Topological twists
of
non-Lagrangian theories

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Topological Quantum Field Theory

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Abstract. A twisted version of four dimensional supersymmetric gauge theory is formulated. The model, which refines a nonrelativistic treatment by Atiyah, appears to underlie many recent developments in topology of low dimensional manifolds; the Donaldson polynomial invariants of four manifolds and the Floer groups of three manifolds appear naturally. The model may also be interesting from a physical viewpoint; it is in a sense a generally covariant quantum field theory, albeit one in which general covariance is unbroken, there are no gravitons, and the only excitations are topological.

- Argyres-Douglas theories

[P.Argyres, M.Douglas]
[D.Xie]
:

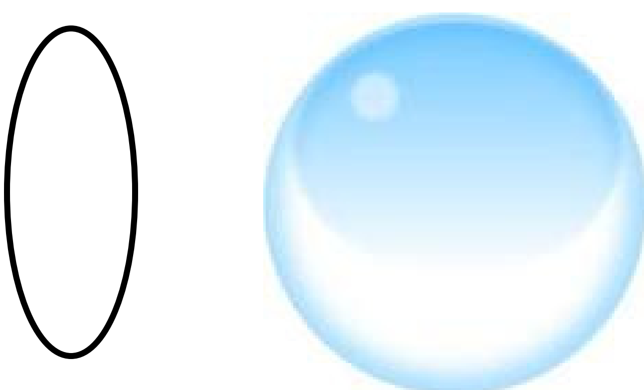
- N=3 theories

[O.Aharony, M.Evtikhiev]
[I.Garca-Etxebarria, D.Regalado]
[O.Aharony, Y.Tachikawa]
:

- Theories with non-freely generated Coulomb branch chiral rings

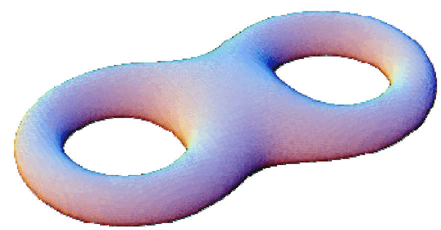
[P.Argyres, M.Lotito, Y.Lu, M.Martone]

Topological $S^1 \times S^3$ index = Coulomb branch index



[M.Dedushenko, S.G., P.Putrov]

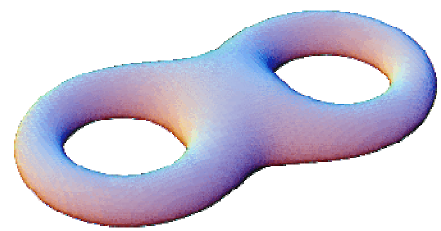
4d theory



2d theory

[M.Bershadsky, A.Johansen, V.Sadov, C. Vafa]
[J.Harvey, G.Moore, A.Strominger]
[A.Losev, N.Nekrasov, S.Shatashvili]
[J.Maldacena, C.Nunez]
[A.Kapustin, E.Witten]
⋮

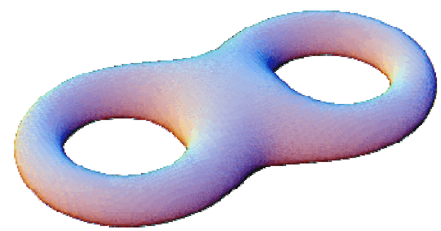
S-folds



T-folds

Self-mirror

$$A_{4d} = (a-c)\left(2c_1(R)p_1(TM_4) - 8c_1(R)^3\right) - (8a-4c)c_1(R)c_2(E)$$



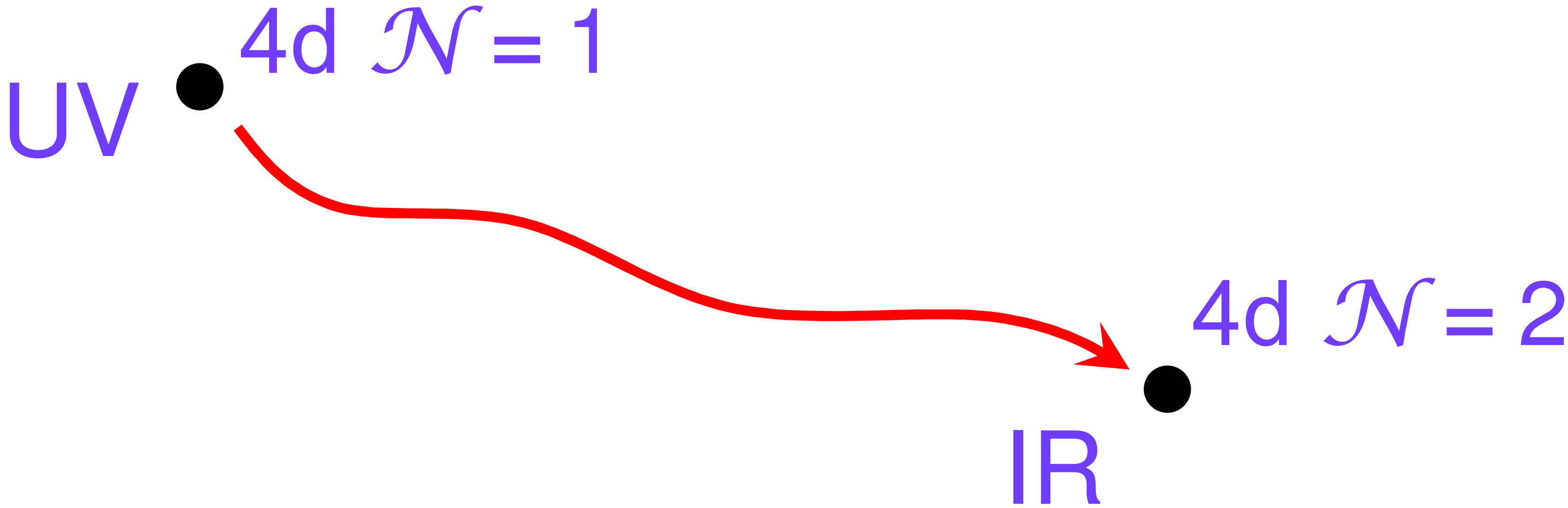
effective dilaton:

$$\frac{1}{\pi i}(1-g)\left(1-\frac{1}{\Delta(u)}\right)\log u$$

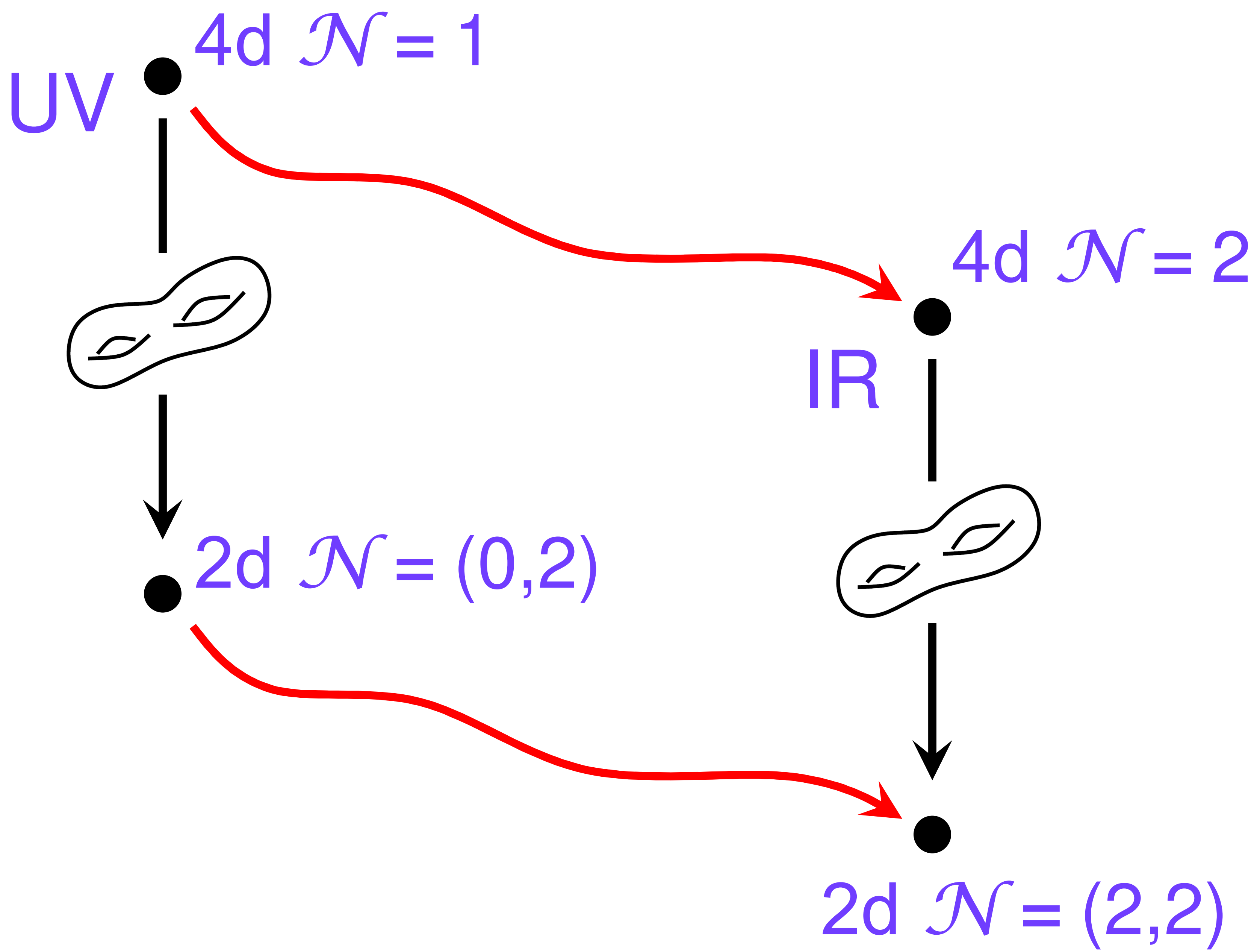
[C.Beem, M.Lemos, P.Liendo, L.Rastelli, B.C.van Rees]

Shortening	Quantum Number Relations			DO	KMMR
$\varnothing$	$\Delta \geqslant \max(\Delta_1, \Delta_2)$			$\mathcal{A}^{\Delta}_{R,r(j_1,j_2)}$	$\mathbf{aa}_{\Delta,j_1,j_2,r,R}$
$\mathcal{B}^1$	$\Delta = 2R + r$	$j_1 = 0$		$\mathcal{B}_{R,r(0,j_2)}$	$\mathbf{ba}_{0,j_2,r,R}$
$\bar{\mathcal{B}}_2$	$\Delta = 2R - r$	$j_2 = 0$		$\bar{\mathcal{B}}_{R,r(j_1,0)}$	$\mathbf{ab}_{j_1,0,r,R}$
$\mathcal{B}^1 \cap \mathcal{B}^2$	$\Delta = r$	$R = 0$		$\mathcal{E}_{r(0,j_2)}$	$\mathbf{ba}_{0,j_2,r,0}$
$\bar{\mathcal{B}}_1 \cap \bar{\mathcal{B}}_2$	$\Delta = -r$	$R = 0$		$\bar{\mathcal{E}}_{r(j_1,0)}$	$\mathbf{ab}_{j_1,0,r,0}$
$\mathcal{B}^1 \cap \bar{\mathcal{B}}_2$	$\Delta = 2R$	$j_1 = j_2 = r = 0$		$\hat{\mathcal{B}}_R$	$\mathbf{bb}_{0,0,0,R}$
$\mathcal{C}^1$	$\Delta = 2 + 2j_1 + 2R + r$			$\mathcal{C}_{R,r(j_1,j_2)}$	$\mathbf{ca}_{j_1,j_2,r,R}$
$\bar{\mathcal{C}}_2$	$\Delta = 2 + 2j_2 + 2R - r$			$\bar{\mathcal{C}}_{R,r(j_1,j_2)}$	$\mathbf{ac}_{j_1,j_2,r,R}$
$\mathcal{C}^1 \cap \mathcal{C}^2$	$\Delta = 2 + 2j_1 + r$	$R = 0$		$\mathcal{C}_{0,r(j_1,j_2)}$	$\mathbf{ca}_{j_1,j_2,r,0}$
$\bar{\mathcal{C}}_1 \cap \bar{\mathcal{C}}_2$	$\Delta = 2 + 2j_2 - r$	$R = 0$		$\bar{\mathcal{C}}_{0,r(j_1,j_2)}$	$\mathbf{ac}_{j_1,j_2,r,0}$
$\mathcal{C}^1 \cap \bar{\mathcal{C}}_2$	$\Delta = 2 + 2R + j_1 + j_2$	$r = j_2 - j_1$		$\hat{\mathcal{C}}_{R(j_1,j_2)}$	$\mathbf{cc}_{j_1,j_2,j_2-j_1,R}$
$\mathcal{B}^1 \cap \bar{\mathcal{C}}_2$	$\Delta = 1 + 2R + j_2$	$r = j_2 + 1$		$\mathcal{D}_{R(0,j_2)}$	$\mathbf{bc}_{0,j_2,j_2+1,R}$
$\bar{\mathcal{B}}_2 \cap \mathcal{C}^1$	$\Delta = 1 + 2R + j_1$	$-r = j_1 + 1$		$\bar{\mathcal{D}}_{R(j_1,0)}$	$\mathbf{cb}_{j_1,0,-j_1-1,R}$
$\mathcal{B}^1 \cap \mathcal{B}^2 \cap \bar{\mathcal{C}}_2$	$\Delta = r = 1 + j_2$	$r = j_2 + 1$	$R = 0$	$\mathcal{D}_{0(0,j_2)}$	$\mathbf{bc}_{0,j_2,j_2+1,0}$
$\mathcal{C}^1 \cap \bar{\mathcal{B}}_1 \cap \bar{\mathcal{B}}_2$	$\Delta = -r = 1 + j_1$	$-r = j_1 + 1$	$R = 0$	$\bar{\mathcal{D}}_{0(j_1,0)}$	$\mathbf{cb}_{j_1,0,-j_1-1,0}$

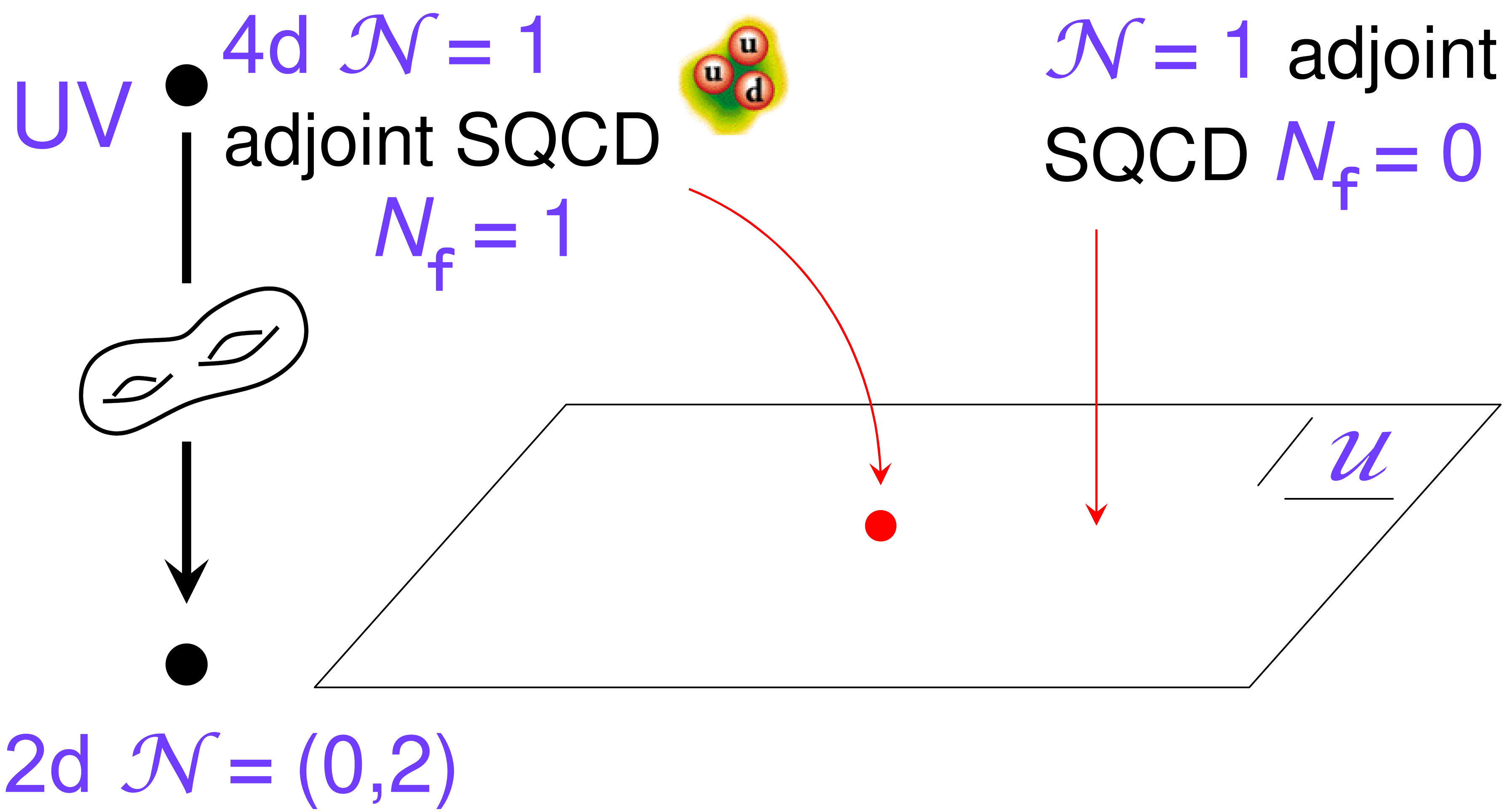
Table 4. Summary of unitary irreducible representations of the $\mathcal{N} = 2$ superconformal algebra.



[A.Gadde, S. Razamat, B.Willett]
[K.Maruyoshi, J.Song]
:



4d $\mathcal{N} = 1$
UV
4d $\mathcal{N} = 2$
IR
2d $\mathcal{N} = (0,2)$
2d $\mathcal{N} = (2,2)$



cf. [D.Kutasov, A.Parnachev, D.Sahakyan]
[K.Intriligator, B.Wecht]
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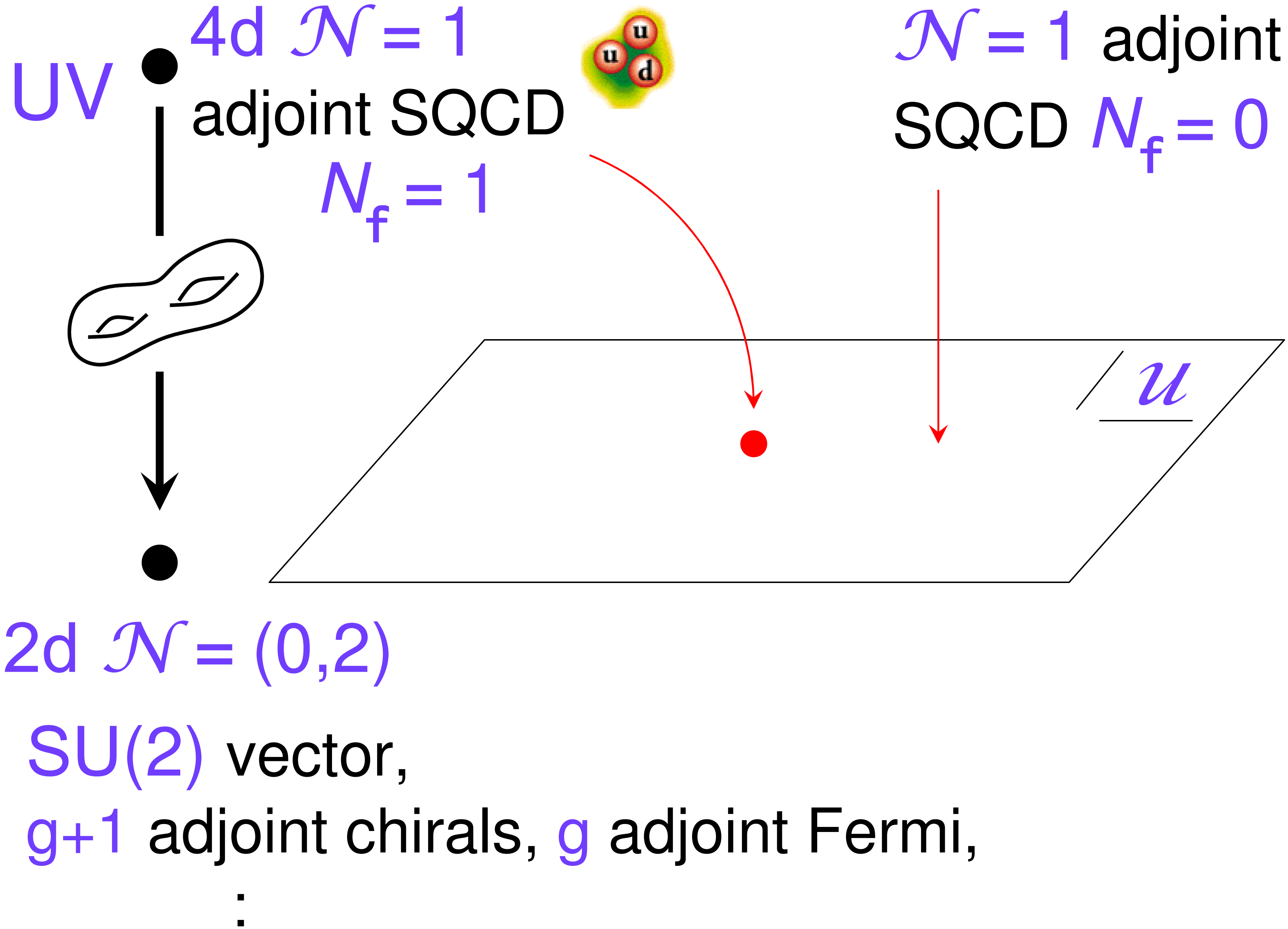
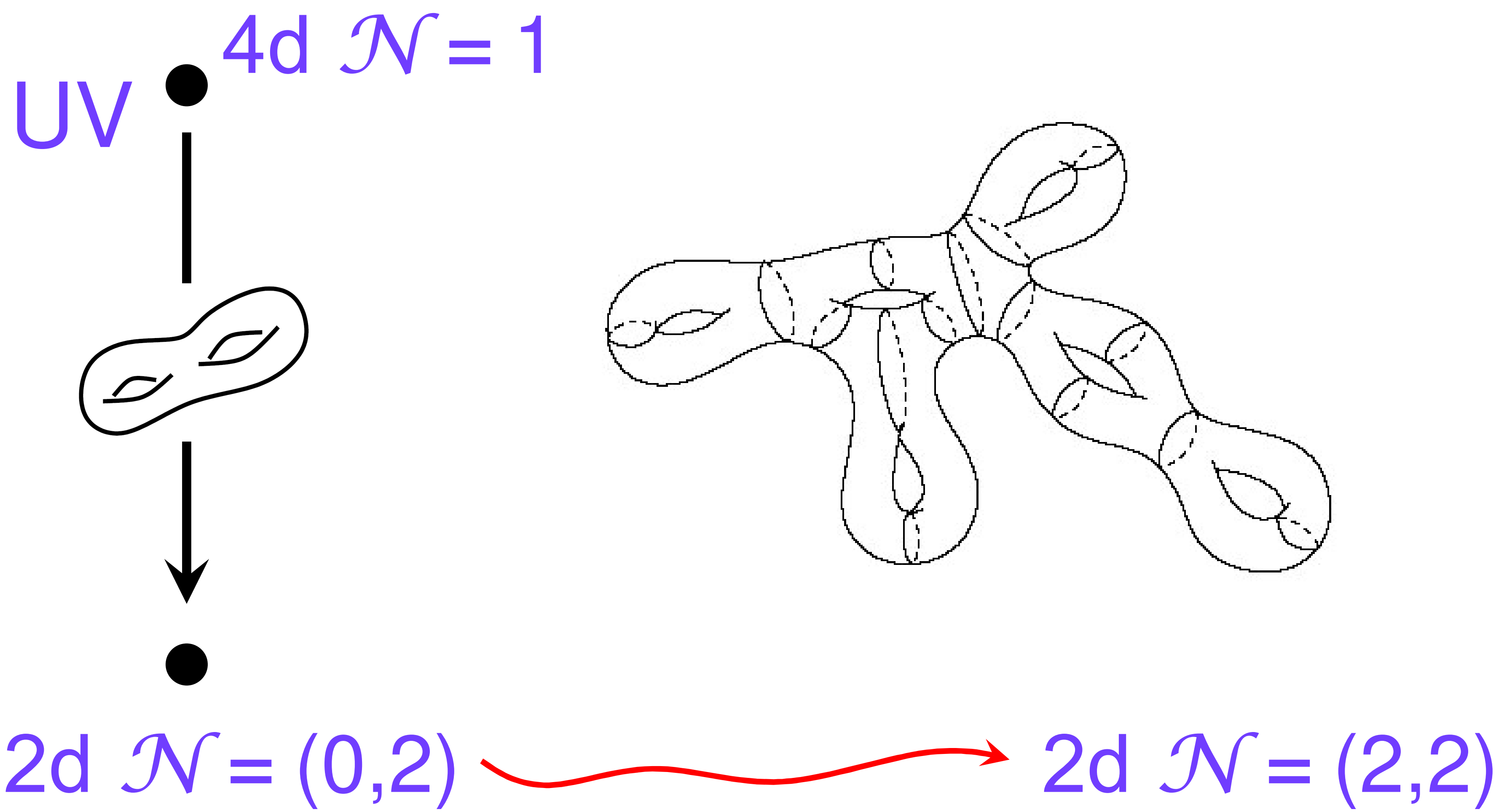
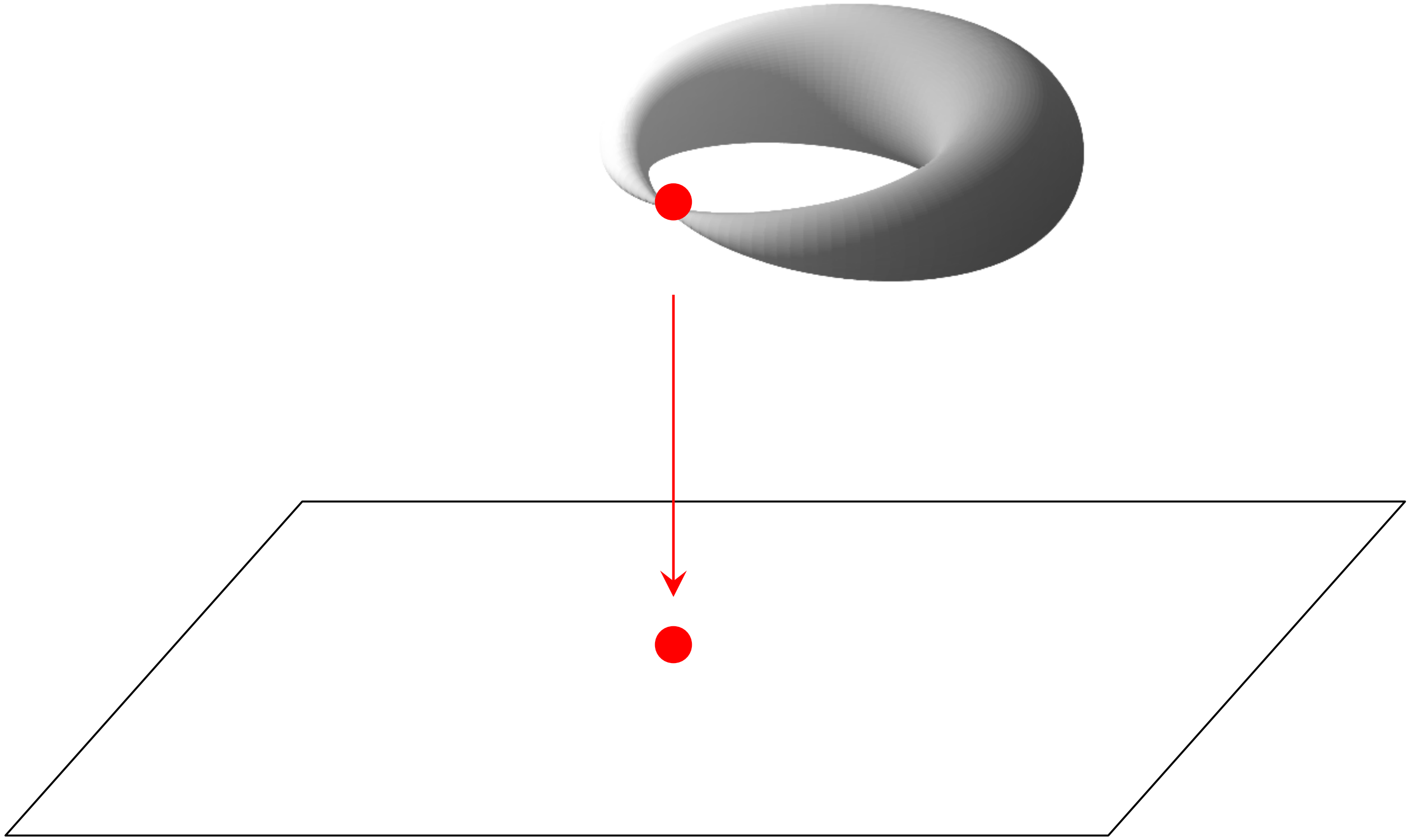

UV
4d $\mathcal{N}=1$
adjoint SQCD
$N_f = 1$
$\mathcal{N}=1$ adjoint
SQCD $N_f = 0$
$\mathcal{U}$
2d $\mathcal{N}=(0,2)$
SU(2) vector,
g+1 adjoint chirals, g adjoint Fermi,
:

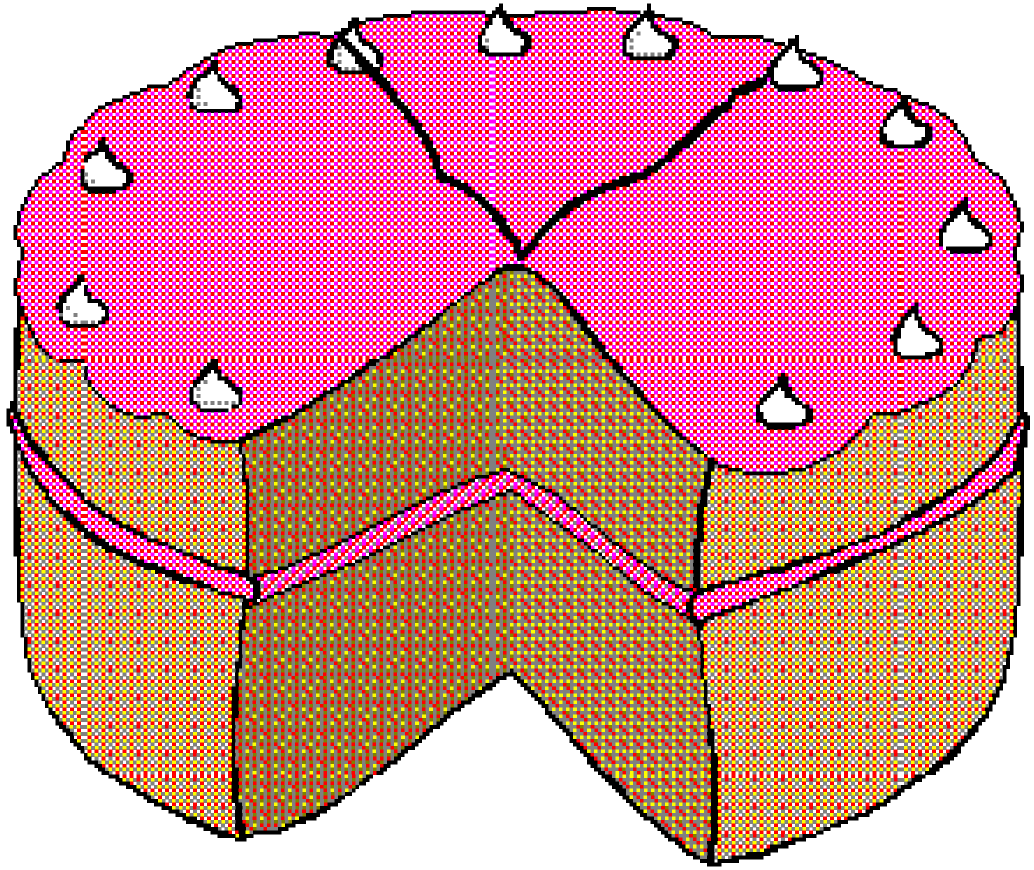


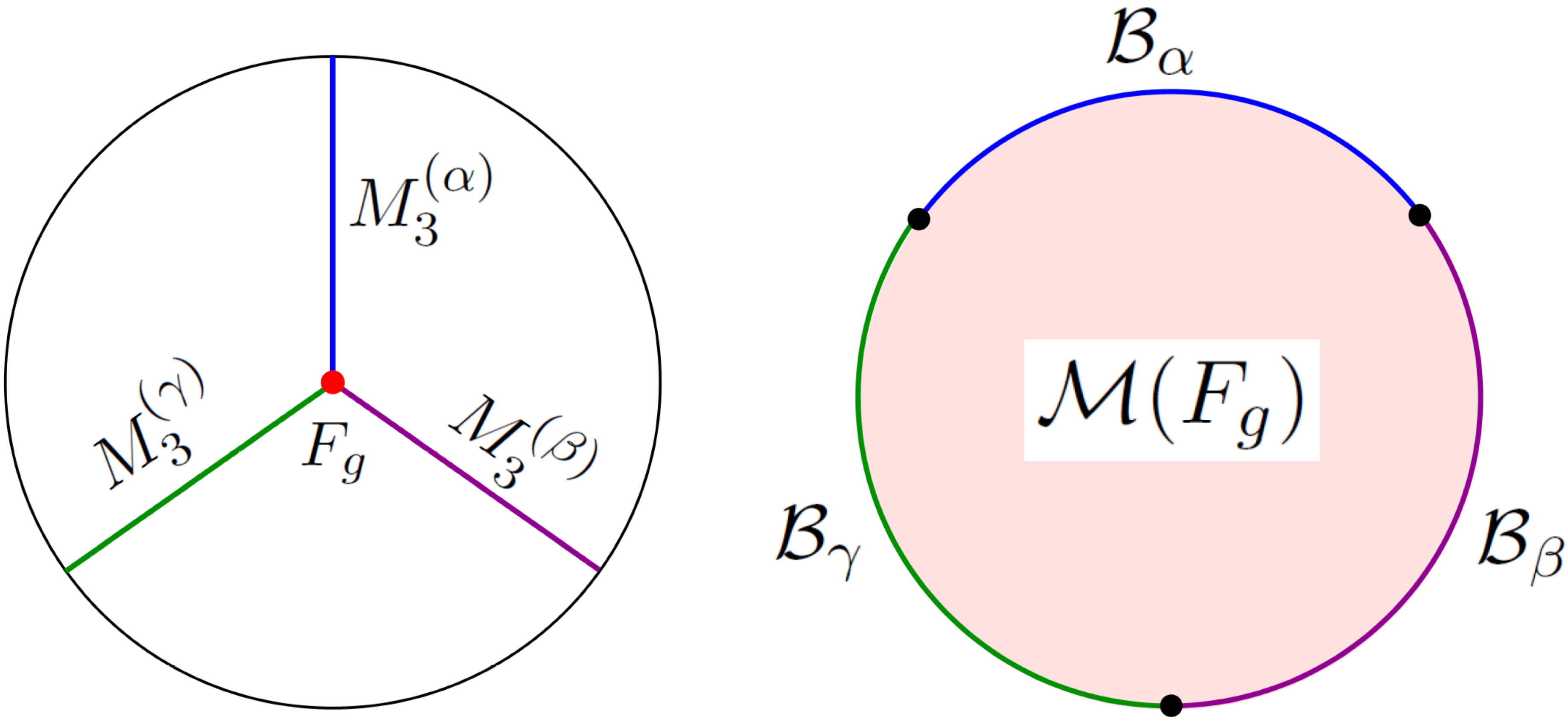
$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle_{\text{A-model}} = \sum_{\text{vacua: } d\widetilde{\mathcal{W}}_{\text{eff}}=0} \mathcal{O}_1(v) \dots \mathcal{O}_n(v) \left(\det\ \text{Hess}\, \widetilde{\mathcal{W}}_{\text{eff}}\right)^{\tilde{g}-1}$$



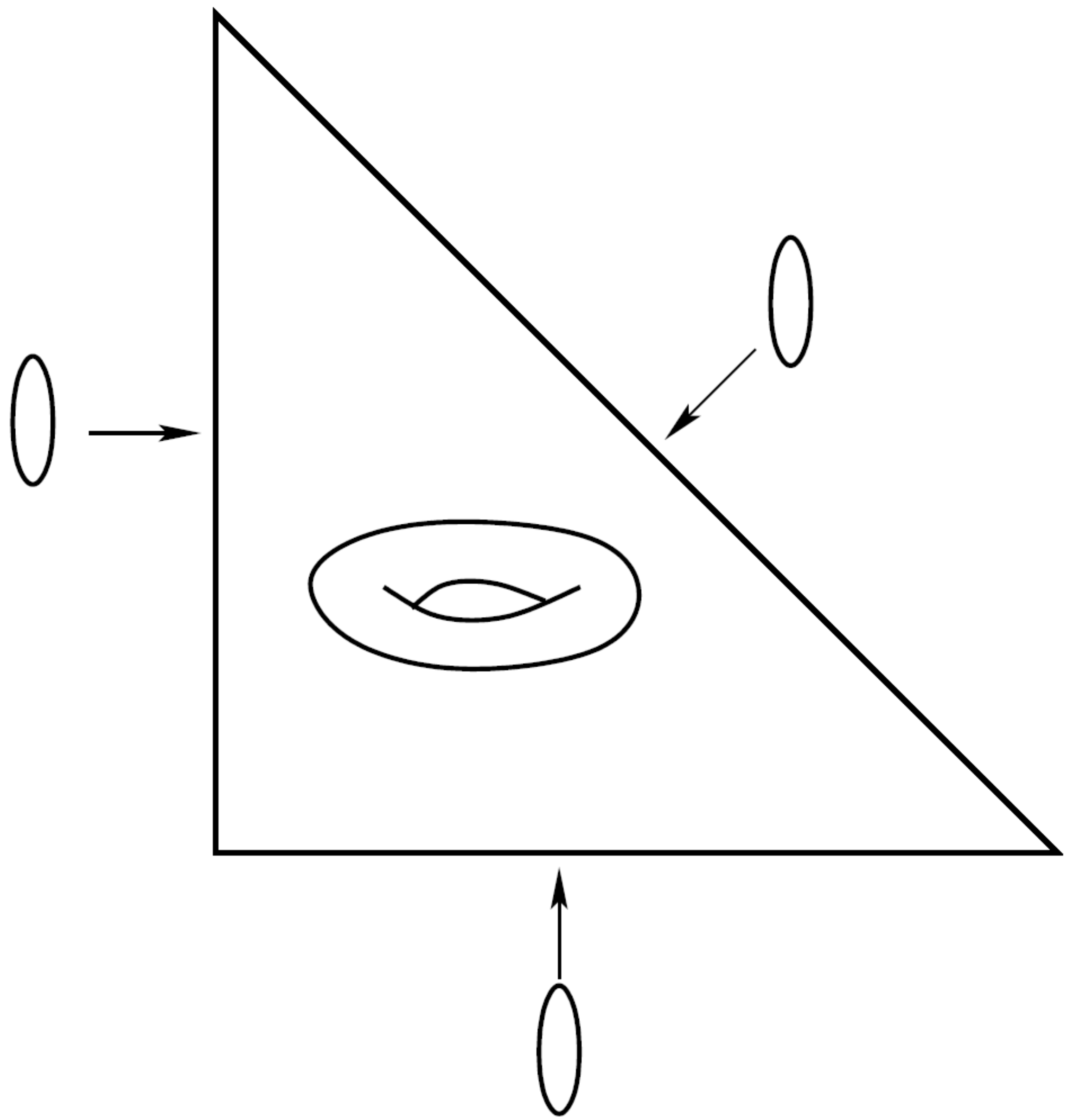
The Witten index of 2d $\mathcal{N} = (2,2)$ theory obtained from reduction of (A_1, A_3) Argyres-Douglas theory

$$
= \begin{cases}
-\frac{2}{(1-\mathfrak{t}^{1/3})(1-\mathfrak{t}^{4/3})}\,, & g = 0 \\
2\,, & g = 1 \\
8(1+\mathfrak{t}^{1/3}+\mathfrak{t}^{2/3}+\mathfrak{t})\,, & g = 2 \\
8(1+6\,\mathfrak{t}^{1/3}+\mathfrak{t}^{2/3})(1+\mathfrak{t}^{1/3}+\mathfrak{t}^{2/3}+\mathfrak{t})^2\,, & g = 3 \\
\quad\vdots &
\end{cases}
$$

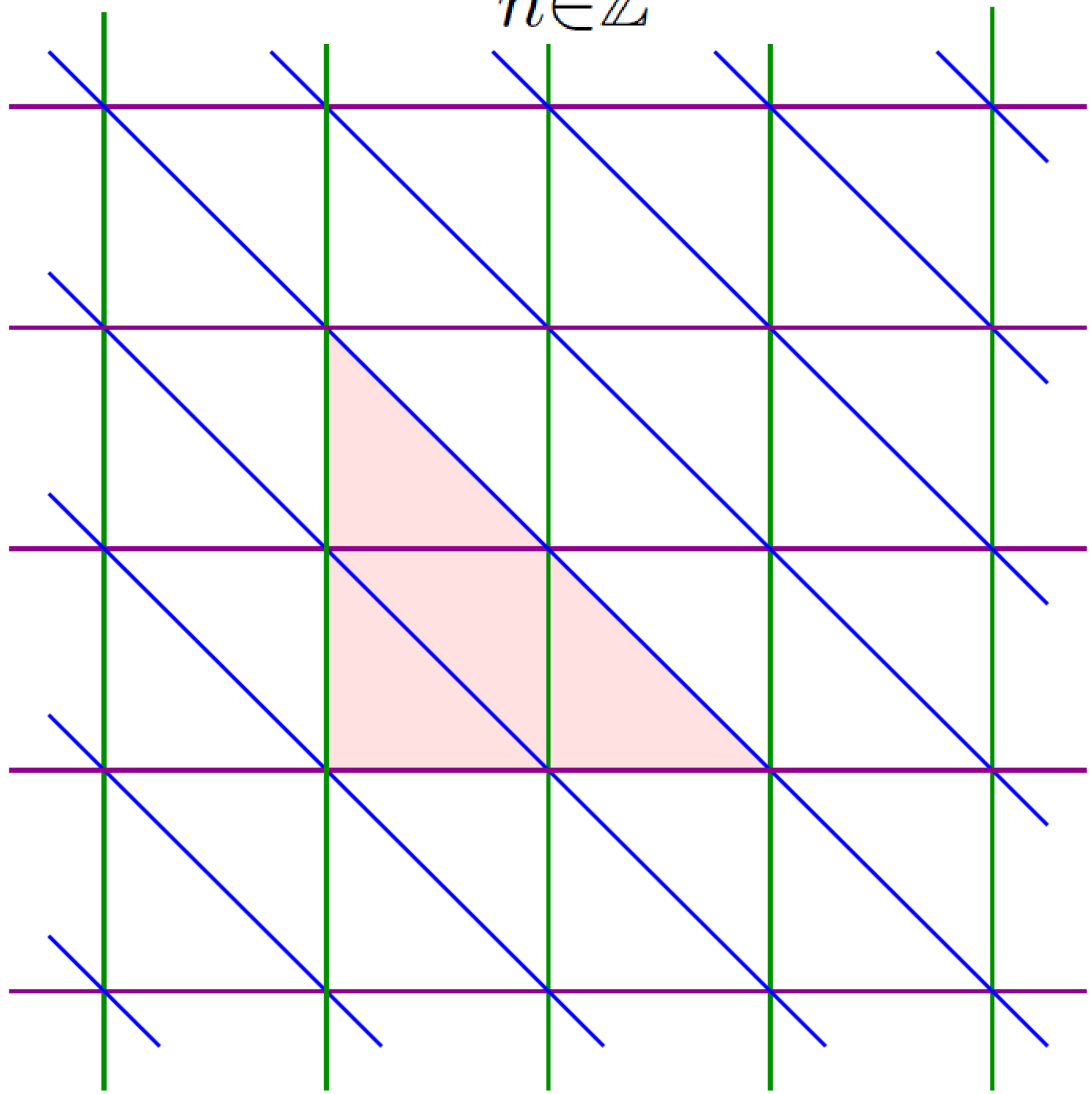


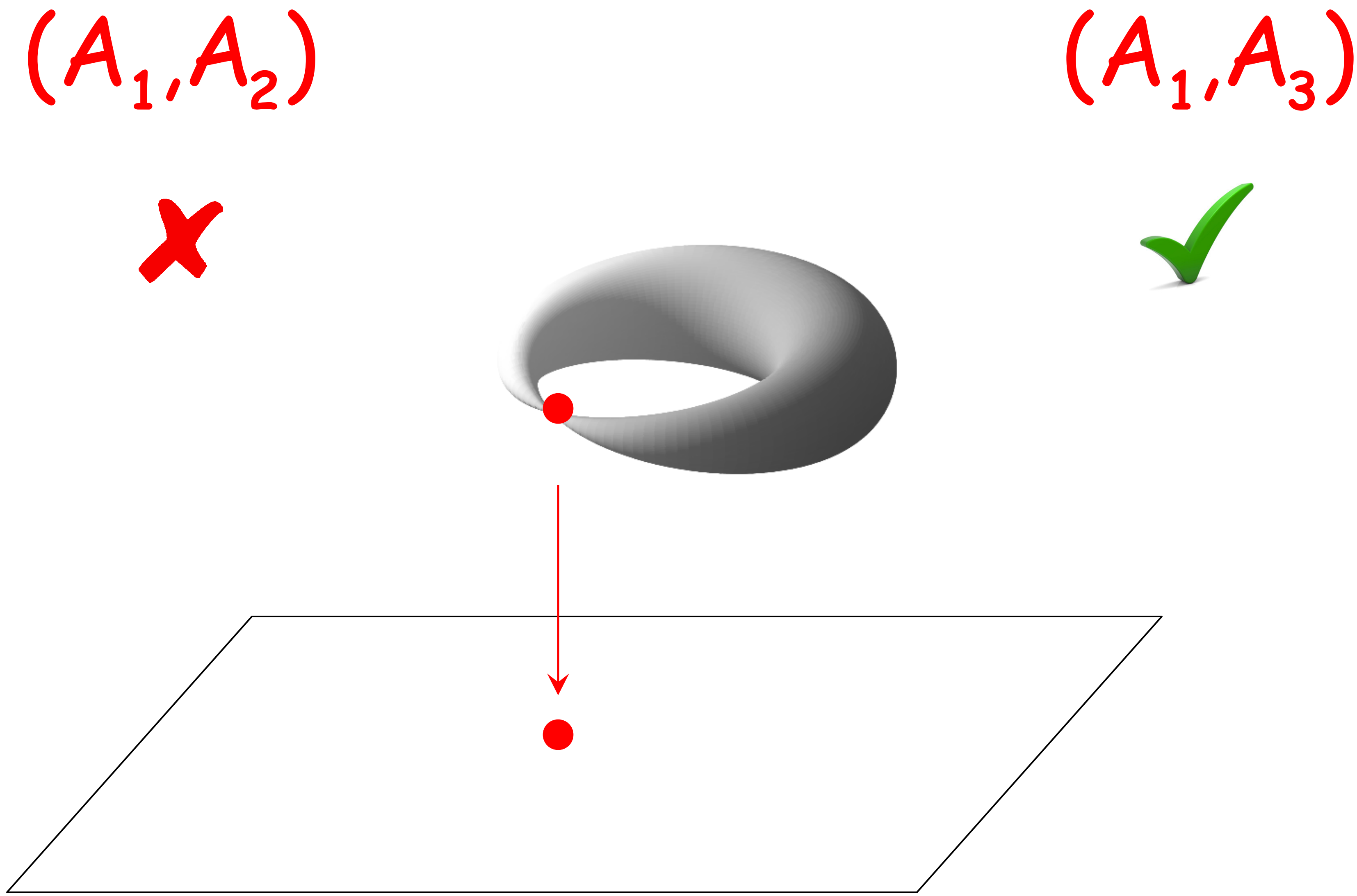


$M_3^{(\alpha)}$
$M_3^{(\gamma)}$
F_g
$M_3^{(\beta)}$
$\mathcal{B}_\alpha$
$\mathcal{M}(F_g)$
$\mathcal{B}_\gamma$
$\mathcal{B}_\beta$

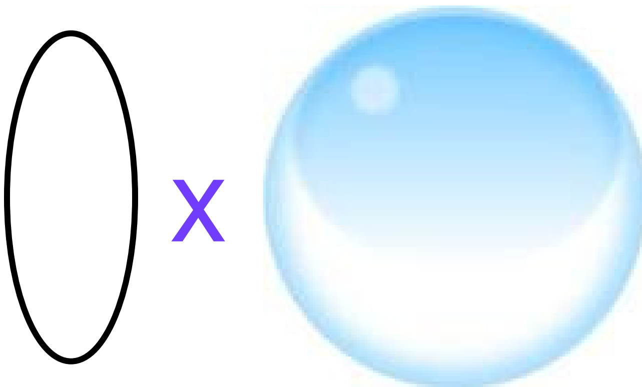


$$\theta(\tau) = \sum_{n\in\mathbb{Z}} q^{n^2/2}$$





(A1,A2)
(A1,A3)



$$\mathcal{B}_\alpha = \mathcal{B}_\beta = \mathcal{B}_\gamma$$

(A_1, A_2)

$$\frac{1}{(1-\mathfrak{t}^{\frac{2}{5}})(1-\mathfrak{t}^{\frac{3}{5}})} + \frac{\mathfrak{t}^{\frac{k}{5}}}{(1-\mathfrak{t}^{\frac{6}{5}})(1-\mathfrak{t}^{-\frac{1}{5}})}$$

(A_1, A_3)

$$\frac{1}{(1-\mathfrak{t}^{\frac{1}{3}})(1-\mathfrak{t}^{\frac{2}{3}})} + \frac{\mathfrak{t}^{\frac{\lambda}{3}} + \mathfrak{t}^{\frac{k-\lambda}{3}}}{(1-\mathfrak{t}^{-\frac{1}{3}})(1-\mathfrak{t}^{\frac{4}{3}})}$$

[L.Fredrickson, D.Pei, W.Yan, K.Ye]