Holographic Complexity Equals Which Action?

Leonel Queimada

King's College London

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Based on

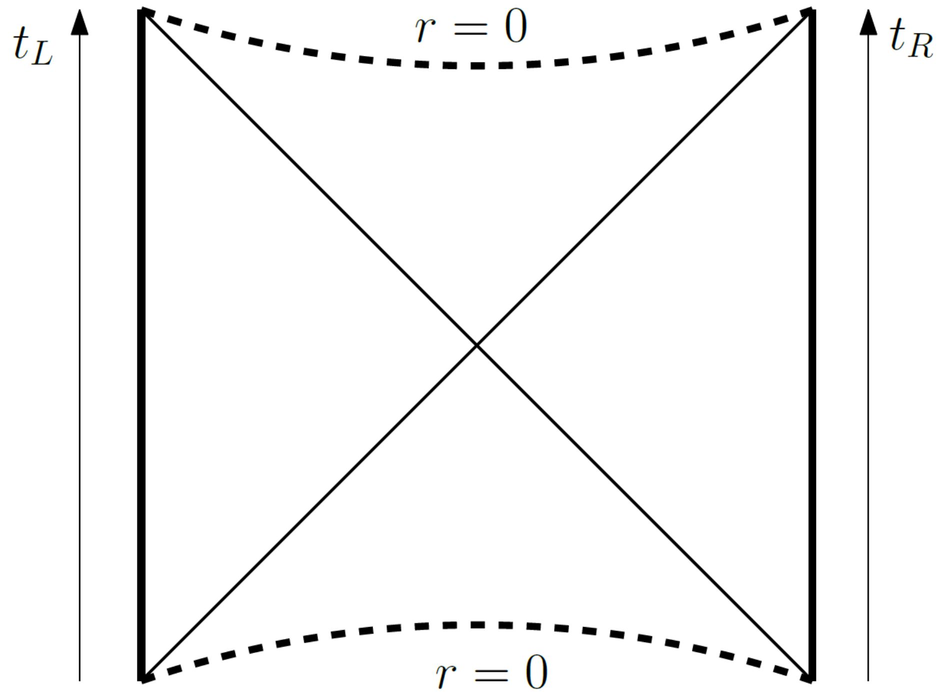
Goto, Marrochio, Myers, LQ, Yoshida
JHEP 1902 (2019) 160, arXiv:1901.00014

See also

Brown, Gharibyan, Lin, Susskind, Thorlacius, Zhao
Phys.Rev. D99 (2019) no.4, 046016, arXiv:1810.08741

Alishahiha
Eur.Phys.J. C79 (2019) no.4, 365, arXiv:1811.09028

Eternal Black Hole = Thermofield Double State



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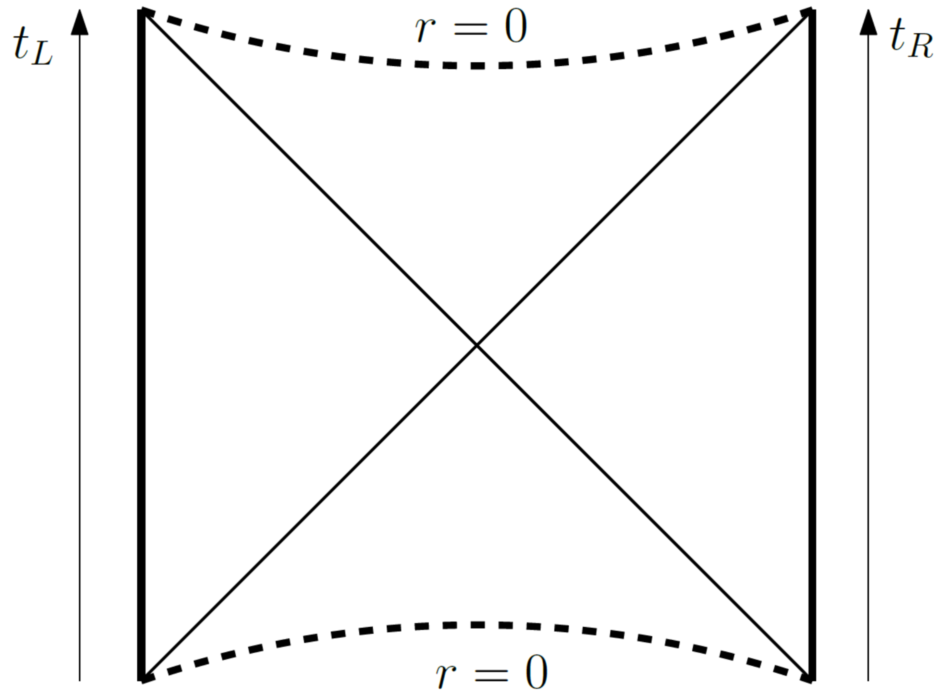

$$\Big\|$$

$$|TFD(t_L, t_R)\rangle = Z^{-1/2} \sum_{\alpha} e^{-E_\alpha/(2T)} e^{-iE_\alpha(t_L+t_R)} \, |E_\alpha\rangle_L \, |E_\alpha\rangle_R$$

(Maldacena, 2001)

(Hartman, Maldacena, 2013)

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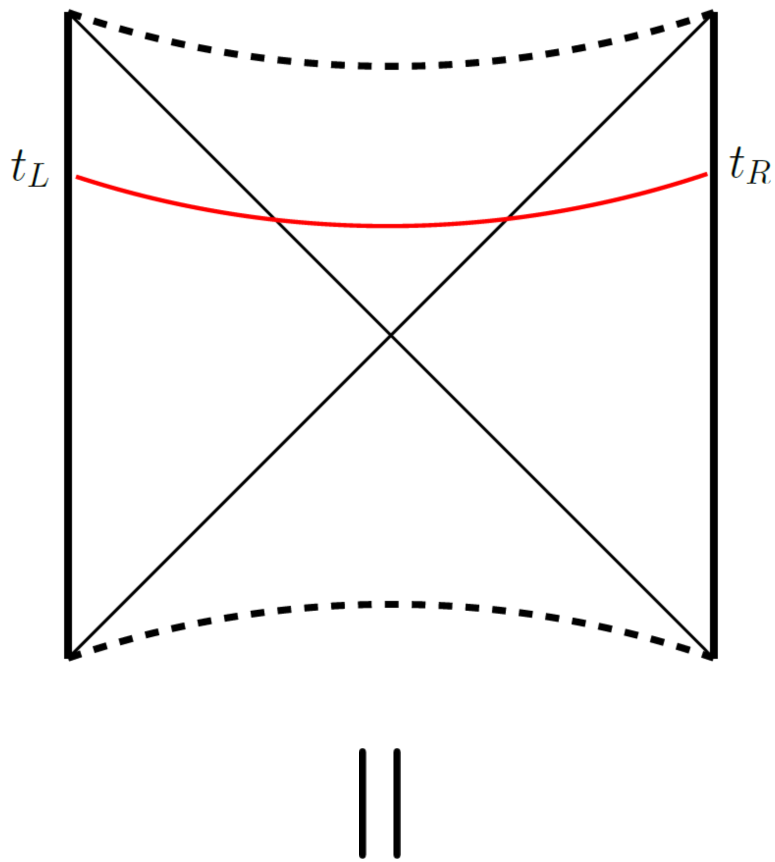

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(Susskind, Stanford, 2014)

What is the Complexity of a State?

$$\mathcal{C}\left(|\Psi\rangle\right) = \text{minimum size of quantum circuit to prepare the state}$$

Computer Science, Simple Quantum Systems

(Aaronson, 2016)

(Watrous, 2008)

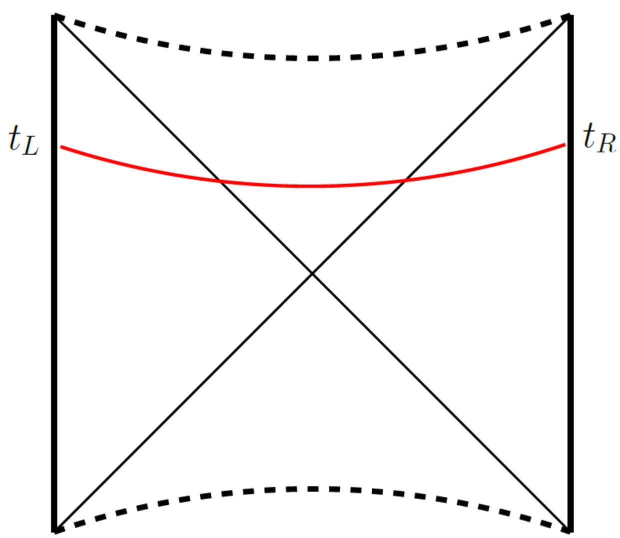
Quantum Field Theories

(Jefferson, Myers, 2017)

(Chapman *et al*, 2017)

(Caputa *et al*, 2017)

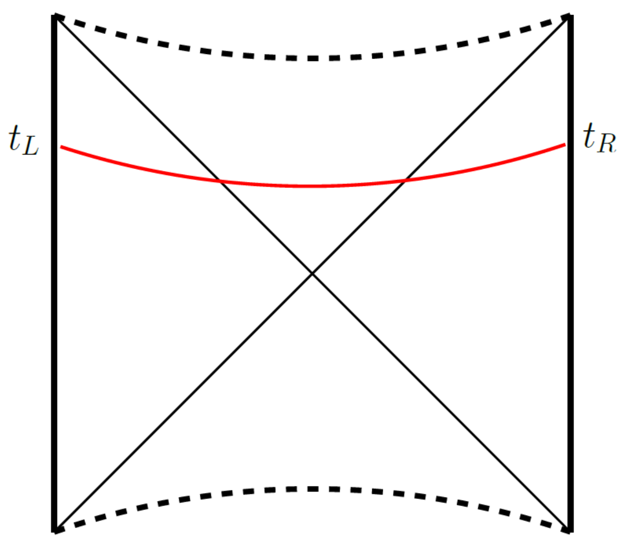
Holographic Complexity Conjectures

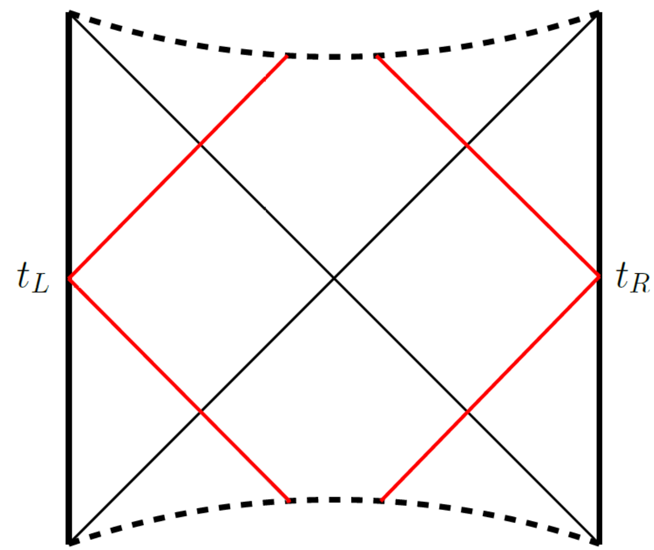


$$\mathcal{C}_{\mathcal{V}} = \max\left[\frac{\mathcal{V}}{Gl}\right]$$

(Susskind, Stanford, 2014)

Holographic Complexity Conjectures





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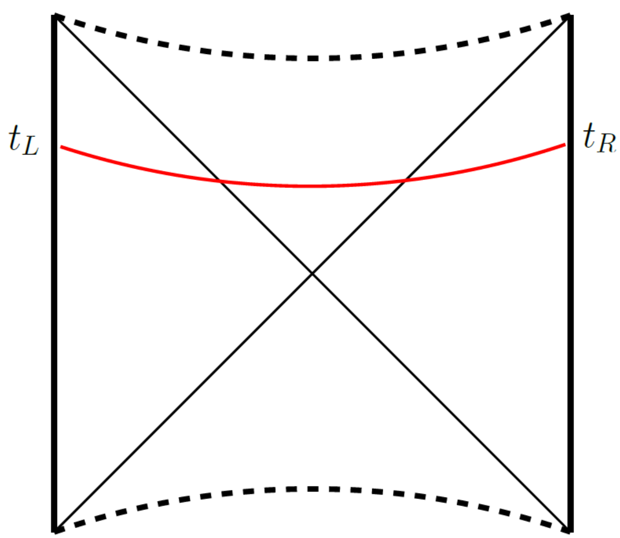
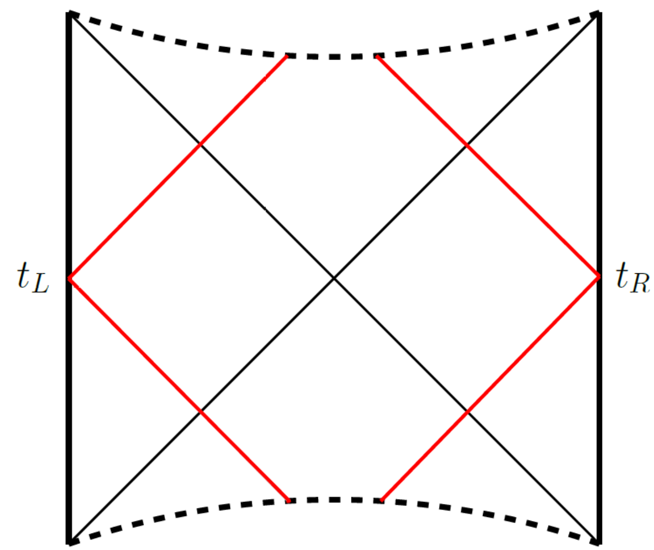
$$\mathcal{C}_{\mathcal{A}} = \frac{I_{\mathrm{WDW}}}{\pi\hbar}$$

(Brown *et al*, 2016)

Holographic Complexity Conjectures





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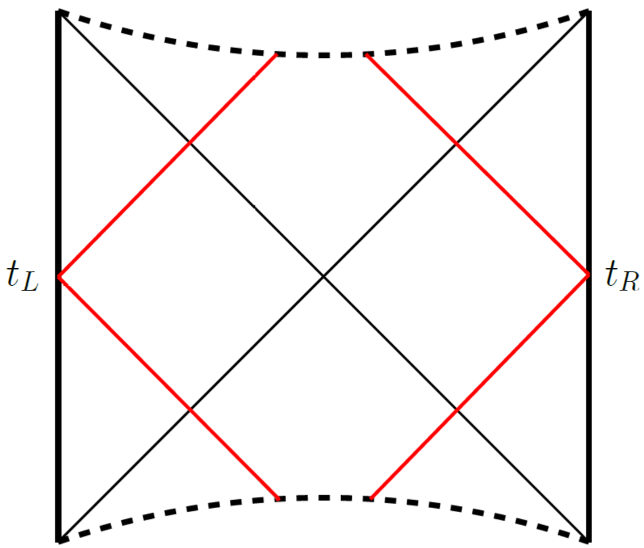
$$\lim_{t\to\infty}\frac{d\mathcal{C}}{dt} \sim TS$$

Uncharged Complexity = Action

Let us first consider a Schwarzschild-AdS black hole



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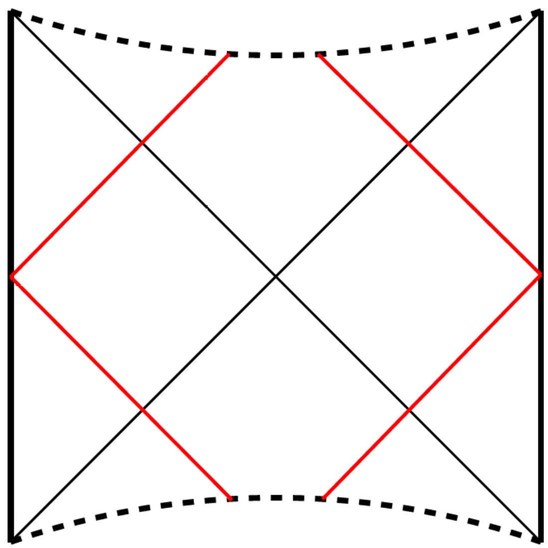

$$I_{\rm WDW} = I_{\rm EH} + I_{\rm surf} + I_{\rm ct}$$

$$I_{\rm EH} = \frac{1}{16\pi G_N} \int_{\mathcal{M}} d^4x \sqrt{-g}\left(\mathcal{R} - 2\Lambda\right)$$

$$I_{\rm surf} = \frac{1}{8\pi G_N} \int_{\mathcal{B}} d^3x \sqrt{|h|} K + \frac{1}{8\pi G_N} \int_{\Sigma} d^2x \sqrt{\sigma}\eta + \frac{1}{8\pi G_N} \int_{\mathcal{B}'} d\lambda\, d^2\theta \sqrt{\gamma}\kappa + \frac{1}{8\pi G_N} \int_{\Sigma'} d^2x \sqrt{\sigma} a$$

$$I_{\rm ct} = \frac{1}{8\pi G_N} \int_{\mathcal{B}'} d\lambda\, d^2\theta \sqrt{\gamma}\, \Theta \,\log\left(l_{ct}\Theta\right)$$

(Lehner *et al*, 2016)

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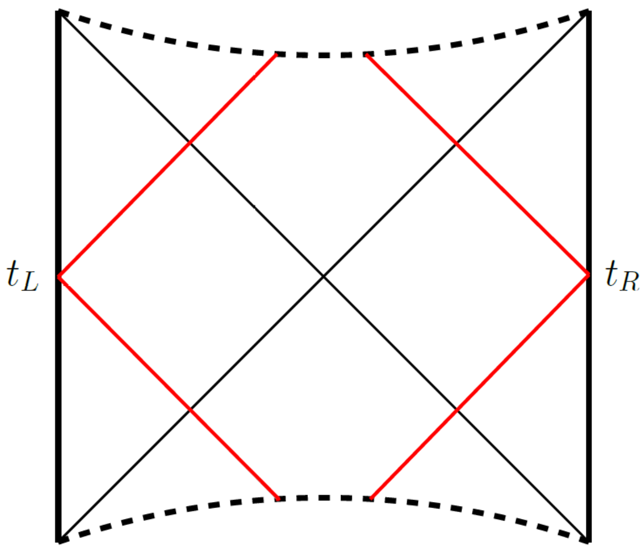
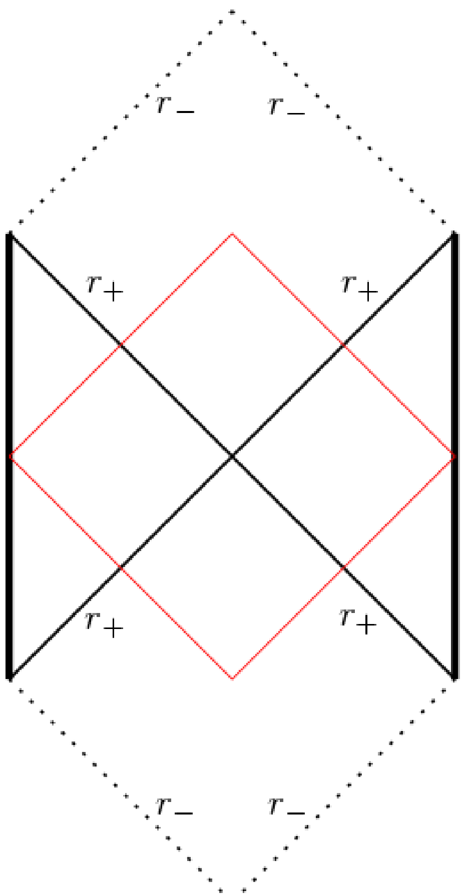


$$\lim_{t\to\infty} \frac{d\mathcal{C}_{\mathcal{A}}}{dt} = \frac{2M}{\pi\hbar}$$

Charged Complexity = Action

Now, we consider a dyonic Reissner-Nordstrom-AdS black hole



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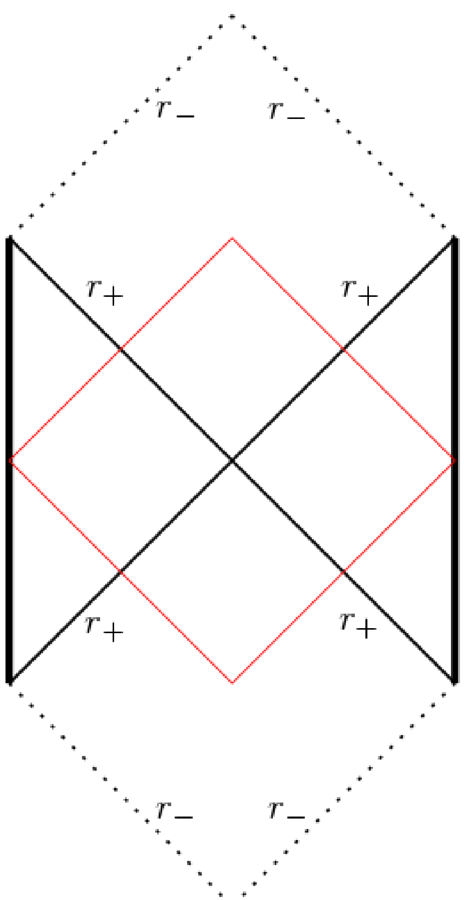

$$I_{\text{WDW}} = I_{\text{Grav}} + I_{\text{Max}}$$

$$I_{\text{Max}} = -\frac{1}{4g^2}\int_{\mathcal{M}} d^4x\,\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu}$$

$$F = \frac{g}{\sqrt{4\pi G_N}}\left(\frac{q_e}{r^2}\,dr\wedge dt + q_m\sin\theta\,d\phi\wedge d\theta\right)$$

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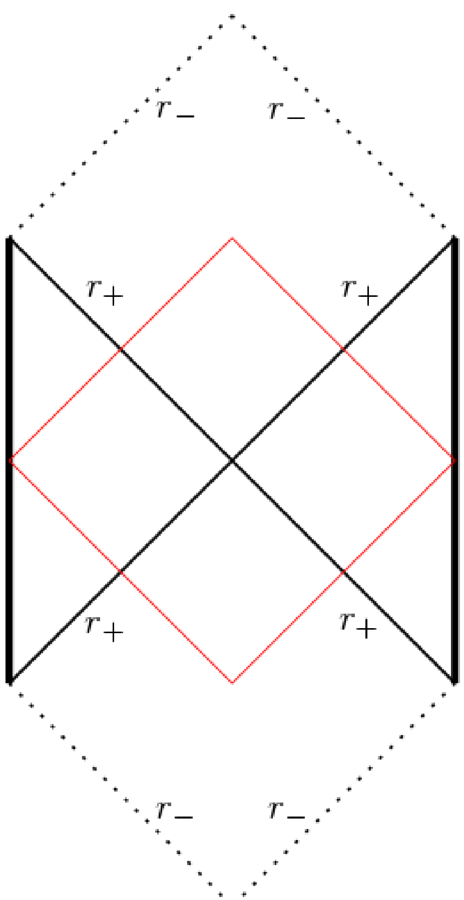

$$\lim_{t\to\infty} \frac{d\mathcal{C}_{\mathcal{A}}}{dt} = \frac{1}{\pi\hbar} \left[\frac{q_e^2}{G_N r} \right]_{r_+}^{r_-}$$

Charged Complexity = Action

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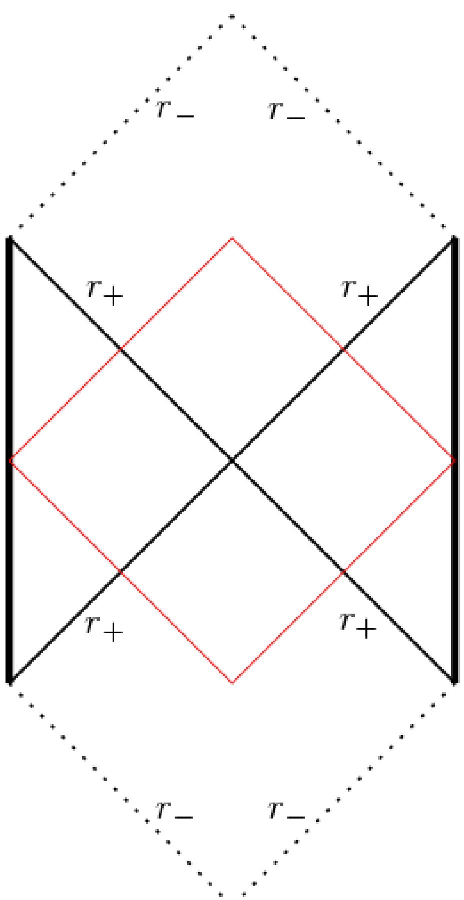
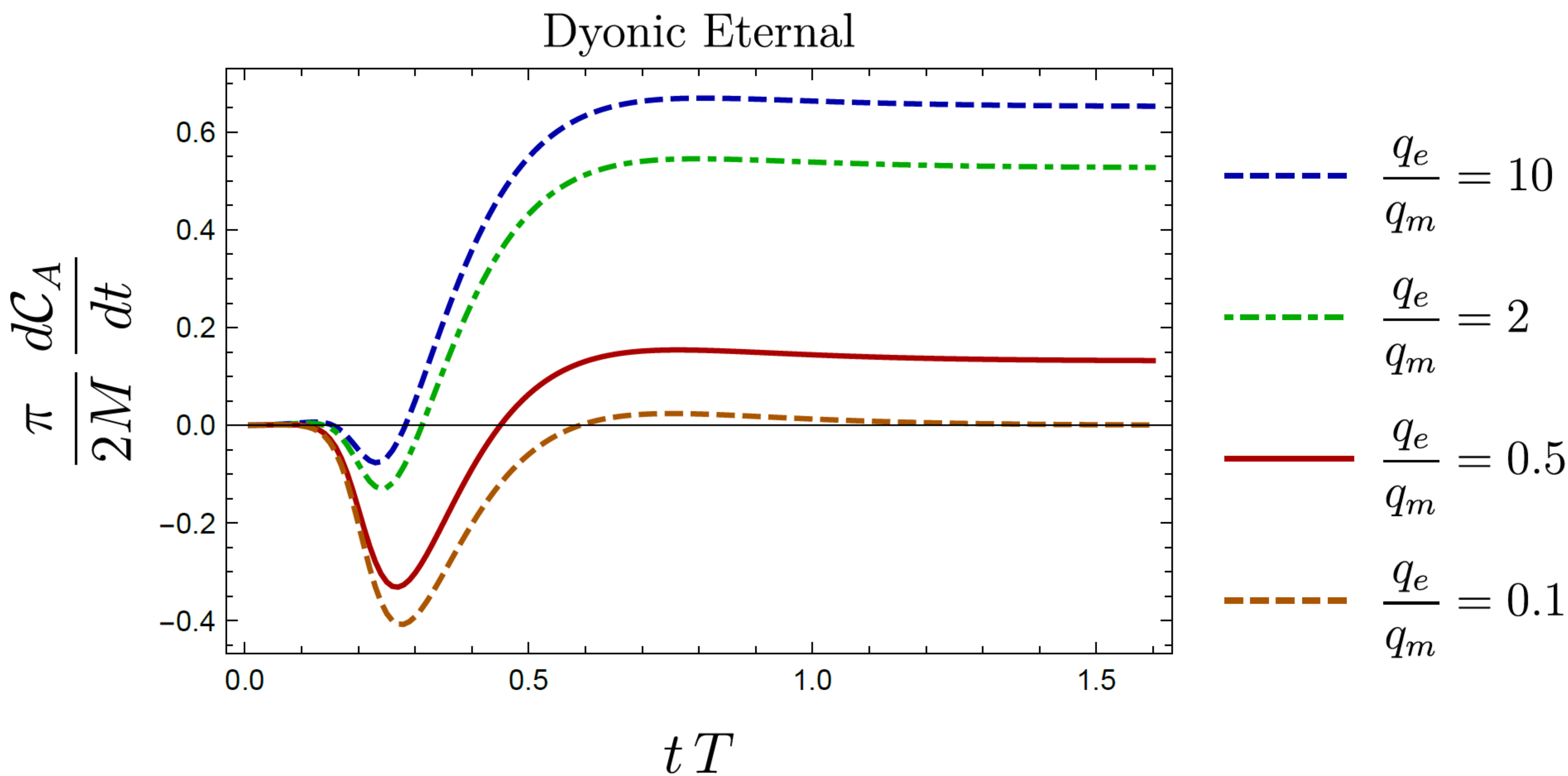


$$\lim_{t\to\infty} \frac{d\mathcal{C}_{\mathcal{A}}}{dt} = \frac{1}{\pi\hbar}\left[\frac{q_e^2}{G_N r}\right]_{r_+}^{r_-}$$

The magnetic charge seems to play no role.

Charged Complexity = Action



Lessons from BH thermodynamics

For simplicity, let us assume a purely electric black hole ($q_m = 0$).

$$I_{\text{Eucl}} = I_{\text{Grav}} + I_{\text{Max}}$$

⇓

Fixed μ ensemble

$$I_{\text{Eucl}} = I_{\text{Grav}} + I_{\text{Max}} + I_{\mu Q}$$

⇓

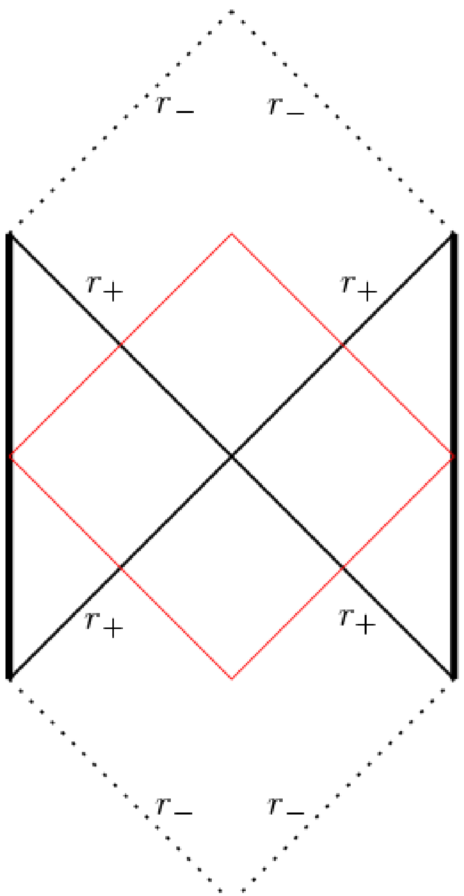
$$I_{\mu Q} = \frac{1}{g^2} \int_{\partial \mathcal{M}} d\Sigma_\mu \, F^{\mu\nu} \, A_\nu$$

Fixed Q ensemble

(Hawking, Ross, 1995)

Complexity = Which Action?

We can consider the addition of a boundary term for the Maxwell field



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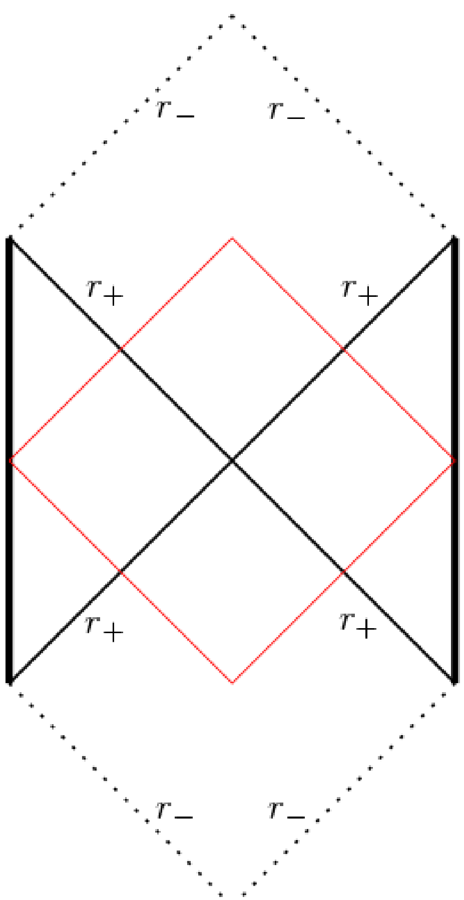

$$I_{\text{WDW}} = I_{\text{Grav}} + I_{\text{Max}} + I_{\mu Q}$$

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$$F = \frac{g}{\sqrt{4\pi G_N}} \left(\frac{q_e}{r^2} \, dr \wedge dt + q_m \sin\theta \, d\phi \wedge d\theta \right)$$

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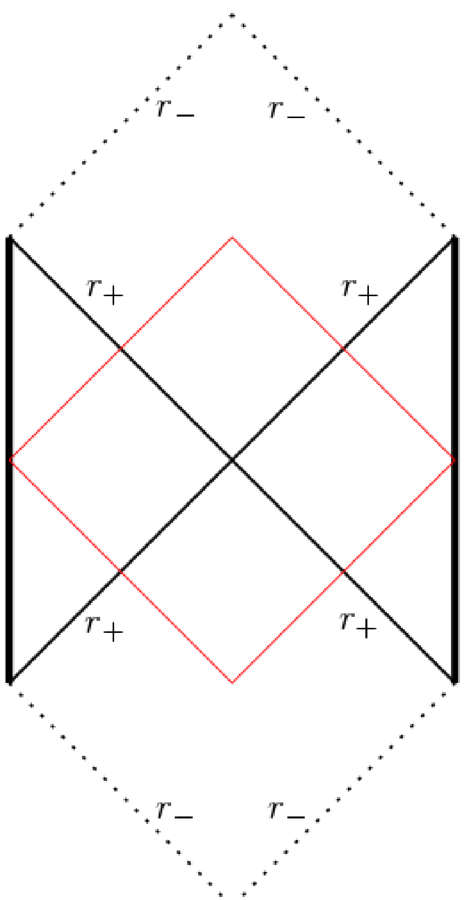

$$\lim_{t\to\infty}\frac{d\mathcal{C}_{\mathcal{A}}}{dt}=\frac{1}{\pi\hbar}\left[\frac{q_m^2}{G_N r}\right]_{r_+}^{r_-}$$

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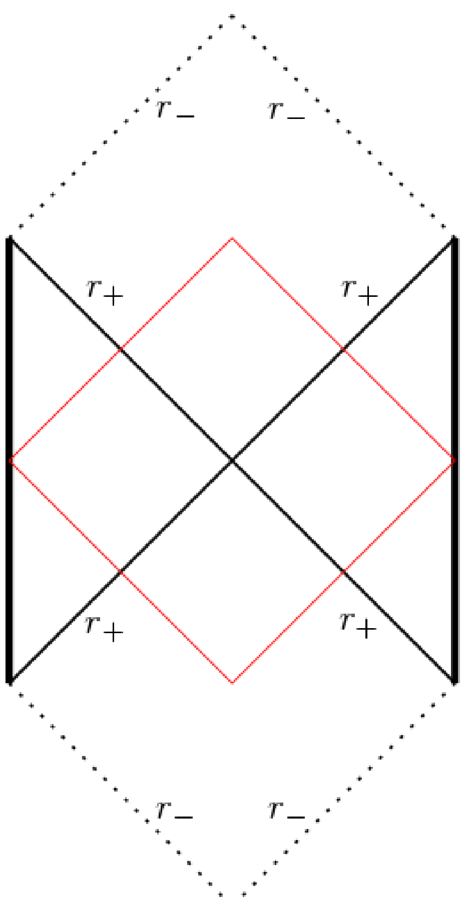

$$\lim_{t\to\infty}\frac{d\mathcal{C}_{\mathcal{A}}}{dt}=\frac{1}{\pi\hbar}\left[\frac{q_m^2}{G_N r}\right]_{r_+}^{r_-}$$

The boundary term changes the type of charge that contributes to the complexity growth rate.

Implications for Jackiw-Teitelboim Model

$$I_{\mathrm{BH}} = I_{\mathrm{Grav}} + I_{\mathrm{Max}}$$

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$$I_{\text{BH}} = I_{\text{Grav}} + I_{\text{Max}}$$

$$ds^2 = g_{ab}dx^a dx^b + r^2 d\Omega^2$$

$$F = \frac{g}{\sqrt{4\pi G_N}} q_m \sin\theta \, d\phi \wedge d\theta$$

$$\Phi = 4\pi r^2$$

$$\Phi = \Phi_0 + \Phi_\epsilon$$

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$$\begin{aligned} I_{\mathrm{JT}} = &\frac{\Phi_0}{16\pi G_N}\left(\int_M d^2x\sqrt{-g}R + 2\int_{\partial M}\sqrt{|\gamma|}K\right) \\ &+ \frac{1}{16\pi G_N}\left(\int_M d^2x\sqrt{-g}\Phi_\epsilon\left(R + \frac{2}{L_2^2}\right) + 2\int_{\partial M}\sqrt{|\gamma|}\Phi_\epsilon K\right) \end{aligned}$$

(Nayak *et al*, 2018)

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⇓

$$\begin{aligned} I_{\rm JT} = & \frac{\Phi_0}{16\pi G_N} \left(\int_M d^2x \sqrt{-g} R + 2 \int_{\partial M} \sqrt{|\gamma|} K \right) \\ & + \frac{1}{16\pi G_N} \left(\int_M d^2x \sqrt{-g} \Phi_\epsilon \left(R + \frac{2}{L_2^2} \right) + 2 \int_{\partial M} \sqrt{|\gamma|} \Phi_\epsilon K \right) \end{aligned}$$

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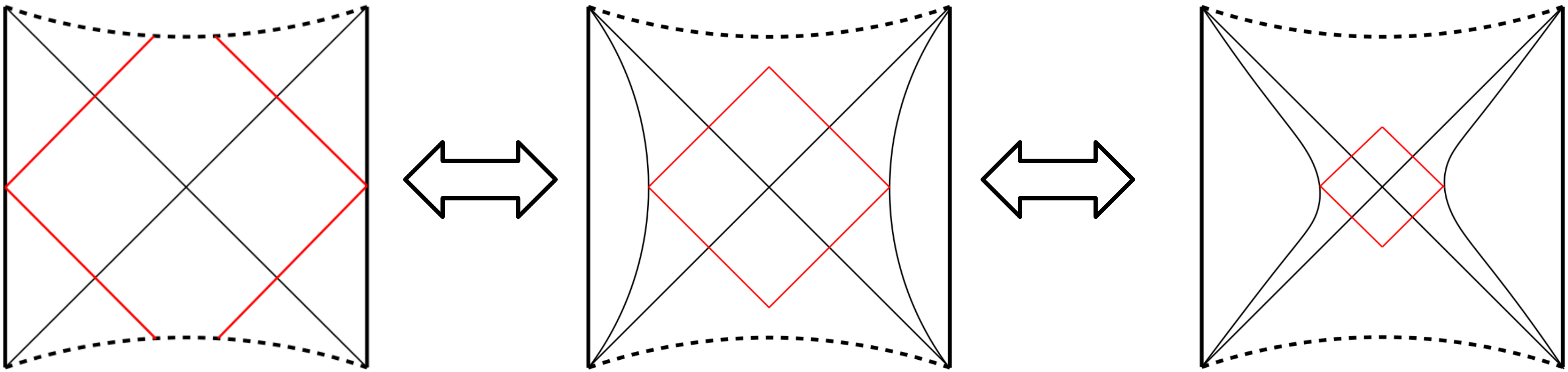
Implications for Jackiw-Teitelboim Model



JT model action does not grow at late times.

Main Results and Outlook

- Late time complexity growth is independent of where we anchor the WDW patch. Possible relation with $T\bar{T}$ deformations? (Akhavan *et al*, 2018)

- Boundary Term Dependence of Holographic Complexity

$$\Downarrow$$

Distinction between CA and CV conjectures

- What is the meaning of the boundary terms associated to the matter fields for the definition of complexity in the dual CFT?

- More generally, given the ambiguities associated to an action, holographic complexity equals which action?