AdS diagrams and holographic global conformal blocks

Gong Show, Strings 2019

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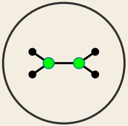
Caltech

July 9, 2019

SP [1901.01267], Jepsen & SP [1906.08405], and [work in progress].

Two objects in AdS/CFT

Bulk: Feynman diagrams

e.g.  $= ?$

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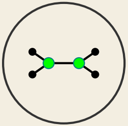
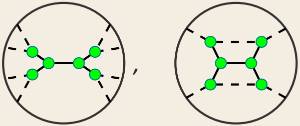
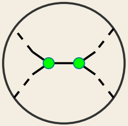
e.g.  = ?

- integrate over AdS
- fixed point

Subdiagrams:



Limiting behavior:



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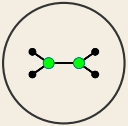
e.g.  = ? ● integrate over AdS • fixed point

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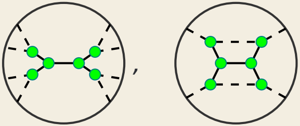

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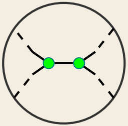

Boundary: Conformal blocks

What holographic object computes the *n*-**point** global conformal block?

Two objects in AdS/CFT

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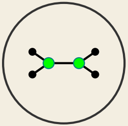
e.g.  = ? ● integrate over AdS
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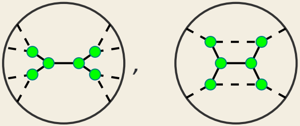

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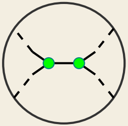

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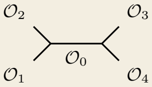
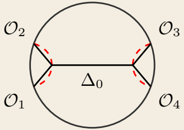
What holographic object computes the *n*-**point** global conformal block?

$n = 4:$ 



$$= \frac{[\text{diagram}]}{B(\Delta_{01,2}, \Delta_{02,1})B(\Delta_{03,4}, \Delta_{04,3})/4}$$

Hijano, Kraus, Perlmutter, Snively [1508.00501]

Quick summary

- A class of such Feynman diagrams closely related with conformal blocks.
- Set up a systematic procedure to study them – possible generalization to general multi-point results
- Today: Illustrate with the help of simple examples.

AdS origins of global conformal blocks

Individual Witten diagrams admit conformal block decomposition.

Every conformal block $\longrightarrow$ canonical tree-level Witten diagram

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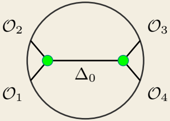
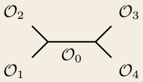
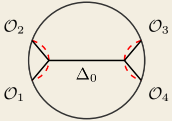
- Four point block:



$= C_{\Delta_1\Delta_2\Delta_0} C_{\Delta_3\Delta_4\Delta_0} \Bigg($



$\Bigg) + \text{double trace}$



$$= C_{\Delta_1\Delta_2\Delta_0} C_{\Delta_3\Delta_4\Delta_0} \frac{[\text{diagram}]}{B(\Delta_{01,2}, \Delta_{02,1}) B(\Delta_{03,4}, \Delta_{04,3})/4} + \text{double trace}$$

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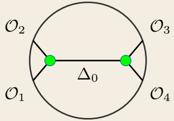
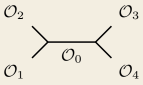
Individual Witten diagrams admit conformal block decomposition.

Every conformal block ⟶ canonical tree-level Witten diagram

- Four point block:

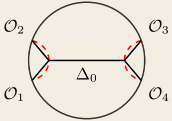


$$= C_{\Delta_1\Delta_2\Delta_0} C_{\Delta_3\Delta_4\Delta_0} \left(\right.$$



$$\left. \right) + \text{double trace}$$

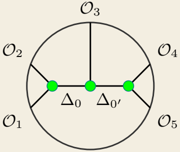
$$= C_{\Delta_1\Delta_2\Delta_0} C_{\Delta_3\Delta_4\Delta_0}$$



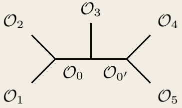
$$\overline{B(\Delta_{01,2}, \Delta_{02,1}) B(\Delta_{03,4}, \Delta_{04,3})/4} + \text{double trace}$$

AdS origins –continued–

- Five-point block:

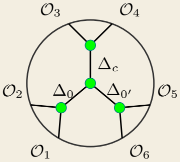


$$= C_{\Delta_1\Delta_2\Delta_0} C_{\Delta_0\Delta_3\Delta_{0'}} C_{\Delta_4\Delta_5\Delta_{0'}} \Bigg($$

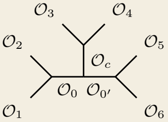


$$\Bigg) + \cdots$$

- Six-point block in the OPE channel:



$$= C_{\Delta_1\Delta_2\Delta_0} C_{\Delta_3\Delta_4\Delta_c} C_{\Delta_{0'}\Delta_5\Delta_6} C_{\Delta_0\Delta_c\Delta_{0'}} \Bigg($$



$$\Bigg) + \cdots$$

AdS origins –continued–

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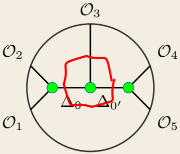
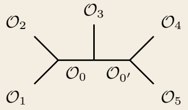
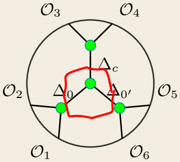


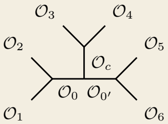
$$= C_{\Delta_1\Delta_2\Delta_0} C_{\Delta_0\Delta_3\Delta_{0'}} C_{\Delta_4\Delta_5\Delta_{0'}} \left(\quad \right) + \cdots$$



- Six-point block in the OPE channel:



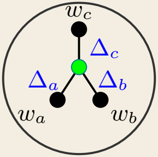
$$= C_{\Delta_1\Delta_2\Delta_0} C_{\Delta_3\Delta_4\Delta_c} C_{\Delta_{0'}\Delta_5\Delta_6} C_{\Delta_0\Delta_c\Delta_{0'}} \left(\quad \right) + \cdots$$



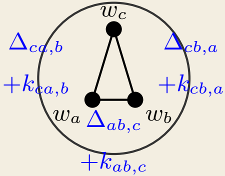
Three-point bulk-to-bulk scattering

*Identity involving three bulk-to-bulk propagators**

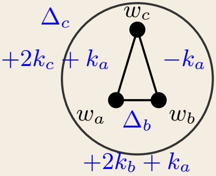
* up to a simple modification unimportant for this talk – see [1906.08405]



$$= C_{\Delta_a\Delta_b\Delta_c} \sum_{k_a,k_b,k_c=0}^{\infty} c^{\Delta_a;\Delta_b;\Delta_c}_{k_a;k_b;k_c}$$



$$+ \left(\sum_{k_a,k_b,k_c=0}^{\infty} d^{\Delta_a;\Delta_b;\Delta_c}_{k_a;k_b;k_c} \right.$$



$$\left. + (a \longleftrightarrow b) + (a \longleftrightarrow c) \right)$$

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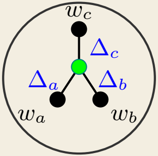

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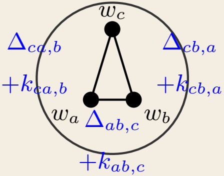

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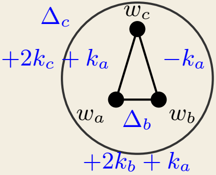


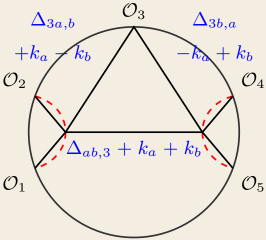
$$\left. + (a \longleftrightarrow b) + (a \longleftrightarrow c) \right)$$

Holographic duals

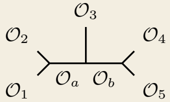
*Five-point block**

Sum over "five-point geodesic diagrams":

$$\sum_{k_a,k_b=0}^{\infty} c_{0;k_a;k_b}^{\Delta_3;\Delta_a;\Delta_b}$$



$$= \frac{1}{4} B(\Delta_{a1,2}, \Delta_{a2,1}) B(\Delta_{b4,5}, \Delta_{b5,4}) \left(\quad \right)$$



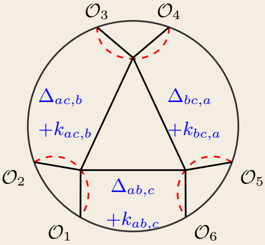
An alternate series representation: Rosenhaus [1810.03244]

Holographic duals

*Six-point block in the "OPE channel" **

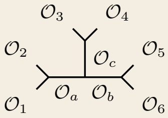
Sum over "six-point geodesic diagrams":

$$\sum_{k_a,k_b,k_c=0}^{\infty} c^{\Delta_a;\Delta_b;\Delta_c}_{k_a;k_b;k_c}$$



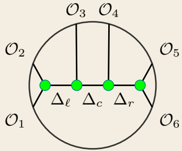
$$= \frac{1}{8} B(\Delta_{a1,2}, \Delta_{a2,1})\, B(\Delta_{c3,4}, \Delta_{c4,3})$$

$$\times\, B(\Delta_{b5,6}, \Delta_{b6,5}) \left(\quad \right)$$

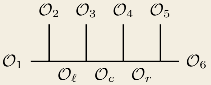


The other six-point conformal block

- Six-point block in the comb channel:



$$= C_{\Delta_1\Delta_2\Delta_\ell} C_{\Delta_\ell\Delta_3\Delta_c} C_{\Delta_c\Delta_4\Delta_r} C_{\Delta_r\Delta_5\Delta_6}$$



$$\left(\,\cdot\,\right) + \cdots$$

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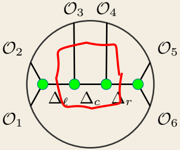
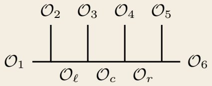
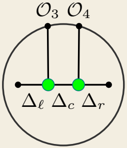


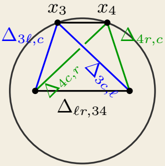
$$= C_{\Delta_1\Delta_2\Delta_\ell} C_{\Delta_\ell\Delta_3\Delta_c} C_{\Delta_c\Delta_4\Delta_r} C_{\Delta_r\Delta_5\Delta_6} \left(\quad \right) + \cdots$$



Schematics of 4-point scattering & six-point block

- 

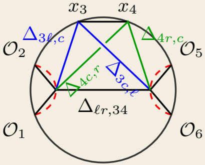
$$\sim C_{\Delta_3 \Delta_\ell \Delta_c} C_{\Delta_4 \Delta_r \Delta_c} \sum_{k_1,k_2,k_3,k_4=0}^{\infty} c^{\Delta_3;\Delta_\ell;\Delta_c;\Delta_r;\Delta_4}_{k_1;k_2;k_3;k_4}$$



$$+ \ldots$$

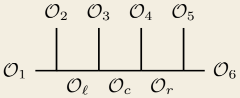
- Block:

$$\sum_{k_1,k_2,k_3,k_4=0}^{\infty} c^{\Delta_3;\Delta_\ell;\Delta_c;\Delta_r;\Delta_4}_{k_1;k_2;k_3;k_4}$$



$$\sim \frac{1}{4} B(\Delta_{\ell 1,2}, \Delta_{\ell 2,1}) B(\Delta_{r5,6}, \Delta_{r6,5})$$

$$\times \Bigg($$



$$\Bigg)$$

Summary

- Systematic procedure to obtain exact expressions for a class of **bulk Feynman diagrams** — reduce to Witten diagrams; manifestly dictate the structure of **holographic duals** of higher-point conformal blocks.
- Application: Obvious generalization to n-point conformal blocks in the comb channel in d dimensions ([Rosenhaus]: alternate series representation for $d = 1, 2$). Other channels as well.
- Desirable: More intuitive, geometric reformulation.
- Relation to (various semi-classical limits of) Virasoro blocks in AdS_3/CFT_2 …[Hijano Kraus Perlmutter Snively]′ [Banerjee Datta Sinha] …[Alkalaev Pavlov] …