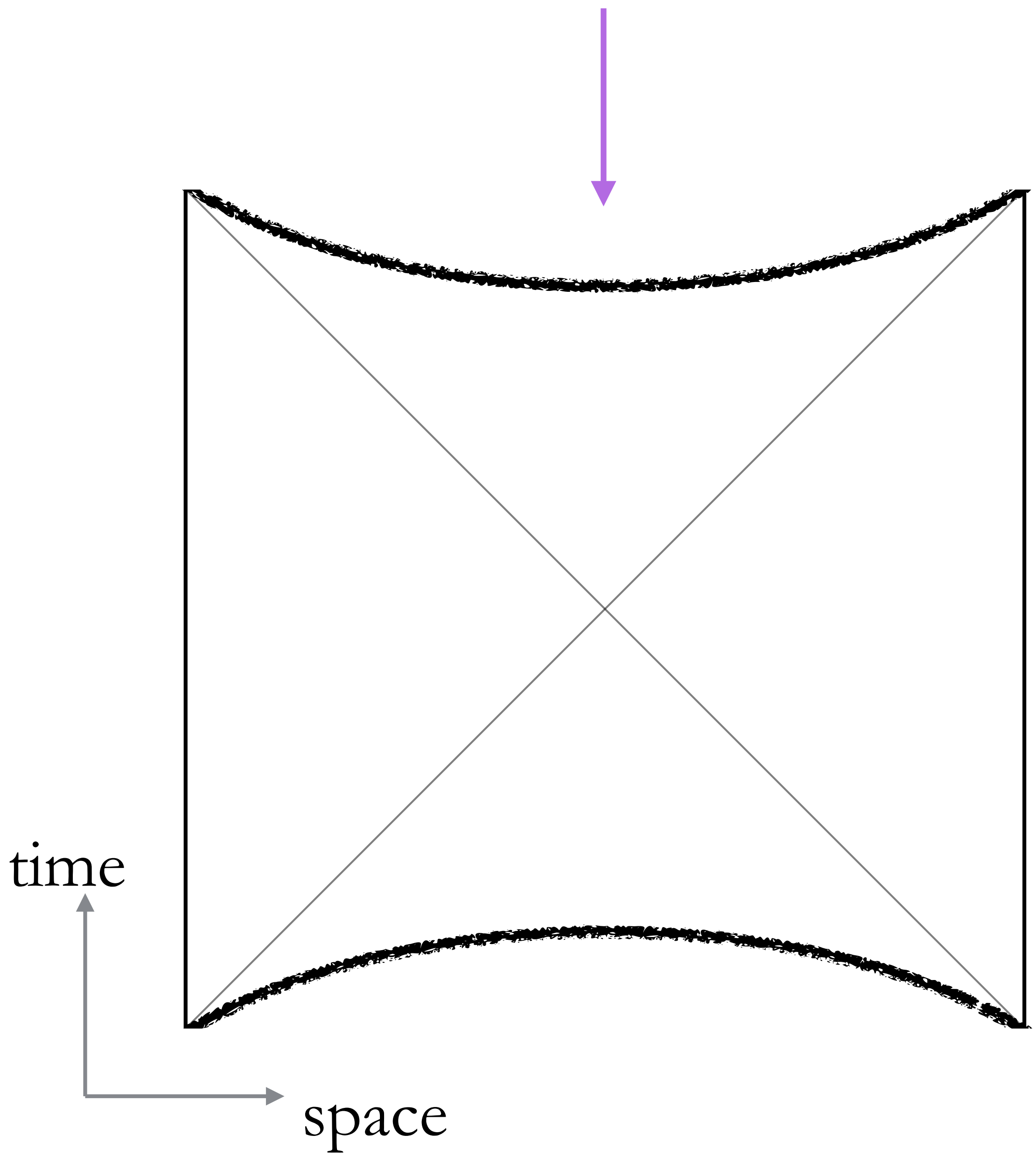

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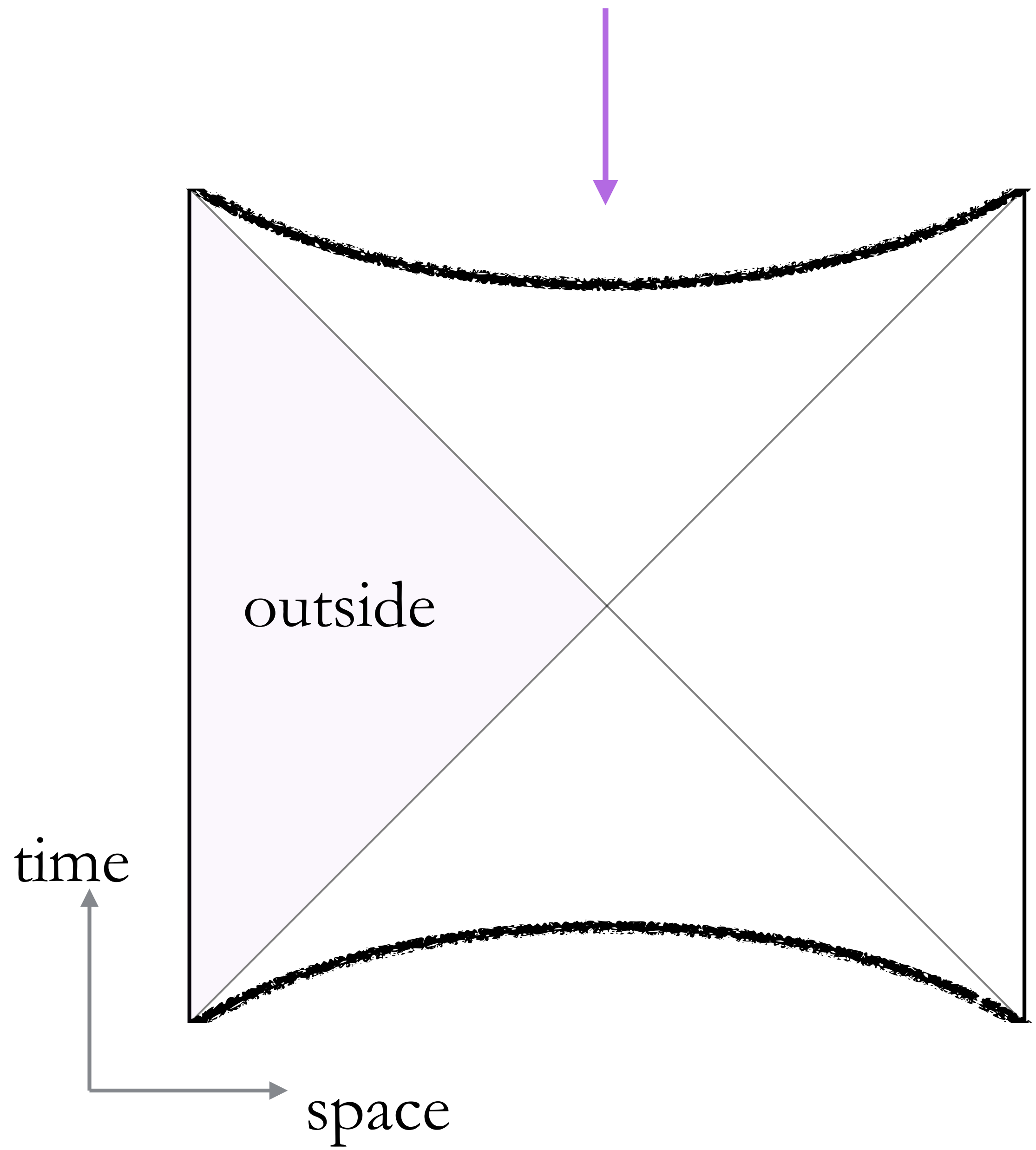

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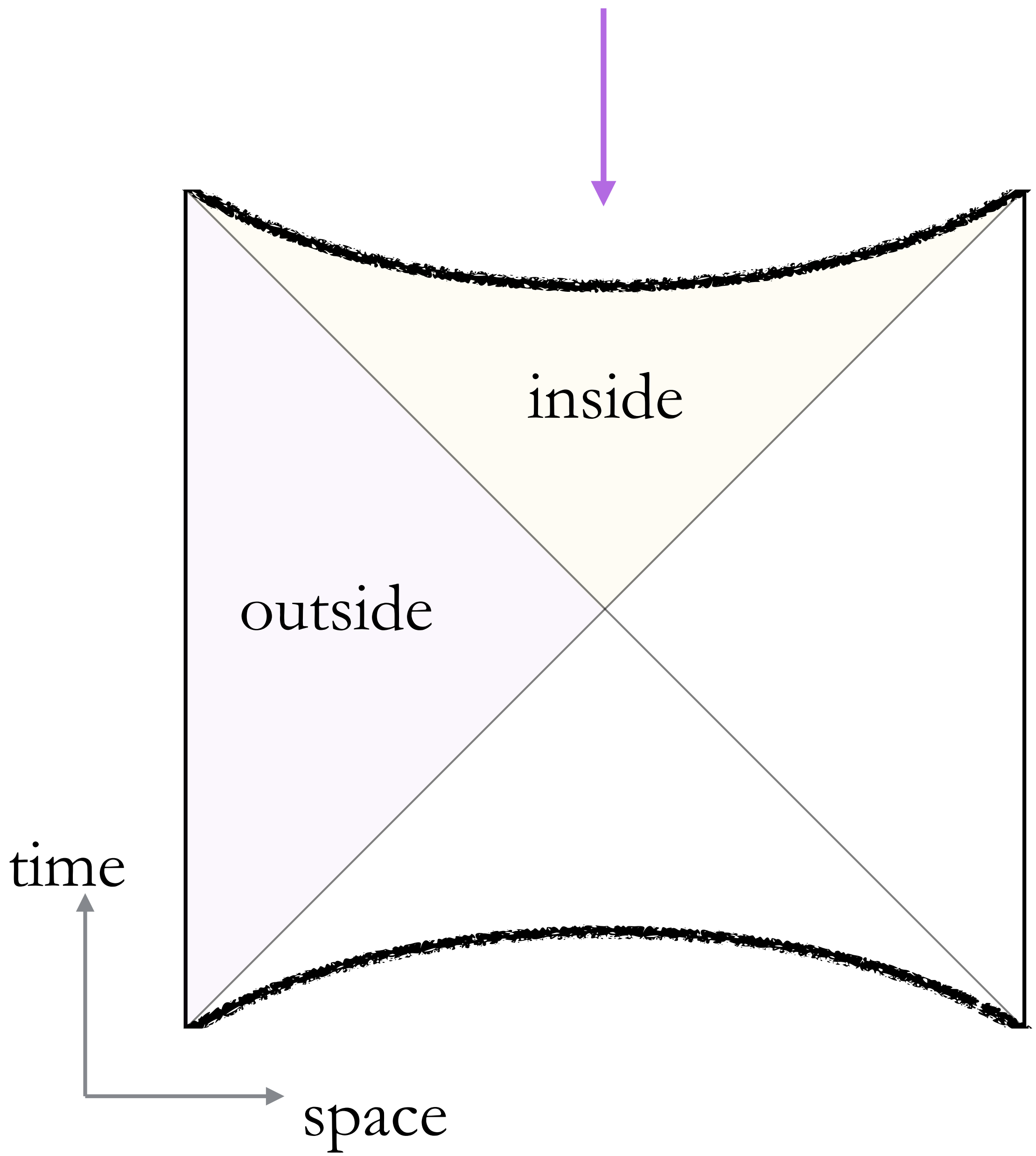

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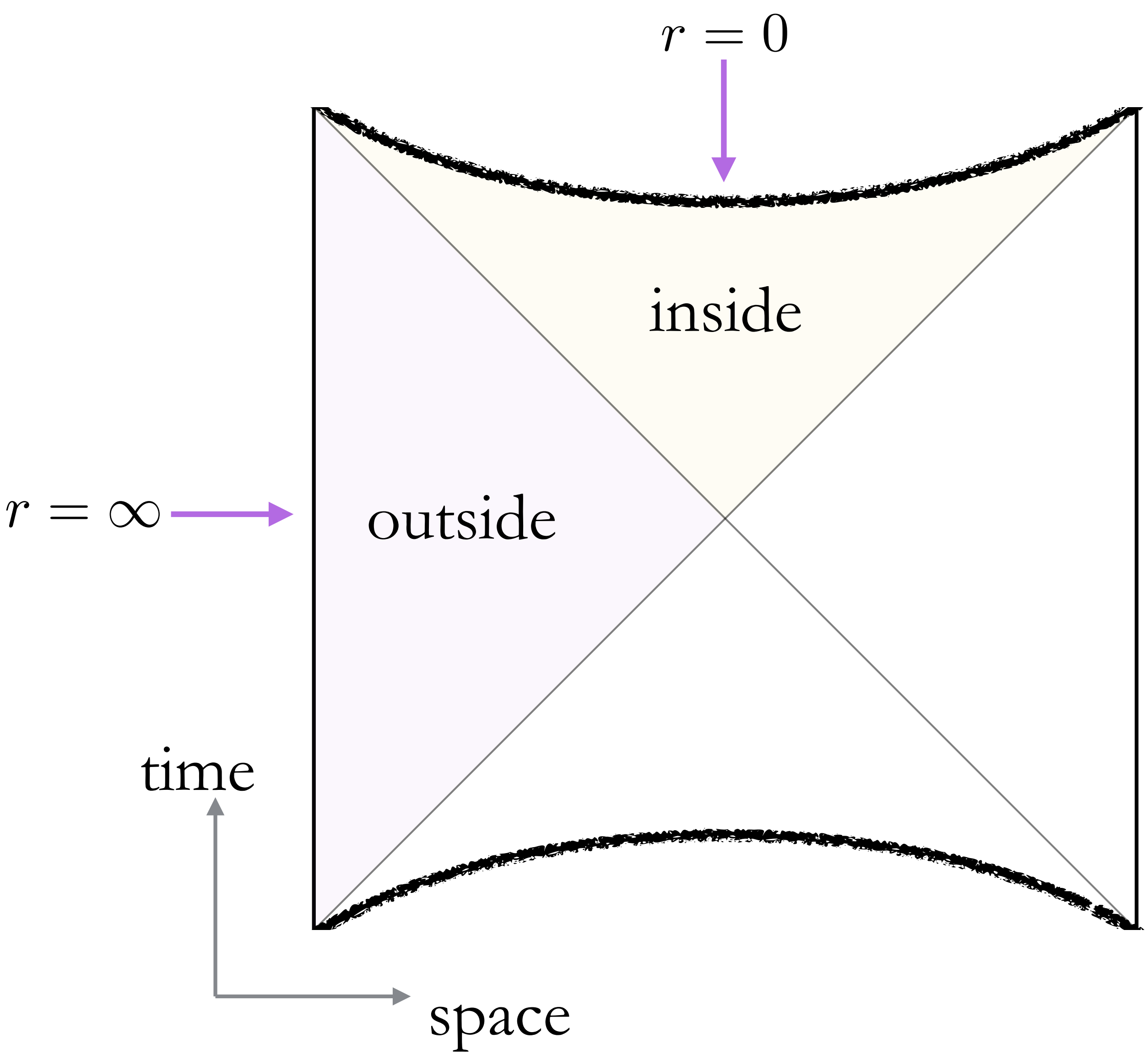

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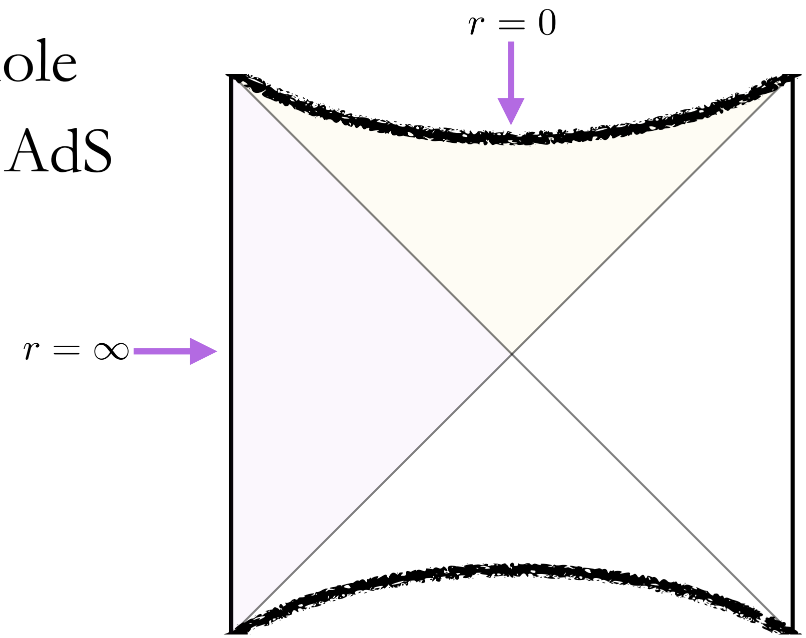

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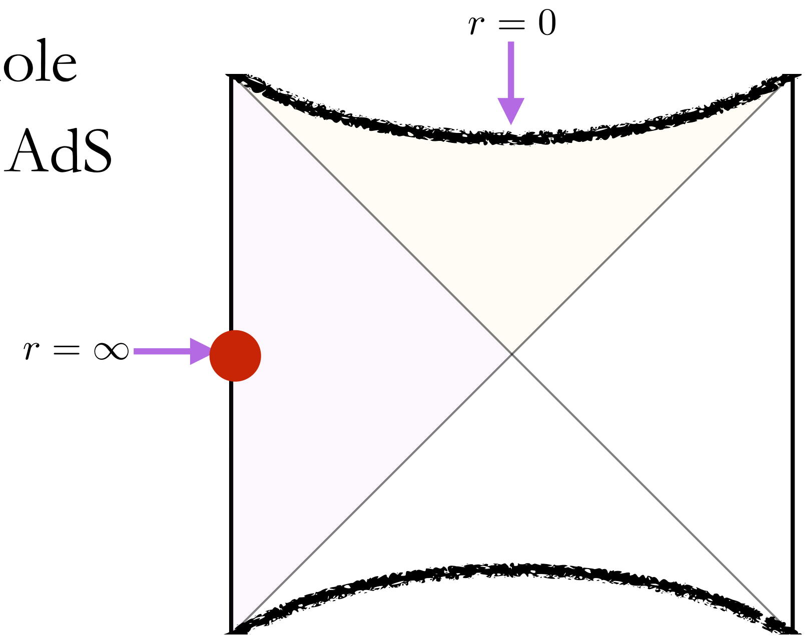

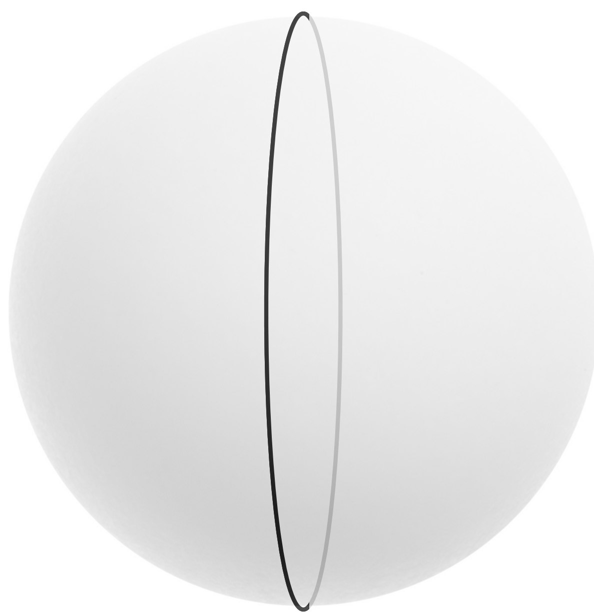

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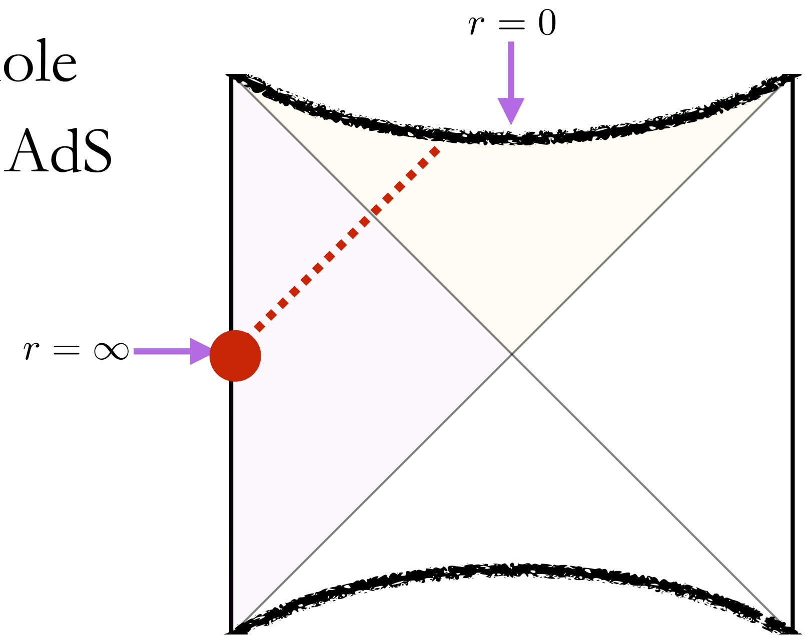

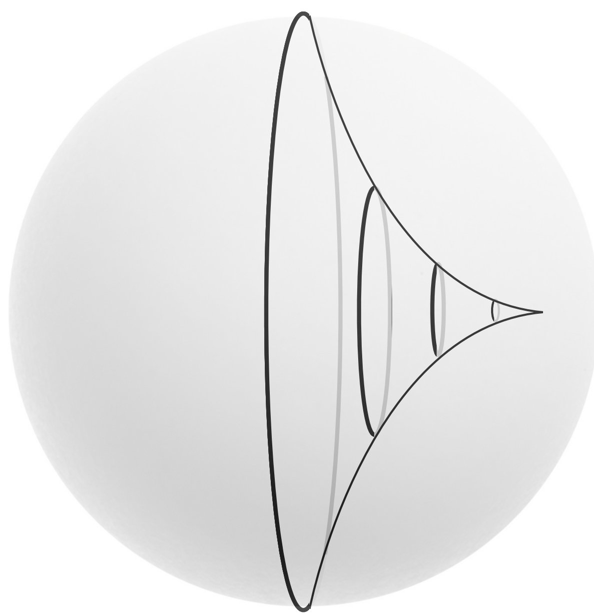

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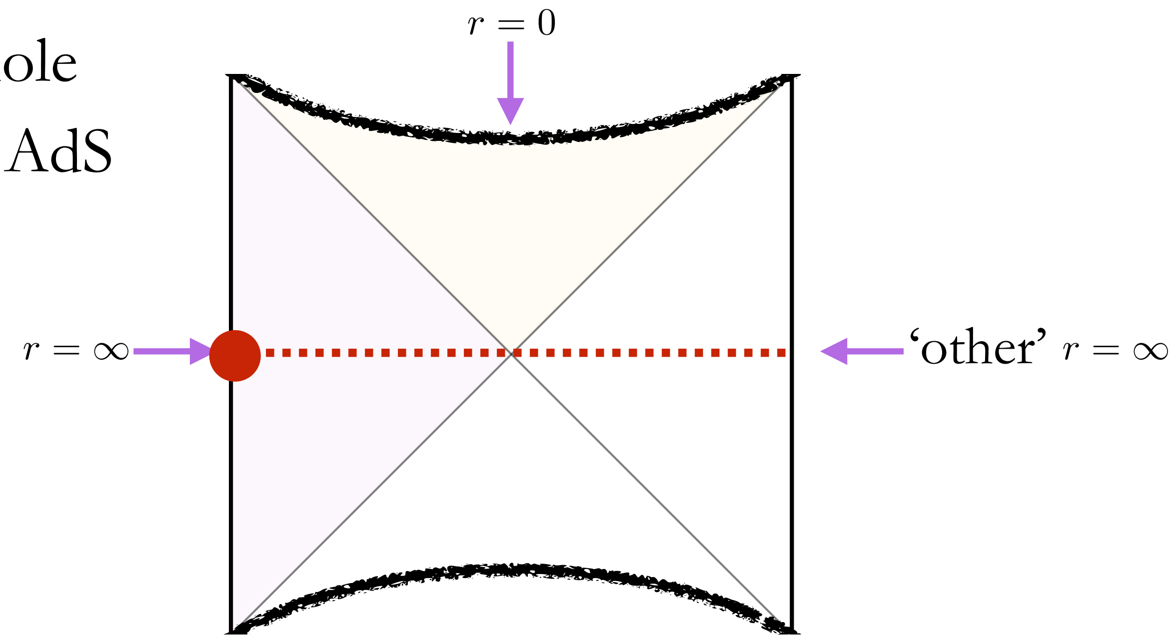

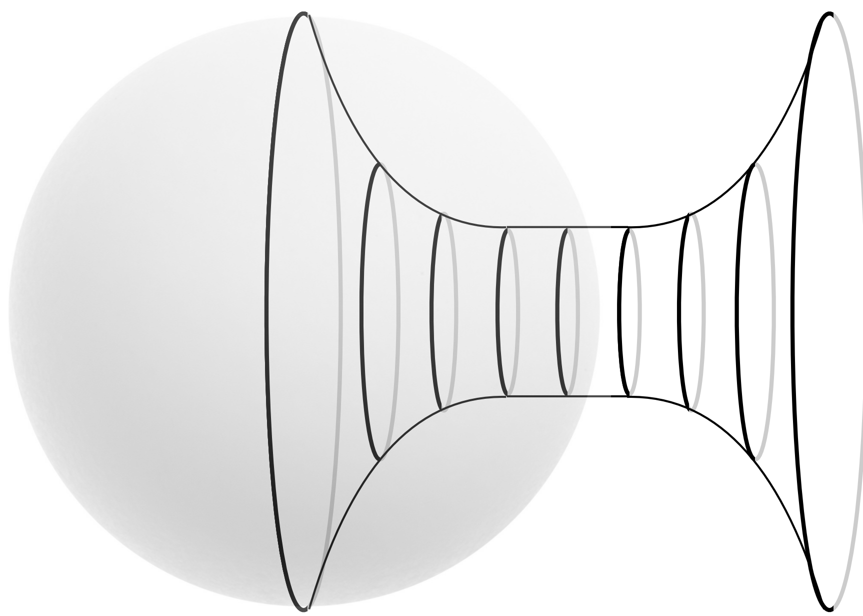

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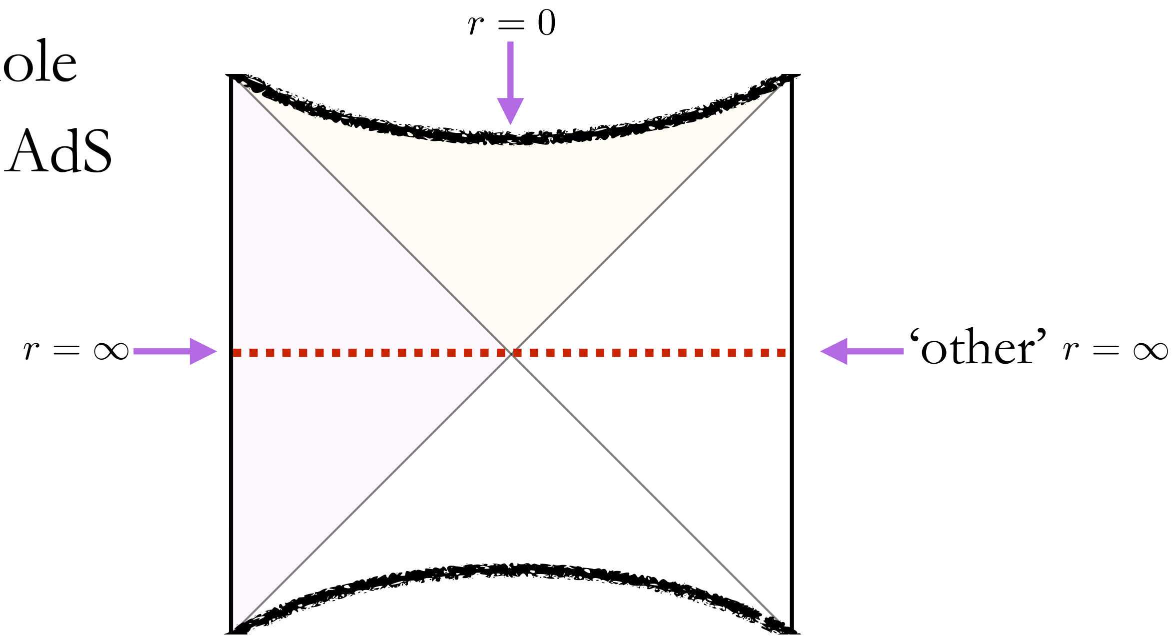

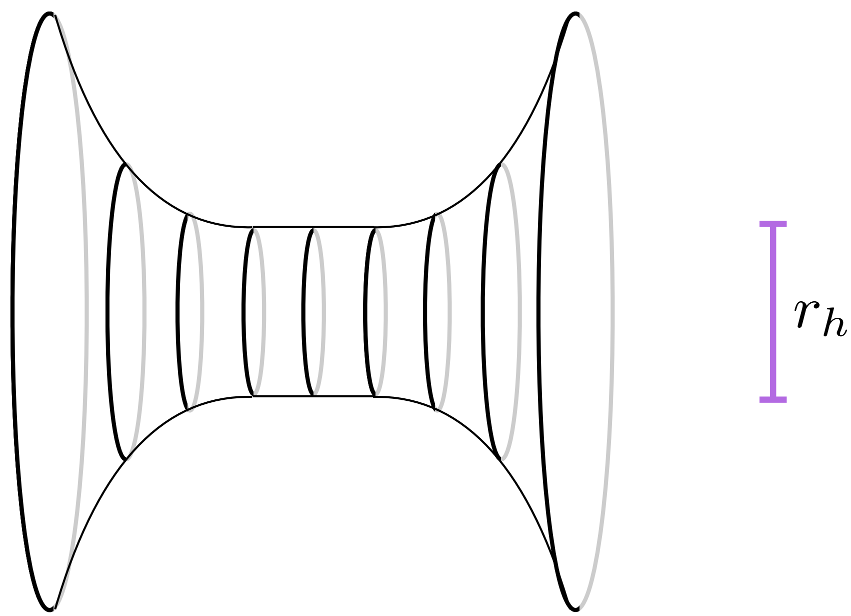

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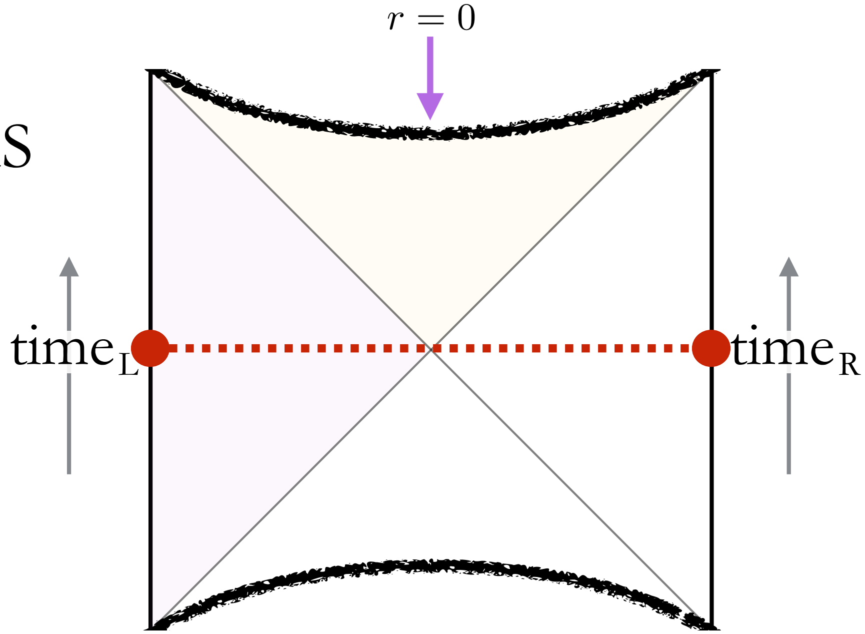

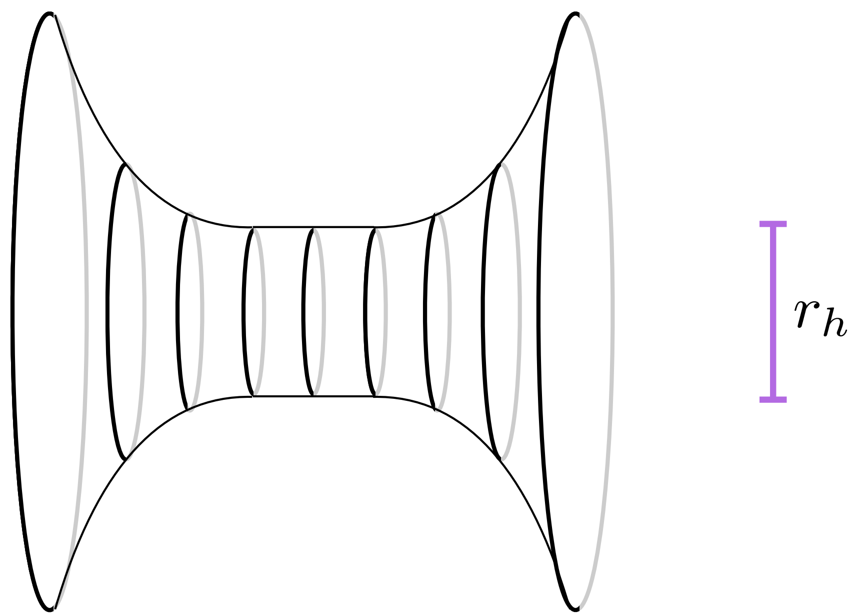

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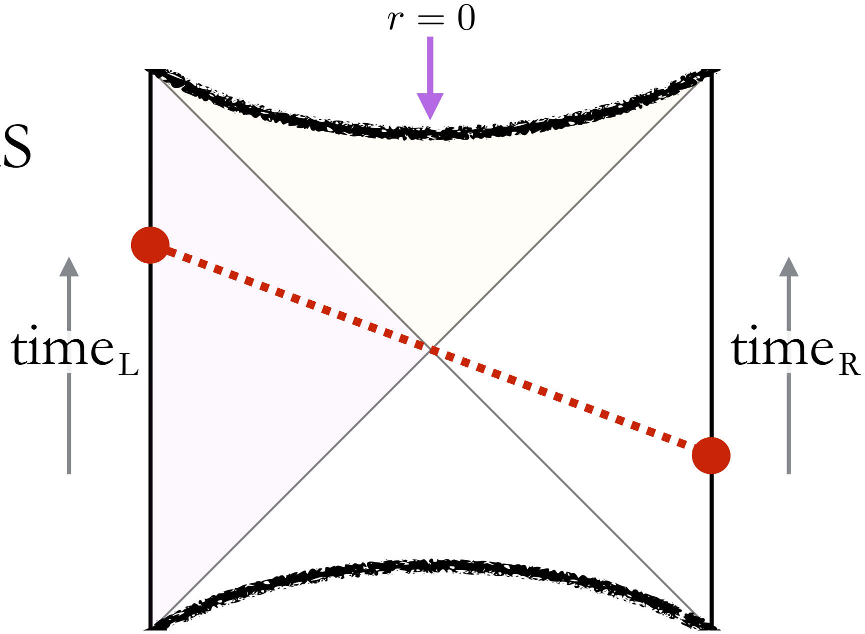

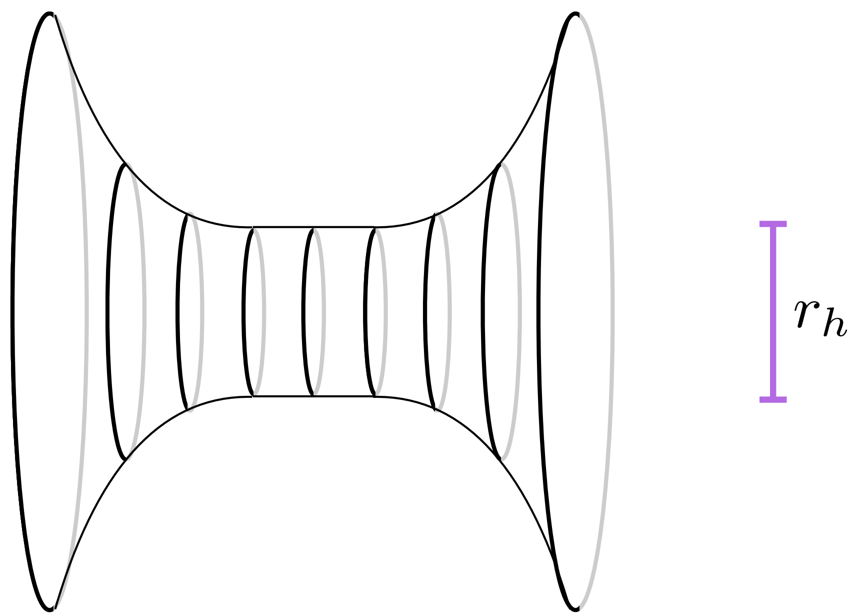

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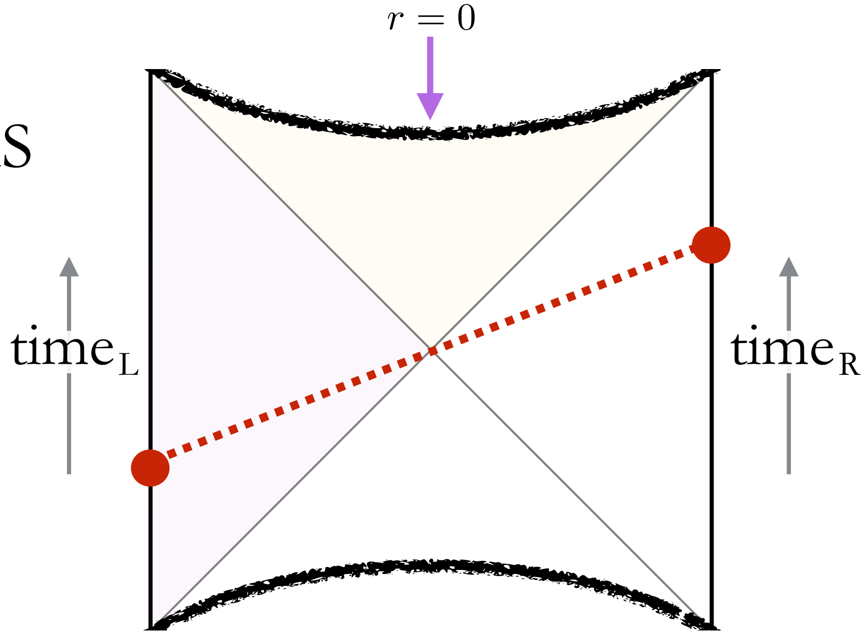

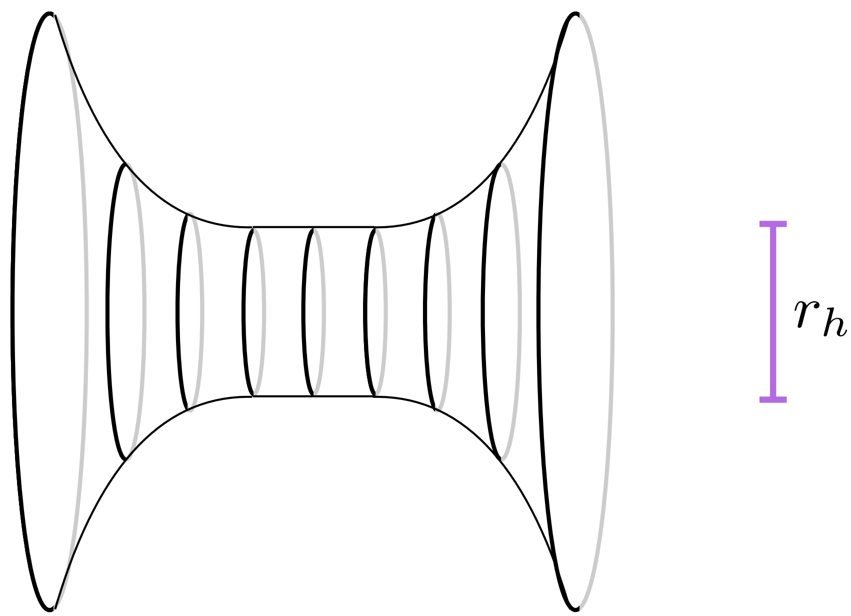

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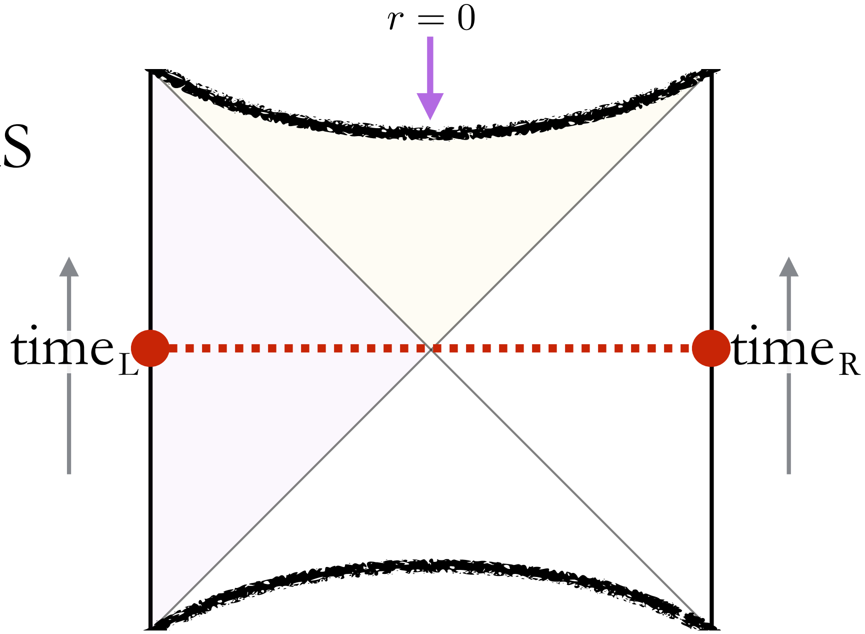

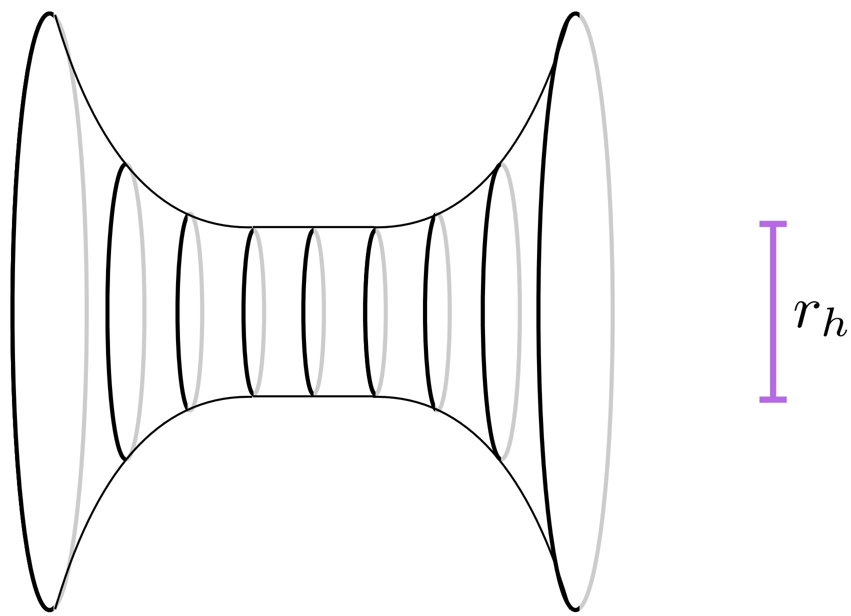

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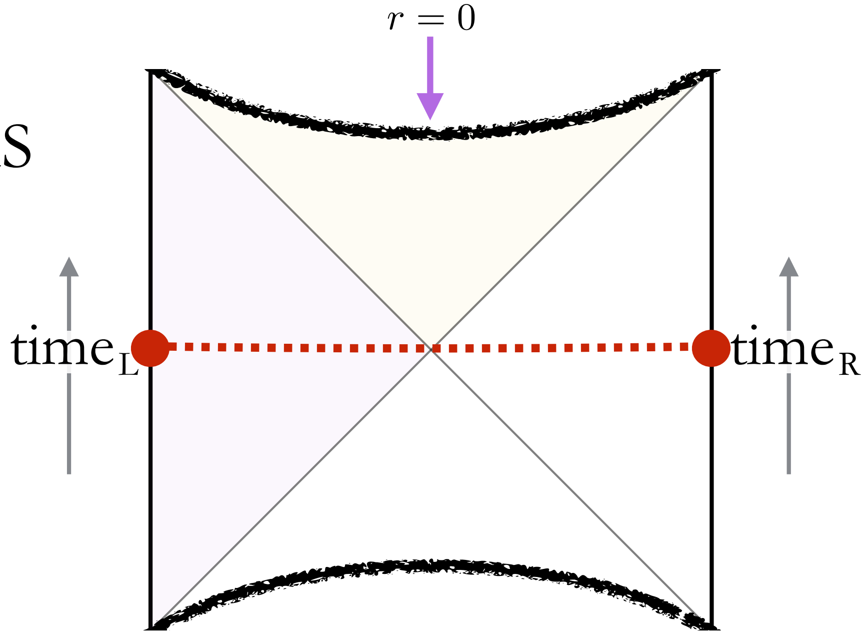

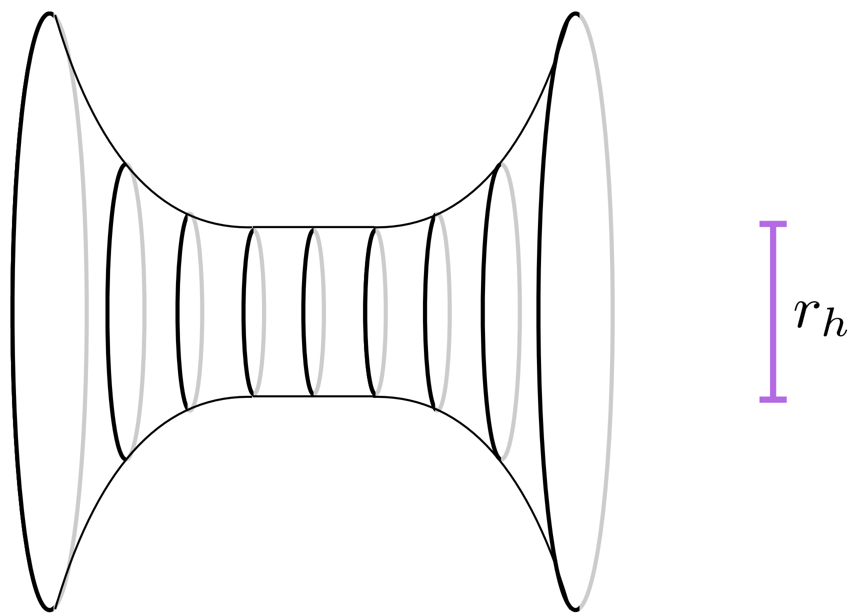

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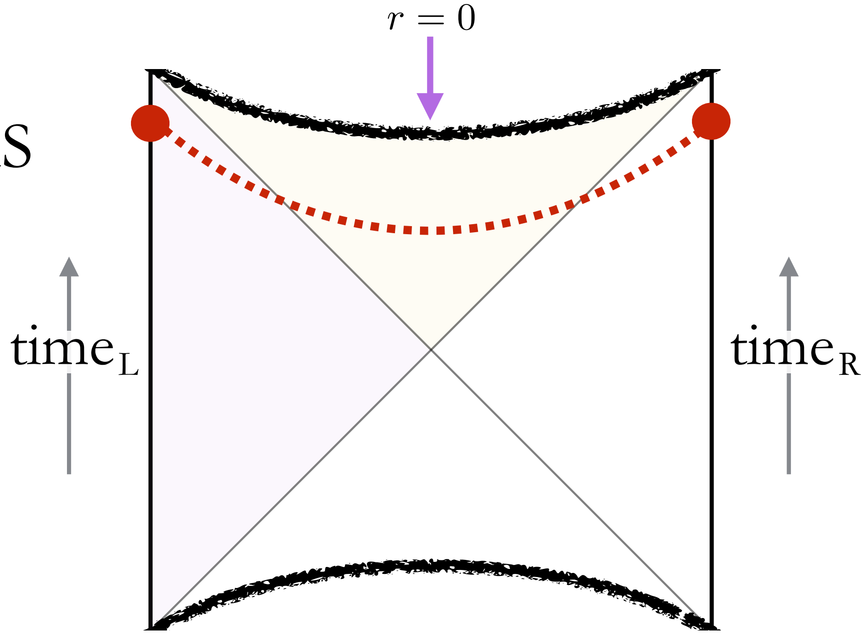
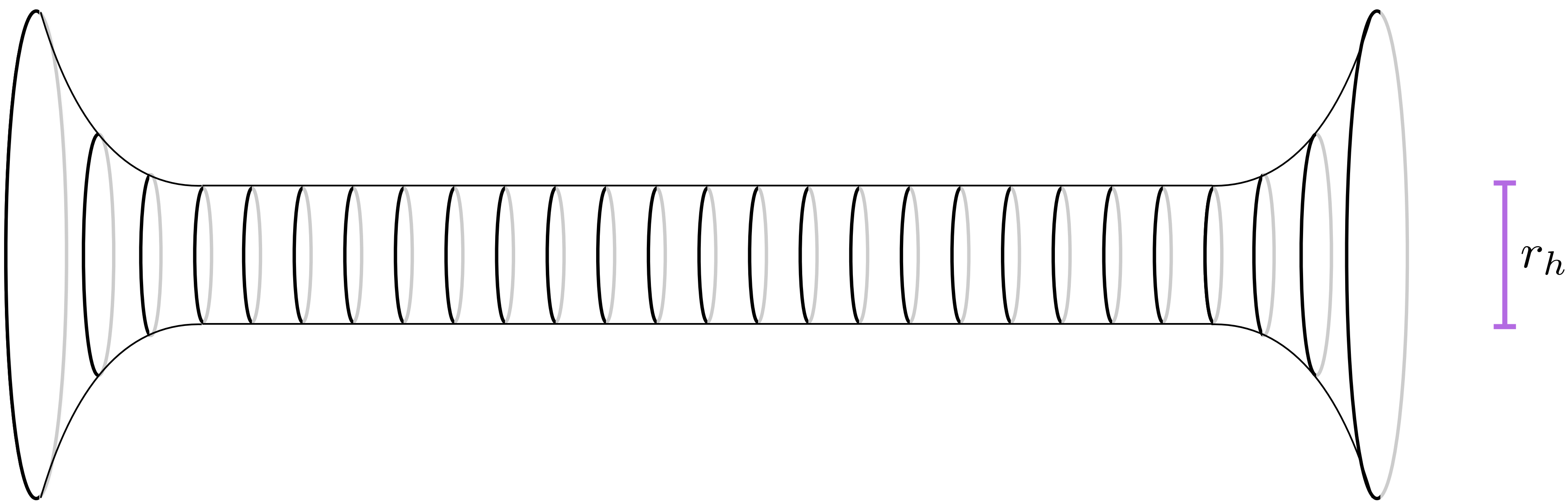
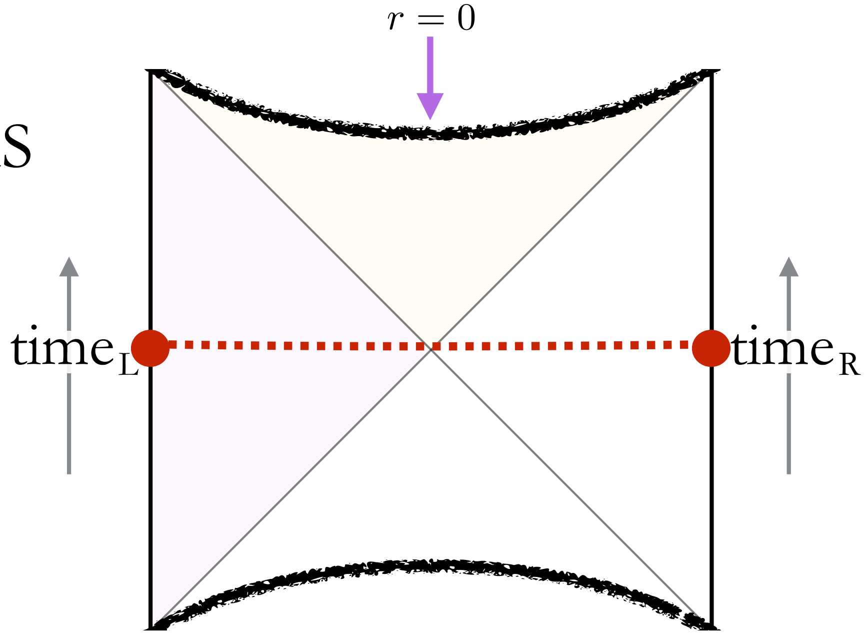
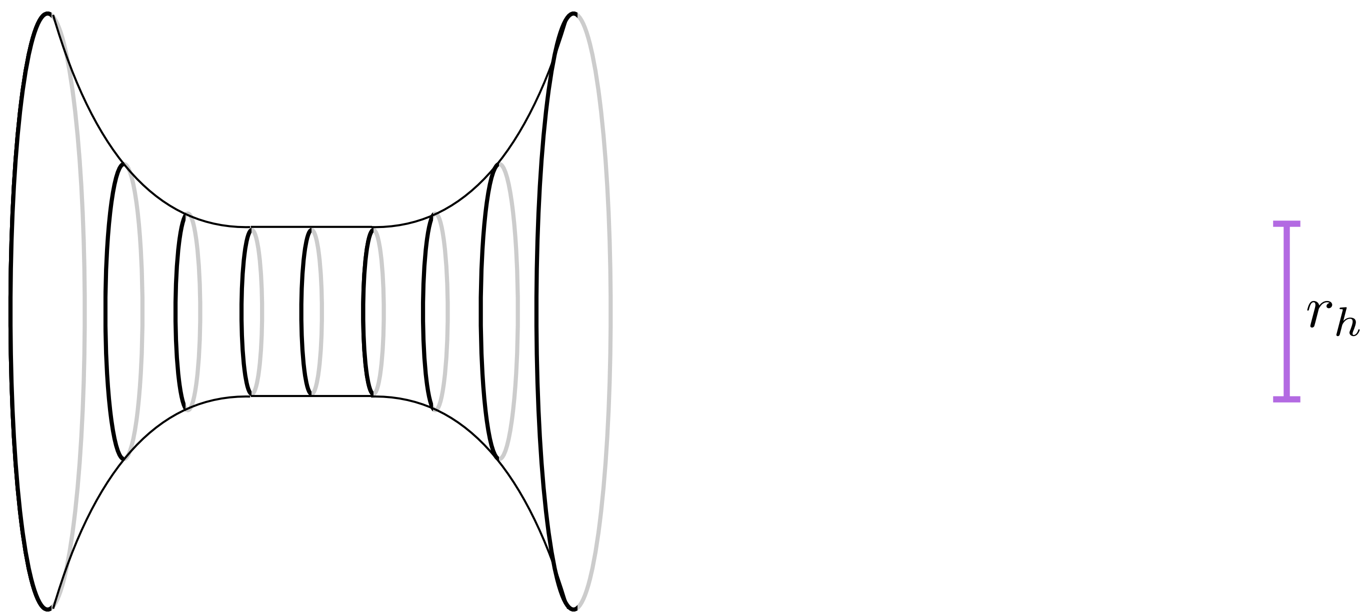




wormhole length $\sim t_L + t_R$

‘size’ of wormhole for black hole in AdS





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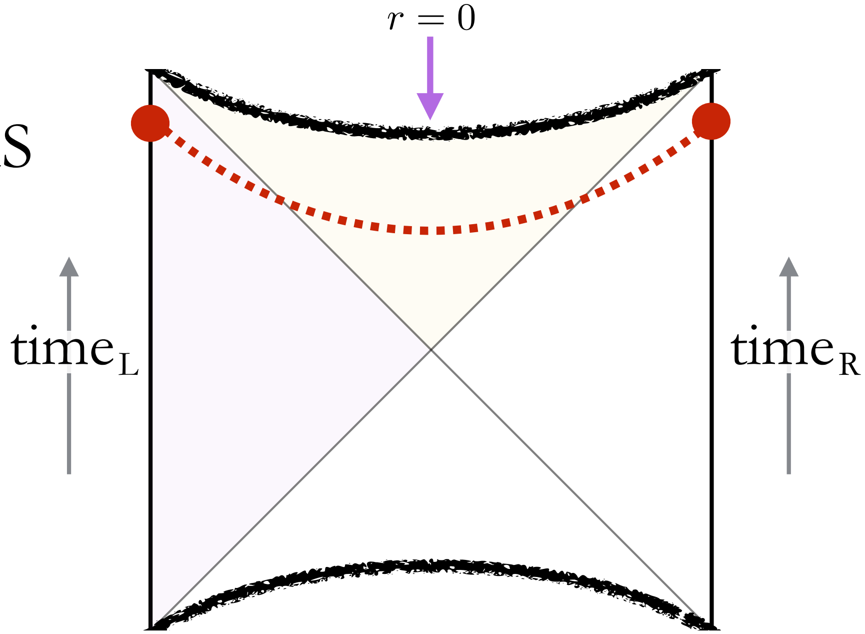

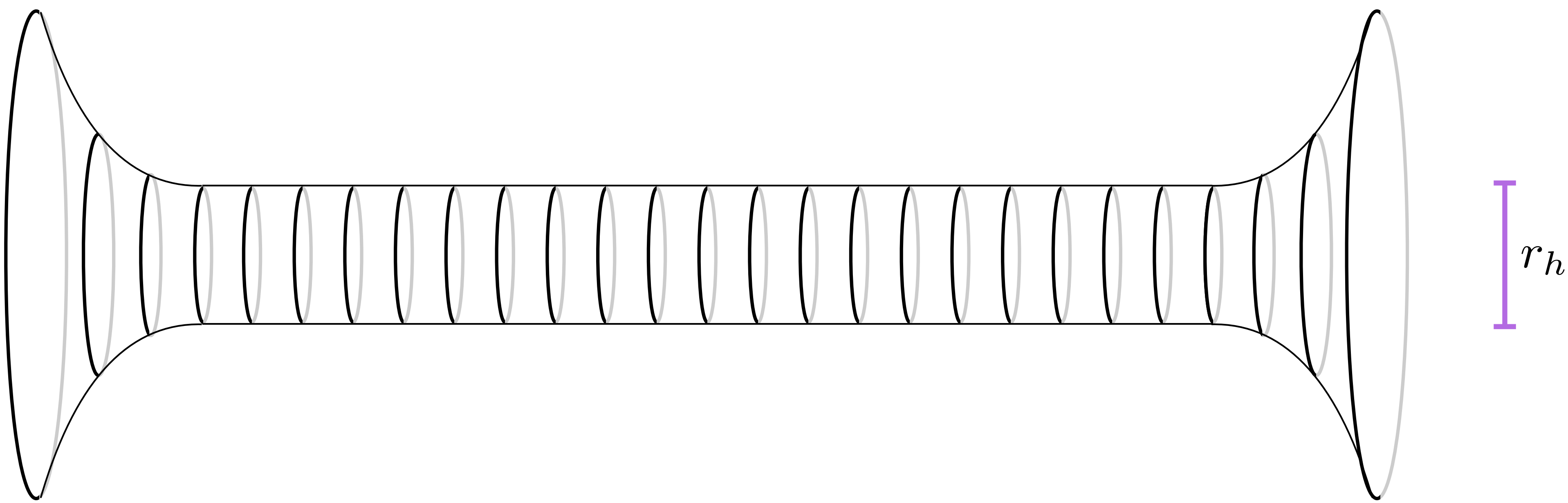

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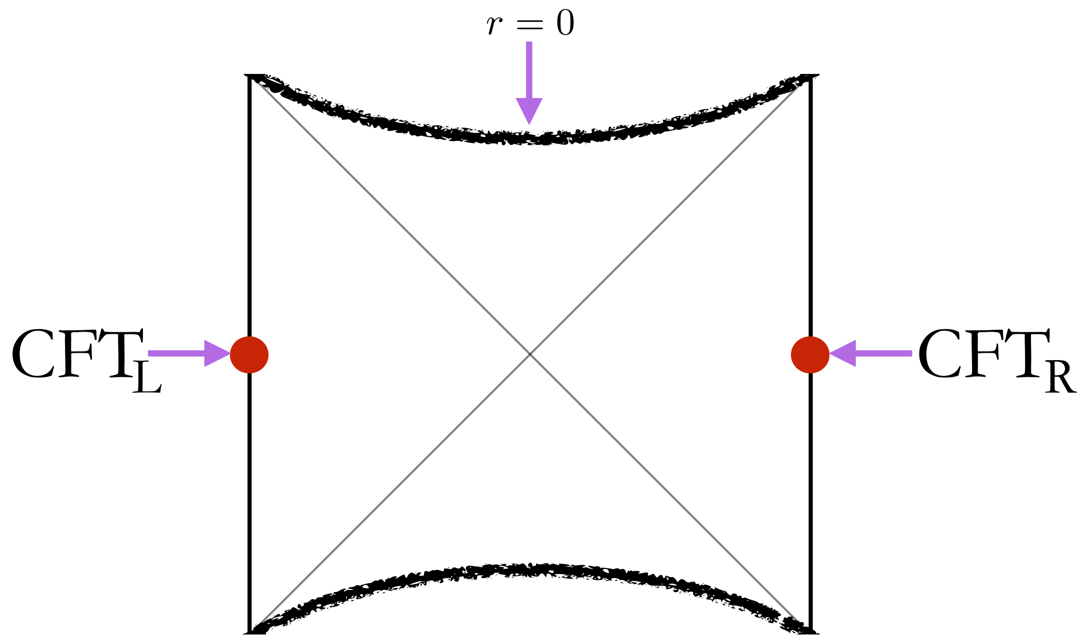

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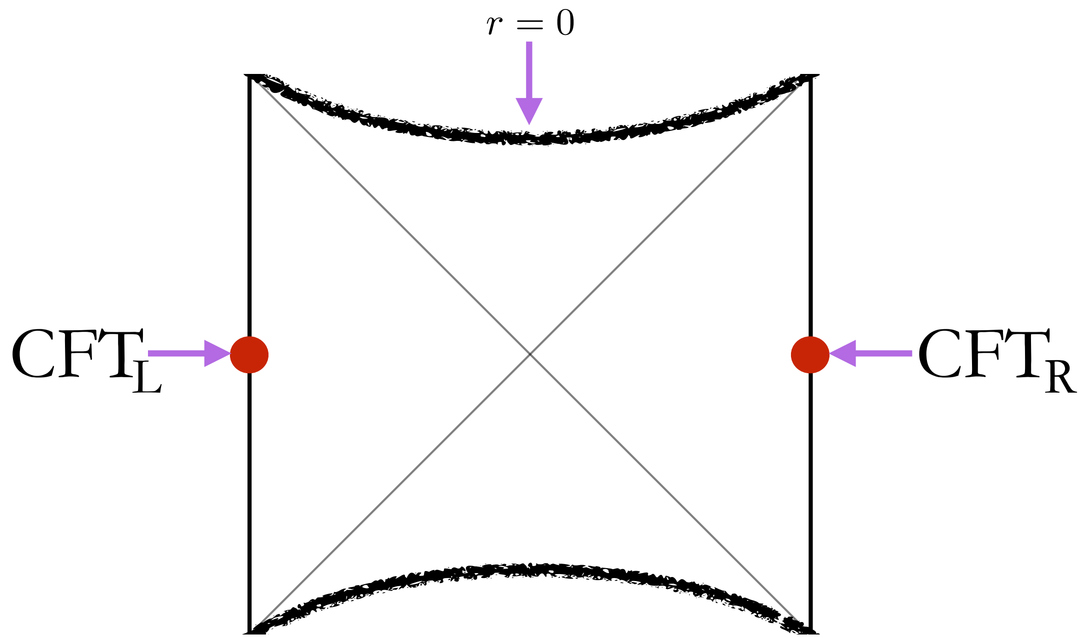
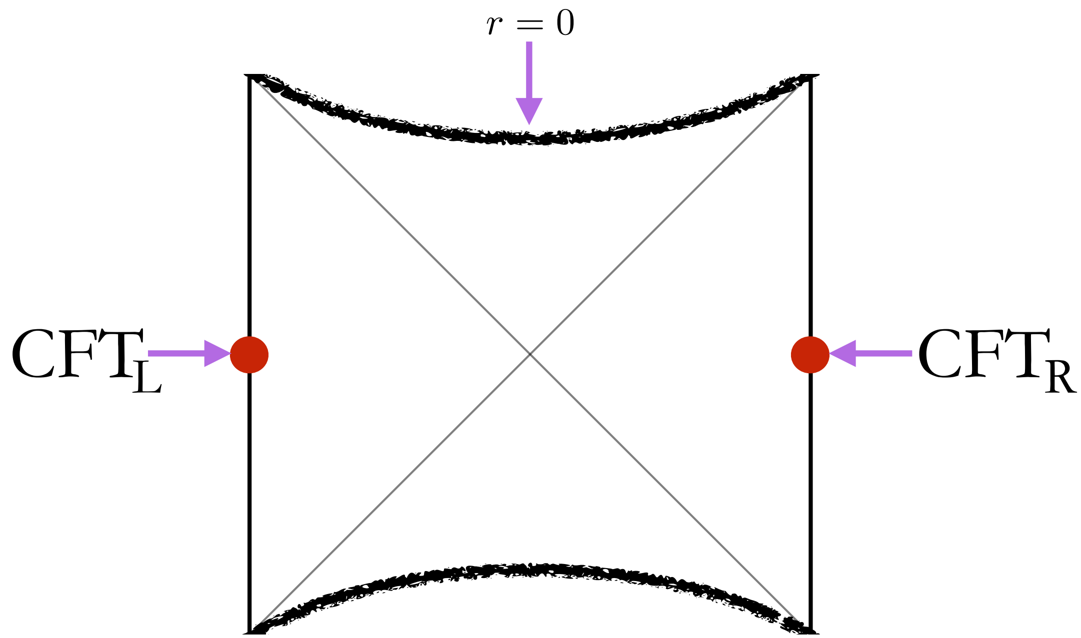


$$|\mathrm{TFD}\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle_L |E_i\rangle_R$$

wormhole length $\sim t_L + t_R$



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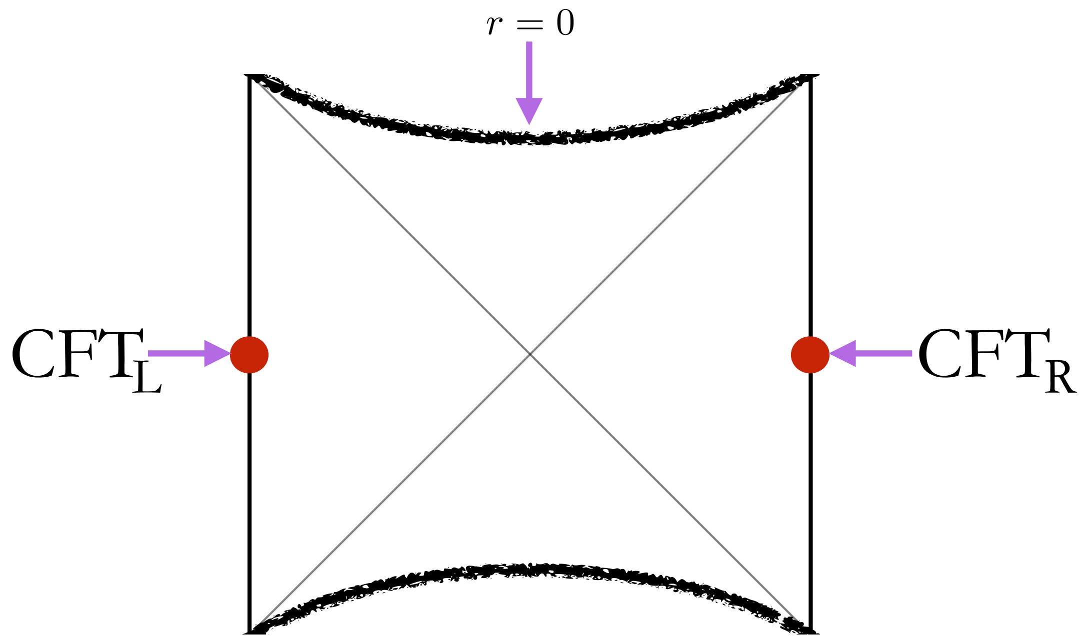

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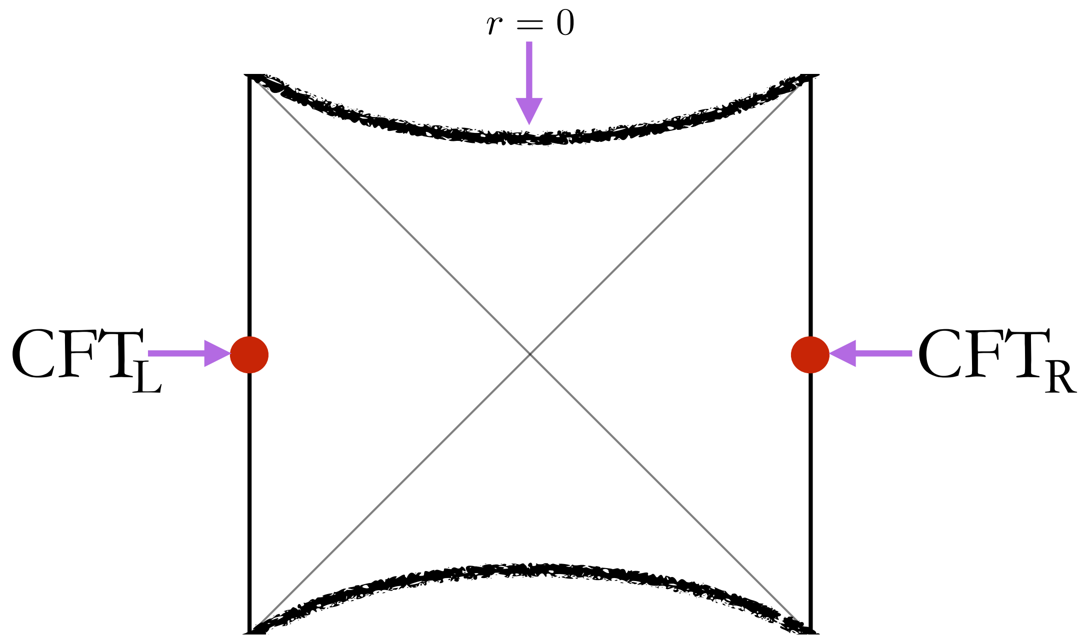
What is CFT dual to linear growth of wormhole?

wormhole length $\sim t_L + t_R$



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What is CFT dual to linear growth of wormhole?

COMPLEXITY?

wormhole length $\sim t_L + t_R$

COMPLEXITY?

computational complexity of a quantum state

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computational complexity of a quantum state

e.g. $|\psi(t_L, t_R)\rangle = \sum_i e^{-\beta E_i/2 + iE_i(t_L+t_R)} |E_i\rangle_L |E_i\rangle_R$

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DEFINITION?

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DEFINITION?

$\otimes|\uparrow\rangle$

$\otimes|\downarrow\rangle$

COMPLEXITY?

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DEFINITION?

NEAR?

$\otimes |\uparrow\rangle$

$\otimes |\downarrow\rangle$

COMPLEXITY?

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DEFINITION? starting in a reference state

e.g. $|\mathrm{TFD}\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle_L |E_i\rangle_R$

how many fundamental gates

e.g. unitaries each of which act only on two-qubits

are needed to make target state

e.g. to within an accuracy ϵ

computational complexity of a quantum state

how many fundamental gates

starting in a reference state are needed to make target state

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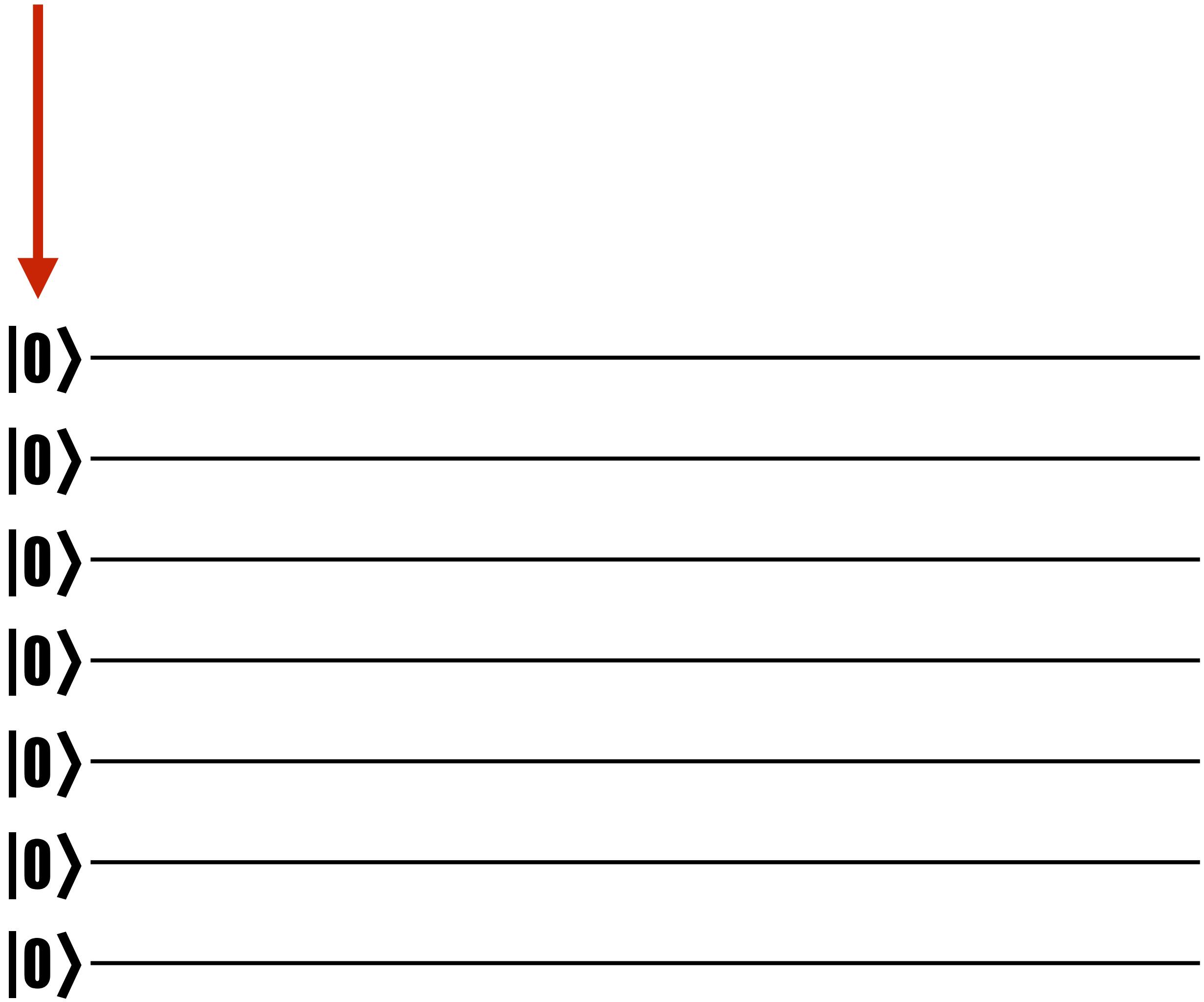

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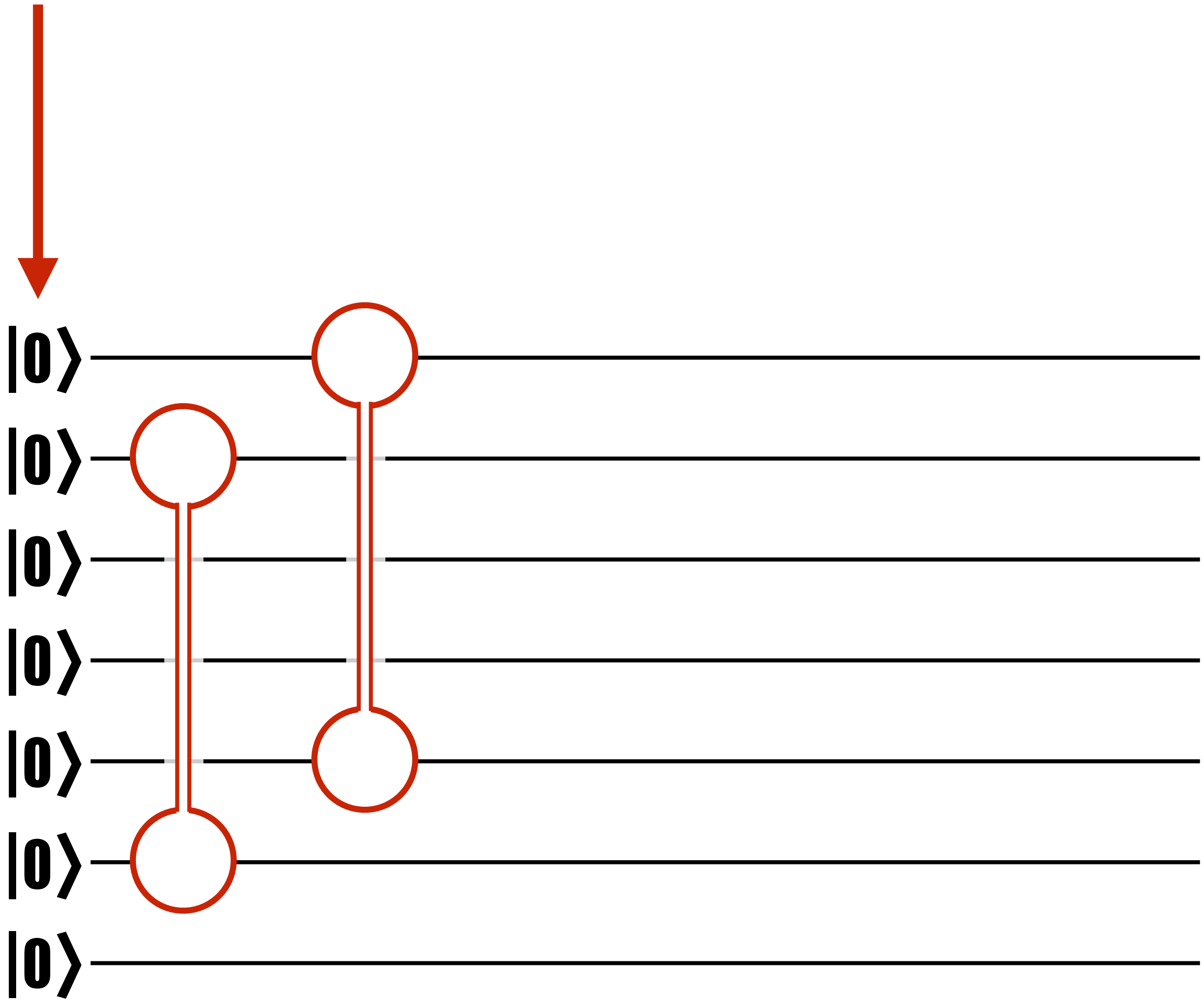

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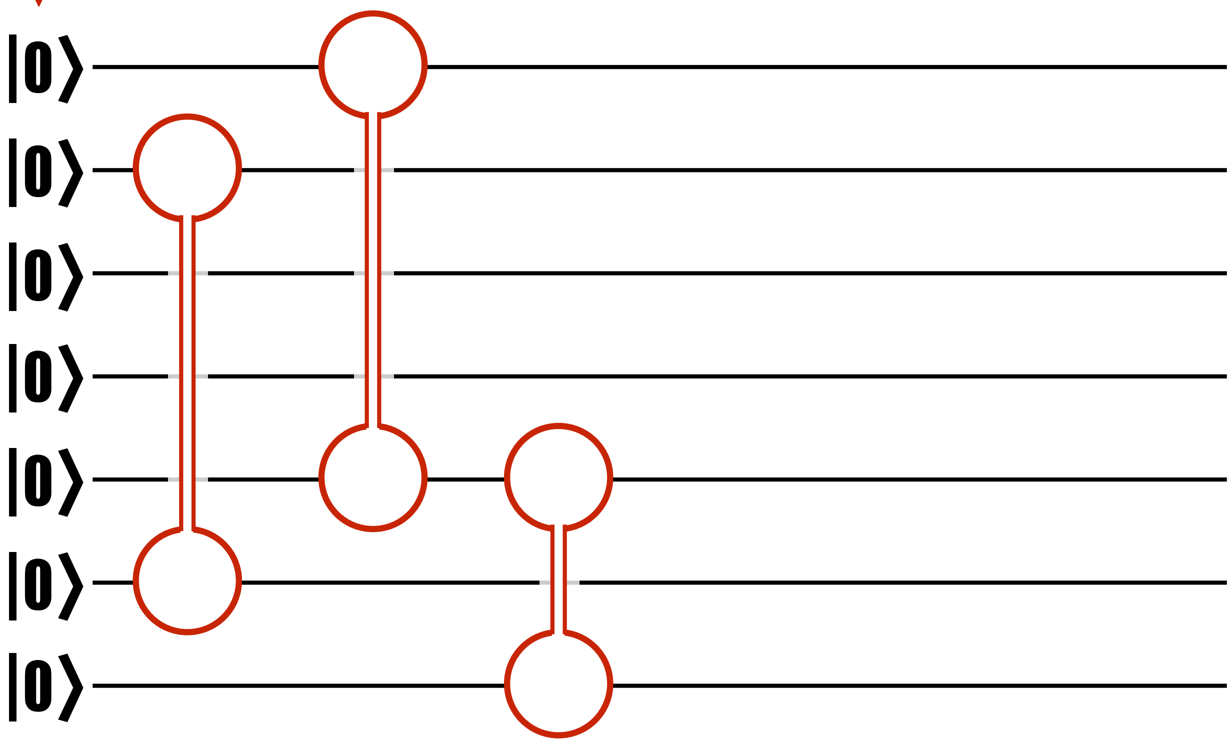

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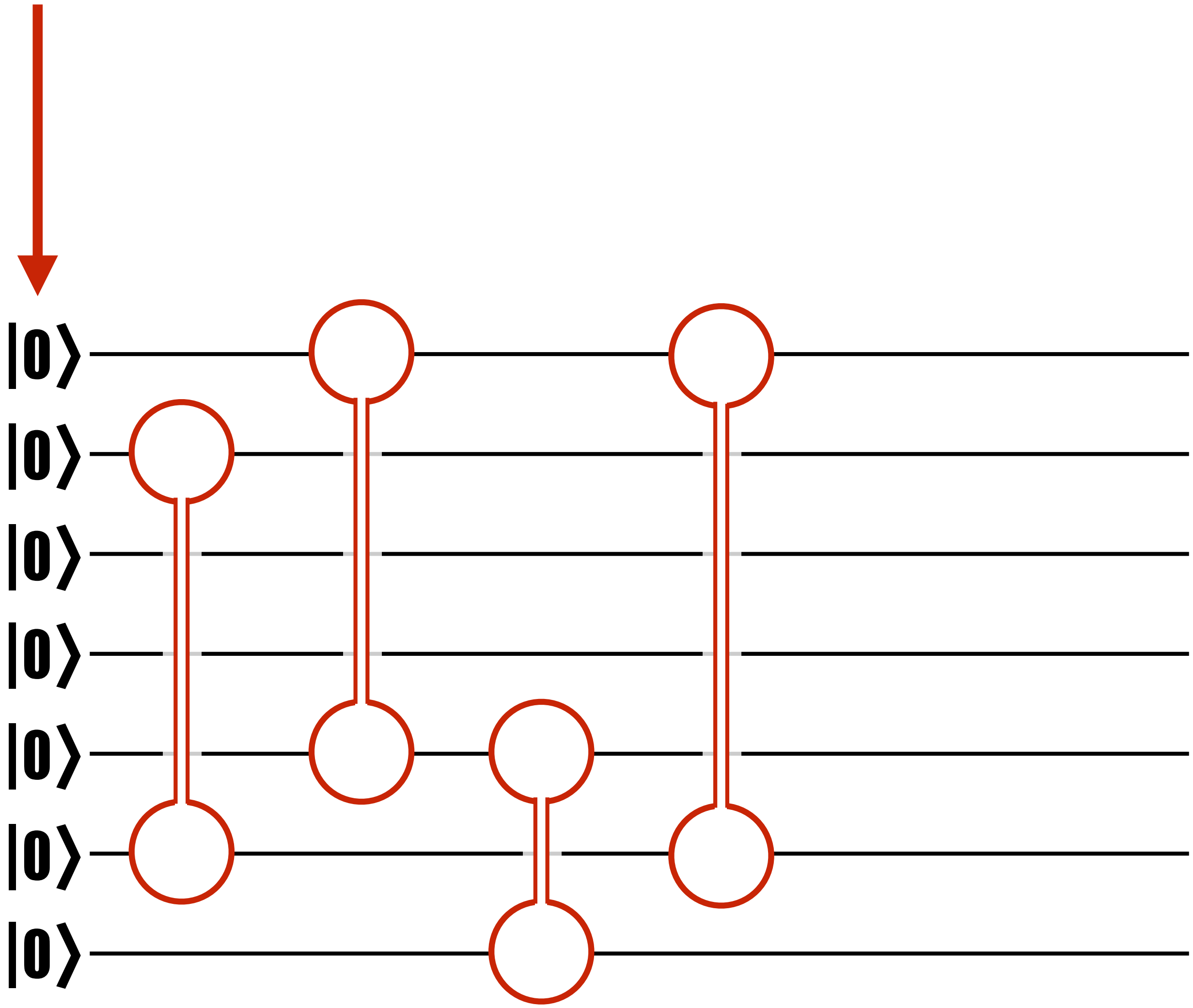

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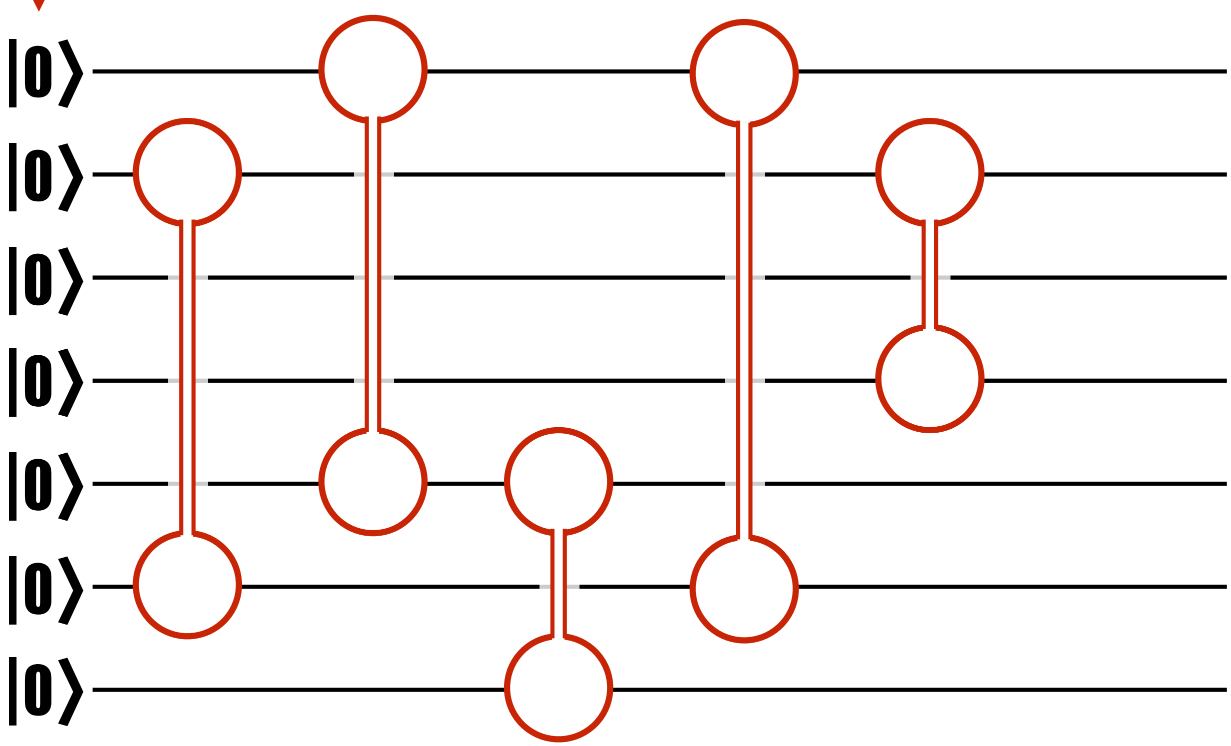

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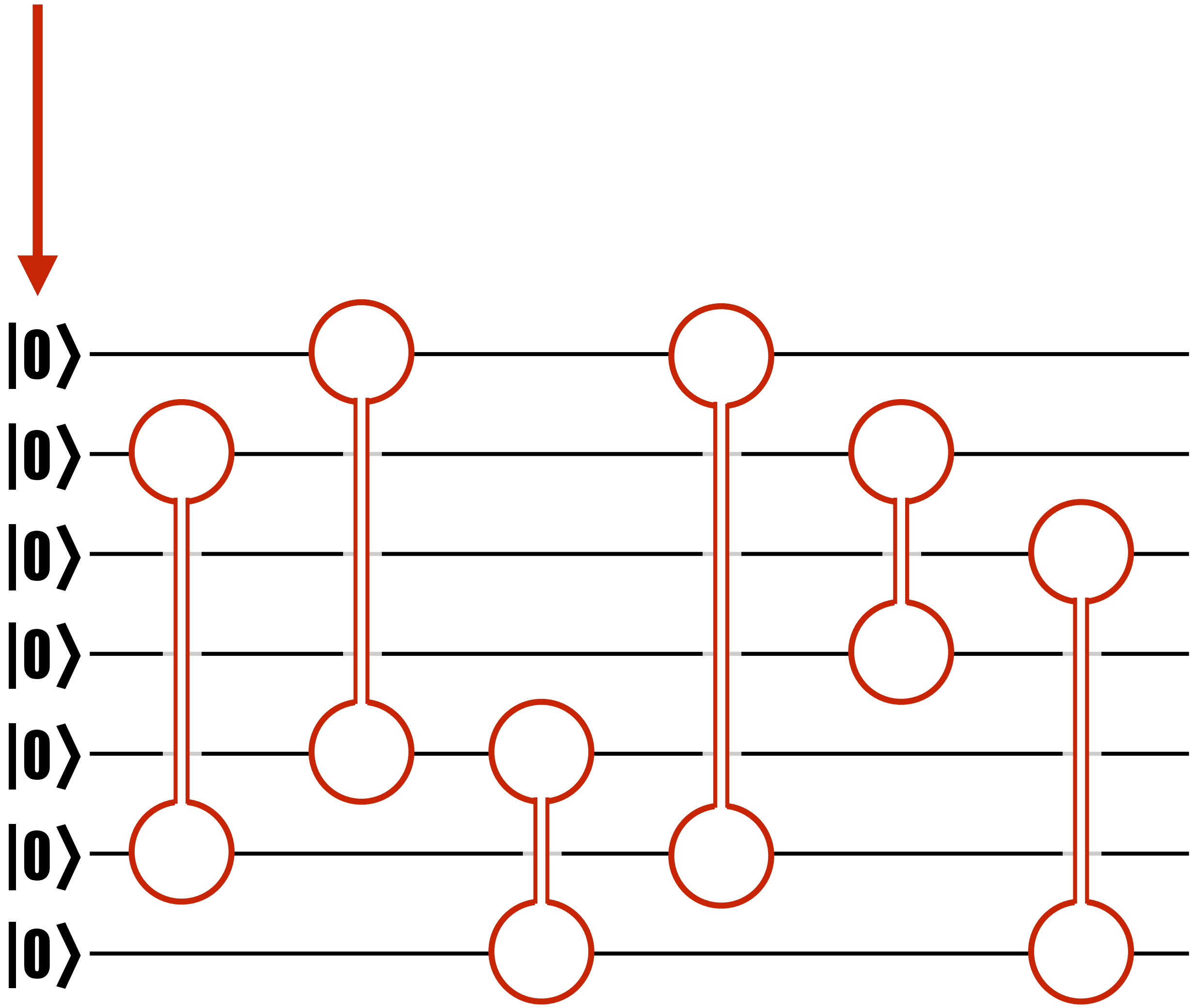

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evolution of complexity?

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QUANTUM (N qubits)

e.g. $\alpha_1|00000000\rangle + \ldots$

$$\ldots + \alpha_{2^N}|11111111\rangle$$

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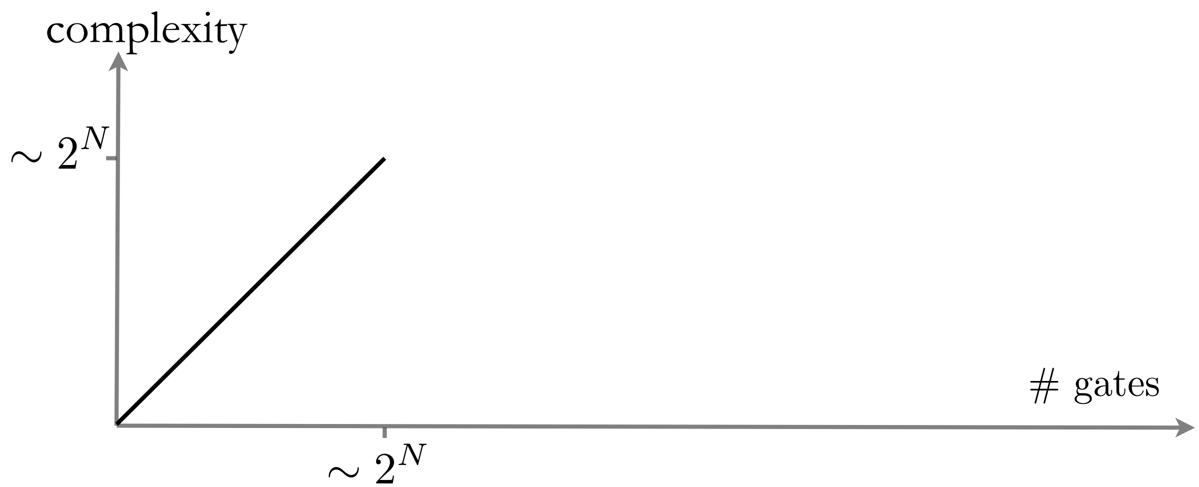

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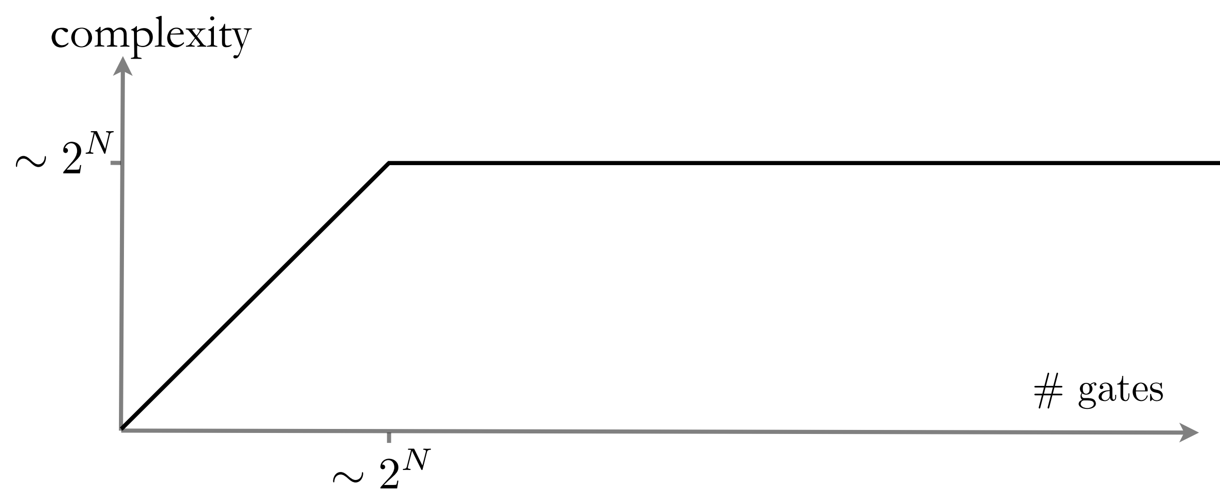

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↓

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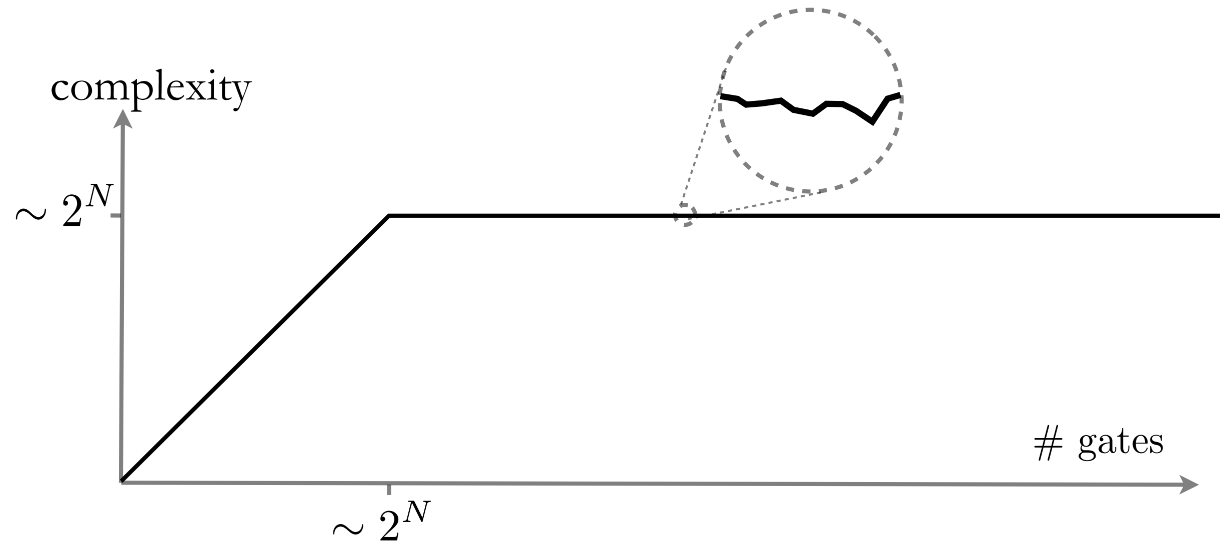

complexity ~ size of wormhole?

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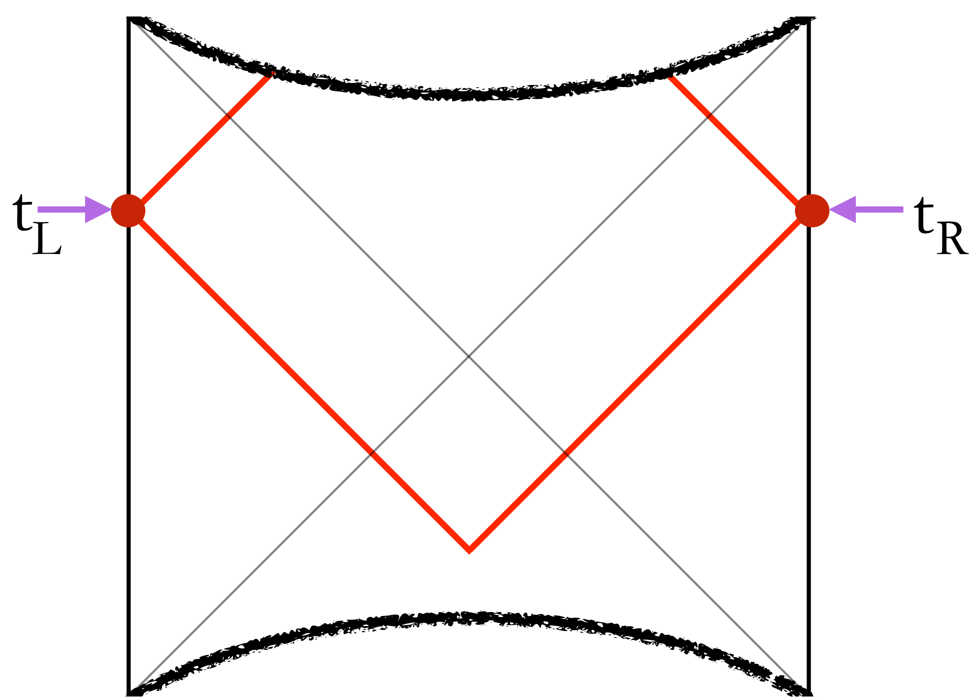

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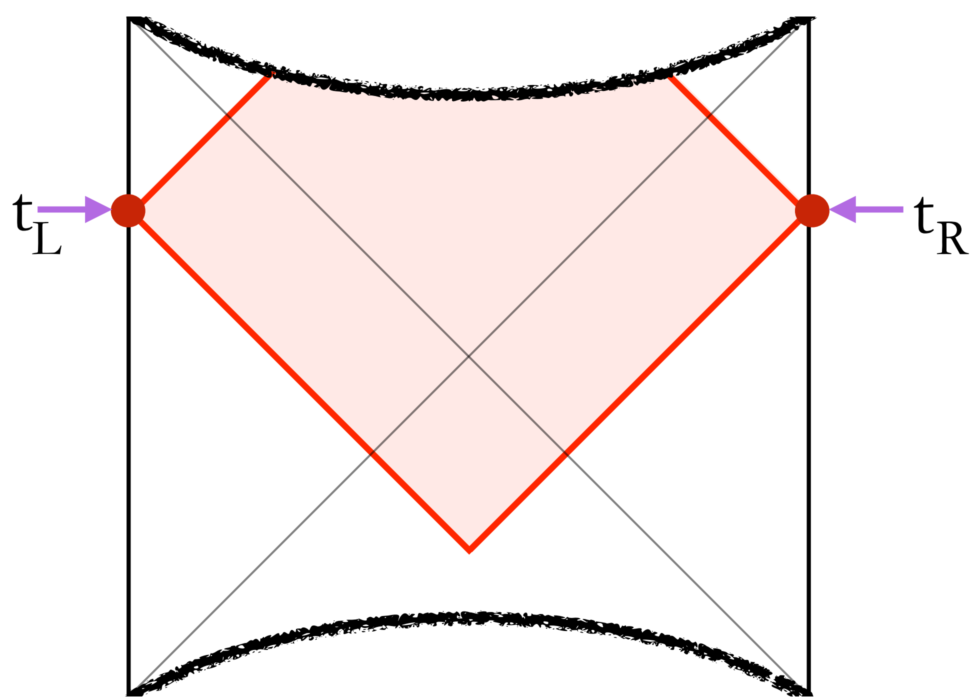

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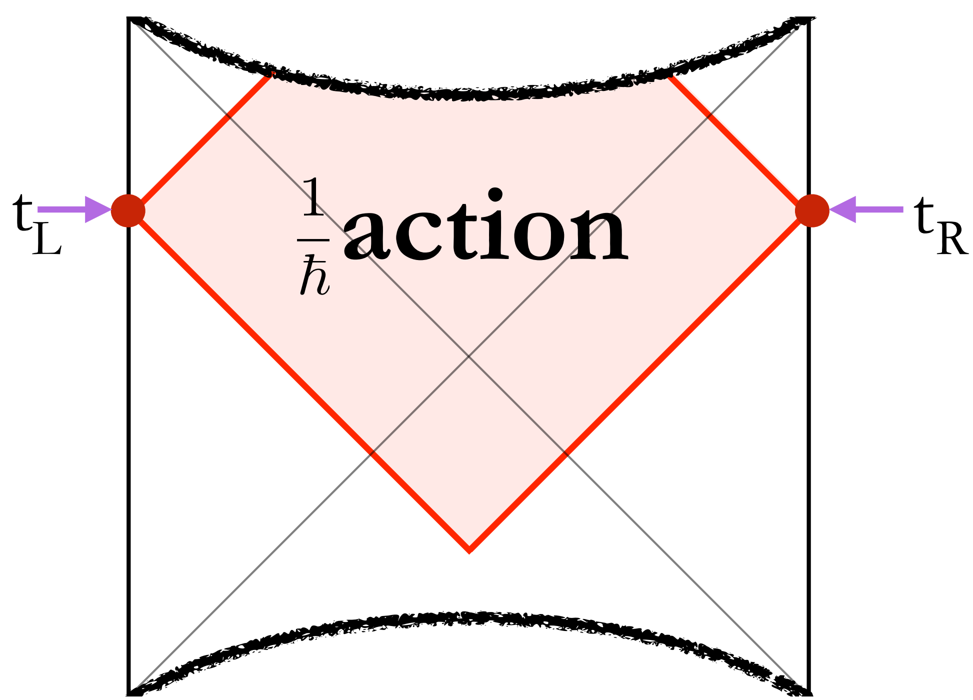

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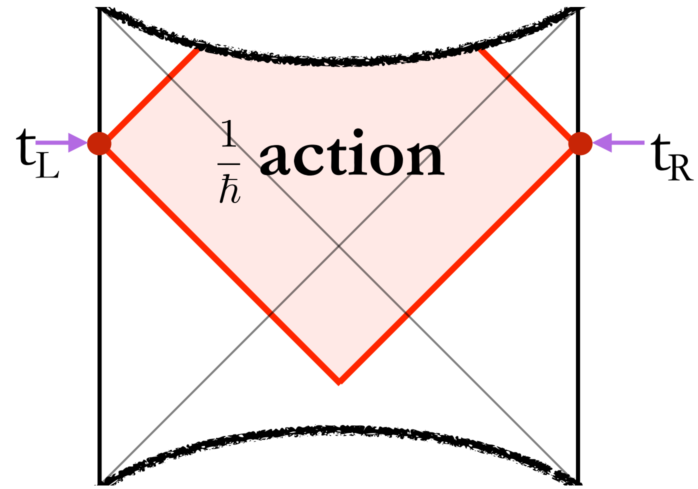

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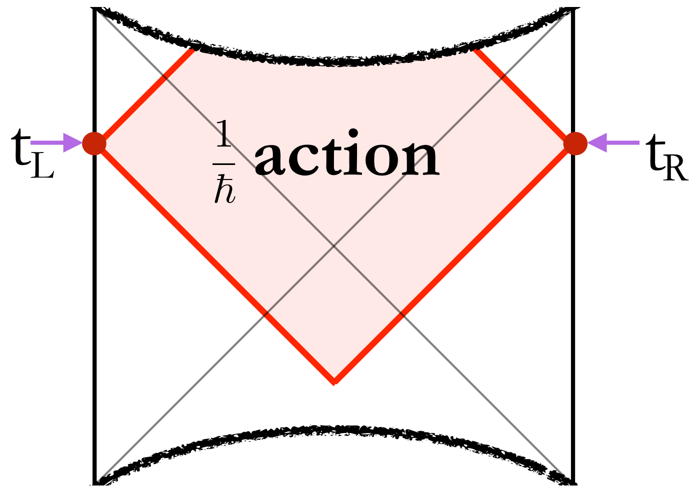

$$\text{Complexity} \sim \frac{\text{Action}}{\hbar}$$

$$\boxed{\text{Complexity} = \frac{\text{Action}}{\pi\hbar}}$$

ASSUME a conjectured bound on rate of computation
AND that black holes saturate that bound

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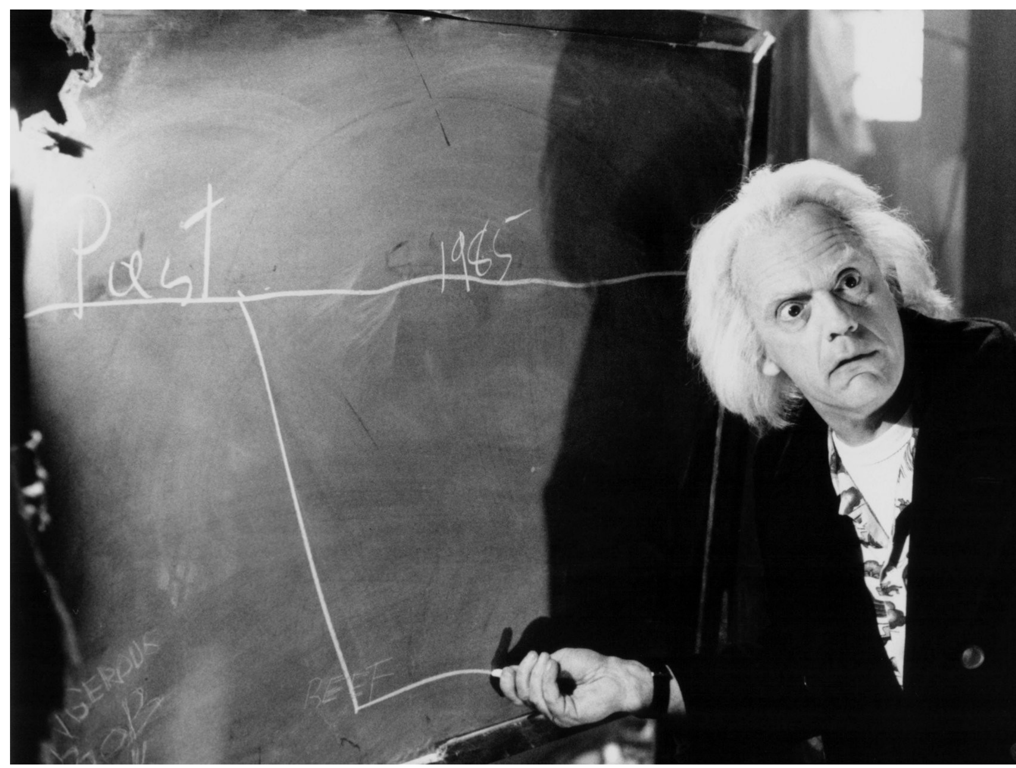

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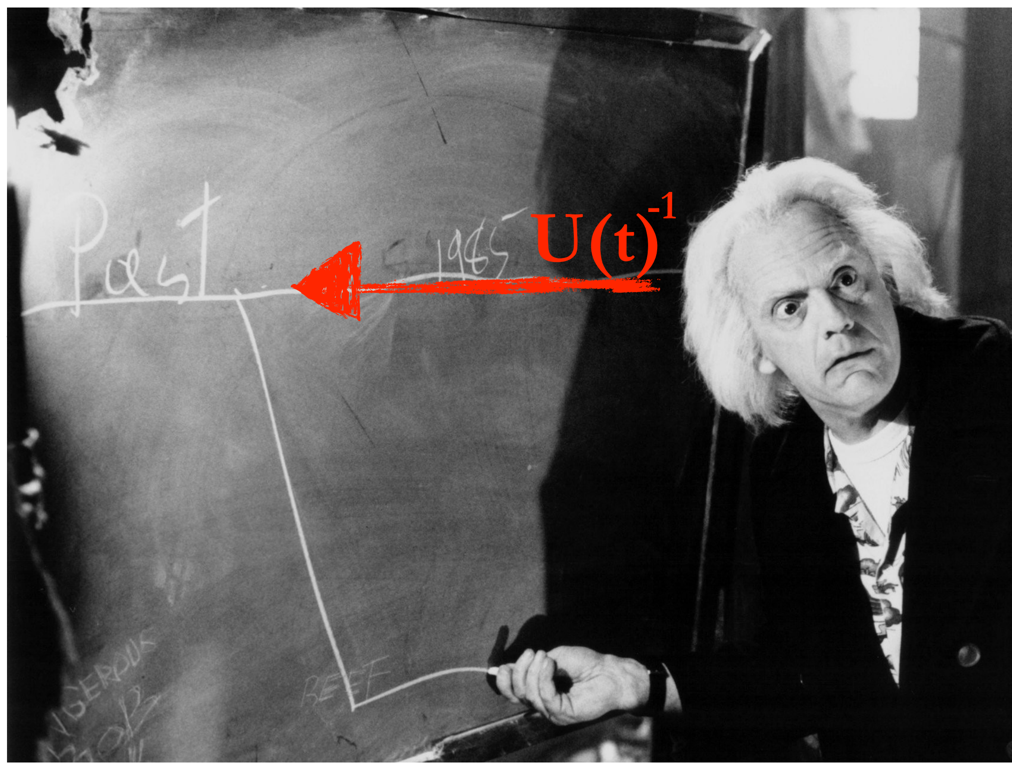

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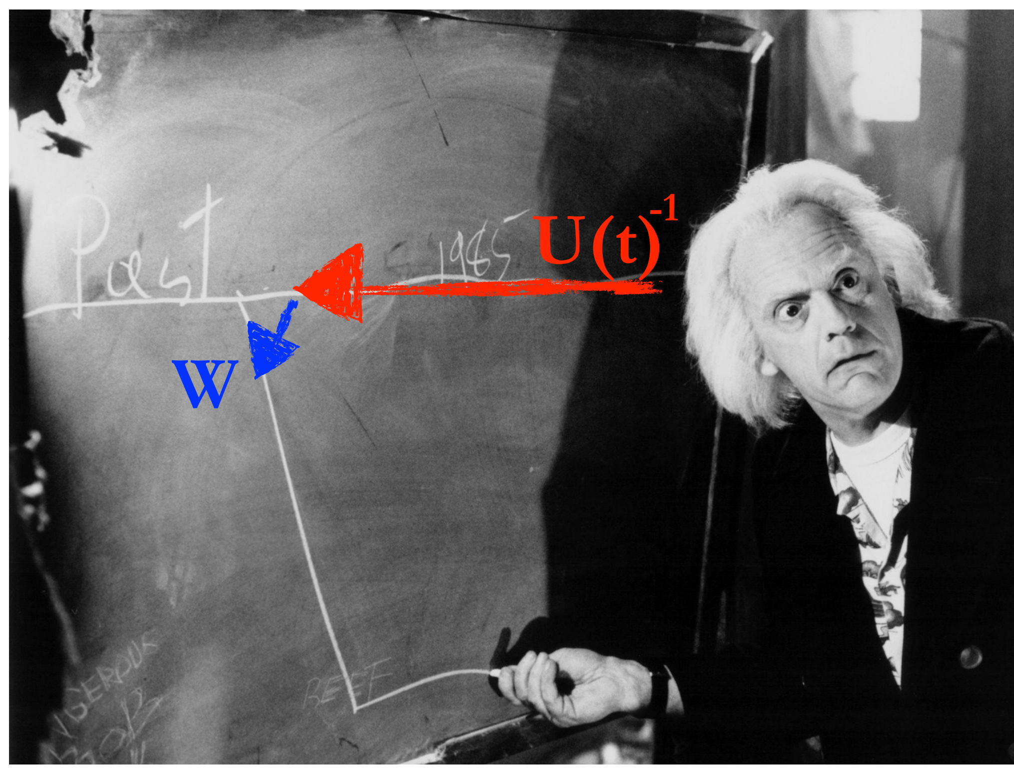

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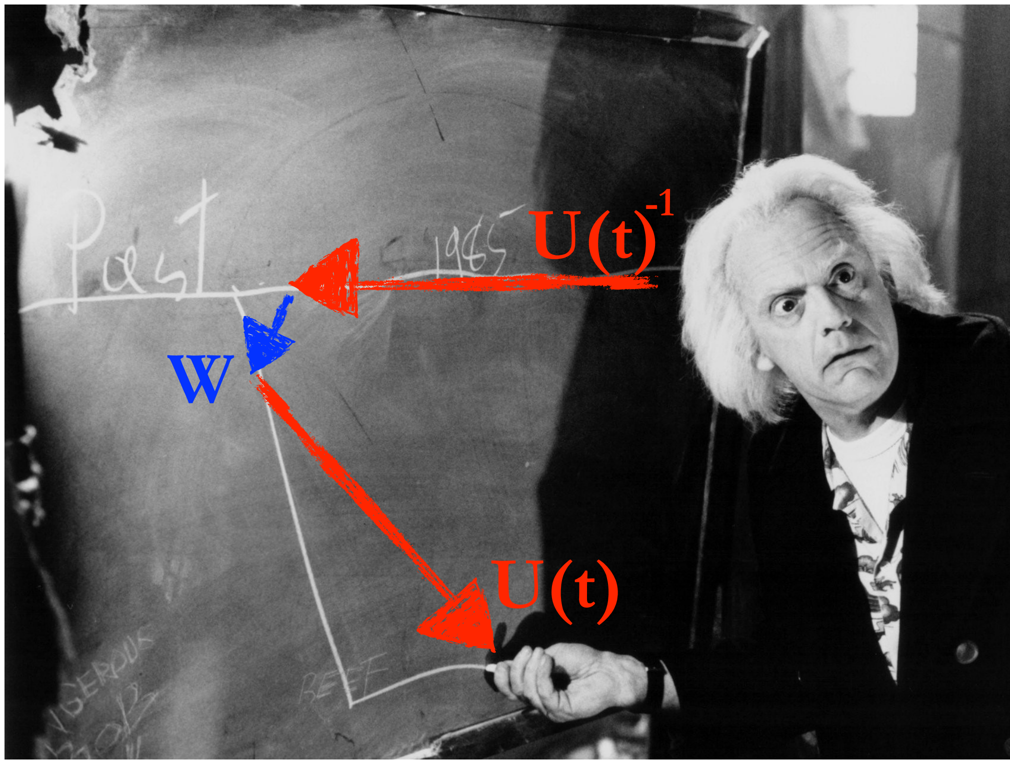

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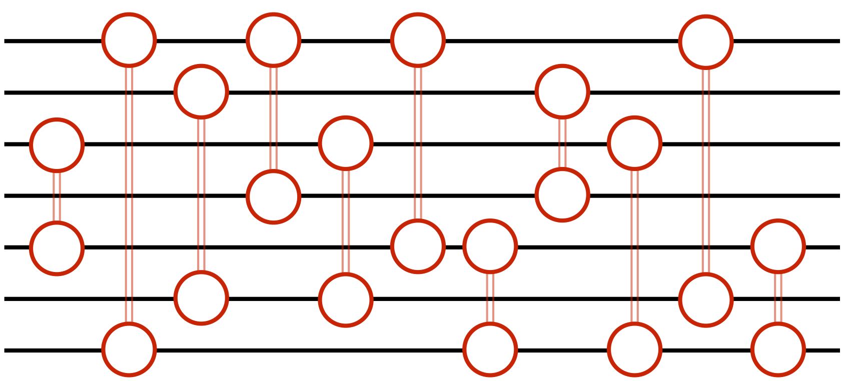

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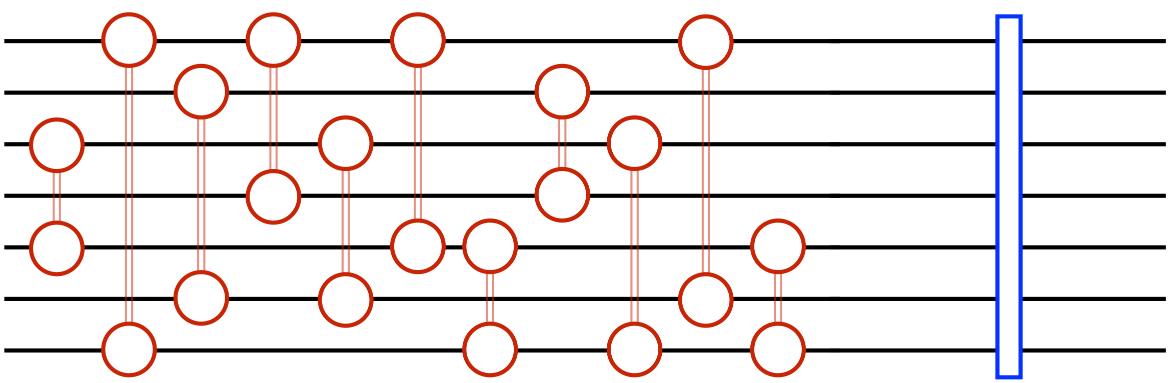

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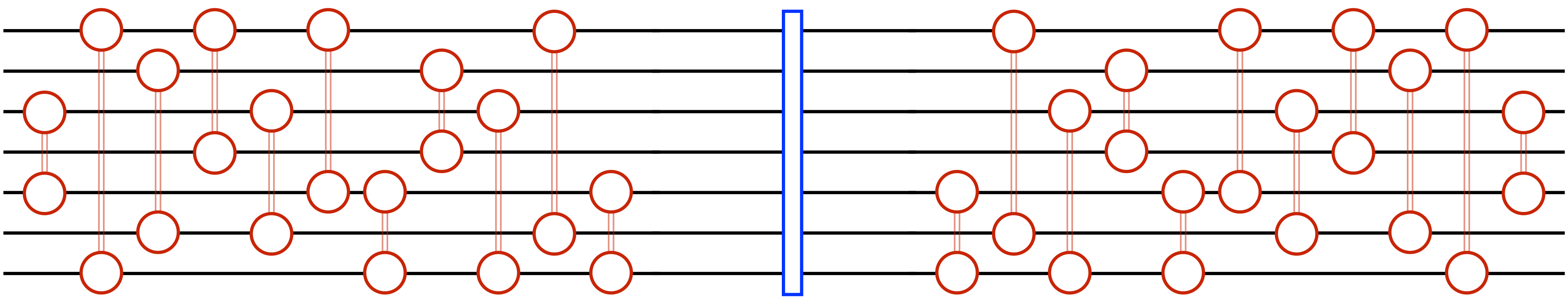

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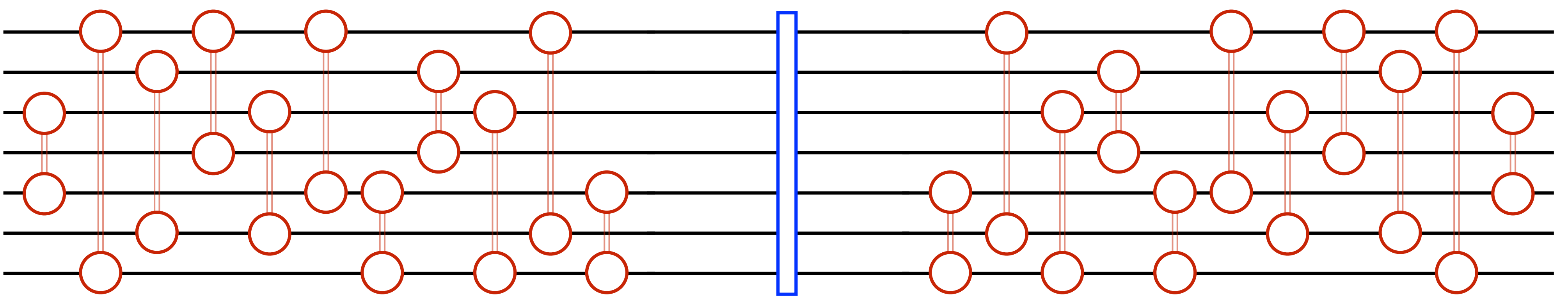

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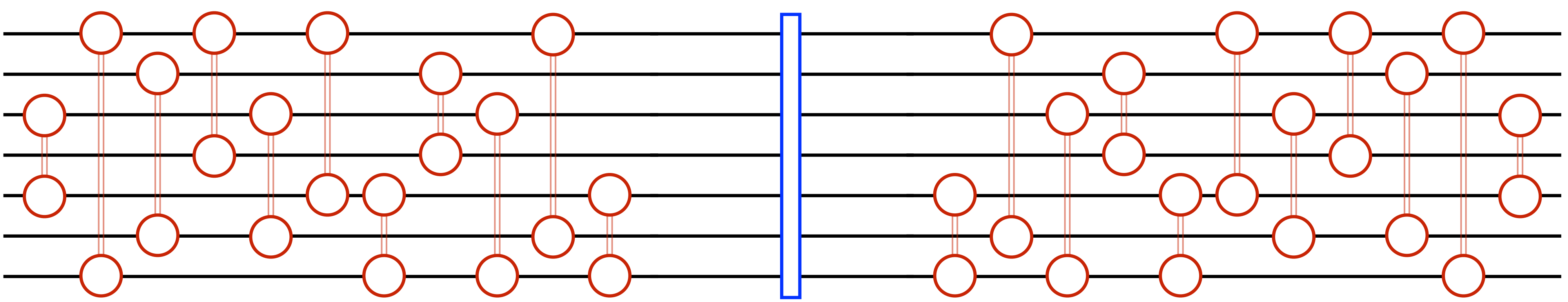

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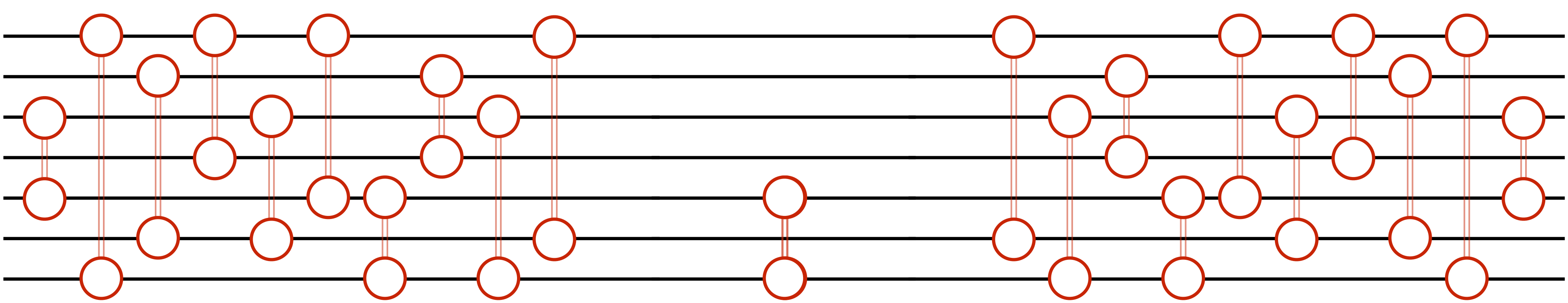

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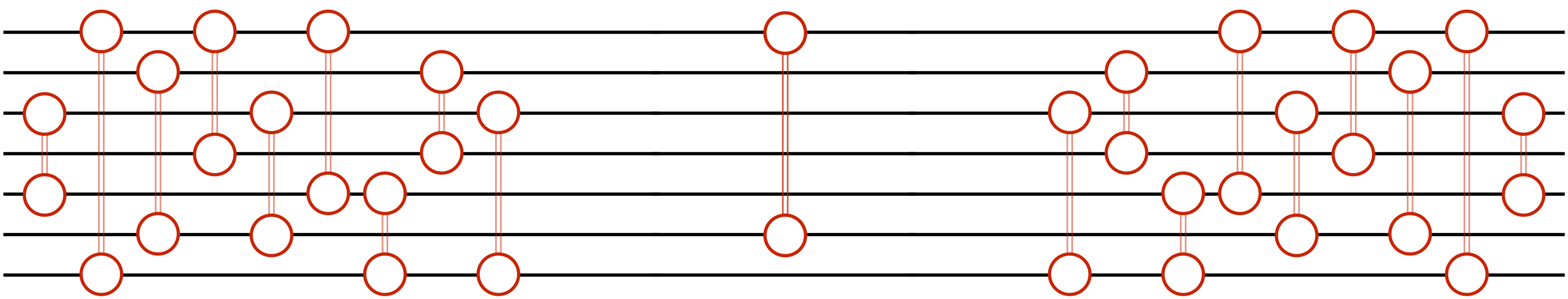

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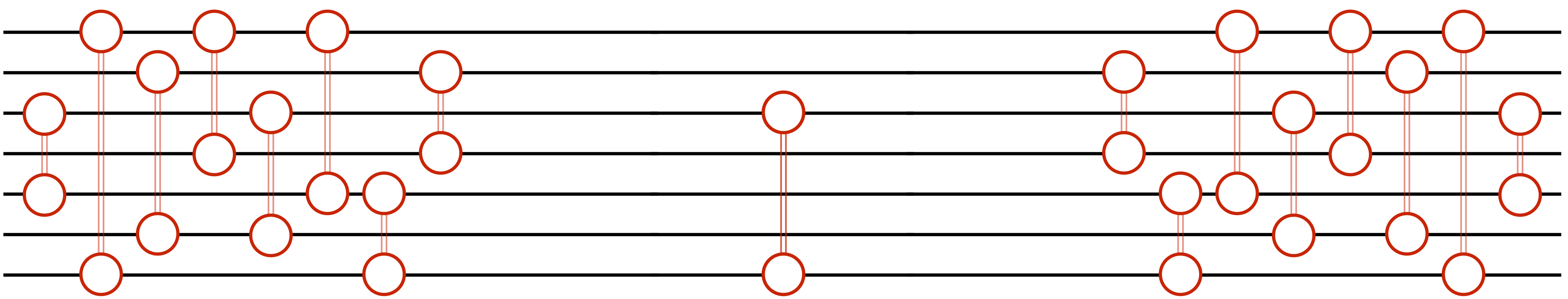

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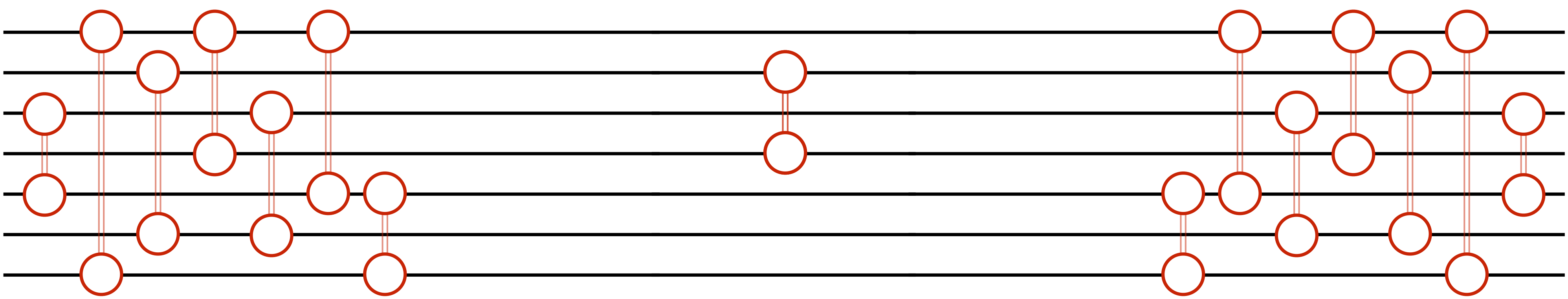

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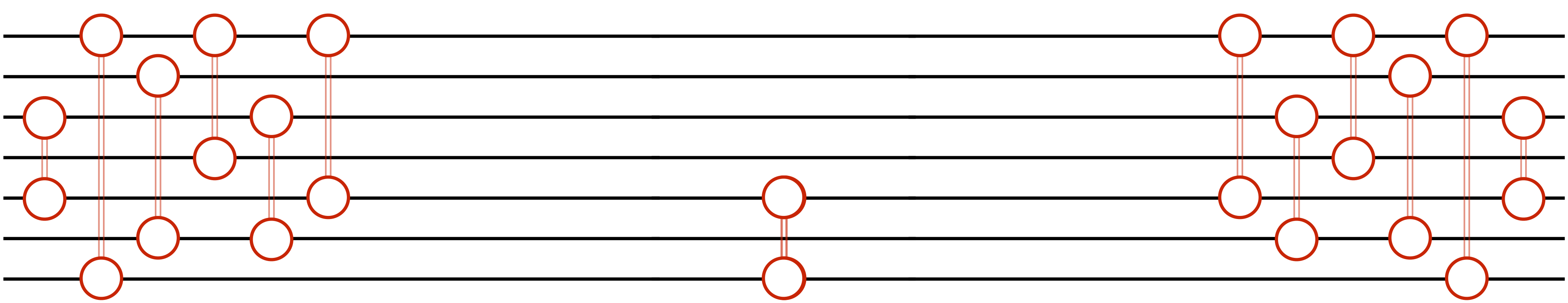

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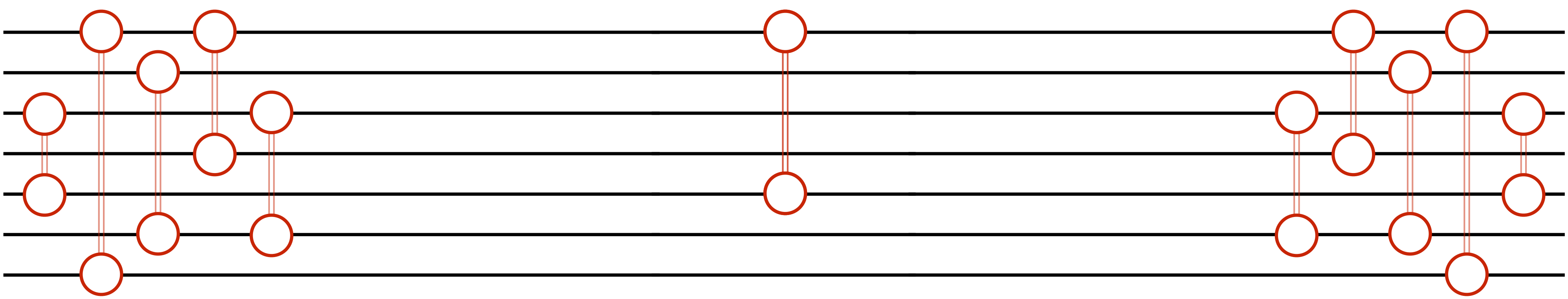

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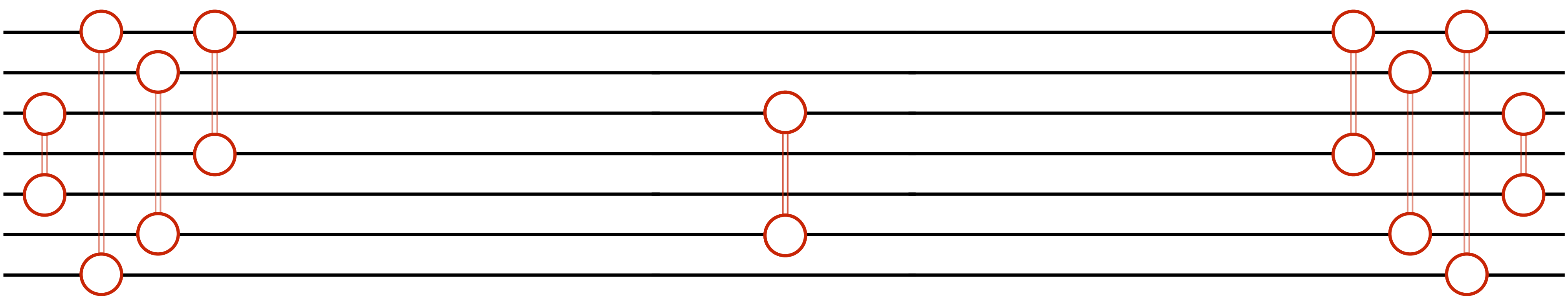

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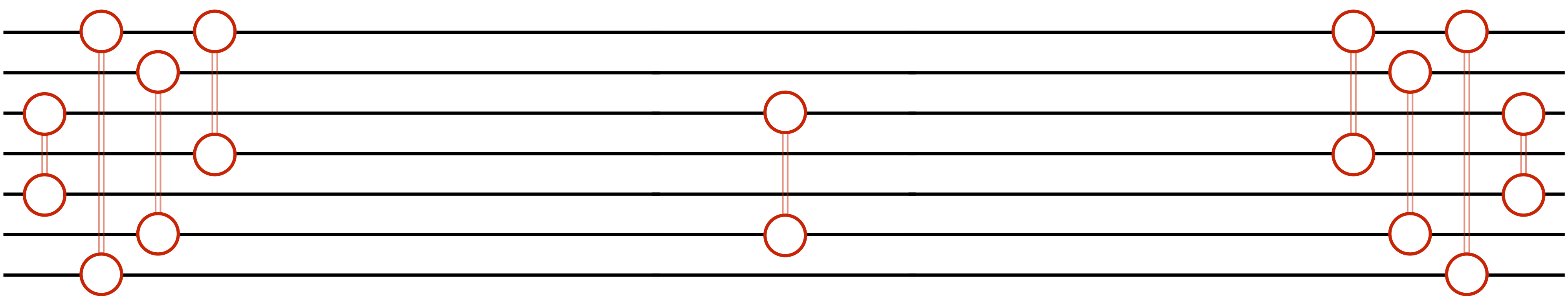

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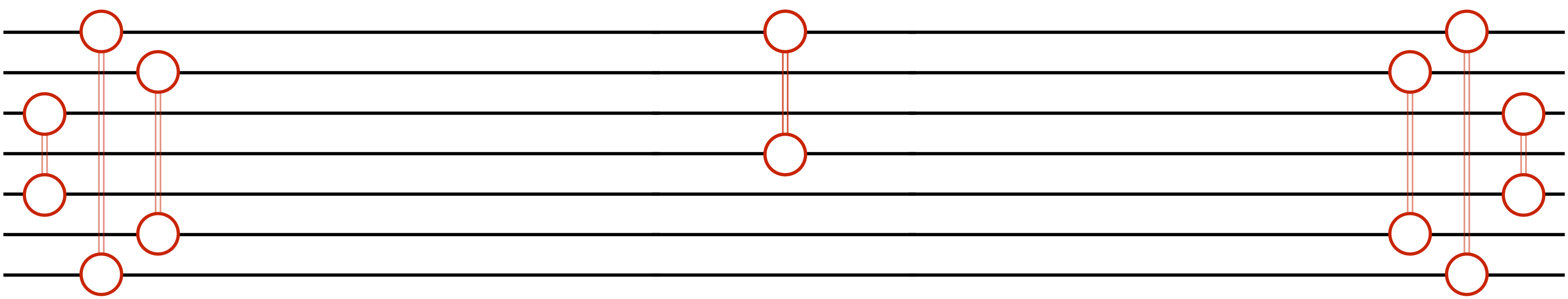

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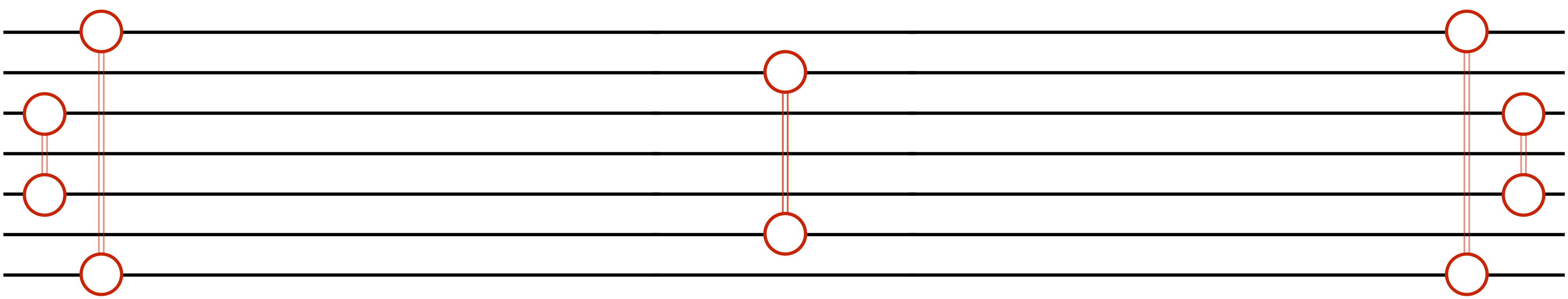

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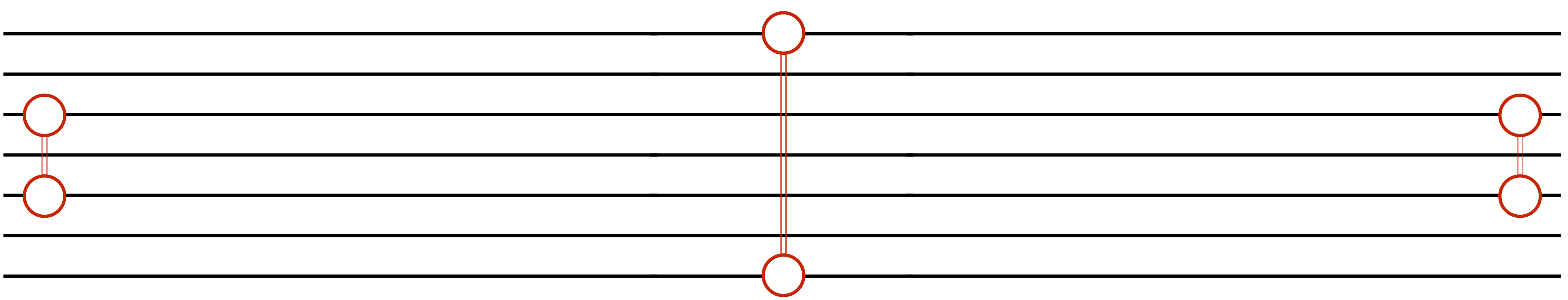

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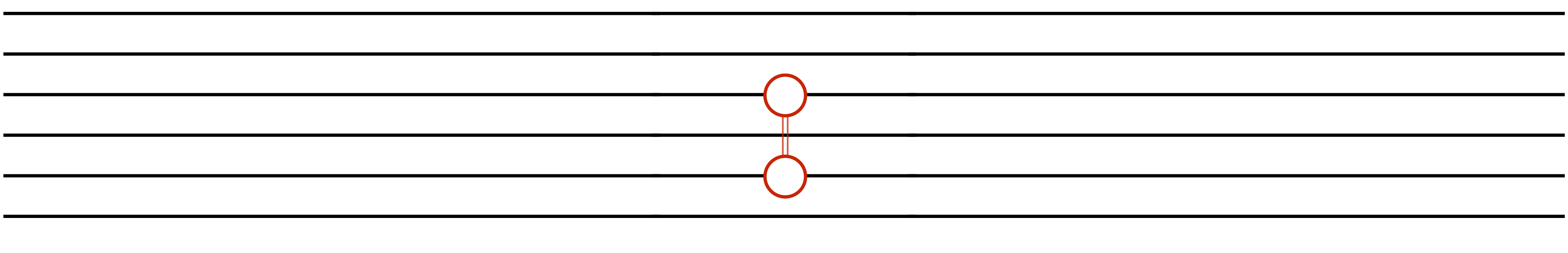

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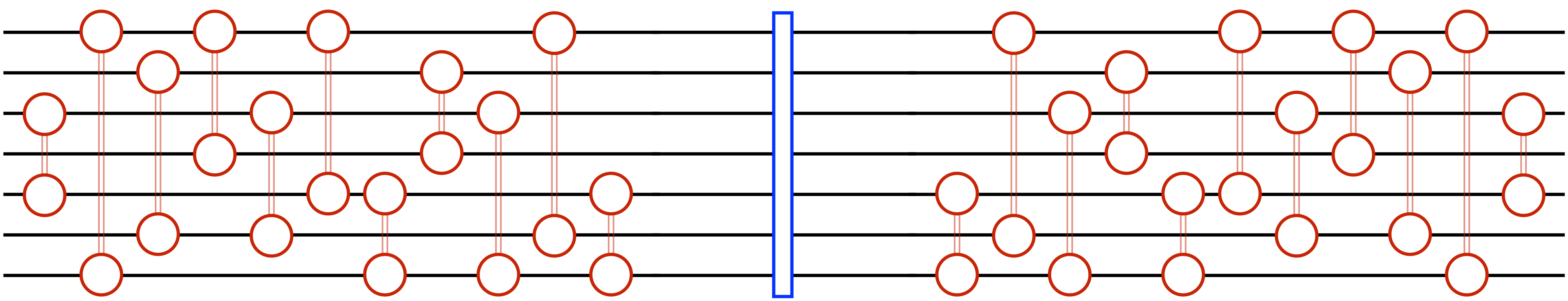

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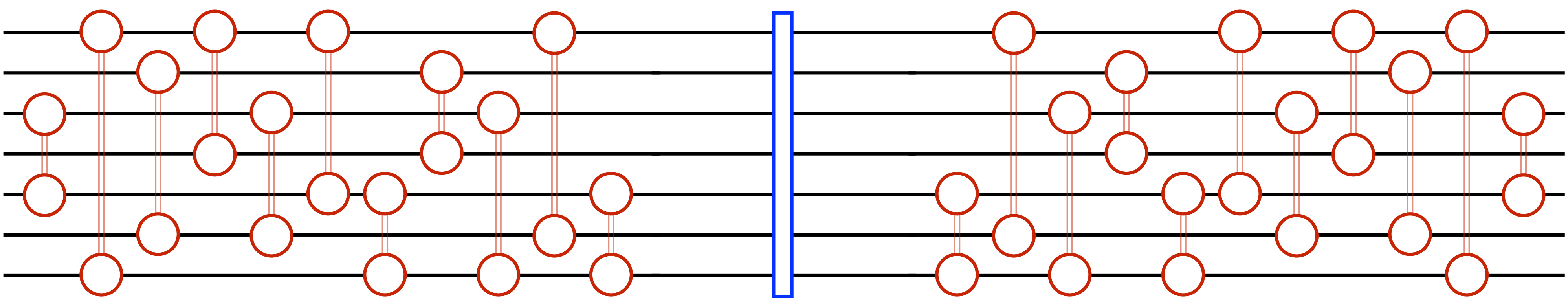

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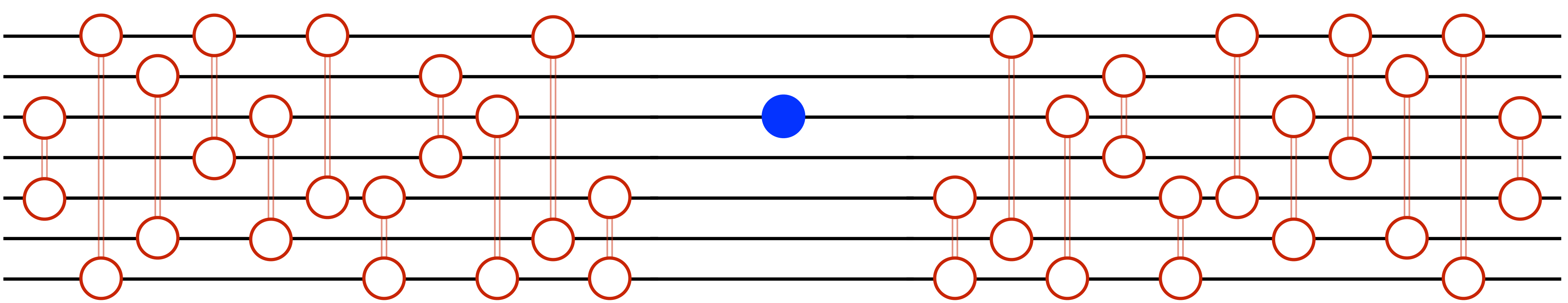

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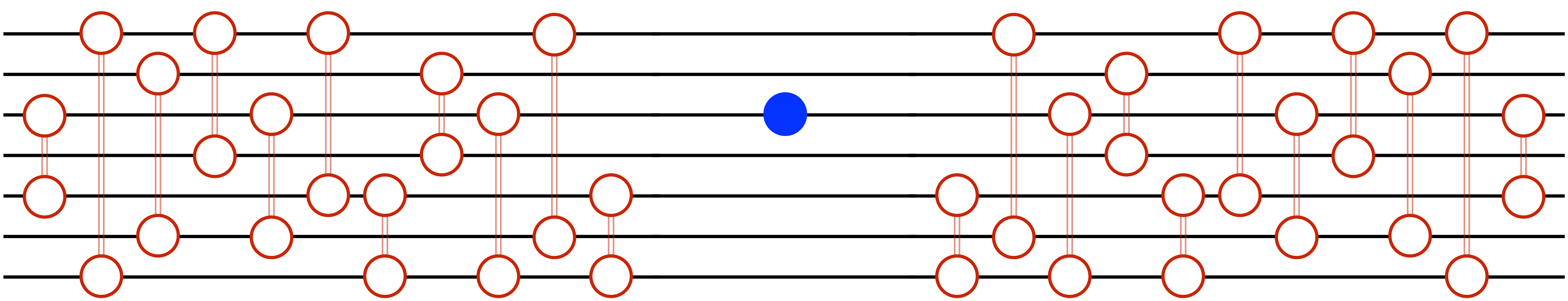

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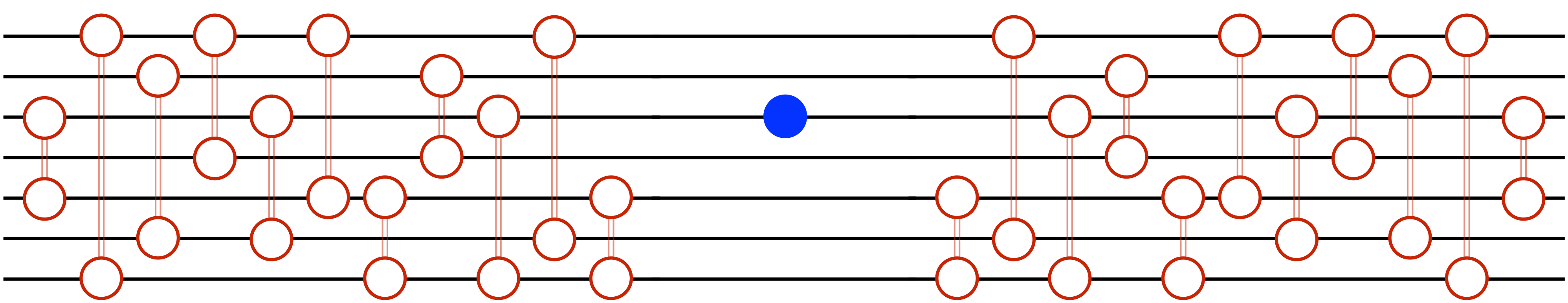

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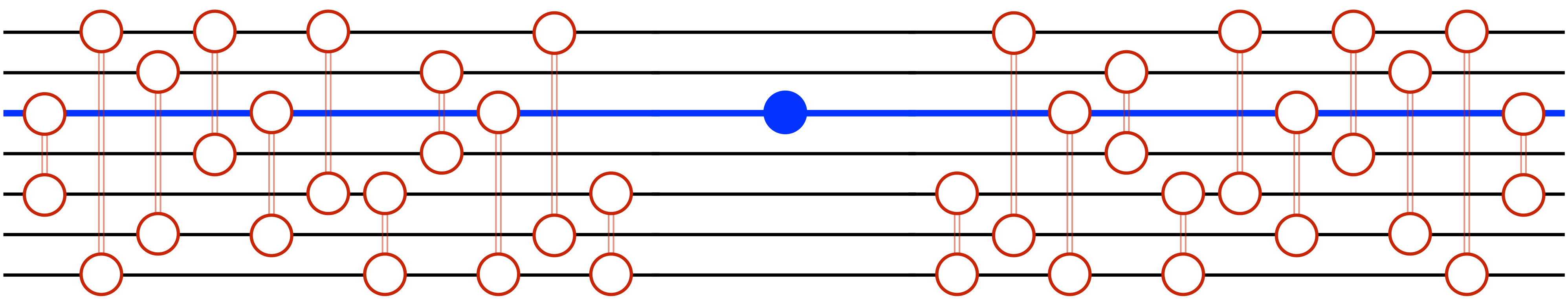

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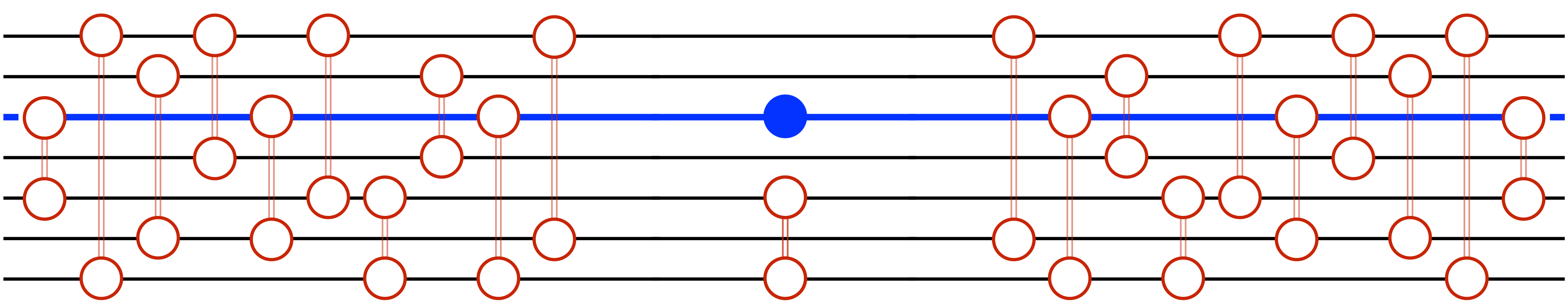

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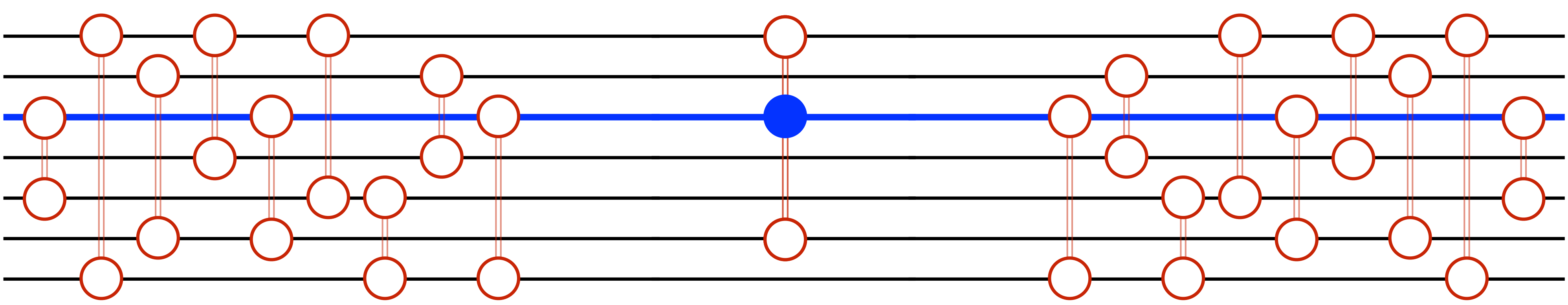

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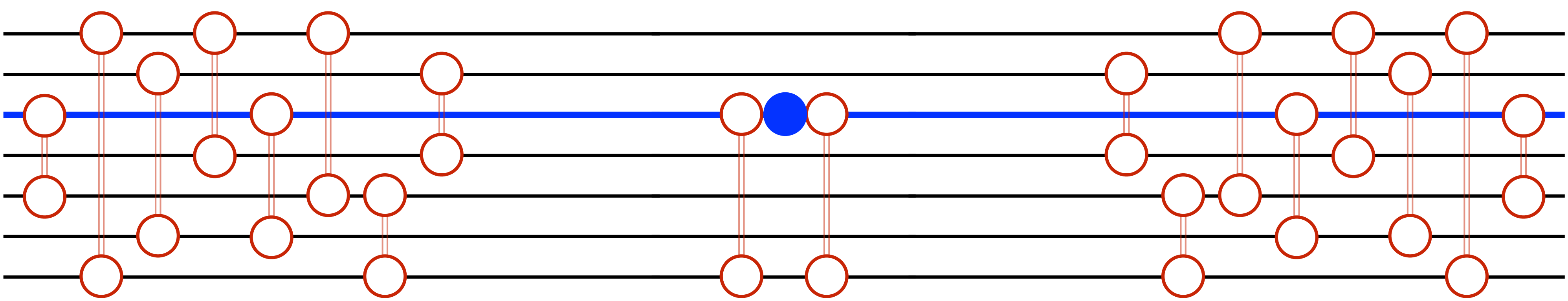

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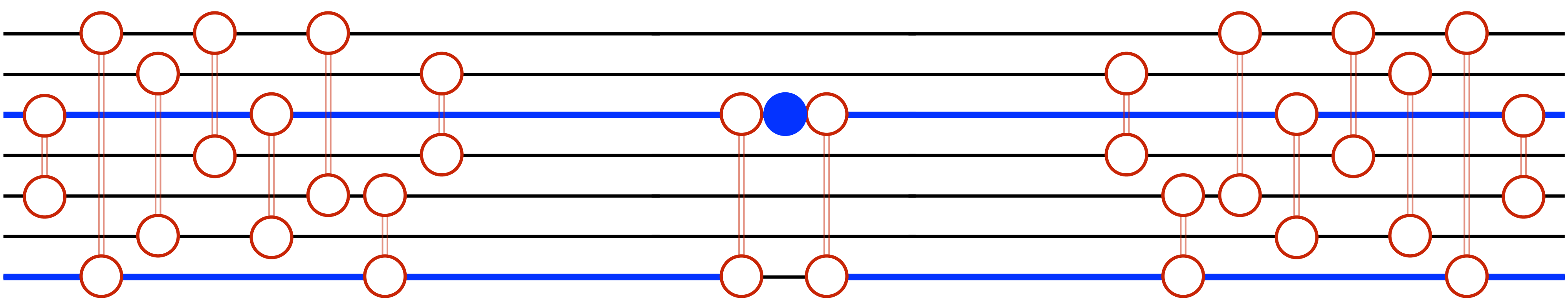

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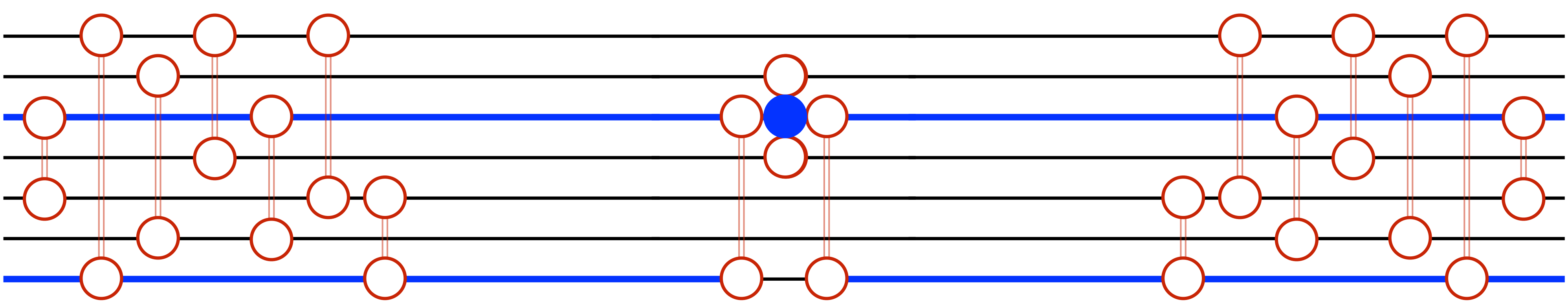

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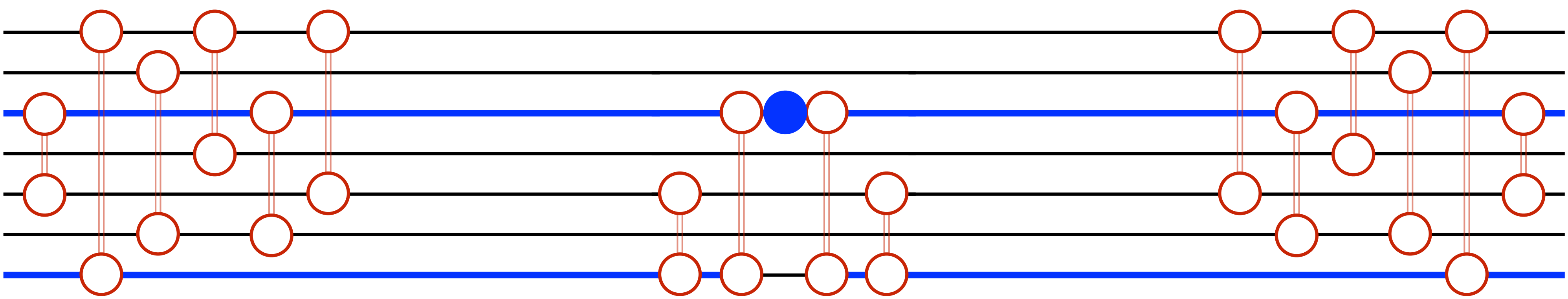

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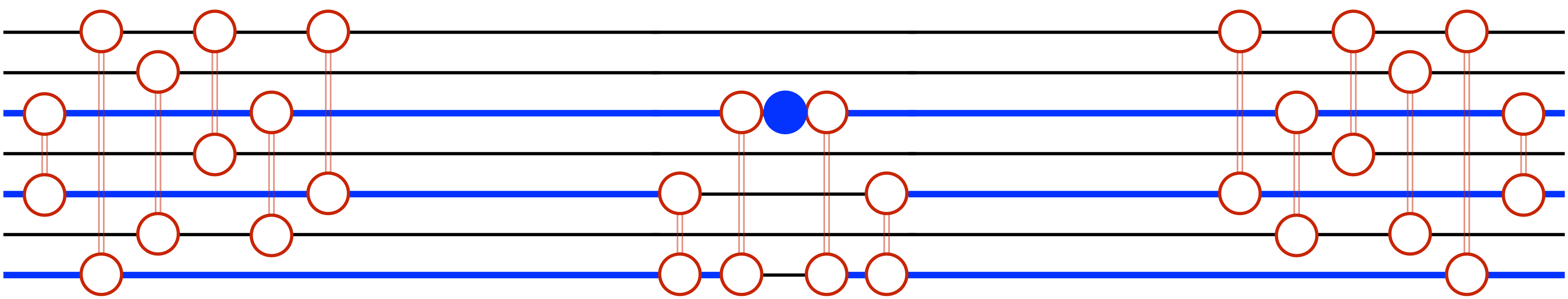

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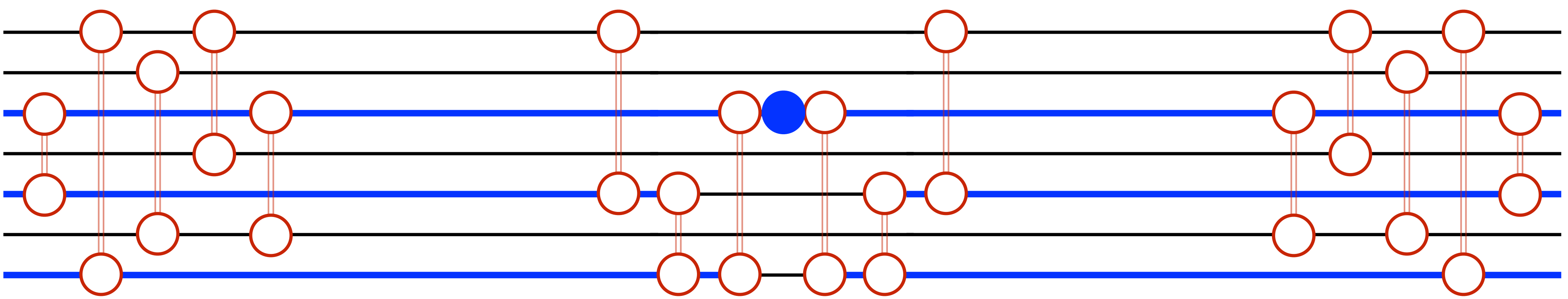

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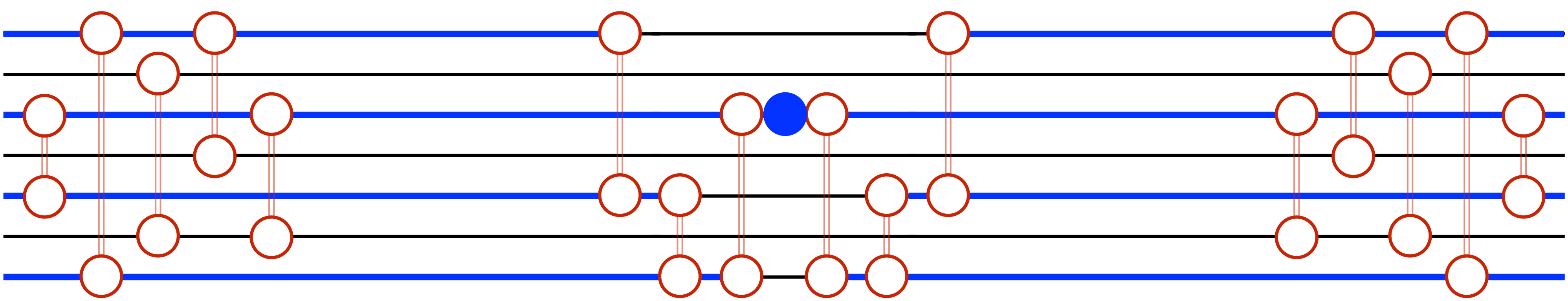

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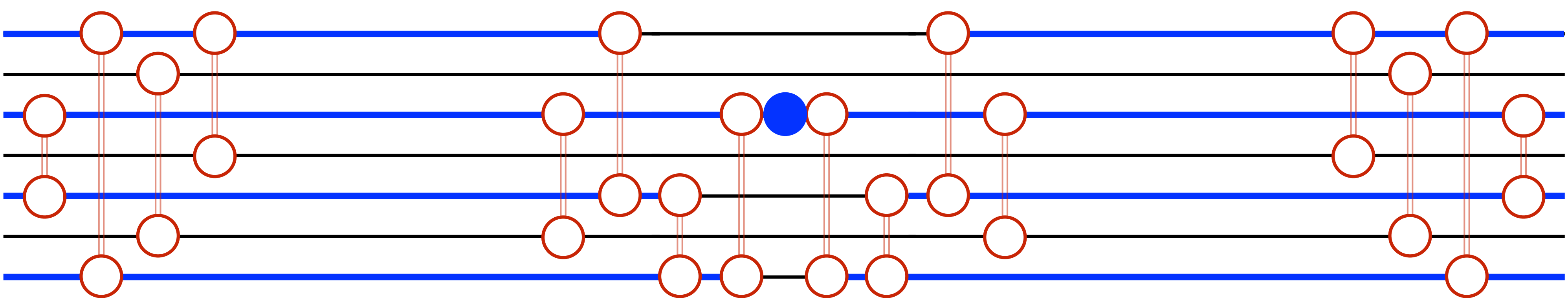

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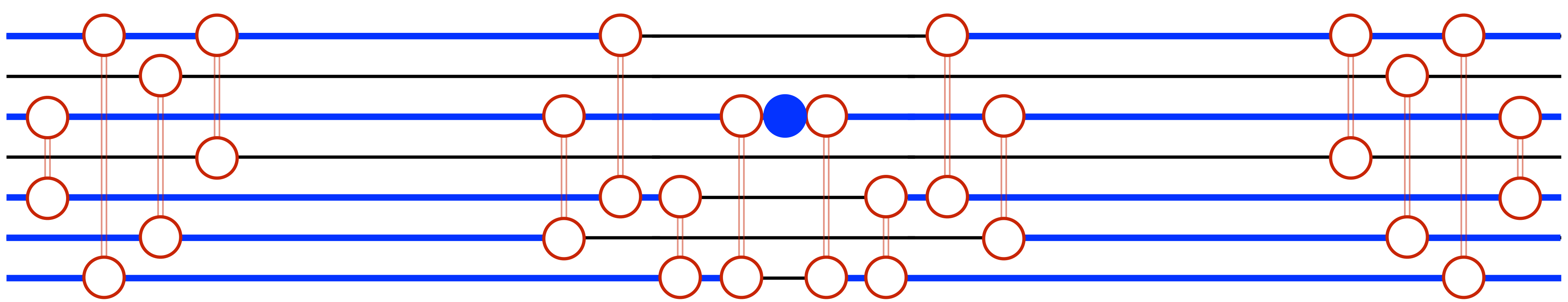

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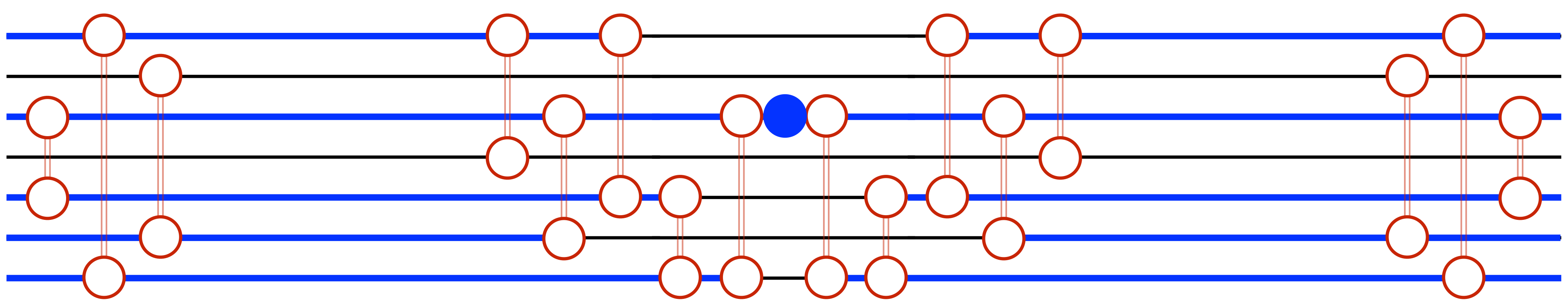

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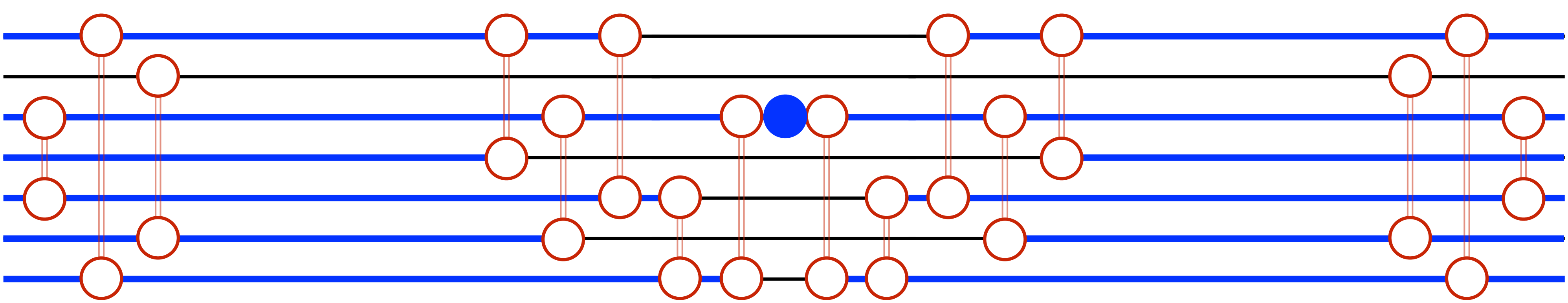

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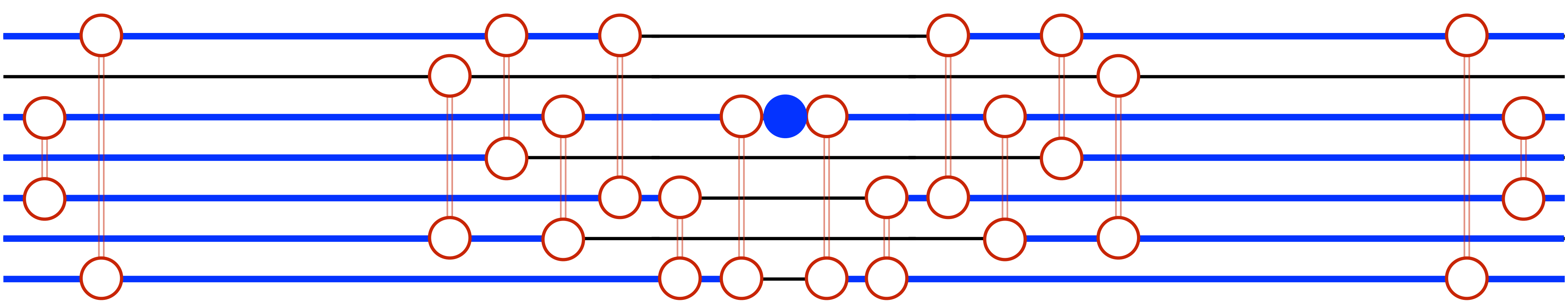

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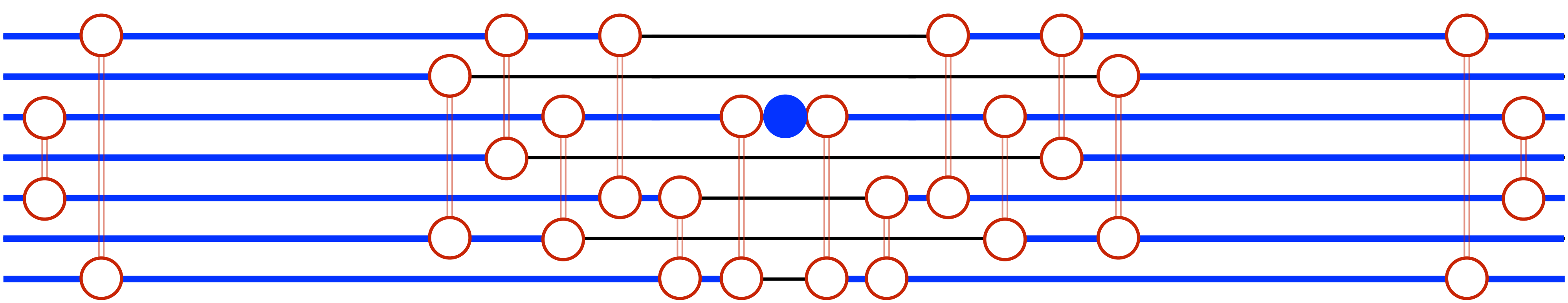

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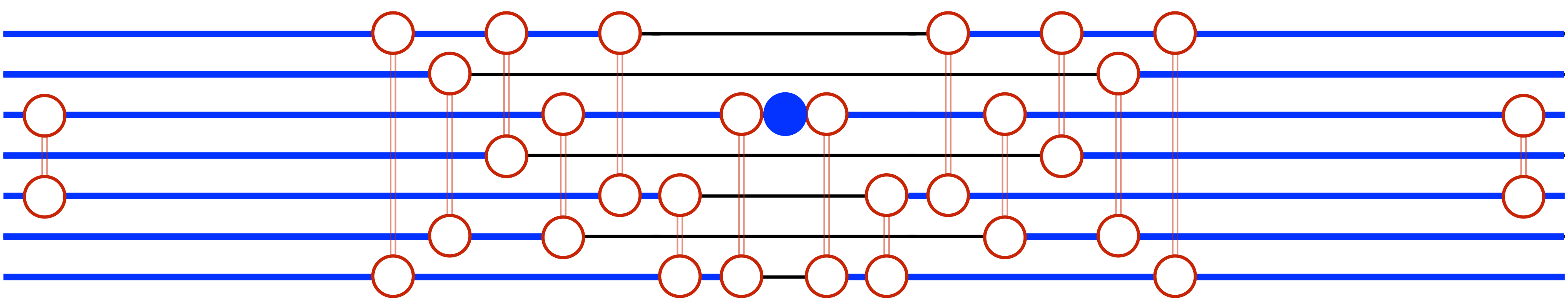

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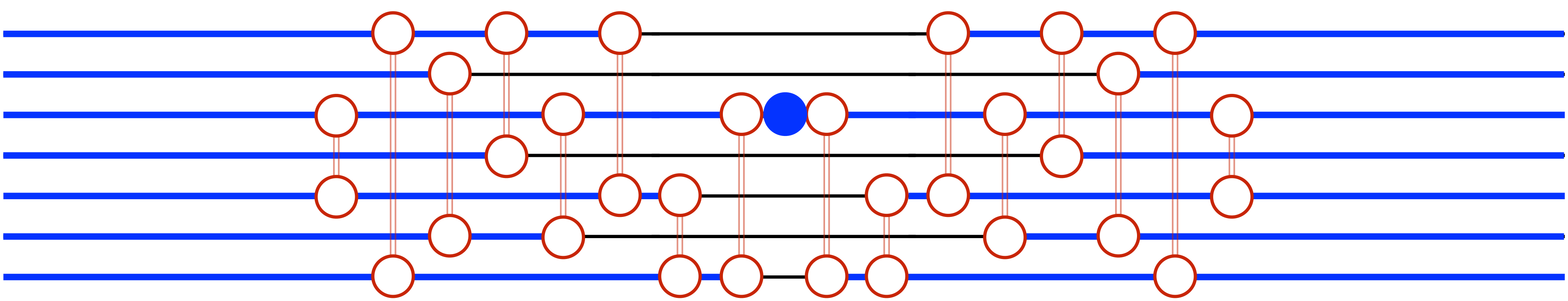

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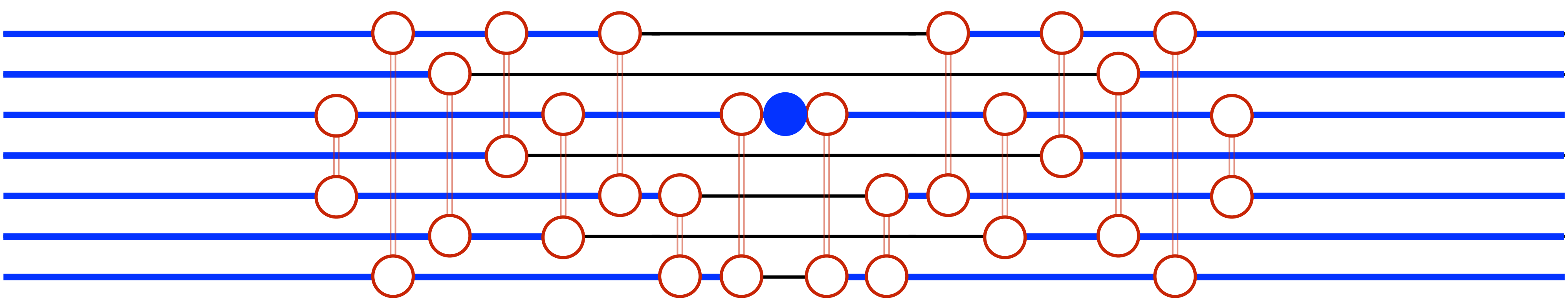
most qubits 'infected' at $t_* \equiv \frac{1}{2} N \log N$



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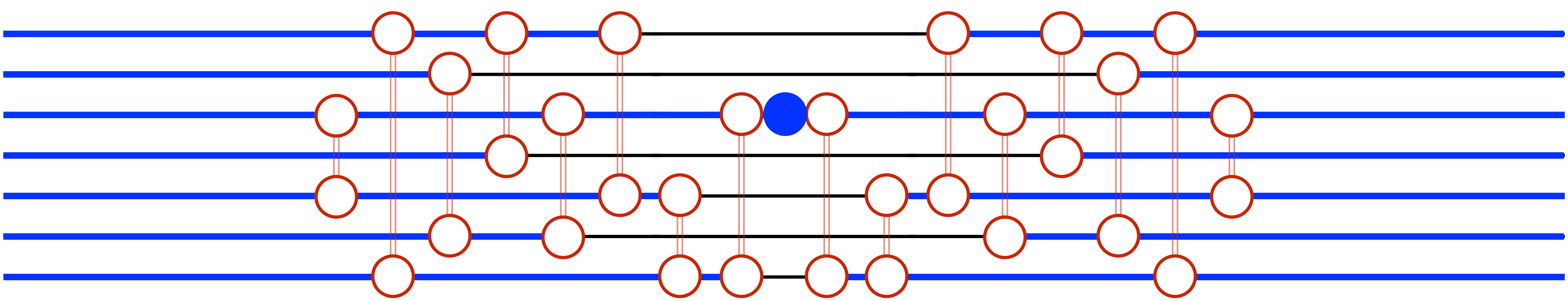
"switchback subtraction"

most qubits 'infected' at $t_* \equiv \frac{1}{2} N \log N$



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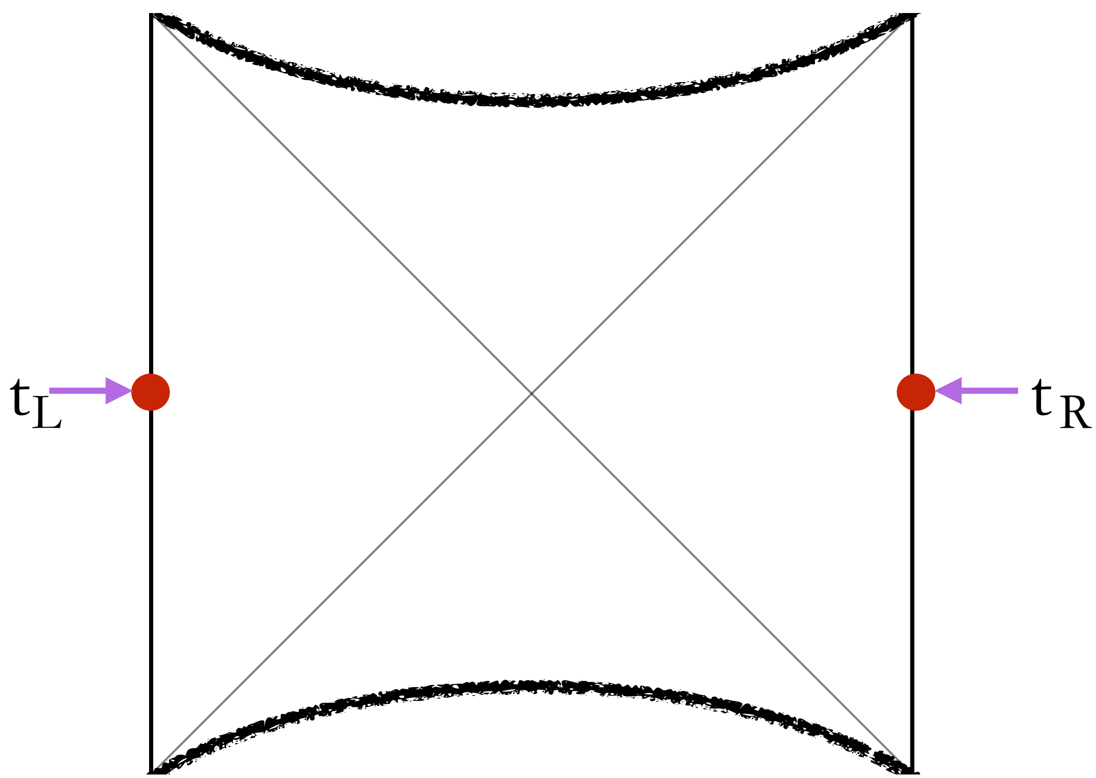

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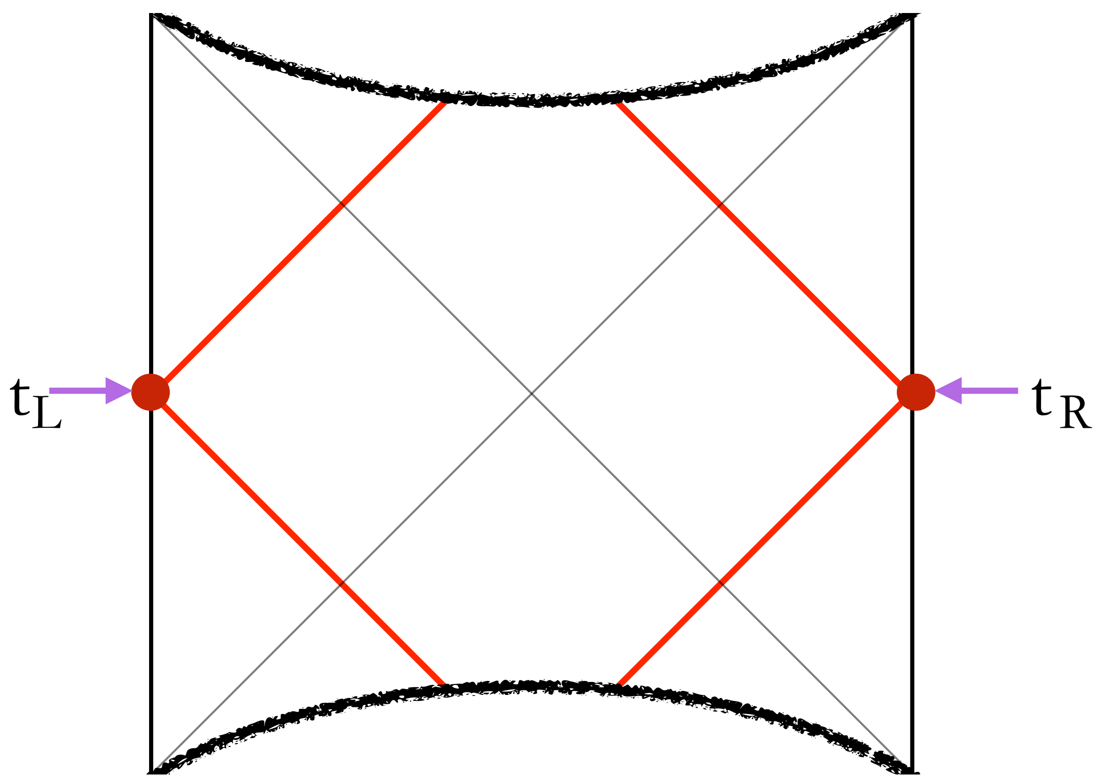

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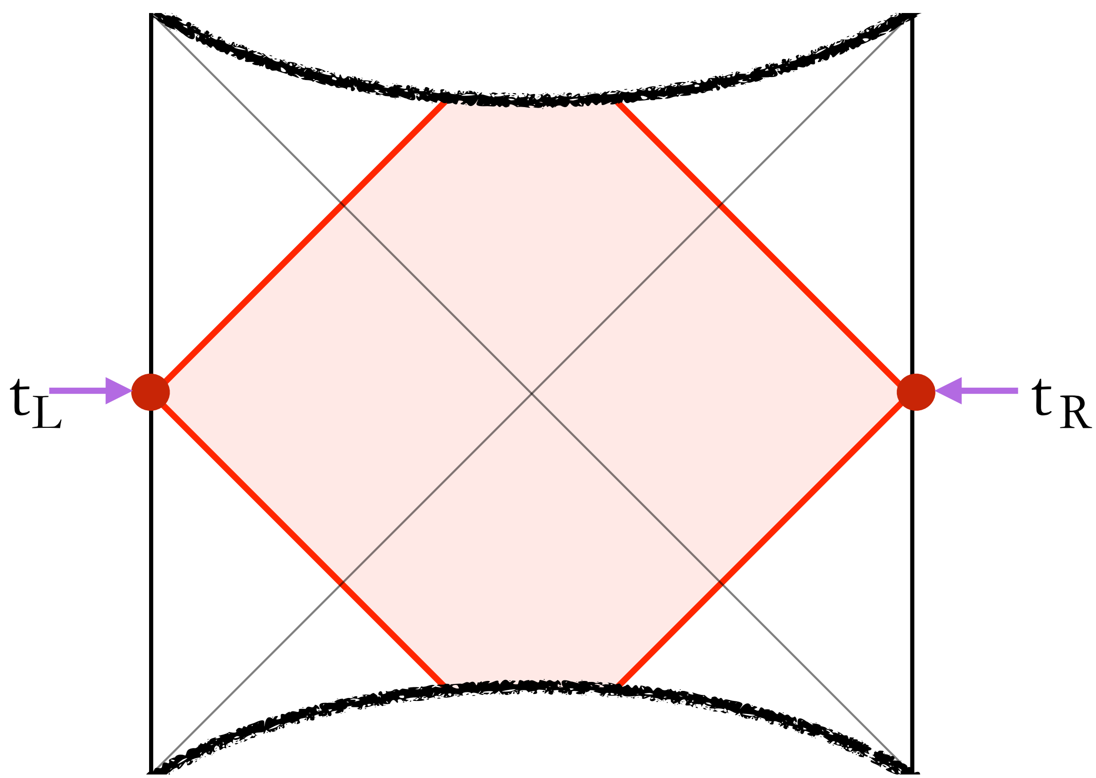

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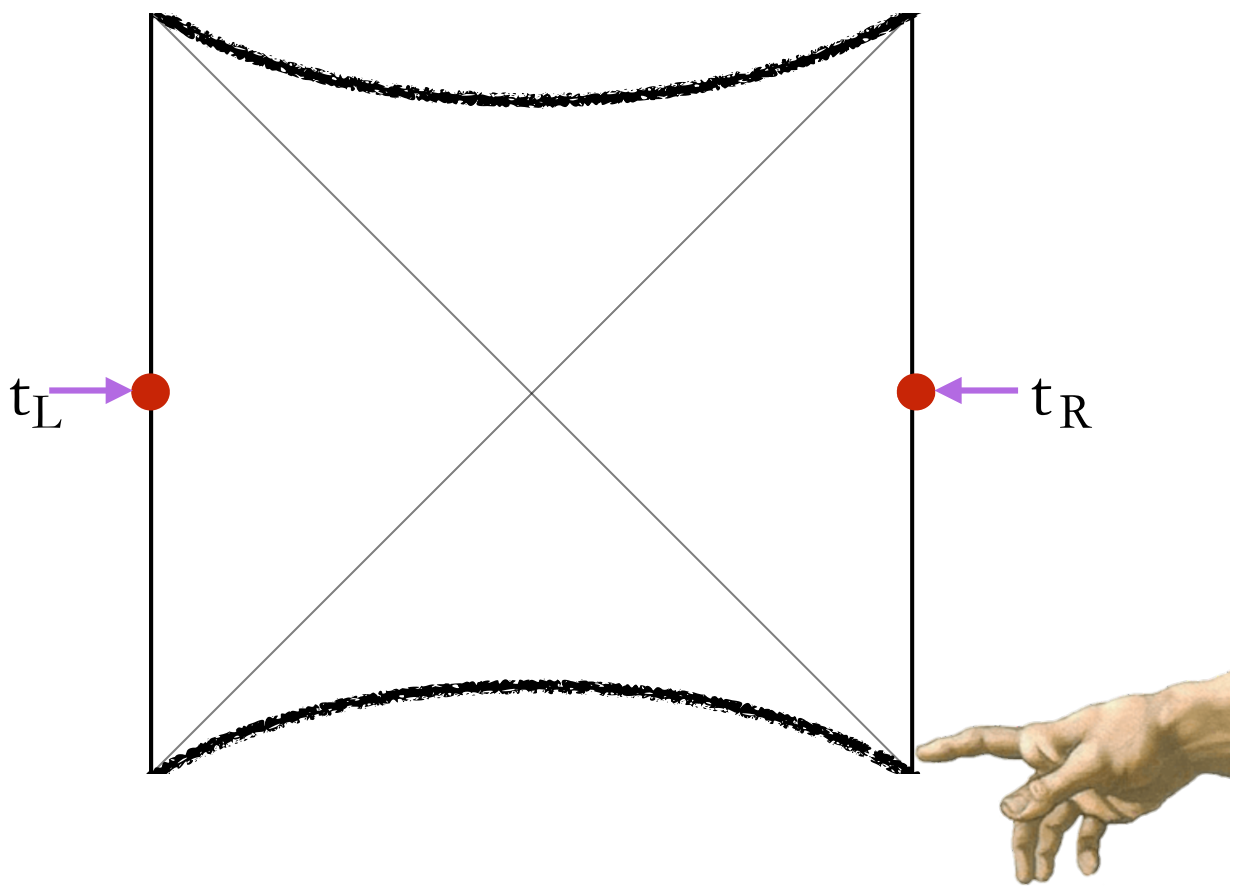

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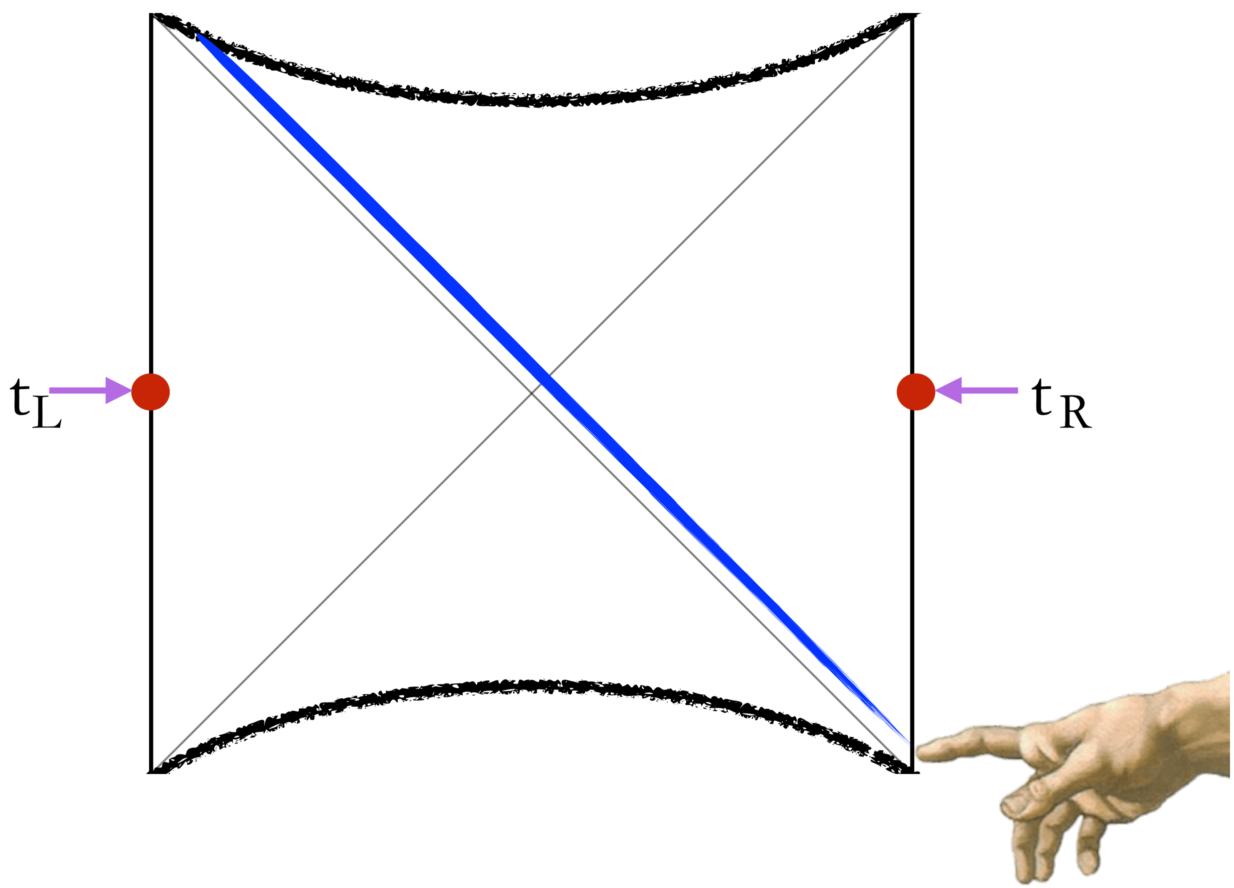

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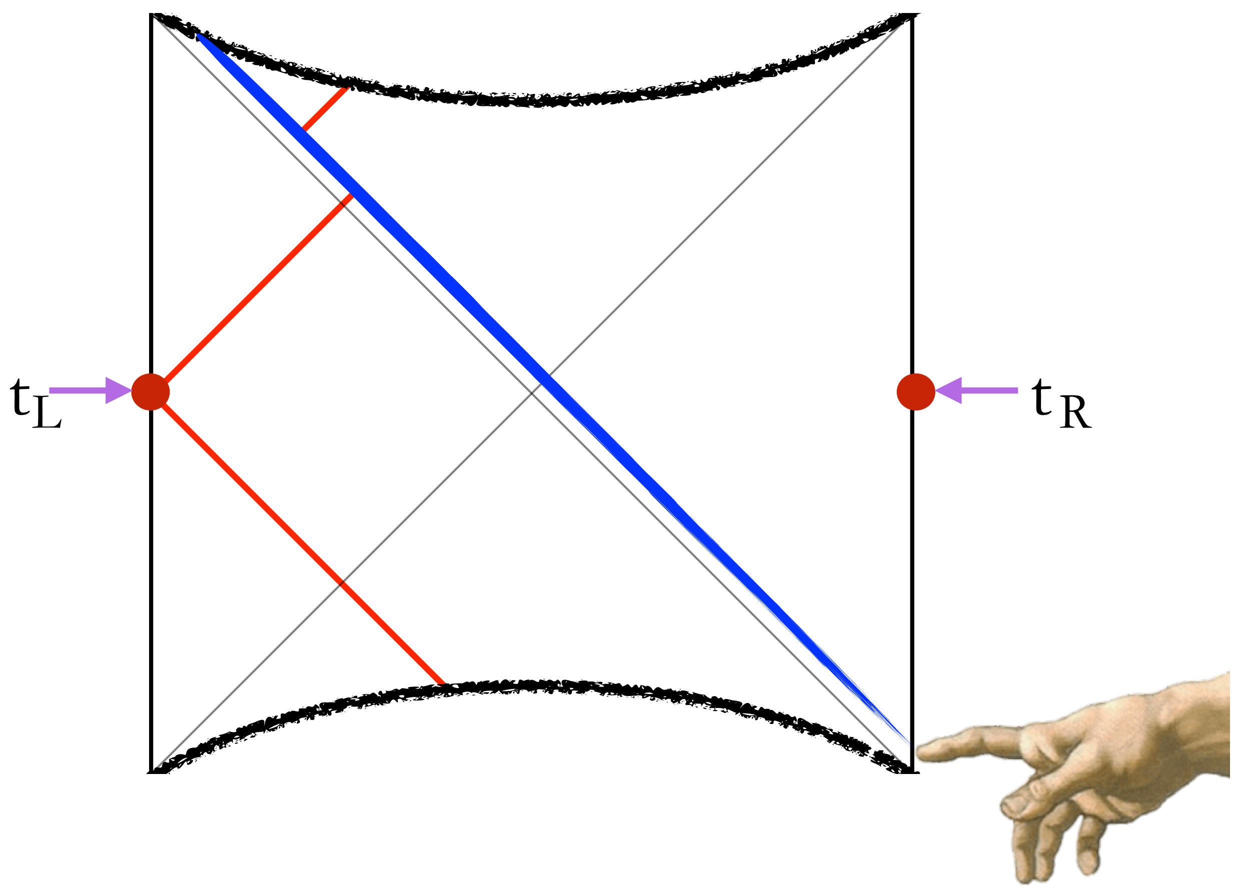

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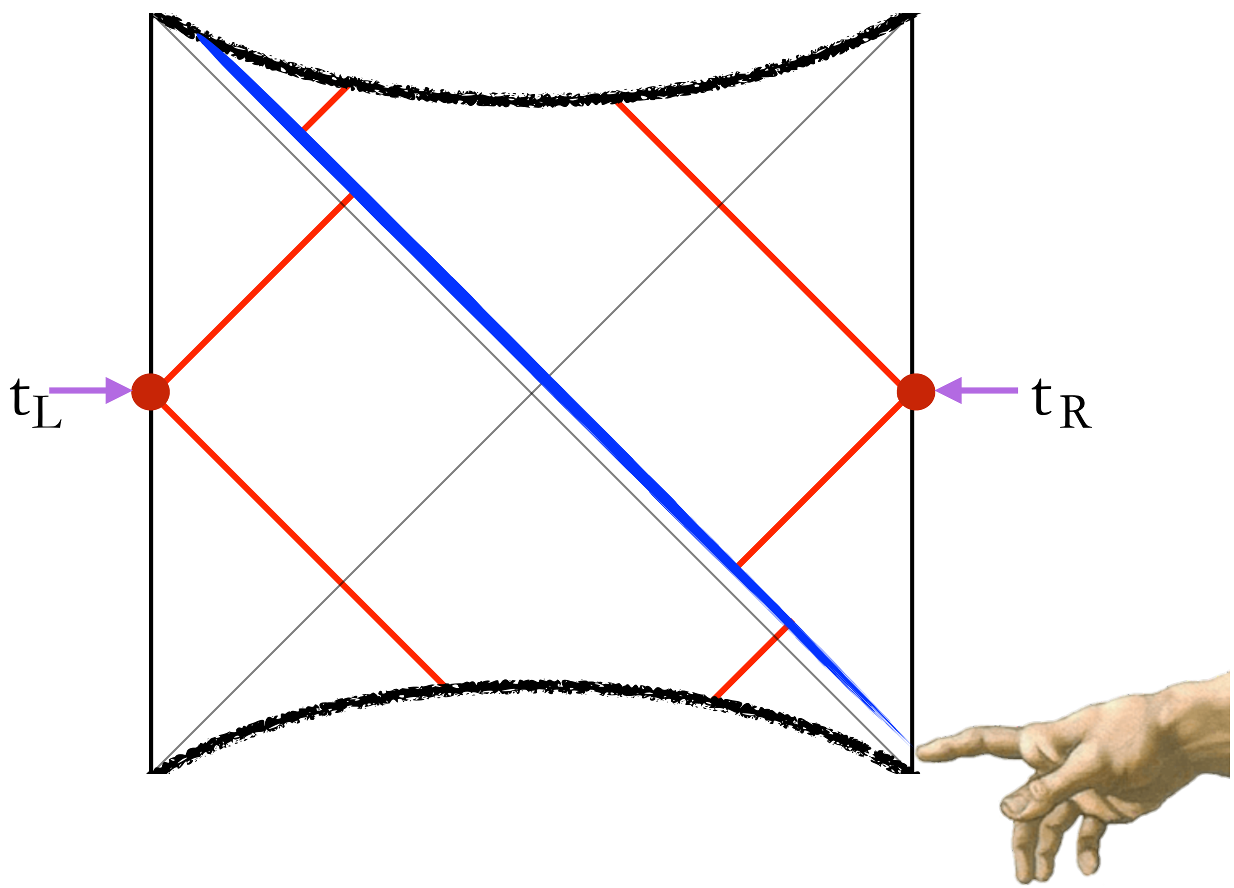

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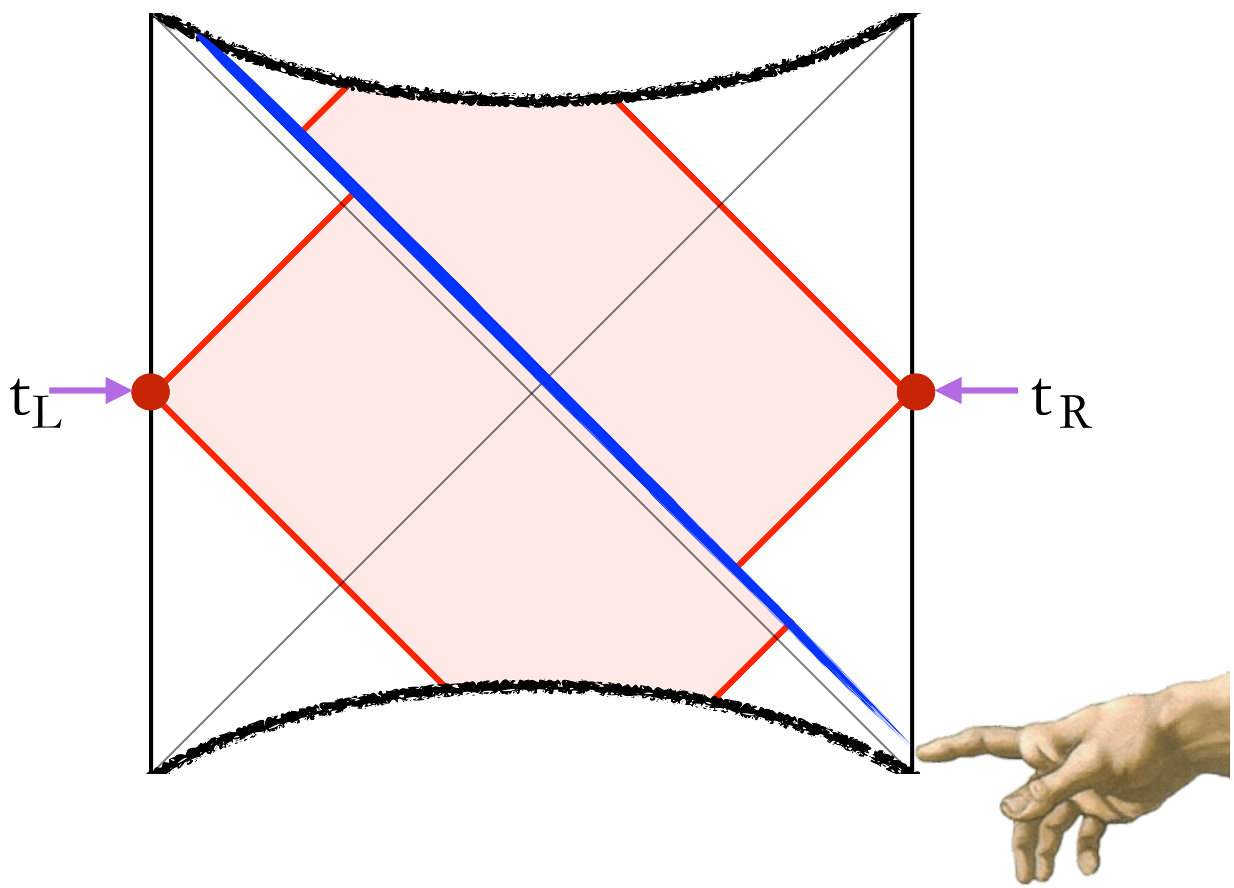

complexity ~ size of wormhole?

EVIDENCE:

- both expected to grow linearly (at early times)
- both can be exponentially large
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  - in CFT, leads to increase in complexity (chaos)
  - in wormhole, leads to shockwave, increases size

the two increases match
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- entomb black hole in inert shell

Complexity and Geometry ?

FURTHER WORK:

- precise definition of complexity?
- precise definition of action?
- relate imprecision in two definitions?
- reference state? (“complexity of formation”)
- classical proof that black holes maximize action?
- more general black holes?
- higher-derivative theories and singularities?
- quantum corrections in the bulk?
- principle of least computation?
- complexity and horizon transparency?
- lots of puzzles!

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- OLD: number of gates
  - need “tolerance”
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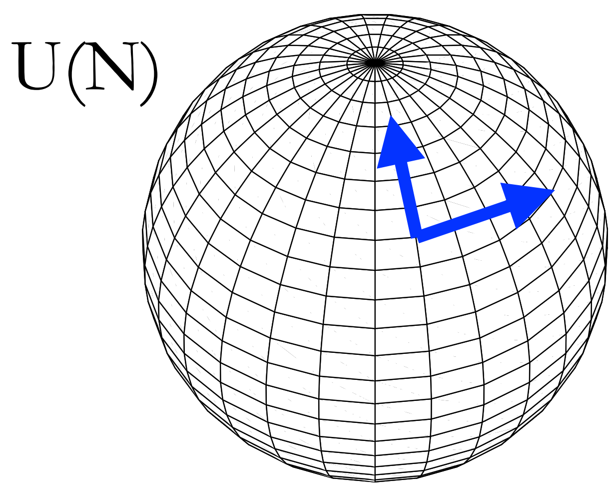

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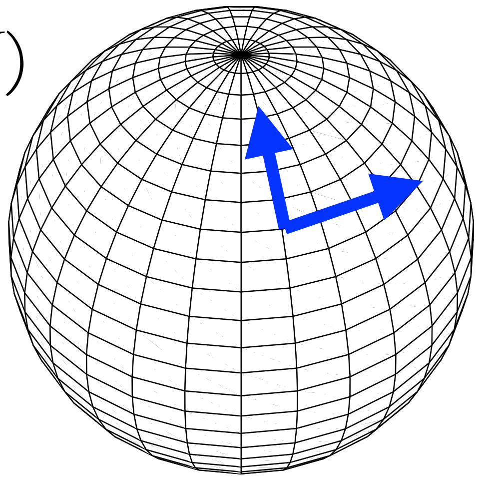

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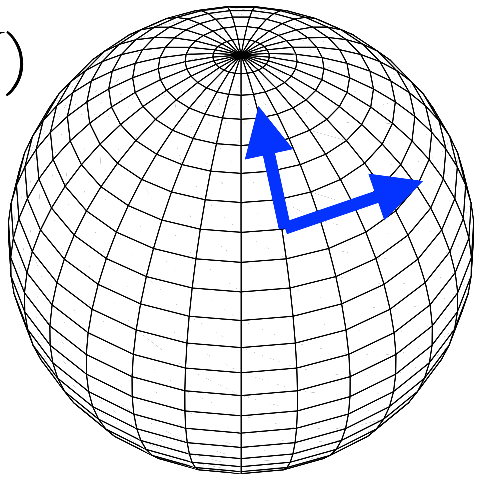

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- inner-product metric $\longrightarrow$ $g_{IJ} = \delta_{IJ}$

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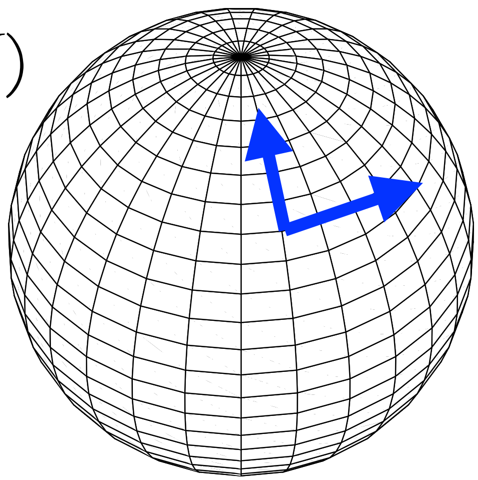

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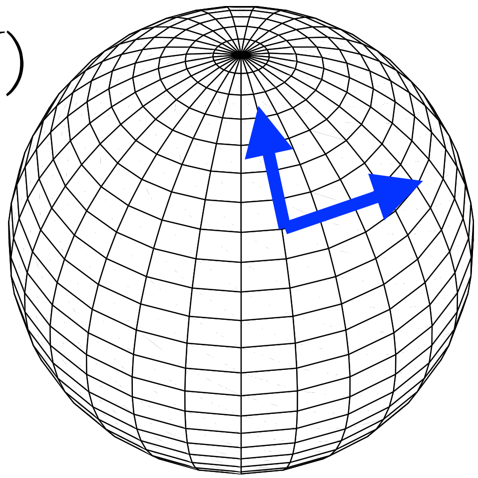

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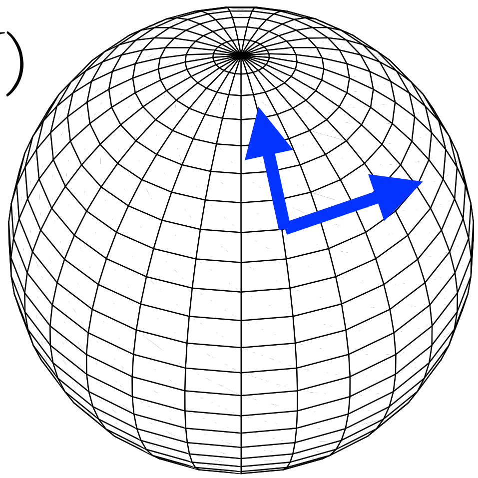
$\downarrow$

k-local sections have negative curvature

- NEW: complexity metric ("Nielsen geometry")



- complexity metric → punish directions that touch more qubits (reward *k*-locality)

↓

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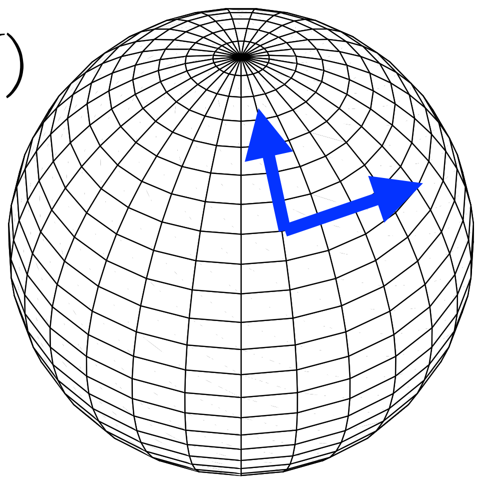
U(N)



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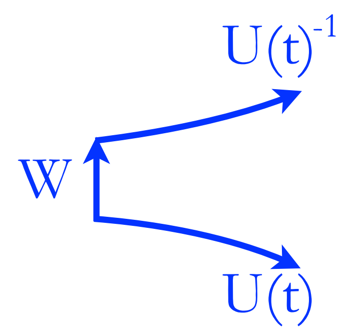
↓

negative curvature makes geodesics diverge



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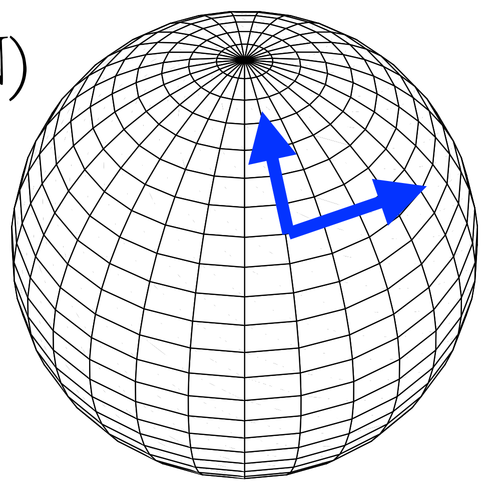
U(N)



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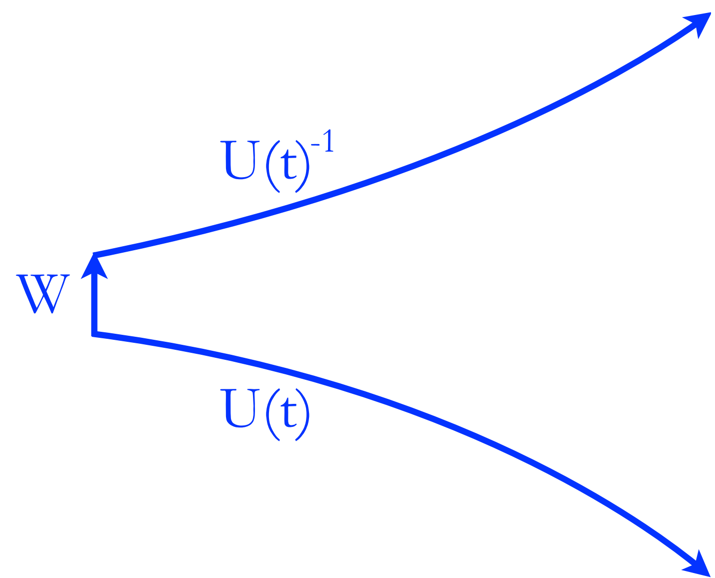

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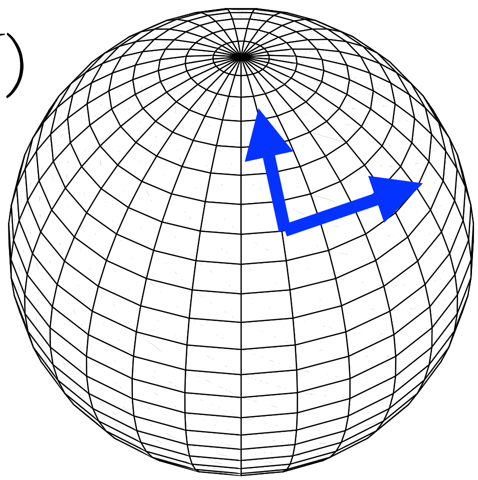

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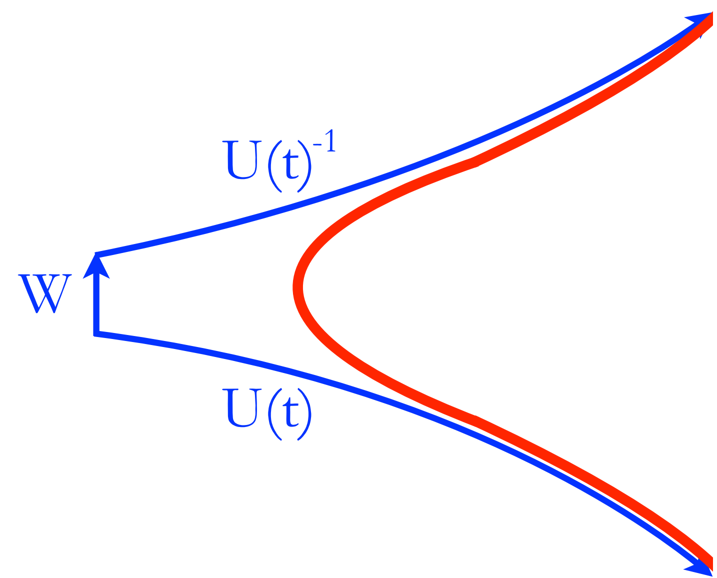

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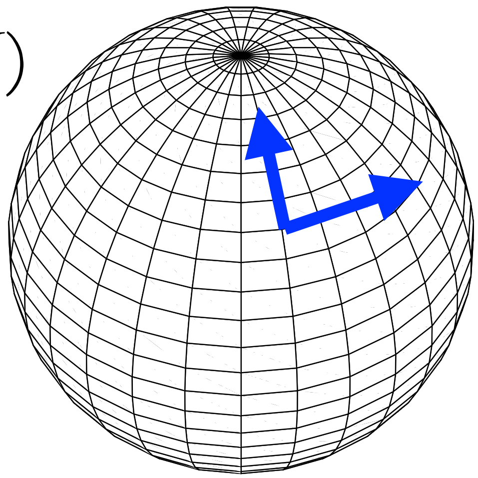

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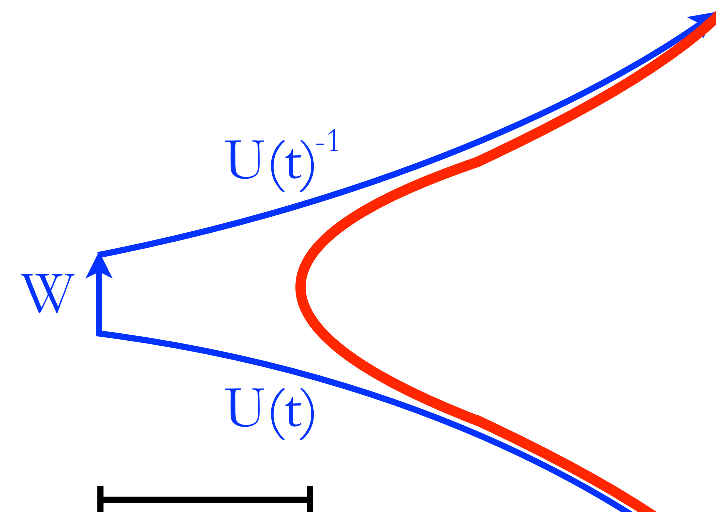

$$\ell \log \frac{W}{\ell}$$

shortcut at corner gives switchback subtraction

- 2^{nd} Law of Complexity?

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$$\text{entropy} \quad \neq \quad \text{complexity}$$

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maxes out at S

maxes out at $\sim 2^S$

◉ 2nd Law of Complexity?

$$\text{entropy} \quad \neq \quad \text{complexity}$$

maxes out at S maxes out at $\sim 2^S$

$\#$ states $\sim e^S$ $\#$ states $\sim e^C$

◉ 2^{nd} Law of Complexity?

$$\text{entropy} \quad \neq \quad \text{complexity}$$

maxes out at S

states $\sim e^S$

entropy tends to increase

maxes out at $\sim 2^S$

states $\sim e^C$

complexity tends to increase

◉ 2^{nd} Law of Complexity?

entropy $\neq$ complexity

maxes out at S

maxes out at $\sim 2^S$

states $\sim e^S$

entropy tends to increase

states $\sim e^C$

complexity tends to increase

submaximal entropy is
resource for doing
useful work

◉ 2^{nd} Law of Complexity?

entropy $\neq$ complexity

maxes out at S	maxes out at $\sim 2^S$
# states $\sim e^S$	# states $\sim e^C$
entropy tends to increase	complexity tends to increase
submaximal entropy is resource for doing useful work	submaximal complexity is resource for doing useful computation?


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