F-theory and AdS_3/CFT_2

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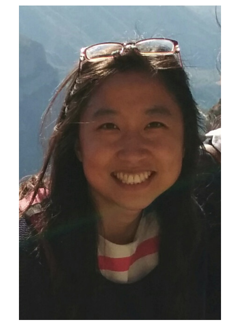


Ben Assel

Related work 1610.03663 with:

and 1612.05640 with:

Craig Lawrie Timo Weigand

1. Motivation

Strings (2d SCFTs) from D3-branes in F-theory;
and the holographic duals: new AdS_3 solutions in F-theory.

Motivation 1: F-theory

F-theory is IIB with varying $\tau = C_0 + ie^{-\phi}$. [Vafa][Morrison, Vafa]

On elliptically fibered Calabi-Yau Y_n: minimal susy in $12 - 2n$ dimensions

$$\mathbb{E}_\tau \; \to \; Y_n \; \to \; B_{n-1} \, .$$

Latest developments: constructions/classifications of new SCFTs:

6d: Classification of 6d (1,0) SCFTs from F-theory on CY three-folds

[Heckman, Morrison, Vafa]

4d: Model Building for $N = 1$ SUSY (see String Pheno 2017); SCFTs?

2d: (0,2) F-theory vacua [SSN, Weigand][Apruzzi, Heckman, Hassler, Melnikov]

Motivation 1: Wrapped D3-branes in F-theory

F-theory is IIB with varying $\tau = C_0 + ie^{-\phi}$. [Vafa][Morrison, Vafa]
On elliptically fibered Calabi-Yau Y_n: minimal susy

$$\mathbb{E}_\tau \ \to \ Y_n \ \to \ B_{n-1} \supset C \, .$$

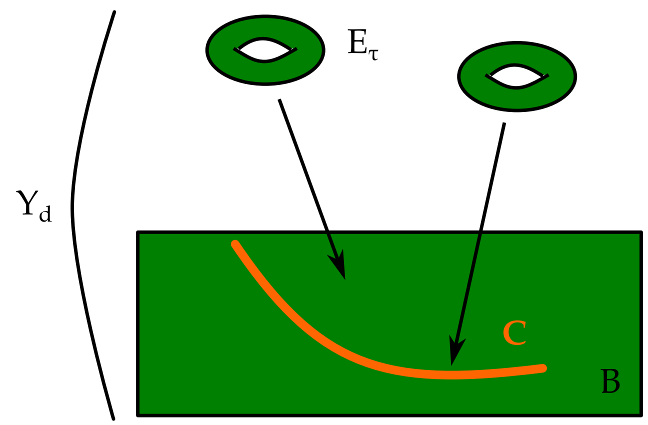
Latest developments: constructions/classifications of new SCFTs:

6d: Classification of 6d (1,0) SCFTs from F-theory on CY three-folds
[Heckman, Morrison, Vafa]
$\to$ tensionless strings are diagnostic for superconformal invariance

4d: Model Building for $N = 1$ SUSY (see String Pheno 2017); SCFTs?

2d: (0,2) F-theory vacua [SSN, Weigand][Apruzzi, Heckman, Hassler, Melnikov]
$\to$ D3s for tadpole cancellation $[C_{D3}] = \frac{1}{24} c_4(CY_5) - \frac{1}{2} G_4 \wedge G_4$

Cartoon of Setup:



Questions:

- # How to characterize wrapped D3-branes in F-theory?
- # What 2d SCFTs do we get?
- # Do these 2d SCFTs have AdS_3 duals in F-theory?

Motivation 2: AdS/CFT

- After 20 years: exploring new AdS/CFT applications/setups still going strong
- Much progress towards classifications of AdS_d solutions in 11d sugra, (massive) IIA, IIB.
- Today: focus on $\text{AdS}_3/\text{CFT}_2$.
  Many known IIB solutions with constant τ:
  D1-D5: $AdS_3 \times S^3 \times T^4/K3$ [Maldacena '97]
  D1-D5-D5′ $AdS_3 \times S^3 \times S^3 \times S^1$ [Giveon, Kutasov, Seiberg]
  D3 [Kim][Benini, Bobev]]

I will discuss new AdS_3 Solutions of IIB supergravity with varying axio-dilaton ("F-theory") dual to 2d (0,4) SCFTs from wrapped D3-branes in F-theory on CY three-folds.

F-theory meets AdS/CFT

- D3-D7-setups in IIB orientifolds
- 4d $\mathcal{N} = 1, 2$ SCFTs from D3s in F-theory at constant τ
  [Fayyazuddin, Spalinski]
- Classification of 6d (1,0) SCFTs
  [Heckman, Morrison, Vafa]
- New $(0, 2)$ and $(0, 4)$ 2d SCFTs from D3s [Haghighat, Murthy, Vafa, Vandoren],[Lawrie, SSN, Weigand]

- AdS/CFT with 7-branes
- $AdS_5 \times S^5/\Gamma$ orbifold, $\Gamma = \mathbb{Z}_n$, $n = 3, 4, 6, 8, 12$
  [Aharony, Fayyazuddin, Maldacena]
- AdS_7 solutions in M/IIA
  [Apruzzi, Gaiotto, Passias, Tomasiello...]
- $AdS_3 \times S^3 \times B$ with varying τ
  [Couzens, Lawrie, Martelli, SSN, Wong]

Plan

1. Motivation ✓
2. 2d SCFTs from F-theory
3. AdS_3 Holography in F-theory
4. Application: Central charges
5. Outlook

2. 2d SCFTs from D3-branes in F-theory

[Martucci][Assel, SSN]

[Haghighat, Murthy, Vafa, Vandoren][Lawrie, SSN, Weigand]

D3-branes in IIB vs. F-theory

D3-brane effective theory in Type IIB string theory: N=4 SYM with
$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} = \tau_1 + i\tau_2$

Perturbative Type IIB:

D3-branes on $\mathbb{R}^{1,1} \times C \subset$CY [Bershadsky, Johanson, Sadov, Vafa][Benini, Bobev]

Topological twist:
$SO(1,3)_L \to SO(1,1) \times U(1)_C$ twisted with $U(1)_R \subset SU(4)_R$
$\Rightarrow$ Sigma-model into Hitchin moduli space

F-theory (varying τ):

\# $N = 4$ SYM with varying coupling τ: undergoes $SL_2\mathbb{Z}$ monodromy $\tau \to \frac{a\tau+b}{c\tau+d}$

\# Under $SL_2\mathbb{Z}$, Q and $\tilde{Q}$ transform under a $U(1)_D$ [Kapustin, Witten]

$$\begin{aligned} Q^{\dot{m}} &\to e^{-\frac{i}{2}\alpha(\gamma)} Q^{\dot{m}} \\ \tilde{Q}^{m} &\to e^{\frac{i}{2}\alpha(\gamma)} \tilde{Q}^{m} \end{aligned} \qquad \text{where} \quad e^{i\alpha(\gamma)} = \frac{c\tau+d}{|c\tau+d|}, \quad \gamma = \begin{pmatrix} a\ b \\ c\ d \end{pmatrix} \in SL_2\mathbb{Z}$$

$$\phi^{\widehat{i}} \to \phi^{\widehat{i}}, \quad \lambda^{\dot{m}}_{\pm} \to e^{\mp\frac{i}{2}\alpha(\gamma)} \lambda^{\dot{m}}_{\pm}, \quad F^{(\pm)}_{\mu\nu} = \sqrt{\tau_2}(F \pm \star F)/2 \to e^{\mp i\alpha(\gamma)} F^{(\pm)}_{\mu\nu}.$$

Cf. bonus symmetry for abelian $N = 4$ SYM [Intrilligator]

$\Rightarrow$ Topologial duality twist:

$U(1)_D$ and $U(1)_C$ twisted with $U(1)_R \subset SU(4)_R$ [Martucci][Assel, SSN]

Fields transform as sections of line bundle $\mathcal{L}_D$, with connection $\mathcal{A}_D = d\tau_1/2\tau_2$.

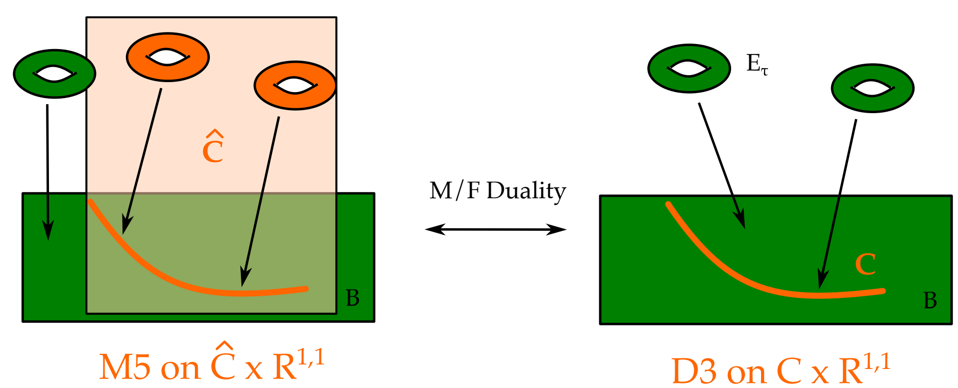
Works explictly for $G = U(1)$, but to generalize: dualize to M5s.

Duality Twist from 6d

{6d $(2,0)$ theory on $\mathbb{E}_\tau \times \mathbb{R}^4$} = {$N = 4$ SYM on $\mathbb{R}^4$ with coupling τ }

Generalization:

{6d $(2,0)$ theory on elliptic fibration} = { 4d $N = 4$ SYM with varying τ}



Standard Topological Twist on Kähler manifold of the 6d (2,0) theory
= Topological duality twist of 4d N=4
Advantage: can be generalized to non-abelian theory [Assel, SSN]

6d $(2,0)$ Theory on Elliptic Surface $\widehat{C}$

Symmetries: $SO(1,5)_L \times Sp(4)_R \subset OSp(6|4)$

Standard topological twist:

$$SO(1,5)_L \to SO(1,1)_L \times SU(2)_\ell \times U(1)_\ell : \qquad \mathbf{4} \to \mathbf{2}_{0,1} \oplus \mathbf{1}_{1,-1} \oplus \mathbf{1}_{-1,-1}$$

$$Sp(4)_R \to SU(2)_R \times U(1)_R : \qquad \mathbf{4} \to \mathbf{2}_1 \oplus \mathbf{2}_{-1}$$

Twist on Kähler surface: $N = (0,4)$, cf. [Maldacena, Strominger, Witten]

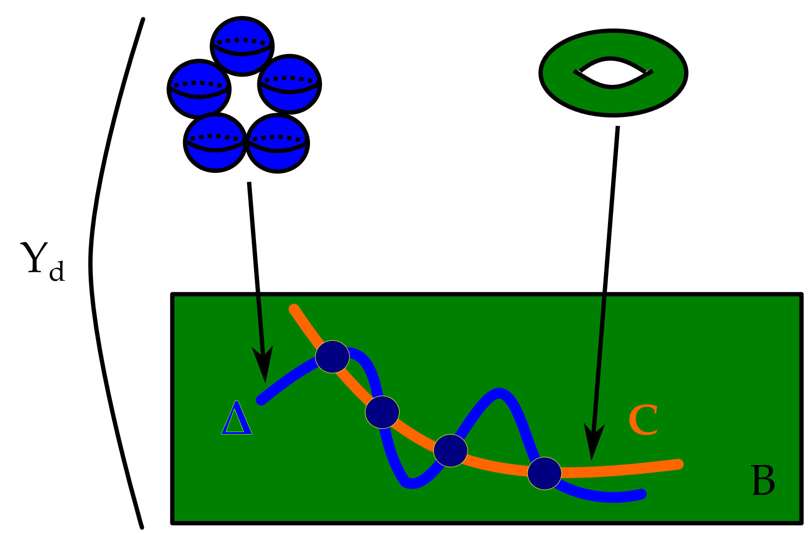
$$T_{U(1)_{\text{twist}}} = T_{U(1)_\ell} + T_{U(1)_R}$$

Specializing to an elliptic Kähler surface $\widehat{C}$, with base C. Fibration:

$$\omega^{\mathbb{E}_\tau} = \frac{d\tau_1}{2\tau_2} = \mathcal{A}_D$$

Thus: $T_{U(1)_\ell} = T_{U(1)_C} + T_{U(1)_D}$ and the top twist for the M5-brane on Kähler surface becomes topological duality twist and can be generalized to non-abelian case. [Assel, SSN]

Including 7-branes



Δ = discriminant locus of elliptic fibration ('7-branes')

Intersections with 7-branes $\Rightarrow$ 3-7 strings.

M5-brane dual:

Extra (1,1)-forms from rational curves in Kodaira fibers above Δ

Localized chiral modes: $d\mathcal{B}_2 = \mathcal{H}_3 = \star\mathcal{H}_3 = \sum \partial_{\bar{z}} b_i d\bar{z} \wedge \omega_i^{(1,1)}$, $\partial_z b_i = 0$

Duality defects, where τ undergoes an $SL_2\mathbb{Z}$ monodromy

[Ganor][Witten][Martucci][Assel, SSN]

Example: Strings in F-theory on CY_3

Description either in terms of duality twisted 4d $N = 4$ SYM on $C \times \mathbb{R}^{1,1}$ [Haghighat, Murthy, Vafa, Vandoren][Lawrie, SSN, Weigand] or dual M5 wrapped:

$$
\begin{array}{rrcl}
SO(1,5)_L \times Sp(4)_R & & \rightarrow & SU(2)_l \times SO(3)_T \times SO(1,1)_L \times U(1)_{\text{twist}} \\
\mathcal{H}_3^+ : & (\overline{\mathbf{10}}, \mathbf{1}) & \rightarrow & (\mathbf{3},\mathbf{1})_{-2,0} \oplus (\mathbf{1},\mathbf{1})_{2,2} \oplus (\mathbf{1},\mathbf{1})_{2,0} \oplus (\mathbf{1},\mathbf{1})_{2,-2} \oplus (\mathbf{2},\mathbf{1})_{0,1} \oplus (\mathbf{2},\mathbf{1})_{0,-1} \\
\Phi^{ij} : & (\mathbf{1}, \mathbf{5}) & \rightarrow & (\mathbf{1},\mathbf{1})_{0,2} \oplus (\mathbf{1},\mathbf{1})_{0,-2} \oplus (\mathbf{1},\mathbf{3})_{0,0} \\
Q^i, \rho^i : & (\overline{\mathbf{4}}, \mathbf{4}) & \rightarrow & (\mathbf{2},\mathbf{2})_{-1,1} \oplus (\mathbf{2},\mathbf{2})_{-1,-1} \oplus (\mathbf{1},\mathbf{2})_{1,2} \\
& & & \oplus (\mathbf{1},\mathbf{2})_{1,0} \oplus (\mathbf{1},\mathbf{2})_{1,0} \oplus (\mathbf{1},\mathbf{2})_{1,-2} \qquad \Rightarrow \text{(0,4) SUSY}
\end{array}
$$

After twist, fields become forms on $\widehat{C}$:
$\mathbf{2}_1$ and $\mathbf{2}_{-1}$ under $SU(2)_l \times U(1)_{\text{twist}}$ correspond to sections of $\Omega^{0,1}(\widehat{C})$ and $\Omega^{1,0}(\widehat{C})$

Spectrum of 2d $(0,4)$ from M5 on $\widehat{C} \subset CY_3$

$\widehat{C}$= $\widehat{C} = \mathbb{E}_\tau \to C$= elliptic fibration restricted to C:

$$y^2 = x^3 + fx + g\,, \quad f \in \Gamma(\mathcal{L}^4)\,,\; g \in \Gamma(\mathcal{L}^6)\,, \qquad \mathcal{L} = K_B^{-1}|_C$$

Multiplicity	$(0,4)$ Multiplet	Complex scalars	R-Weyl	L-Weyl
$h^{0,0}(\widehat{C}) = 1$	Hyper	2	2	–
$h^{0,1}(\widehat{C}) = \frac{1}{2}(C \cdot C - c_1(B_2) \cdot C)$	Fermi	–	–	2
$h^{0,2}(\widehat{C}) = \frac{1}{2}(C \cdot C + c_1(B_2) \cdot C)$	Hyper	2	2	–
$h^{1,1}(\widehat{C}) - 2h^{0,2}(\widehat{C}) - 2 = 8\, c_1(B_2) \cdot C$	half-Fermi	–	–	1

Central Charges

Direct computation from 6d $(2,0)$ on the elliptic surface $\widehat{C} = \mathbb{E}_\tau \to C$ times $\mathbb{R}^{1,1}$:

$$c_R = 3C \cdot C N^2 + 3c_1(B) \cdot C N + 6$$

$$c_L = 3C \cdot C N^2 + 9c_1(B) \cdot C N + 6$$

From spectrum computation for one M5 [MSW][Vafa][Lawrie, SSN, Weigand]

Likewise: can be derived from duality twisted $N = 4$ SYM. Requires incorporating the duality defect modes (3-7 strings) by hand

$$\delta c_L^{\text{defect}} = 8c_1(B) \cdot C$$

From the M5 approach: automatically included from reduction of $\mathcal{B}$.

Clearly: main open question here is: non-abelian generalization.

Strings in 4d and 2d F-theory Compactifications

[Lawrie, SSN, Weigand]

CY_4 Duality twist $N = (0,2)$:

$$
\begin{aligned}
c_R =& 3(g + c_1(B_3) \cdot C + h^0(C, N_{C/B_3})) \\
c_L =& 3(g + h^0(C, N_{C/B_3})) + c_1(B_3) \cdot C + 8c_1(B) \cdot C
\end{aligned}
$$

CY_5 Duality twist $N = (0,2)$: No M5 picture, but M2

$$
\begin{aligned}
c_L &= 3(g + h^0(C, N_{C/B_4}) - 1) + 9c_1(B_4) \cdot C \\
c_R &= 3(g + c_1(B_4) \cdot C + h^0(C, N_{C/B_4}) - 1)
\end{aligned}
$$

Application to 2d $(0,2)$ vacua from CY_5 compactifications of F-theory [SSN, Weigand], [Apruzzi, Hassler, Heckman, Melnikov]. Tadpole cancellation requires D3-branes wrapped on curves in the class (for $G_4 = 0$)

$$
C = \frac{1}{24} c_4(Y_5)|_{B_4}
$$

BPS-equations and Hitchin moduli space

For τ constant, $N = 4$ SYM on $C \times \mathbb{R}^{1,1}$ with Vafa-Witten twist, gives rise to a sigma-model into the Hitchin moduli space, which for the abelian case is just flat connections. [Bershadsky, Johansen, Sadov, Vafa]

In duality-twisted theories the BPS equations along C that we find are

$$\mathcal{F}_{\mathcal{A}} = \frac{1}{2}\Big(\bar{\partial}_{\mathcal{A}}(\sqrt{\tau_2}a) - \partial_{\mathcal{A}}(\sqrt{\tau_2}\bar{a})\Big) = 0$$

where $\mathcal{A} = \mathcal{A}_D$, and the internal components of the gauge field $a, \bar{a}$ are

$$\sqrt{\tau_2}\bar{a} \in \Gamma(\Omega^{0,1}(C, \mathcal{L}_D^{-1}))$$
$$\sqrt{\tau_2}a \in \Gamma(\Omega^{0,0}(C, K_C \otimes \mathcal{L}_D))$$

In particular, for this abelian setup, the theory is a sigma-model into $U(1)_D$-twisted flat connections.
$\rightarrow$ ”duality twisted Hitchin moduli space”

3. AdS_3/CFT_2 in F-theory

[Couzens, Lawrie, Martelli, SSN, Wong]


WORLD FOOTBALL TOURNAMENT
F-theory vs. AdS/CFT

AdS_3 in F-theory

Starting point: IIB with $G_3 \equiv 0$ but $d\tau \neq 0$

$$\mathrm{d}s^2 = \mathrm{d}s^2(\mathrm{AdS}_3) + \mathrm{d}s^2(M_7) \qquad F_5 = (1+\star)\mathrm{vol}(\mathrm{AdS}_3) \wedge F^{(2)}$$

Implies: Killing vector ∂_ψ

$$\mathrm{d}s^2(M_7) = (\mathrm{d}\psi + \rho)^2 + \mathrm{d}s^2(M_6)$$

where M_6 and τ combine into "auxiliary" M_8

$$\mathrm{d}s^2(M_8) = \frac{1}{\tau_2}((\mathrm{d}x + \tau_1 y)^2 + \tau_2^2 \mathrm{d}y^2) + \mathrm{d}s^2(M_6)$$

such that

$$\Box_8 R_8 - \frac{1}{2} R_8^2 + R_{8ij} R_8^{ij} = 0$$

General solution unknown, but we can impose additional supersymmetry.

AdS_3 dual to $(0,4)$ in F-theory

Imposing $(0,4)$ supersymmetry implies:

$$M_8 = S^2 \times Y_3\,, \qquad Y_3 = \text{elliptically fibered Calabi-Yau 3-fold}$$

In summary: the most general F-theory solution dual to $(0,4)$ in 2d is

$$\text{AdS}_3 \times S^3/\Gamma \times (\mathbb{E}_\tau \hookrightarrow Y_3 \to B_2)\,, \qquad F^{(2)} = J_B$$

- # τ = complex structure of $\mathbb{E}_\tau$
  J_B = Kähler form on B_2, discrete $\Gamma \subset SU(2)$
- # Physical type IIB compactification space is $\text{AdS}_3 \times S^3/\Gamma \times B_2$
  B_2= Kähler surface
- # B_2 constrained by the existence of an elliptic fibration with
  Weierstrass model, dP_n, F_n, blowups thereof or Enriques [Grassi][Gross]

Properties of the Solution

$$\text{AdS}_3 \times S^3 \times (\mathbb{E}_\tau \hookrightarrow Y_3 \to B_2)$$

- Supersymmetry: Killing spinors transform as **2** of $SU(2)_r \subset SO(4)_T$ acting on S^3
  $\Rightarrow$ R-symmetry is $SU(2)_r$ of the (0,4) small SCA
- Can allow also for S^3/Γ retaining $(0,4)$ supersymmetry.
- $\Gamma = \mathbb{Z}_M$: additional M KK-monopoles
  $\Rightarrow$ F-theory brane-setup in: $Y_3 \times \text{TN}_M \times \mathbb{R}^{1,1}$.
  $\Rightarrow$ Special case of F-theory on CY 5-folds

4. Central Charges

[Couzens, Lawrie, Martelli, SSN, Wong]

Holographic Central Charges in IIB/F

- Leading order by Brown-Henneaux

$$c_L^{(2)} = c_R^{(2)} = 3\frac{R_{\mathrm{AdS}_3}}{2G_N^{(3)}}$$
$$= 3N^2 \frac{\mathrm{vol}(S^3/\mathbb{Z}_M)\mathrm{vol}(B_2)32\pi^2}{\mathrm{vol}(S^3/\mathbb{Z}_M)^2} = 6N^2 M \mathrm{vol}(B_2)$$

- N = 5-form flux quantum through $S^3/\mathbb{Z}_M \times C$, $C \subset B_2$
- Computation of volume of B_2:
  # Fact: The metric on B_2 is singular (cf. Stringy Cosmic Strings [Greene, Shapere, Vafa, Yau]), as τ of the elliptic fibration can become singular. The metric on Y_3 is smooth.
  # Compute in Y_3: B_2 is a divisor (section of the fibration) and its volume is

$$\mathrm{vol}(B_2) = \frac{1}{2}\int_{Y_3} \omega_0 \wedge \pi^* J_B \wedge \pi^* J_B$$

  where $\omega_0 = (1,1)$ form dual to B_2 and $\pi : Y_3 \to B_2$.

- Algebro-geometrically: $\text{vol}(B_2) = \frac{1}{2}\int_B J_B \wedge J_B = \frac{1}{2} C \cdot_B C$, where C = curve dual (in B) to J_B

$$c_L^{(2)} = c_R^{(2)} = 3N^2 M C \cdot C$$

- Subleading order: CS-coupling of 7-branes:

$$c_L^{(1)} - c_R^{(1)} = 6N c_1(B_2) \cdot C\,.$$

  and level of R-symmetry $k_R = c_R/6$ from gauging of the $SO(4)_T$ isometry of the S^3 ($M = 1$)

$$k_R^{(1)} = \frac{1}{2} N c_1(B) \cdot C$$

- For $M = 1$:

$$c_L^{AdS} = 3N^2 C \cdot C + 9N c_1(B) \cdot C$$
$$c_R^{AdS} = 3N^2 C \cdot C + 3N c_1(B) \cdot C\,.$$

NB: manifestly $c_R \in 6\mathbb{Z}$ using adjunction.

- Comparison to spectrum of $N=4$ SYM on $\mathbb{R}^{1,1}\times C$ with duality twist

  $$c_L^{spec} = 3C\cdot C + 9c_1(B)\cdot C + 6\,, \qquad c_R^{spec} = 3C\cdot C + 3c_1(B)\cdot C + 6\,.$$

  Spectrum computation includes center of mass mode $(c_L, c_R) = (4,6)$, which decouples in the IR:

  $$c_L^{AdS}|_{N=1} = 3C\cdot C + 9c_1(B)\cdot C + O(N^0)$$
  $$c_R^{AdS}|_{N=1} = 3C\cdot C + 3c_1(B)\cdot C\,.$$

  matches spectrum for $N=1$ in first two leading orders. c_R exact result (see also match with self-dual string anomaly in 6d), but c_L gets corrections of $O(1)$.

- What is M? $S^3/\mathbb{Z}_M$ is the near-horizon of TN_M
  $\Rightarrow$ Dual brane-setup is N D3-branes + M KK-monopoles
  $\Rightarrow$ $M=1$ has same near horizon as no KK monopole
  $\Rightarrow$ Central charge for general M: M-theory dual.

Cross-Check 1: Anomalies of Self-dual Strings in 6d

F-theory on CY_3 gives a 6d (1,0) theory. Wrapped D3s are self-dual strings in this 6d theory. (Here: no KK monopoles). Global symmetry transverse rotations and R-symmetry: $SU(2)_R \times SU(2)_L \times SU(2)_I$.
Anomaly polynomial in our setup: [Berman, Harvey][Shimizu, Tachikawa]

$$I_4 = c_2(R)\left[\frac{1}{2}N^2 C\cdot C + \frac{1}{2}Nc_1(B)\cdot C\right] + c_2(L)\left[-\frac{1}{2}N^2 C\cdot C + \frac{1}{2}Nc_1(B)\cdot C\right] + c_2(I)\,[N] - \frac{1}{24}p_1(T)\,[6Nc_1(B)\cdot C]\ ,$$

Coefficients determine $c_L - c_R = 6Nc_1(B)\cdot C$ and levels $k_I = N$ and

$$k_R = \frac{1}{2}N^2 C\cdot C + \frac{1}{2}Nc_1(B)\cdot C$$
$$k_L = -\frac{1}{2}N^2 C\cdot C + \frac{1}{2}Nc_1(B)\cdot C$$

No mixing between $SU(2)$'s and $SU(2)_R$ is IR R-symmetry so that $c_R = 6k_R$ by the standard $N = 4$ SCA.
Would be interesting to include KK monopole to extend comparison.

Cross-check 2: M-theory Dual

M/F-duality:

$$\{\text{M-theory on elliptic CY } Y\} = \{\text{ F-theory on } Y \times S^1\ \}$$
$$= \{\text{IIB on } B \times S^1 \text{ with varying } \tau\}$$

T-duality along the Hopf-fiber of $S^3/\mathbb{Z}_M$ and M-theory uplift of our solution yields

$$\text{M-theory on} \qquad \text{AdS}_3 \times S^2 \times Y_3\,, \qquad G_4 = \text{vol}(S^2) \wedge J_Y$$

where Y_3 has a smooth Ricci-flat Kähler metric. Falls into 'classification' of $\text{AdS}_3 \times S^2$ solutions of M-theory in [Colgain, Wu, Yavartanoo]
Flux is given by Kähler form, so M5s wrap ample divisor (dual to $(1,1)$ form in the Kähler cone)

- N= flux through $\widehat{C} = \mathbb{E}_\tau \to C \to$ D3s on C
- M = flux through base $B_2 \to$ KK monopoles

$\Rightarrow$ M5s wrap ample divisor $P = M B_2 + N\widehat{C}$

Central Charge from M-theory

Two related approaches:

M5-brane anomaly polynomial: $I_4 = \int_P I_8$

[Harvey],[± Freed, Harvey, Minasian, Moore]

Holographic central charge for $AdS_3 \times S^2 \times Y_3$

- Leading order from Brown-Henneaux:

$$(c_L^{11})^{(3)} = (c_R^{11})^{(3)} = 3N^2 MC \cdot C - 3NM^2 c_1(B) \cdot C + M^3(10 - h^{1,1}(B))$$

- Subleading order from CS coupling in 11d

$$\int_{M_{11}} C_3 \wedge X_8 \,, \qquad X_8 = \mathrm{Tr}[\mathcal{R}^4] - \frac{1}{4}(\mathrm{Tr}[\mathcal{R}^2])^2$$

$$\Rightarrow (c_L - c_R)^{(1)} = 6Nc_1(B) \cdot C + M(h^{1,1}(B) - 4)\,.$$

- Level of small $N=4$ SCA for $AdS_3 \times S^2 \times Y_3$

[Kraus, Larsen][Hansen, Kraus]

$$k_r = \frac{N^3}{6} C_{IJK} k^I k^J k^K + \frac{N}{12} \int_{Y_3} J_{Y_3} \wedge c_2(Y_3)$$

where C_{IJK}= triple intersection numbers, $J_Y = \sum k^I \omega_I^{(1,1)}$

$$= \frac{1}{2} M N^2 C \cdot C + \frac{1}{2} N (2 - M^2) c_1(B) \cdot C + \frac{1}{6} \left(M^3 (10 - h^{1,1}(B)) + M(h^{1,1}(B) - 4) \right)$$

Central charges for general N, M:

$$c_L^{11} = 3N^2 MC \cdot C + 3N(4 - M^2) c_1(B) \cdot C + M^3 (10 - h^{1,1}(B)) + 2M(h^{1,1}(B) - 4)$$

$$c_R^{11} = 3N^2 MC \cdot C + 3N(2 - M^2) c_1(B) \cdot C + M^3 (10 - h^{1,1}(B)) + M(h^{1,1}(B) - 4)$$

Agrees for $M = 1$ with F-theory/D3 analysis. Would be interesting to generalize the latter to include the KK-monopole and compare with this result.

Summary

Started exploring new testing ground for AdS/CFT within F-theory:

Holography in F-theory, with focus on AdS_3/CFT_2

Particularly nice setup:
wrapped D3s in F-theory on elliptic CY3, where both sides are computationally accessible

- $N = 4$ SYM with varying τ + duality defects: chiral 2d SCFT
- Holographic dual within F-theory, i.e. AdS solutions with holomorphically varying τ, that undergoes $SL_2\mathbb{Z}$ monodromy
- Checks: dual M-theory solution and comparison of central charges via spectrum and anomalies

Outlook

Future directions:

- Field theory side:
  Duality defects, in particular for non-abelian $N = 4$ SYM
- Holography:
  AdS$_3$ dual to $(0, 2)$ (dual to D3s on CY4/5), $(2, 2)$ susy, three-form fluxes.
- AdS$_6$:
  recent work on 5d SCFTs and IIB solutions with non-trivial τ monodromy [D'Hoker, Gutperle, Uhlemann → Uhlemann's talk]
- New AdS_5 solutions? Generalizing known F-theory at constant coupling ones [Aharony, Fayyazuddin, Maldacena]

Backup Slides: M5 anomaly

Anomaly polynomial for N M5s: anomaly from chiral field $\mathcal{B}_2$, has to be cancelled from the bulk $\Rightarrow$ CS term in 11d action:

$$I_8[N] = N I_8^{\text{free}}[1] + \frac{1}{24}(N^3 - N)p_2(\mathcal{N})$$

$$I_8^{\text{free}}[1] = \frac{1}{48}\left[p_2(\mathcal{N}) - p_2(W) + \frac{1}{4}(p_1(W) - p_1(\mathcal{N}))^2\right]$$

TW = tangent bundle of worldvolume W of the M5s

$\mathcal{N}$ = $SO(5)_R$ normal bundle transverse to W in 11d

Anomaly polynomial I_4 for string from M5 on P: integrate over $P \subset Y$ splitting

$$TW = TP \oplus TW_2\,, \qquad \mathcal{N} = \mathcal{N}_P \oplus \mathcal{N}_{SO(3)_T}$$

The anomaly of the strings is upon integration over P

$$I_4[N] = \int_P I_8[N] = N I_4[1] + \frac{1}{24}(N^3 - N) P^3 p_1(\mathcal{N}_3)$$

$$I_4[1] = \int_P I_8[1] = \frac{1}{48}\left[2P^3 p_1(\mathcal{N}_3) + c_2(Y_3)\cdot P(p_1(W_2) + p_1(\mathcal{N}_3))\right]$$

From this we can be read off:

- Gravitational Anomaly:

$$I_4 \supset \frac{c_L - c_R}{24} p_1(W_2) \qquad \Rightarrow \qquad c_L - c_R = \frac{1}{2} N c_2(Y_3)\cdot P$$

- Level k_3 of the $SO(3)_T$

$$I_4 \supset \frac{k_3 p_1(\mathcal{N}_3)}{4} \qquad \Rightarrow \qquad k_3 = \frac{1}{6} N^3 P^3 + \frac{1}{12} N c_2(Y_3)\cdot P$$

Imposing that $[P] = M[B] + N[\widehat{C}]$ we obtain

$$c_L - c_R = 6N c_1(B) \cdot C + M(h^{1,1}(B) - 4)$$

$$k_3 = \frac{1}{2} M N^2 C \cdot C + \frac{1}{2} N(2 - M^2) c_1(B) \cdot C + \frac{1}{6} \left(M^3 (10 - h^{1,1}(B)) + M(h^{1,1}(B) - 4) \right)$$

$\Rightarrow$ Agrees for $M = 1$ at leading and subleading order in N with F-theory.

A note on ampleness: ●●●

A bundle over X is very ample if the global sections define embedding of X to projective space. Ample if some tensor power of the bundle does so (‘D ample if dual (1,1) form is inside the Kähler cone’).

- MSW: If P is very ample, then $SO(3)_T$ is R-symmetry and $c_R = 6k_3$.
- LSMSW: From AdS dual we see this is the case more generally for any ample divisor
- Implies: we cannot have $M = 0$ as this would not result in an ample divisor. I.e. there is no AdS dual to M5-brane on $\widehat{C}$ only.