QCD Strings and Jackiw-Teitelboim Gravity

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SD, Victor Gorbenko and Mehrdad Mirbabayi 1706.06604,
SD, Guzman Hernandez-Chifflet 1611.09796
SD, Victor Gorbenko 1511.01908

continuation of
SD, Raphael Flauger, Victor Gorbenko, 1404.0037, 1301.2325, 1205.6805, 1203.1054
SD, Victor Gorbenko and Mehrdad Mirbabayi 1305.6939
Patrick Cooper, SD, Victor Gorbenko, Ali Mohsen, Stefano Storace 1411.0703
SD, Peter Conkey 1603.00719

The main goal is to read off from the experiment the answer to

$\boxed{\text{What is } SU(\infty) \text{ Yang-Mills?}}$

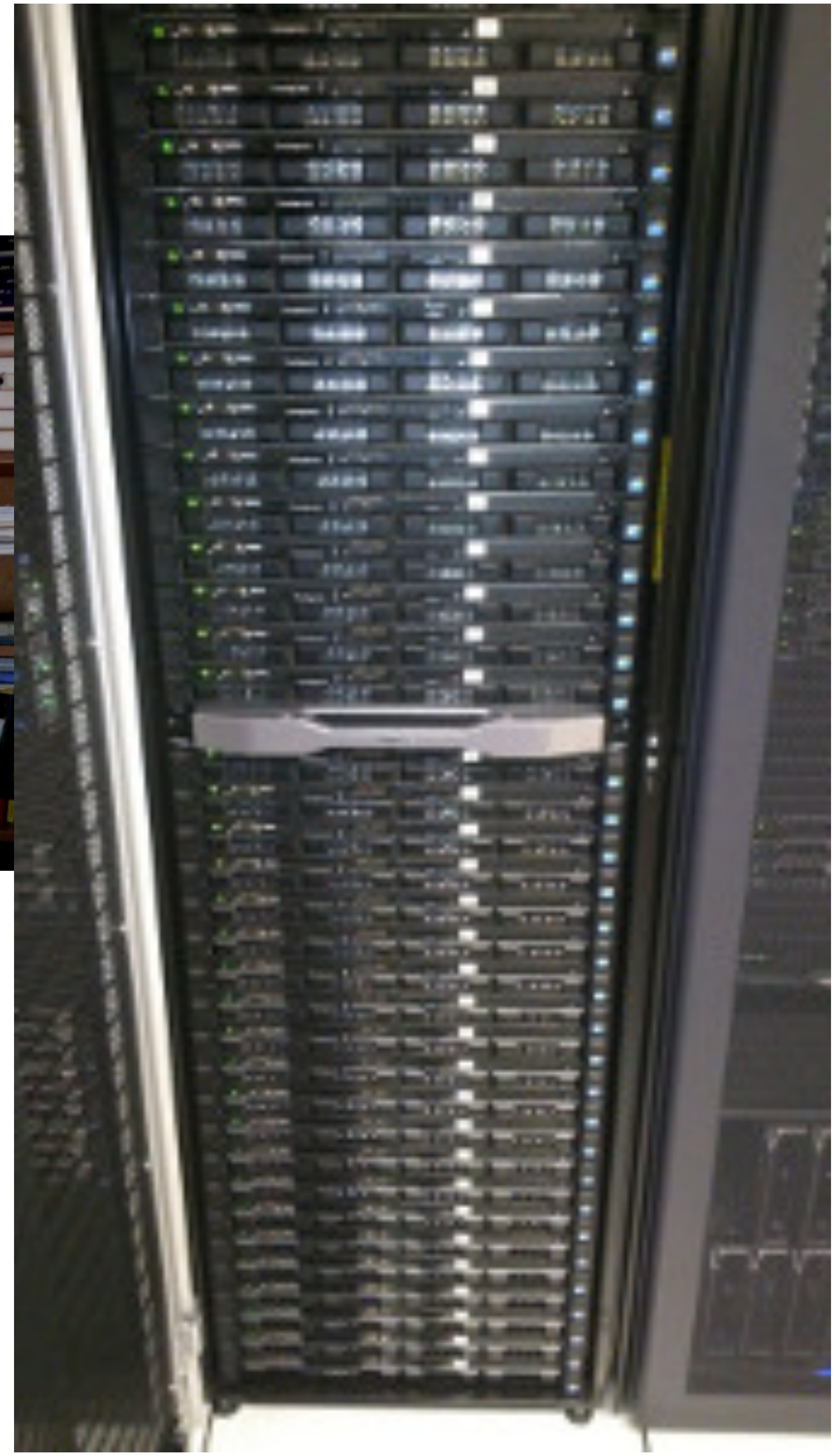
More precisely, we know that large N QCD is a theory of free strings.

$\boxed{\text{Can we solve this free string theory?}}$

A typical particle physics experiment

In our case we have

Mike Teper



Barak Bringoltz

Andreas Athenodorou

1702.03717, 1609.03873, 1602.07634, 1303.5946, 1103.5854, 1007.4720, ...

Let's divide the question into two parts:

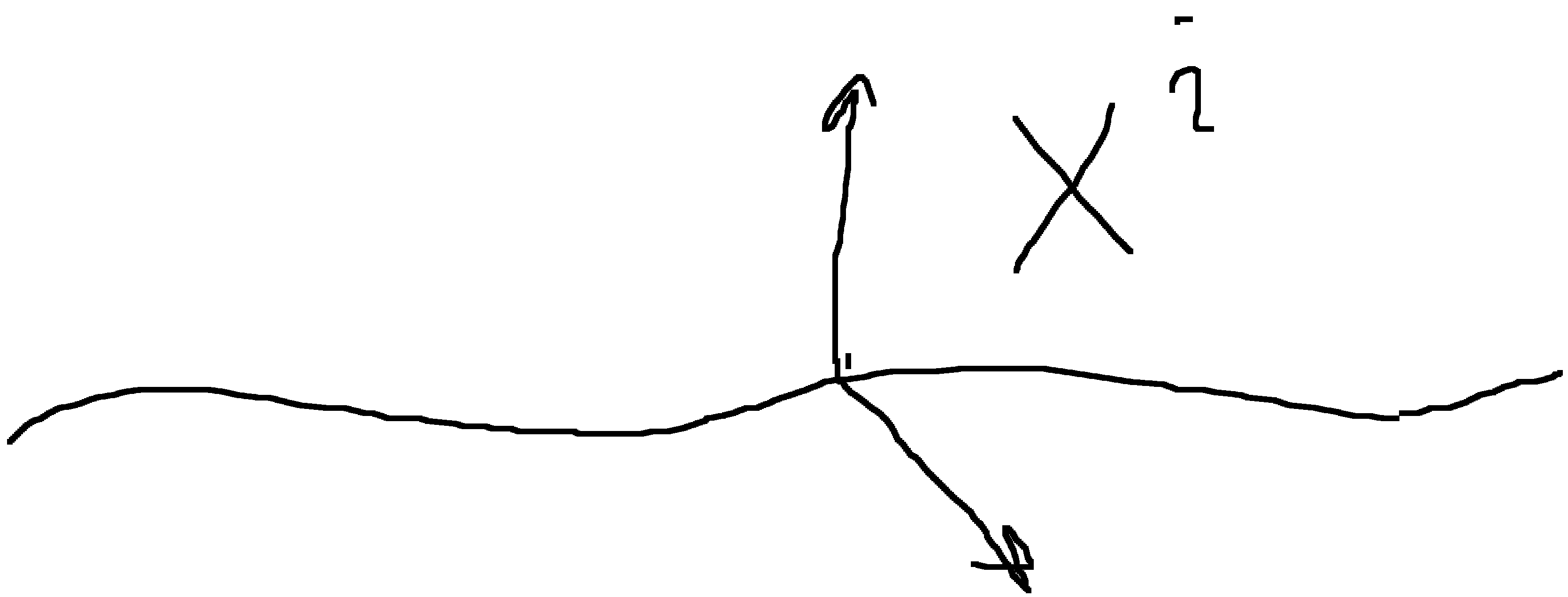
1) What is the worldsheet theory of an infinitely long string?

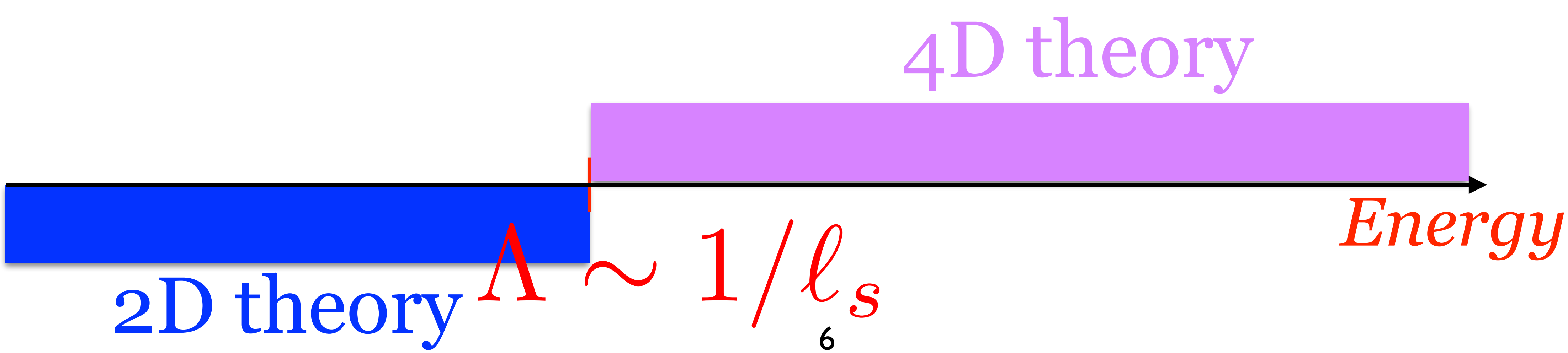
2) If we know the answer to 1) what can we say about short strings (glueballs)?

SETUP

✓Confining gauge theory with a gap

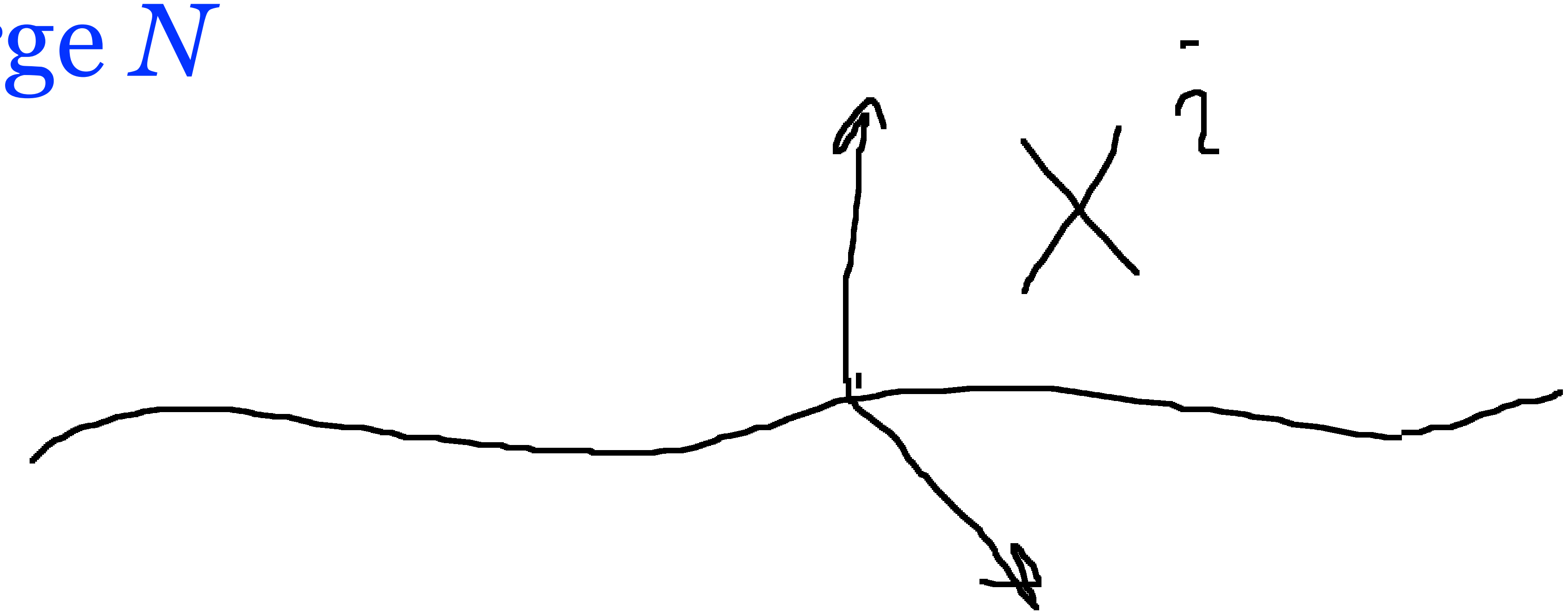
✓Unbroken center symmetry

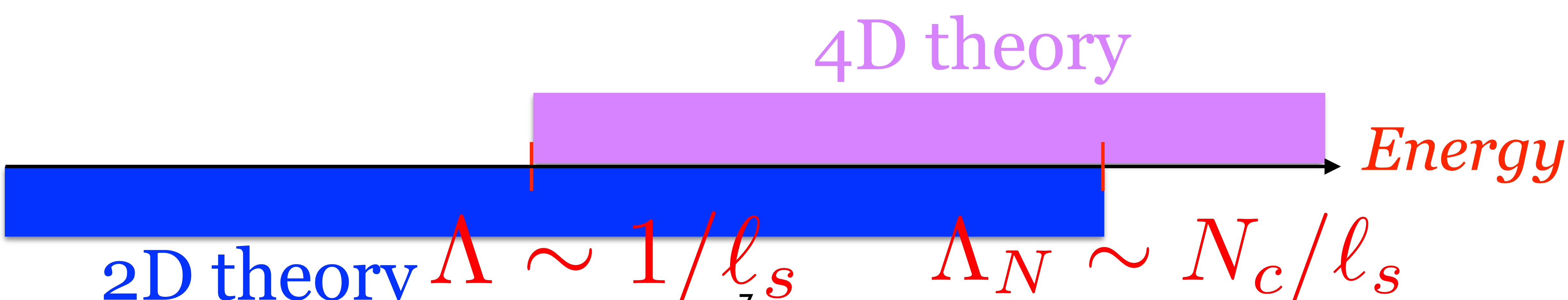




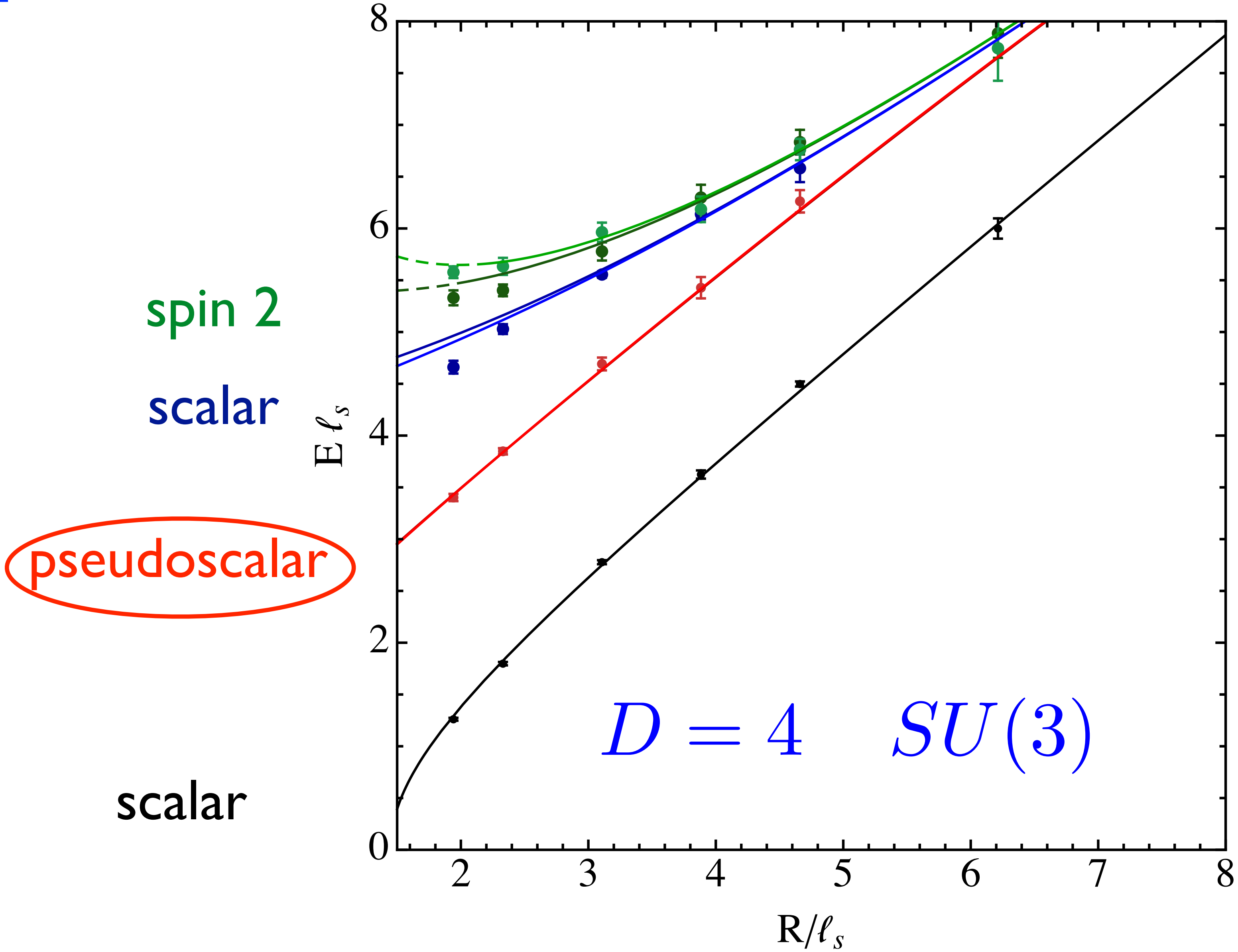
SETUP

- ✓ Confining gauge theory with a gap
- ✓ Unbroken center symmetry
- ✓ Large N





Lattice provides us with a finite volume spectrum of the worldsheet theory



Theory curves:

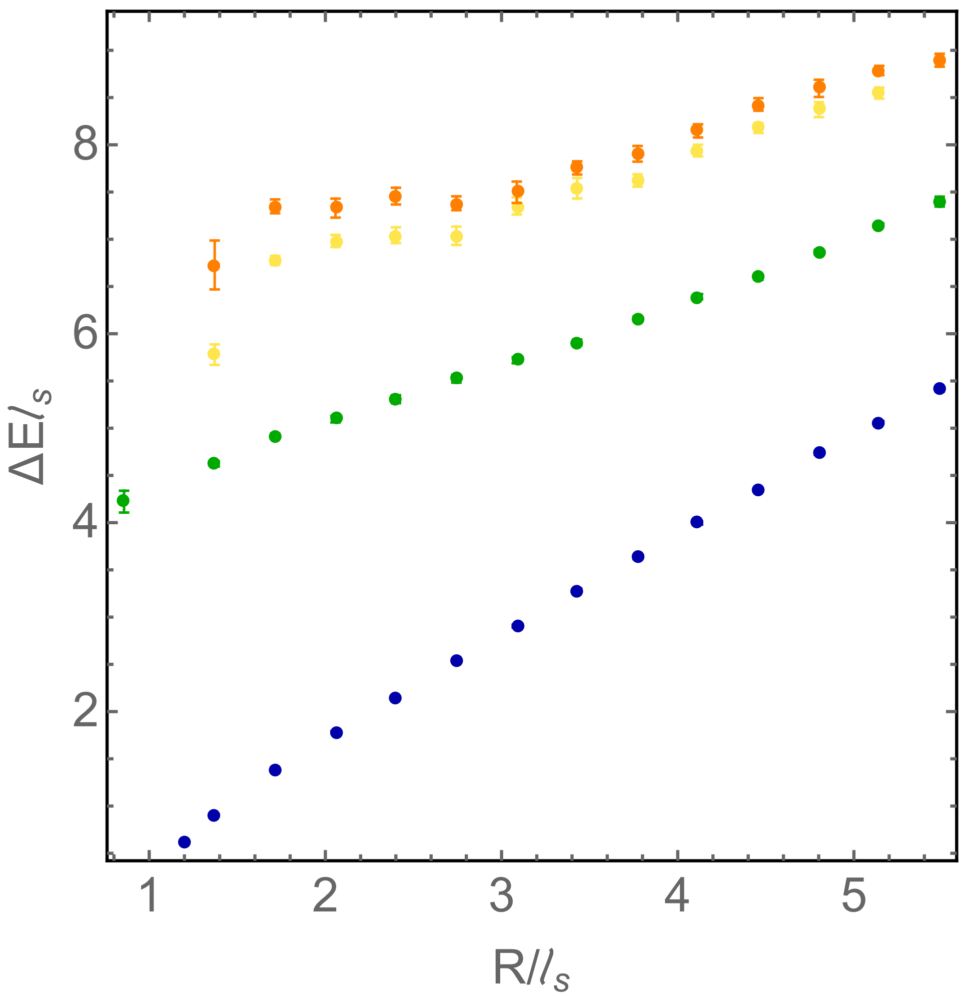
ground state: sum of universal ℓ_s/R terms

Lüscher '81
Lüscher, Weisz '04
Aharony, Komargodski et al '09-13

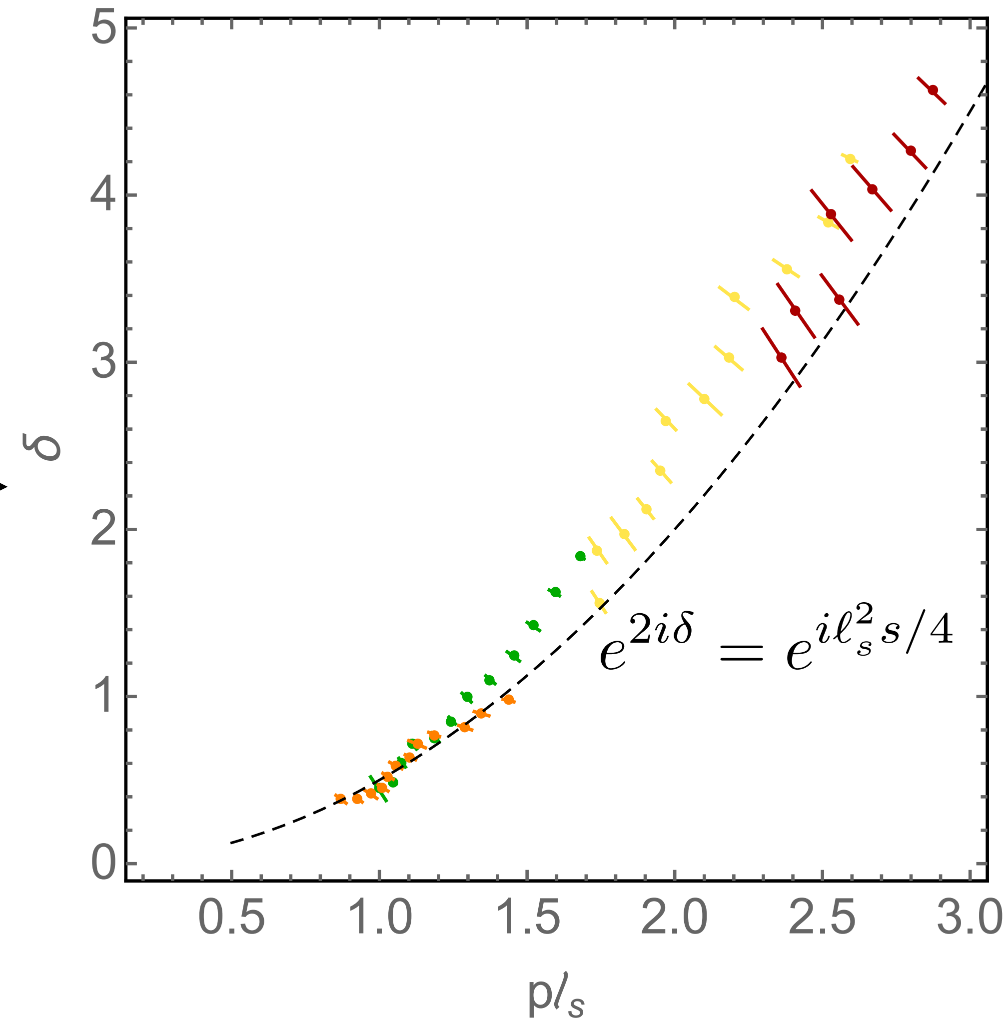
excited states: TBA calculations

SD, Flauger, Gorbenko'13

TBA allows to reconstruct scattering phase shift



TBA

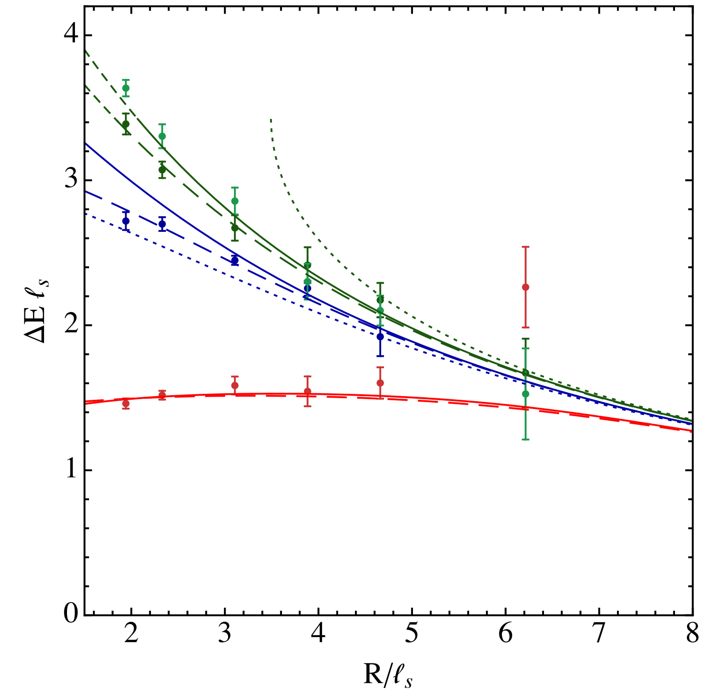


$$D = 3 \quad SU(6)$$

Experimental summary (long strings)

D=4

field content $X^1 \,,\ X^2$ (Goldstones) ϕ (pseudoscalar "worldsheet axion")



action

$$S = S_{NG}[X^1, X^2] + \int d^2\sigma \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}m^2\phi^2 + Q\phi\epsilon^{\alpha\beta}\epsilon_{ij}K^i_{\alpha\gamma}K^{j\,\gamma}_{\beta} + \dots$$

$$m\ell_s \approx 1.85^{+0.02}_{-0.03} \qquad\qquad Q \approx 0.38 \pm 0.04$$

D=3

field content X (Goldstone) action $S = S_{NG}[X] + \dots$

Can a worldsheet theory be integrable?

why this question?

- ✓To get an idea of what one might expect
- ✓By now we have a few examples of integrable higher dimensional conformal theories (N=4 SYM, ABJM). This looks as a natural definition of an integrable higher-dimensional *confining* theory
- ✓One may expect QCD string to be somewhat simple in the UV. Simple=Integrable?

Can a theory of Goldstones only be integrable?

Yes, at D=3 or 26

Ward identities of non-linearly realized Poincare plus integrability determine

$$e^{2i\delta(s)} = e^{is\ell_s^2/4}$$

Finite volume spectrum from TBA

$$E(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell^4} + \frac{4\pi}{\ell^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

At D=26 this a critical bosonic string

A simple option to restore integrability at general D

$$S_{string} = -\ell_s^{-2} \int d^2\sigma \sqrt{-det(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i + \partial_\alpha \phi \partial_\beta \phi)} + Q \int d^2\sigma \phi R[X] + \ldots$$

$$Q = \sqrt{\frac{25 - D}{48\pi}}$$

$$e^{2i\delta(s)} = e^{is\ell^2/4}$$

This is also known as a linear dilaton background.

A conventional path to non-critical strings.

Another simple option to restore integrability at D=4

$$\boxed{S_{string} = -\ell_s^{-2}\int d^2\sigma\sqrt{-det(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i + \partial_\alpha\phi\partial_\beta\phi)} + Q\int d^2\sigma\phi K\tilde{K} + \ldots}$$

$$Q = \sqrt{\frac{25-D}{48\pi}} = \sqrt{\frac{7}{16\pi}} \approx 0.373176\ldots$$

$$\boxed{e^{2i\delta(s)} = e^{is\ell^2/4}}$$

Compare to

$$Q_{lattice} \approx 0.38 \pm 0.04$$

???

Experimental summary (long strings)

- D=4

field content $X^1 \,,\, X^2$ ϕ

Goldstones pseudoscalar

Both at D=3,4 the worldsheet theory looks as a deformation of an integrable theory.

Correct matter content+suggestive numerology at D=4.

- D=3

field content X action $S = S_{NG}[X] + \dots$

Goldstone

Axionic String Ansatz

QCD String is a deformation of an integrable theory of

$$X^i \qquad \text{and} \qquad A_{ij} = -A_{ji}$$

D=4: Goldstones + axion

D=3: Goldstones only

Let us focus on this. What can we tell about short strings?
For now will try to predict quantum numbers only :

Spin J, Parity P, Charge conjugation C

$$J \neq 0: \quad J^C \qquad\qquad J = 0: \quad 0^{PC}$$

Tensor Square Structure

$$\mathcal{H}_{closed\ short} = \sum_N \mathcal{H}_L(N) \otimes \mathcal{H}_R(N).$$

$$C:\ \mathcal{H}_L \otimes \mathcal{H}_R \leftrightarrow \mathcal{H}_R \otimes \mathcal{H}_L$$

If massive worldsheet excitations were present:

$$\mathcal{H}_{massive} = \sum_n \mathcal{H}_n$$

n is a number of massive particles at rest

Multiplication Table

$$0^P \otimes 0^P = 0^{++}$$

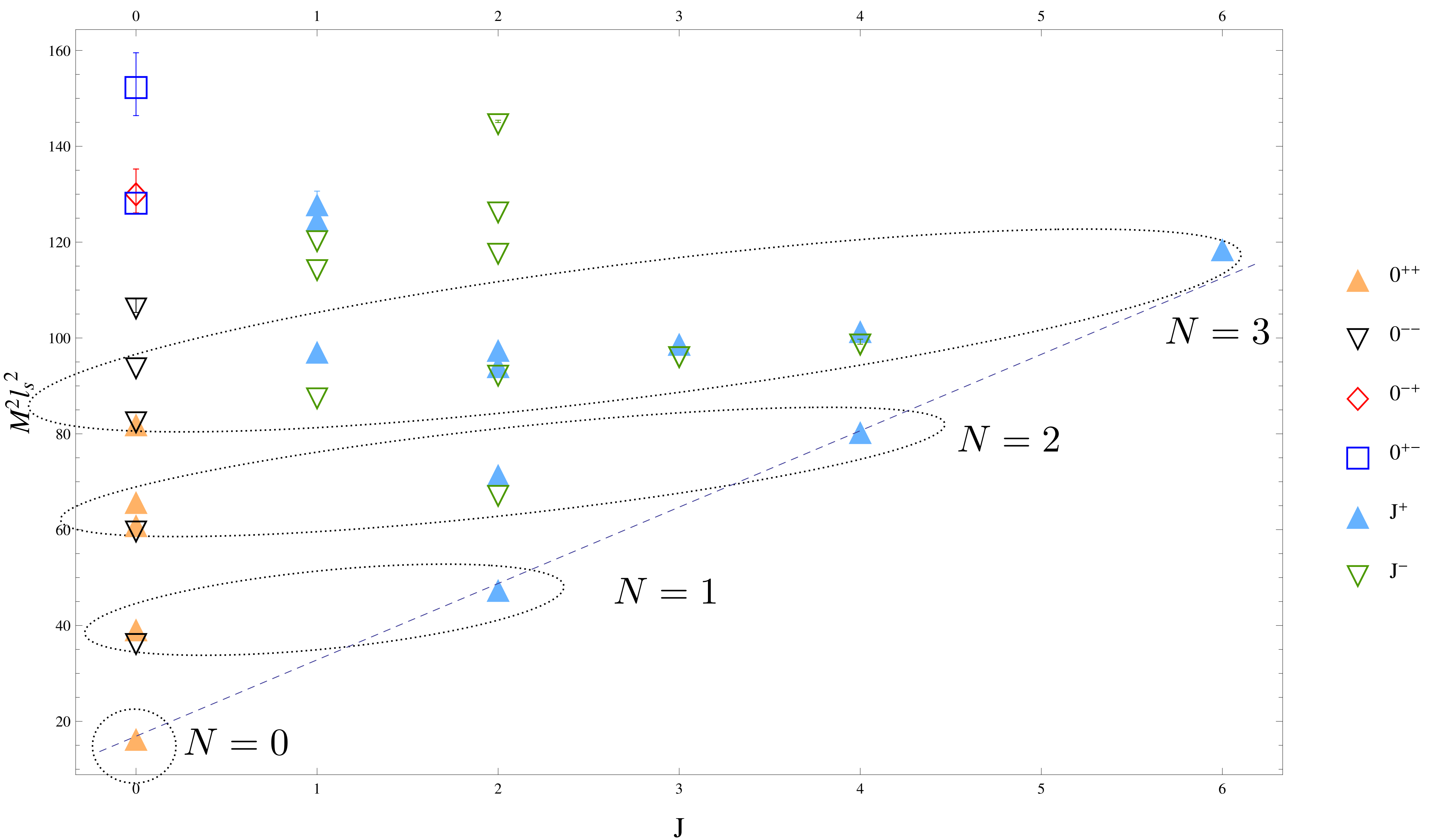
$$0^{P_1} \otimes 0^{P_2} + 0^{P_2} \otimes 0^{P_1} = 0^{(P_1 P_2)+} + 0^{(P_1 P_2)-}$$

$$0^P \otimes J + J \otimes 0^P = J^+ + J^-$$

$$J \otimes J = (2J)^+ + 0^{++} + 0^{--}$$

$$J_1 \otimes J_2 + J_2 \otimes J_1 = (J_1 + J_2)^+ + (J_1 + J_2)^- + |J_1 - J_2|^+ + |J_1 - J_2|^-$$

Lattice Data



Glueball Spin Content

N	Glueball States	# of states
0	$0 \otimes 0 = 0^{++}$	1
1	$1 \otimes 1 = 0^{++} + 0^{--} + 2^{+}$	4
2	$(0+2) \otimes (0+2) = 2 \cdot 0^{++} + 0^{--} + 2^{+} + 2^{-} + 4^{+}$	9
3	✓ ?! $(0+1+3) \otimes (0+1+3) =$ $3 \cdot 0^{++} + 2 \cdot 0^{--} + 1^{+} + 1^{-} + 2 \cdot 2^{+} + 2^{-} + 3^{+} + 3^{-} + 4^{+} + 4^{-} + 6^{+}$	25

N=3: 3 states are missing yet, some
spin determinations are to be performed/confirmed

Semiclassical Ansatz

$$\mathcal{H}_L(N) = \mathcal{H}_R(N) = \mathcal{H}_{open}(N)$$

where

$$\mathcal{H}_{open} \equiv \sum_N \mathcal{H}_{open}(N) = \sum_{J \in \mathbf{Z}} \mathcal{H}_J$$

at large J one can use perturbation theory around the "rotating rod"

$$X^0 = \frac{\ell_s^2 E}{\pi} \tau \qquad X^1 + iX^2 = \frac{\ell_s}{2i} \sqrt{\frac{2J}{\pi}} \left(e^{i\sigma^+} - e^{i\sigma^-} \right)$$

extending the semiclassical answer down to J=0 reproduces observed quantum numbers

suggestive of an underlying localization story?

To go beyond this heuristics we need a tractable path integral formalism for a single massless boson with

$$\boxed{e^{2i\delta(s)} = e^{is\ell_s^2/4}}$$

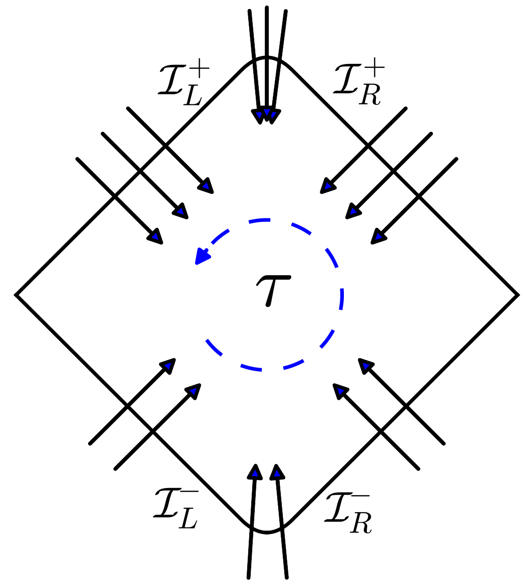
of course, we have the Nambu-Goto action

$$S_{NG} = -\ell_s^{-2}\int d^2\sigma\sqrt{1+\ell_s^2(\partial X)^2}$$

but it does not look tractable

More general construction

SD, Gorbenko, Mirbabayi
1305.6939



"dressed" S-matrix

S-matrix of *any* 2D QFT

$$\boxed{\hat{S}_n(p_i) = e^{i\ell^2/4\sum_{i<j} p_i * p_j} S_n(p_i)}$$

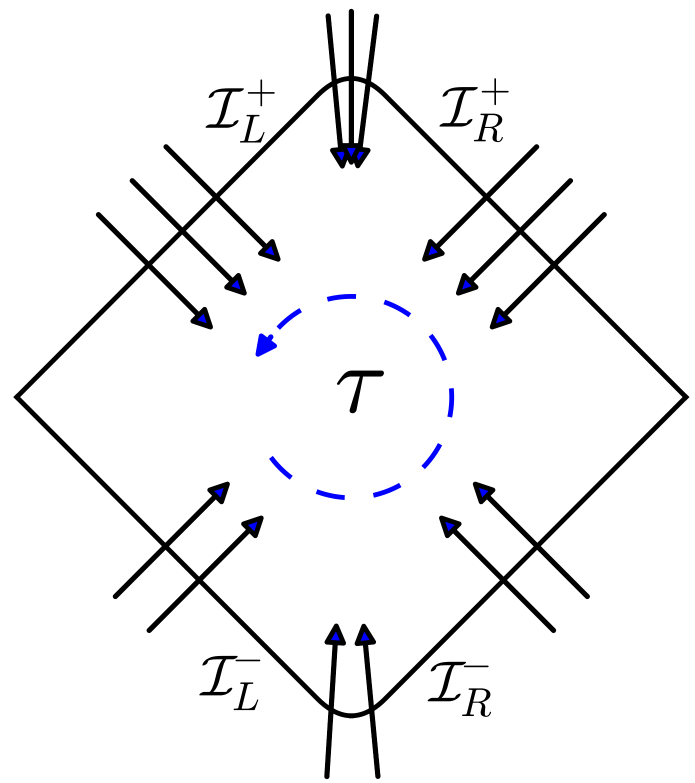
$$p_i * p_j = \epsilon_{\alpha\beta} p_i^\alpha p_j^\beta$$

recently, formulated in the operator language as

$T\bar{T}$ deformation

Smirnov, Zamolodchikov, 1608.05499
Cavaglia, Negro, Szecsenyi, Tateo, 1608.05534

“Holographic” Form of the Dressing Factor



$$e^{i\ell^2/4\sum_{i<j} p_i * p_j} = \int \mathcal{D}X^\alpha e^{iS_{CS}[X^\alpha]+\sum_i p_{i\alpha}X^\alpha}$$

Chern-Simons boundary quantum mechanics

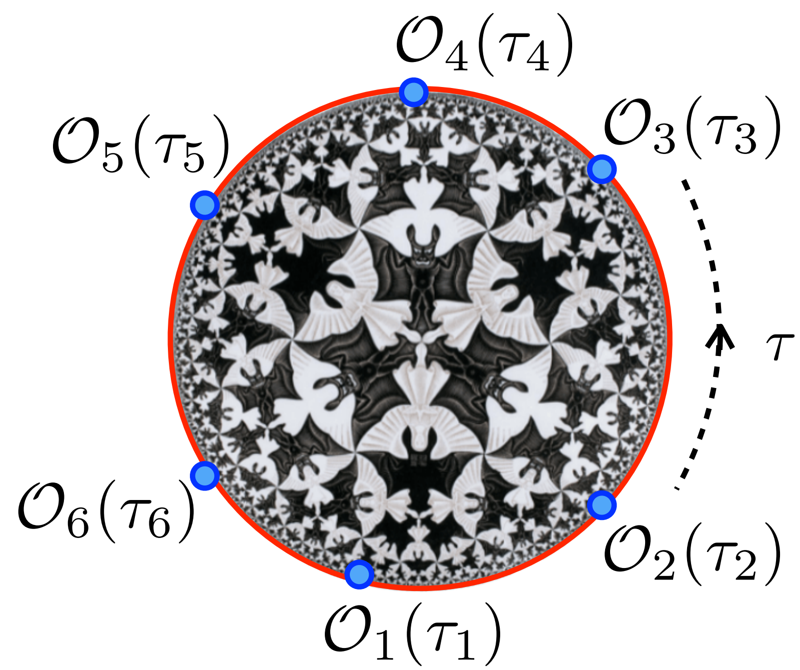
$$S_{CS} = \ell^{-2} \oint d\tau \epsilon_{\alpha\beta} X^\alpha \partial_\tau X^\beta$$

$N\,AdS_2$ Holography

- consider *any* 2D QFT in a rigid AdS_2 (=Poincare disc)

...
Jensen, 1605.06098
Maldacena, Stanford, Yang, 1606.06098
Engelsoy, Mertens, Verlinde, 1606.03438
...

$$ds^2 = dr^2 + L^2 \sinh^2 \frac{r}{L} d\tau^2$$



- calculate generating functional $Z_0[\lambda_{\mathcal{O}_a}(\tau)]$ for boundary conformal correlators

- introduce dynamical Jackiw—Teitelboim gravity

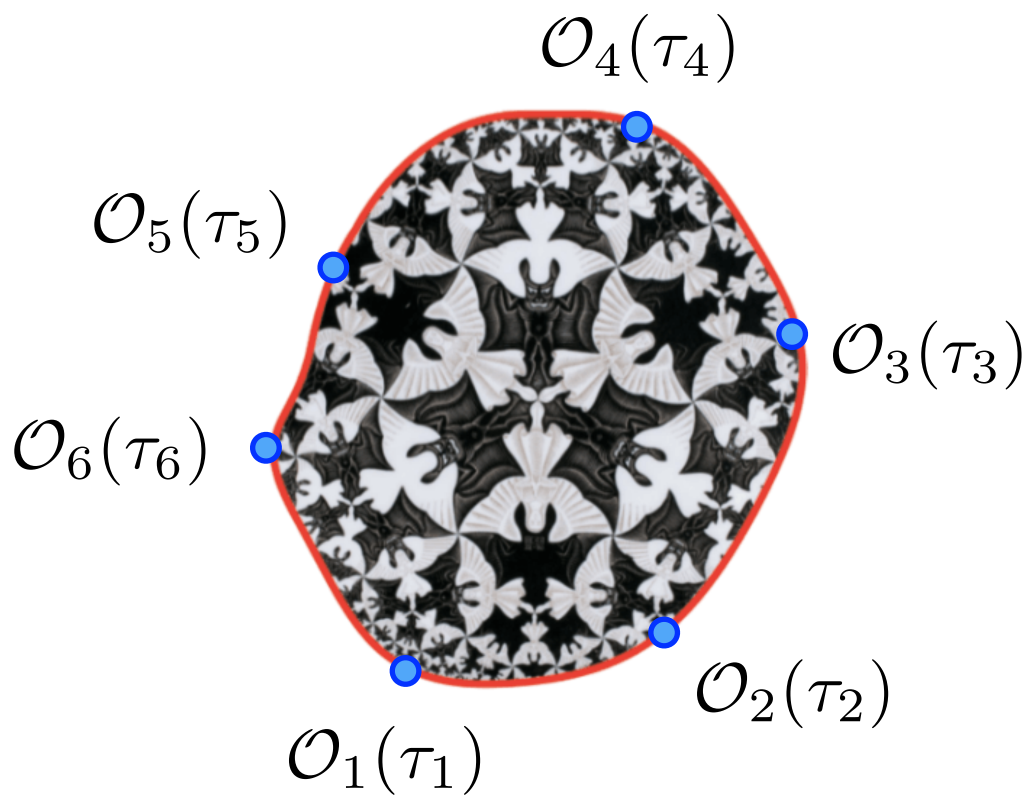
$$S = \int \sqrt{-g} \left(\phi(R + \frac{2}{L^2}) - \Lambda + \mathcal{L}_m(g, \psi) \right)$$

Dressed generating functional

$$Z_{JT}[\lambda_{\mathcal{O}_a}(\tau)] = \int \mathcal{D}\tau e^{\Lambda L^2 \int du Sch(\tau(u))} Z[\lambda_{\mathcal{O}_a}(\tau(u))]$$

Schwarzian boundary quantum mechanics

$$Sch(\tau(u)) = \frac{\partial_u^3 \tau}{\partial_u \tau} - \frac{3}{2}\left(\frac{\partial_u^2 \tau}{\partial_u \tau}\right)^2$$



Heuristics of $e^{i\ell_s^2 \sum p_i * p_j/4} = \lim_{L\to\infty} N\, AdS_2$

$$S = \int \sqrt{-g}\,(\phi R - \Lambda + \mathcal{L}_m(g,\psi))$$

JT dilaton is a Lagrange multiplier, which forces metric to be flat, hence we can write

$$Z = \int \mathcal{D}X^a \mathcal{D}\psi e^{i\int d^2\sigma \sqrt{-g_f}\left(-\Lambda + \mathcal{L}_m(\psi, g_f)\right)}$$

with

$$g_{f\alpha\beta} = \partial_\alpha X^a \partial_\beta X^b \delta_{ab}$$

gives

$$-\Lambda \int d^2\sigma \sqrt{-g_f} = -\frac{\Lambda}{2}\oint d\tau \epsilon_{ab} X^a \partial_\tau X^b$$

Chern-Simons boundary quantum mechanics

Direct Flat Space Derivation

Minkowski vacuum

$$g_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \qquad \phi = -\frac{\Lambda}{4}\eta_{\alpha\beta}\sigma^\alpha\sigma^\beta = \frac{\Lambda}{2}\sigma^+\sigma^-$$

Appears inhomogeneous. Why do we expect Poincare invariant *S*-matrix in the first place?

Conformal gauge

$$g_{\alpha\beta} = e^{2\Omega}\eta_{\alpha\beta}$$

JT action

$$S_{JT} = \int d\sigma^+ d\sigma^- \left(4\phi\partial_+\partial_-\Omega - \Lambda e^{2\Omega}\right)$$

is invariant under

$$\sigma^\pm \to \sigma^\pm + a^\pm \qquad \phi \to \phi - \frac{\Lambda}{2}(a^+\sigma^- + a^-\sigma^+)$$

which is a symmetry of the vacuum as well

Dynamical coordinates of JT gravity

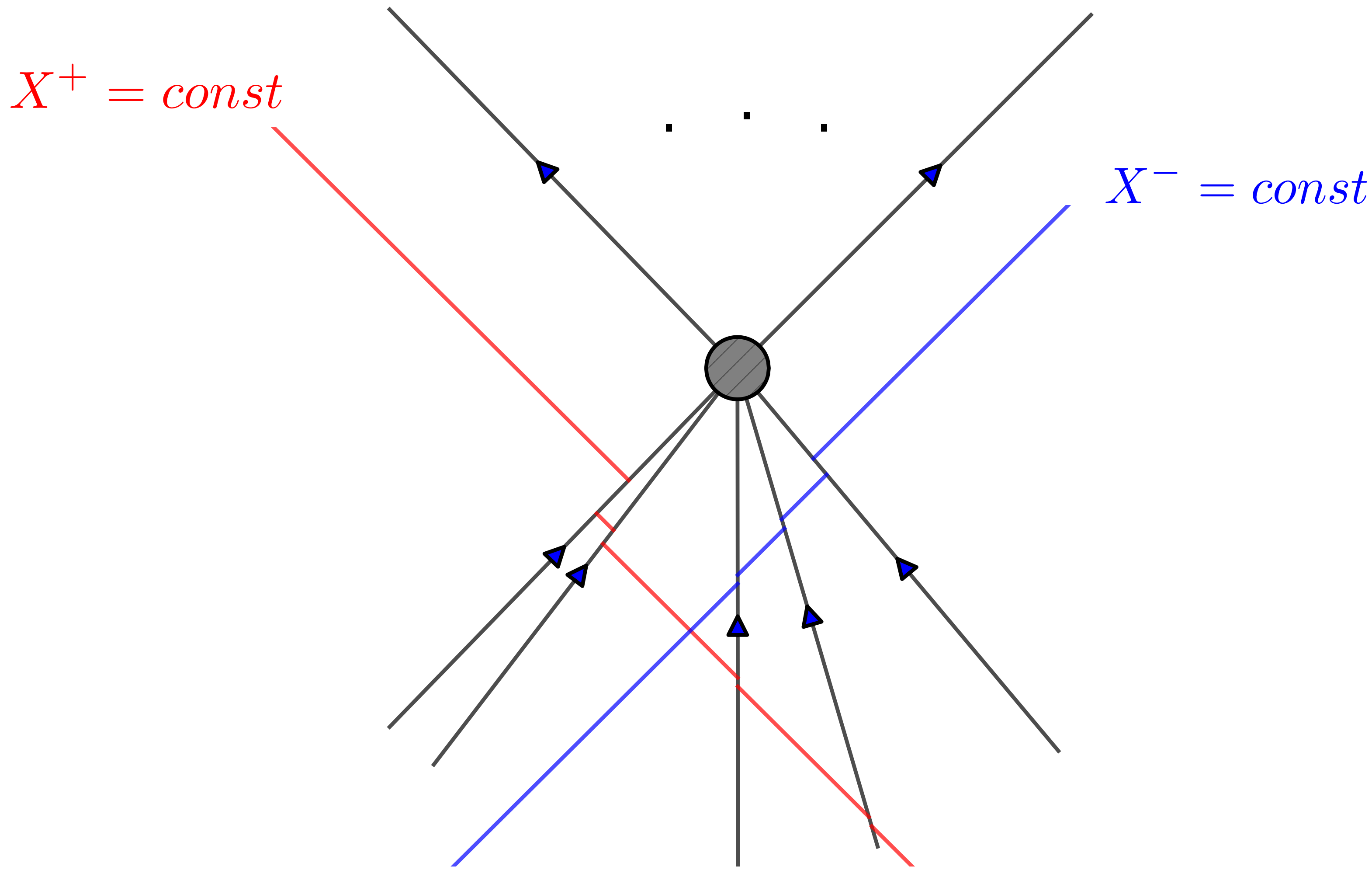
$$X^\pm = 2\frac{\partial_\mp \phi}{\Lambda} \equiv \sigma^\pm + Y^\pm$$ (conformal gauge)

Field equations

$$\partial_+ Y^- = -\frac{T_{++}}{\Lambda}\,,$$
$$\partial_- Y^+ = -\frac{T_{--}}{\Lambda}\,,$$
$$\partial_+ Y^+ = \partial_- Y^- = \frac{T_{+-}}{\Lambda}$$

Matter is unperturbed in σ- coordinates

Let’s focus on the asymptotic *in*-region



$$Y^-(p_i) = \frac{1}{2\Lambda}\left(\mathcal{P}^-_<(p_i) - \mathcal{P}^-_>(p_i)\right)$$

$$Y^+(p_i) = \frac{1}{2\Lambda}\left(\mathcal{P}^+_>(p_i) - \mathcal{P}^+_<(p_i)\right)$$

$\mathcal{P}^\alpha_<(p_i)\ (\mathcal{P}^\alpha_>(p_i))$ are *operators* which add up momenta of all particles with smaller (larger) rapidities

Before gravity

$$\psi = \int_{-\infty}^{\infty} \frac{dp}{\sqrt{2\pi}} \frac{1}{\sqrt{2E}} \left(a_{in}^{\dagger}(p) e^{-ip_\alpha \sigma^\alpha} + h.c. \right)$$

After gravity

$$A_{in}^{\dagger}(p) = a_{in}^{\dagger}(p) e^{ip_\alpha Y^\alpha(p)} = a_{in}^{\dagger}(p) e^{-i\left(p^+ Y^-(p) + p^- Y^+(p)\right)}$$

$$[A_{in}^{\dagger}(p), A_{in}^{\dagger}(p')] = 0$$

Dressed *in*-states

$$|\{p_i\}, in\rangle_{dressed} = \prod_{i=1}^{n_{in}} A_{in}^{\dagger}(p_i)|0\rangle = e^{-\frac{i}{2\Lambda}\sum_{i<j} p_i * p_j} |\{p_i\}, in\rangle$$

Dressed *out*-states

$$|\{q_i\}, out\rangle_{dressed} = \prod_{i=1}^{n_{out}} A_{out}^{\dagger}(q_i)|0\rangle = e^{\frac{i}{2\Lambda}\sum_{i<j} q_i * q_j} |\{q_i\}, out\rangle$$

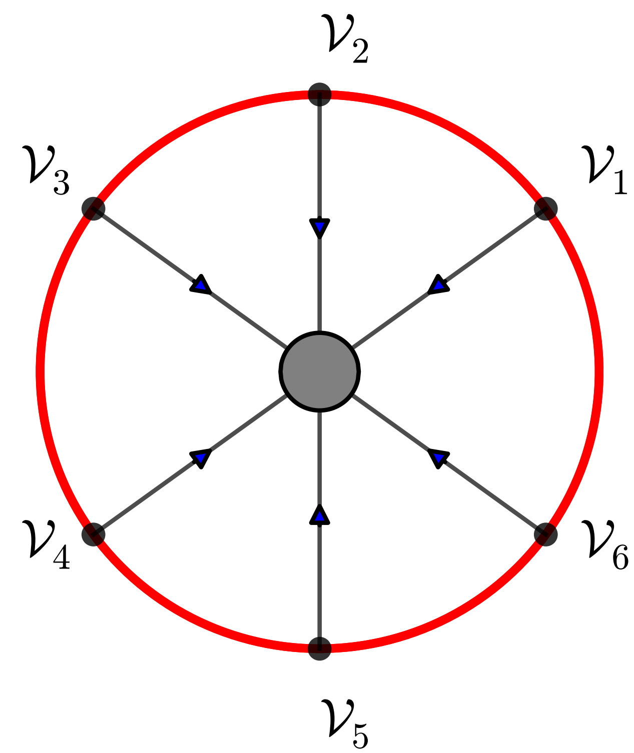
Dressed *S*-matrix

$$\hat{S} \equiv {}_{dressed}\langle out, \{q_i\} | \{p_i\}, in\rangle_{dressed} = e^{-\frac{i}{2\Lambda}\sum_{i<j} p_i * p_j} S$$

Q.E.D.

The same result can be obtained by taking the flat space limit of the Schwarzian dressing

a nice toy: one can explicitly take the flat limit of AdS holography



$$S(\{\beta_i\}) = \lim_{\Delta_i \to \infty} \langle \Delta_1^{1/2} \mathcal{V}_1(\beta_1) \dots \Delta_n^{1/2} \mathcal{V}_n(\beta_n) \rangle$$

holographic LSZ

Schwarzian dressing needs to be modified to reproduce the correct (and unitary) *S*-matrix

$$S_b = \frac{\Lambda L^2}{2}\left(e^{\frac{R-R_0}{L}} + e^{-\frac{R-R_0}{L}} S_{Sch}\right)$$

R is *dynamical*. Corresponds to integrating over total length of the boundary. This action appears non-local (cf Euclidean wormholes?). However, locality is back in the static gauge.

$$S_b = \Lambda L^2 \oint du \left(\cosh\frac{r-R_0}{L} - \frac{1}{2} e^{\frac{R_0-r}{L}} r'^2\right)$$

JT gravity

$$S = \int \sqrt{-g}\,(\phi R - \Lambda + \mathcal{L}_m(g,\psi))$$

is an example of a "degravitating" theory. Vacuum energy does not affect the space-time geometry. Instead, it results in the UV modification of flat space scattering

$$\boxed{\ell_s^{-2} = -\Lambda/2}$$

Summary

✓Lattice QCD provides a rare experimental window into quantum gravity (in 2D)

✓YM glueballs walk, talk and quack as closed strings (at least in 3D)

✓We finally have a promising tractable action for an integrable approximation to the worldsheet theory

$$S_{3D} = \int \sqrt{-g} \left(\phi R + 2\ell_s^{-2} - \frac{1}{2}(\partial X)^2 \right)$$

✓Hopefully, we are approaching the stage when the progress can be made by theorists on their own. However, an input from lattice is very welcome.