



AdS diagrams and holographic global conformal blocks

Gong Show, Strings 2019

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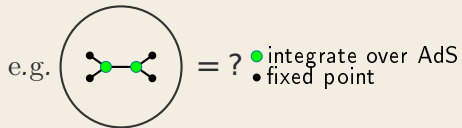
Caltech

July 9, 2019

SP [1901.01267], Jepsen & SP [1906.08405], and [work in progress].

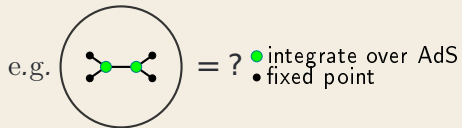
Two objects in AdS/CFT

Bulk: Feynman diagrams

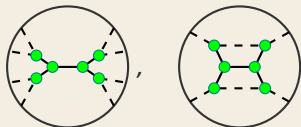


Two objects in AdS/CFT

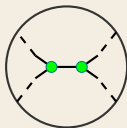
Bulk: Feynman diagrams



Subdiagrams:

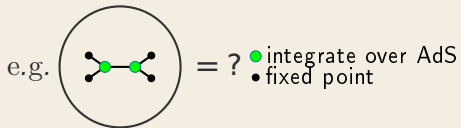


Limiting behavior:

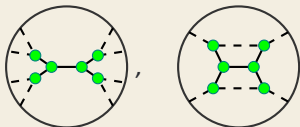


Two objects in AdS/CFT

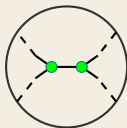
Bulk: Feynman diagrams



Subdiagrams:



Limiting behavior:

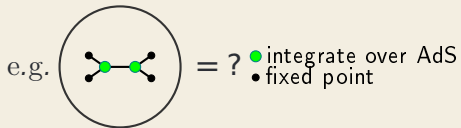


Boundary: Conformal blocks

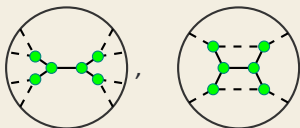
What holographic object computes the n -point global conformal block?

Two objects in AdS/CFT

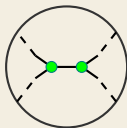
Bulk: Feynman diagrams



Subdiagrams:



Limiting behavior:



Boundary: Conformal blocks

What holographic object computes the n -point global conformal block?

$$\begin{aligned}
 n = 4 : & \quad \begin{array}{c} \mathcal{O}_2 \qquad \qquad \mathcal{O}_3 \\ \diagdown \quad \diagup \\ \mathcal{O}_0 \\ \diagup \quad \diagdown \\ \mathcal{O}_1 \qquad \qquad \mathcal{O}_4 \end{array} \\
 & \quad \begin{array}{c} \mathcal{O}_2 \qquad \qquad \mathcal{O}_3 \\ \diagdown \quad \diagup \\ \Delta_0 \\ \diagup \quad \diagdown \\ \mathcal{O}_1 \qquad \qquad \mathcal{O}_4 \end{array} \\
 & = \frac{B(\Delta_{01,2}, \Delta_{02,1})B(\Delta_{03,4}, \Delta_{04,3})/4}{}
 \end{aligned}$$

Hijano, Kraus, Perlmutter, Snively
[1508.00501]

Quick summary

- A class of such Feynman diagrams closely related with conformal blocks.
- Set up a systematic procedure to study them – possible generalization to general multi-point results
- Today: Illustrate with the help of simple examples.

AdS origins of global conformal blocks

Individual Witten diagrams admit conformal block decomposition.

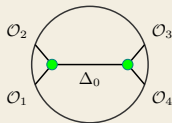
Every conformal block \rightarrow canonical tree-level Witten diagram

AdS origins of global conformal blocks

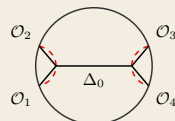
Individual Witten diagrams admit conformal block decomposition.

Every conformal block \rightarrow canonical tree-level Witten diagram

- Four point block:



$$= C_{\Delta_1 \Delta_2 \Delta_0} C_{\Delta_3 \Delta_4 \Delta_0} \left(\begin{array}{c} \mathcal{O}_2 \qquad \mathcal{O}_3 \\ \diagdown \quad \diagup \\ \mathcal{O}_0 \\ \diagup \quad \diagdown \\ \mathcal{O}_1 \qquad \mathcal{O}_4 \end{array} \right) + \text{double trace}$$



$$= C_{\Delta_1 \Delta_2 \Delta_0} C_{\Delta_3 \Delta_4 \Delta_0} \frac{1}{B(\Delta_{01,2}, \Delta_{02,1}) B(\Delta_{03,4}, \Delta_{04,3}) / 4} + \text{double trace}$$

AdS origins of global conformal blocks

Individual Witten diagrams admit conformal block decomposition.

Every conformal block \rightarrow canonical tree-level Witten diagram

- Four point block:

$$\begin{aligned}
 & \text{Diagram 1: A circle with four external legs labeled } \mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4. \text{ Two internal vertices are marked with green dots and connected by a horizontal line labeled } \Delta_0. \\
 & = C_{\Delta_1 \Delta_2 \Delta_0} C_{\Delta_3 \Delta_4 \Delta_0} \left(\begin{array}{c} \mathcal{O}_2 \qquad \mathcal{O}_3 \\ \diagdown \quad \diagup \\ \mathcal{O}_1 \qquad \mathcal{O}_0 \\ \diagup \quad \diagdown \\ \mathcal{O}_4 \end{array} \right) + \text{double trace} \\
 & \\
 & \text{Diagram 2: A circle with four external legs labeled } \mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4. \text{ Two internal vertices are marked with red dashed lines and connected by a horizontal line labeled } \Delta_0. \\
 & = C_{\Delta_1 \Delta_2 \Delta_0} C_{\Delta_3 \Delta_4 \Delta_0} \frac{B(\Delta_{01,2}, \Delta_{02,1}) B(\Delta_{03,4}, \Delta_{04,3})}{4} + \text{double trace}
 \end{aligned}$$

AdS origins –continued–

- Five-point block:

A circular diagram representing a five-point block. The boundary has five external operators labeled \mathcal{O}_1 through \mathcal{O}_5 . Inside the circle, there are two internal operators, Δ_0 and $\Delta_{0'}$, each marked with a green dot. Lines connect Δ_0 to \mathcal{O}_1 and \mathcal{O}_2 , and $\Delta_{0'}$ to \mathcal{O}_4 and \mathcal{O}_5 . A vertical line connects Δ_0 and $\Delta_{0'}$, and another vertical line connects Δ_0 to \mathcal{O}_3 .

$$= C_{\Delta_1 \Delta_2 \Delta_0} C_{\Delta_0 \Delta_3 \Delta_{0'}} C_{\Delta_4 \Delta_5 \Delta_{0'}} \left(\begin{array}{c} \mathcal{O}_2 \quad \mathcal{O}_3 \quad \mathcal{O}_4 \\ \diagdown \quad | \quad \diagup \\ \mathcal{O}_0 \quad \mathcal{O}_{0'} \\ \diagup \quad | \quad \diagdown \\ \mathcal{O}_1 \quad \mathcal{O}_5 \end{array} \right) + \dots$$

- Six-point block in the OPE channel:

A circular diagram representing a six-point block in the OPE channel. The boundary has six external operators labeled \mathcal{O}_1 through \mathcal{O}_6 . Inside the circle, there are three internal operators, Δ_0 , $\Delta_{0'}$, and Δ_c , each marked with a green dot. Lines connect Δ_0 to \mathcal{O}_1 and \mathcal{O}_2 , $\Delta_{0'}$ to \mathcal{O}_4 and \mathcal{O}_5 , and Δ_c to \mathcal{O}_3 and \mathcal{O}_6 . Lines also connect Δ_0 to $\Delta_{0'}$ and $\Delta_{0'}$ to Δ_c .

$$= C_{\Delta_1 \Delta_2 \Delta_0} C_{\Delta_3 \Delta_4 \Delta_c} C_{\Delta_{0'} \Delta_5 \Delta_6} C_{\Delta_0 \Delta_c \Delta_{0'}} \left(\begin{array}{c} \mathcal{O}_3 \quad \mathcal{O}_4 \\ \diagdown \quad \diagup \\ \mathcal{O}_2 \quad \mathcal{O}_5 \\ \diagup \quad \diagdown \\ \mathcal{O}_c \\ \diagdown \quad | \quad \diagup \\ \mathcal{O}_0 \quad \mathcal{O}_{0'} \\ \diagup \quad | \quad \diagdown \\ \mathcal{O}_1 \quad \mathcal{O}_6 \end{array} \right) + \dots$$

AdS origins –continued–

- Five-point block:

$$= C_{\Delta_1 \Delta_2 \Delta_0} C_{\Delta_0 \Delta_3 \Delta_0'} C_{\Delta_4 \Delta_5 \Delta_0'} \left(\begin{array}{c} O_2 \quad O_3 \quad O_4 \\ \diagdown \quad | \quad \diagup \\ O_0 \\ \diagup \quad | \quad \diagdown \\ O_1 \quad O_0' \quad O_5 \end{array} \right) + \dots$$

- Six-point block in the OPE channel:

$$= C_{\Delta_1 \Delta_2 \Delta_0} C_{\Delta_3 \Delta_4 \Delta_c} C_{\Delta_0' \Delta_5 \Delta_c} C_{\Delta_0 \Delta_c \Delta_0'} \left(\begin{array}{c} O_3 \quad O_4 \\ \diagdown \quad \diagup \\ O_c \\ \diagup \quad \diagdown \\ O_2 \quad O_5 \\ \diagdown \quad | \quad \diagup \\ O_1 \quad O_0 \quad O_0' \quad O_6 \end{array} \right) + \dots$$

Three-point bulk-to-bulk scattering

*Identity involving three bulk-to-bulk propagators**

* up to a simple modification unimportant for this talk - see [1906.08405]

$$\begin{aligned}
 & \text{Diagram} = C_{\Delta_a \Delta_b \Delta_c} \sum_{k_a, k_b, k_c=0}^{\infty} C_{k_a; k_b; k_c}^{\Delta_a; \Delta_b; \Delta_c} \\
 & + \left(\sum_{k_a, k_b, k_c=0}^{\infty} d_{k_a; k_b; k_c}^{\Delta_a; \Delta_b; \Delta_c} \right. \\
 & \quad \left. + 2k_c + k_a \quad -k_a \quad + 2k_b + k_a \quad + (a \leftrightarrow b) + (a \leftrightarrow c) \right)
 \end{aligned}$$

Three-point bulk-to-bulk scattering

*Identity involving three bulk-to-bulk propagators**

* up to a simple modification unimportant for this talk - see [1906.08405]

$$\begin{aligned}
 & \left(\text{Diagram with } w_c, \Delta_c, \Delta_a, \Delta_b, w_a, w_b \right) = C_{\Delta_a \Delta_b \Delta_c} \sum_{k_a, k_b, k_c=0}^{\infty} C_{k_a; k_b; k_c}^{\Delta_a; \Delta_b; \Delta_c} \\
 & \left(\text{Diagram with } w_c, \Delta_c, \Delta_a, \Delta_b, w_a, w_b, +k_{ca,b}, +k_{cb,a}, +k_{cb,a}, +k_{ab,c} \right) \\
 & + \left(\sum_{k_a, k_b, k_c=0}^{\infty} d_{k_a; k_b; k_c}^{\Delta_a; \Delta_b; \Delta_c} \left(\text{Diagram with } w_c, \Delta_c, +2k_c, +k_a, -k_a, w_a, \Delta_b, w_b, +2k_b + k_a \right) + (a \leftrightarrow b) + (a \leftrightarrow c) \right)
 \end{aligned}$$

Holographic duals

Five-point block*

Sum over “five-point geodesic diagrams”:

$$\sum_{k_a, k_b=0}^{\infty} c_{0; k_a; k_b}^{\Delta_3; \Delta_a; \Delta_b}$$

$$= \frac{1}{4} B(\Delta_{a1,2}, \Delta_{a2,1}) B(\Delta_{b4,5}, \Delta_{b5,4}) \left(\begin{array}{c} \mathcal{O}_3 \\ \mathcal{O}_2 \quad | \quad \mathcal{O}_4 \\ \mathcal{O}_1 \quad \mathcal{O}_a \quad \mathcal{O}_b \quad \mathcal{O}_5 \end{array} \right)$$

An alternate series representation: [Rosenhaus \[1810.03244\]](#)

Holographic duals

Six-point block in the “OPE channel” *

Sum over “six-point geodesic diagrams”:

$$\sum_{k_a, k_b, k_c=0}^{\infty} c_{k_a; k_b; k_c}^{\Delta_a; \Delta_b; \Delta_c}$$

$$= \frac{1}{8} B(\Delta_{a1,2}, \Delta_{a2,1}) B(\Delta_{c3,4}, \Delta_{c4,3})$$

$$\times B(\Delta_{b5,6}, \Delta_{b6,5})$$

The other six-point conformal block

- Six-point block in the comb channel:

$$\begin{array}{c} \mathcal{O}_3 \quad \mathcal{O}_4 \\ \circlearrowleft \\ \mathcal{O}_2 \quad \mathcal{O}_5 \\ \mathcal{O}_1 \quad \mathcal{O}_6 \end{array} = C_{\Delta_1 \Delta_2 \Delta_l} C_{\Delta_l \Delta_3 \Delta_c} C_{\Delta_c \Delta_4 \Delta_r} C_{\Delta_r \Delta_5 \Delta_6} \left(\begin{array}{c} \mathcal{O}_2 \quad \mathcal{O}_3 \quad \mathcal{O}_4 \quad \mathcal{O}_5 \\ | \quad | \quad | \quad | \\ \mathcal{O}_1 \text{---} \mathcal{O}_\ell \quad \mathcal{O}_c \quad \mathcal{O}_r \text{---} \mathcal{O}_6 \end{array} \right) + \dots$$

The other six-point conformal block

- Six-point block in the comb channel:

$$\begin{aligned}
 & \text{Diagram of a circle with six external operators } \mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5, \mathcal{O}_6 \text{ and a red comb structure with three green dots and triangles } \Delta_l, \Delta_c, \Delta_r. \\
 & = C_{\Delta_1 \Delta_2 \Delta_3} C_{\Delta_l \Delta_3 \Delta_c} C_{\Delta_c \Delta_4 \Delta_r} C_{\Delta_r \Delta_5 \Delta_6} \left(\begin{array}{c} \mathcal{O}_2 \quad \mathcal{O}_3 \quad \mathcal{O}_4 \quad \mathcal{O}_5 \\ | \quad | \quad | \quad | \\ \mathcal{O}_1 \text{---} \mathcal{O}_l \quad \mathcal{O}_c \quad \mathcal{O}_r \quad \mathcal{O}_6 \end{array} \right) + \dots
 \end{aligned}$$

Schematics of 4-point scattering & six-point block

- $$\sim C_{\Delta_3 \Delta_\ell \Delta_c} C_{\Delta_4 \Delta_r \Delta_c} \sum_{k_1, k_2, k_3, k_4=0}^{\infty} C_{k_1; k_2; k_3; k_4}^{\Delta_3; \Delta_\ell; \Delta_c; \Delta_r; \Delta_4} + \dots$$

- Block:

$$\sum_{k_1, k_2, k_3, k_4=0}^{\infty} C_{k_1; k_2; k_3; k_4}^{\Delta_3; \Delta_\ell; \Delta_c; \Delta_r; \Delta_4} \times \left(\text{Diagram} \right) \sim \frac{1}{4} B(\Delta_{\ell 1,2}, \Delta_{\ell 2,1}) B(\Delta_{r 5,6}, \Delta_{r 6,5})$$

The diagram in the middle is a circle with six vertices labeled O_1 through O_6 . Vertices O_3 and O_4 are at the top, O_1 and O_6 are at the bottom, O_2 is on the left, and O_5 is on the right. Inside the circle, there are vertices x_3 and x_4 at the top. Blue lines connect x_3 to O_2 and O_5 , labeled $\Delta_{3\ell,c}$ and $\Delta_{3c,c}$. Green lines connect x_4 to O_2 and O_5 , labeled $\Delta_{4r,c}$ and $\Delta_{4c,c}$. A horizontal line connects O_2 and O_5 , labeled $\Delta_{\ell r,34}$. Red dashed lines connect O_1 to O_2 and O_5 to O_6 .

$$\times \left(\begin{array}{cccc} O_2 & O_3 & O_4 & O_5 \\ | & | & | & | \\ O_1 & \text{---} & \text{---} & \text{---} & O_6 \\ O_\ell & O_c & O_r & \end{array} \right)$$

Summary

- Systematic procedure to obtain exact expressions for a class of **bulk Feynman diagrams** — reduce to Witten diagrams; manifestly dictate the structure of **holographic duals** of higher-point conformal blocks.
- Application: Obvious generalization to n -point conformal blocks in the comb channel in d dimensions ([Rosenhaus]: **alternate series representation for $d = 1, 2$**). Other channels as well.
- Desirable: More intuitive, geometric reformulation.
- Relation to (various semi-classical limits of) Virasoro blocks in AdS_3/CFT_2 ... [Hijano Kraus Perlmutter Snively]' [Banerjee Datta Sinha] ... [Alkalaev Pavlov] ...