Fractional quantum Hall effect and duality

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Plan

- Fractional quantum Hall effect
- Halperin-Lee-Read (HLR) theory
- Problem of particle-hole symmetry
- Dirac composite fermion theory
- Consequences, relationship to field-theoretic duality

References

DTS, arXiv:1502.03446
Wang, Senthil, 1505.05141
Metlitski, Vishwanath, 1505.05142
Geraedts et al. 1508.04140
Karch, Tong, 1606.01893
Seiberg, Senthil, Wang, Witten, 1606.01989
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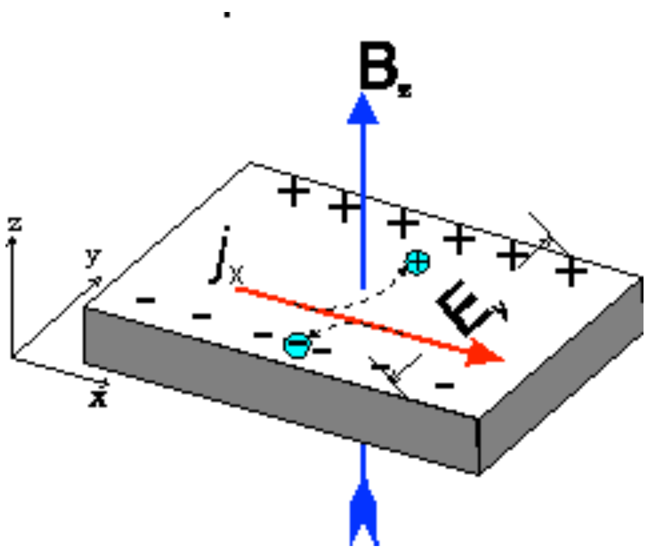
Fractional QHE



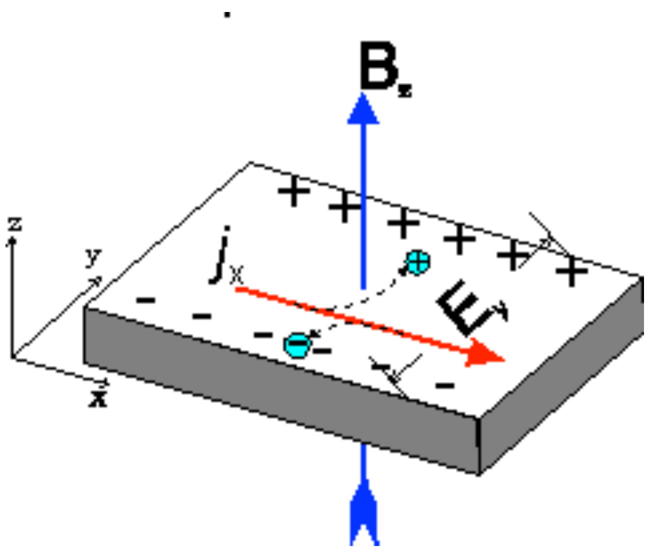
Landau levels of 2D electron in B field

n=3

n=2

n=1

Fractional QHE



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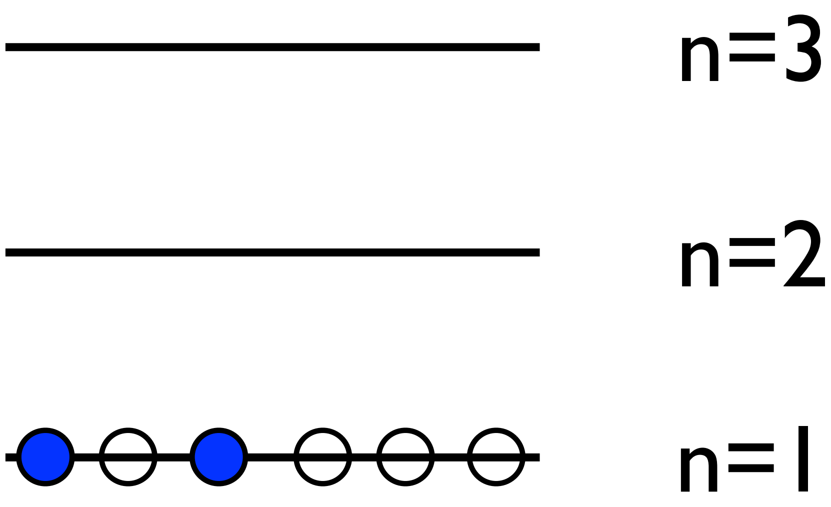

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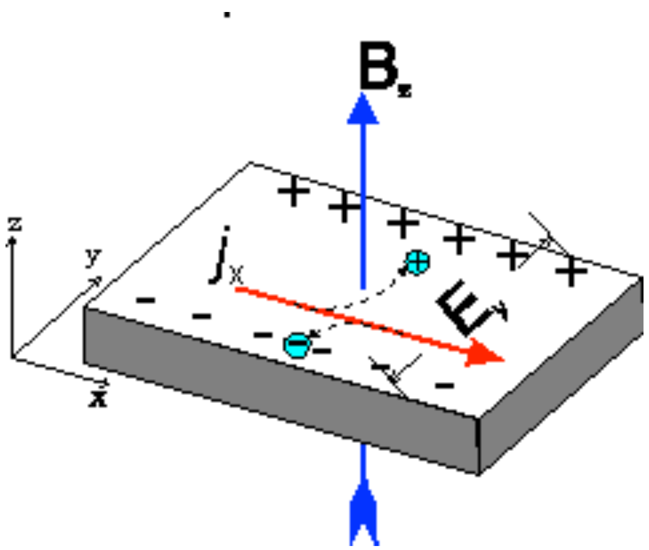

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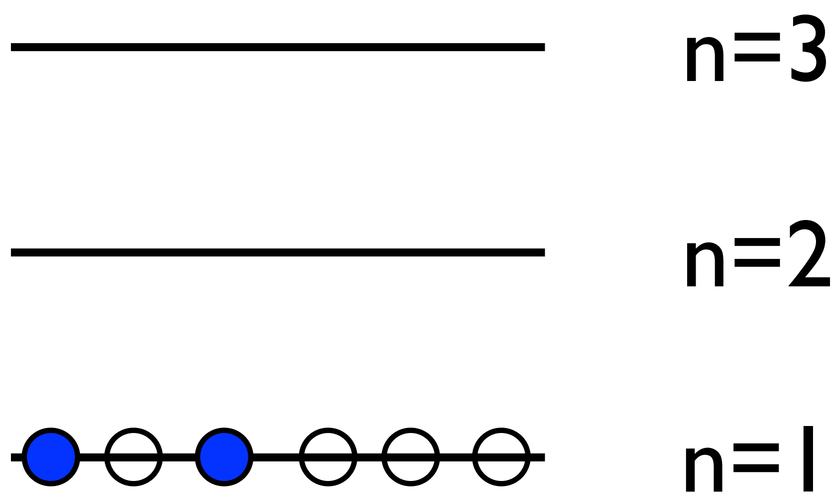
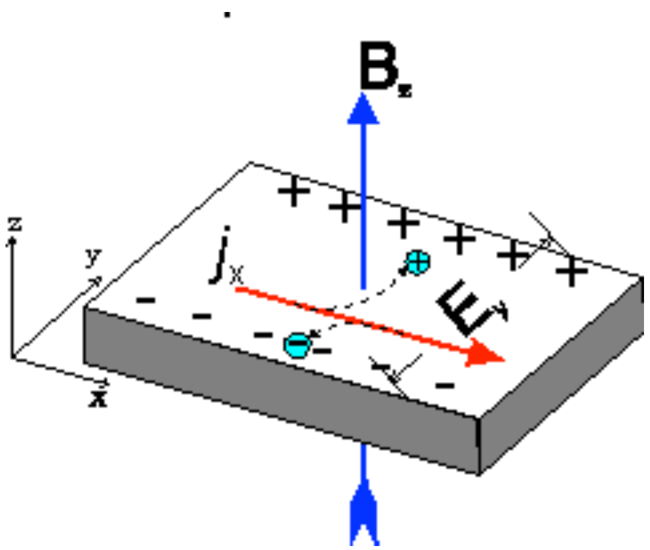


Filling fraction $$\nu = \frac{n}{B/2\pi}$$

Fractional QHE



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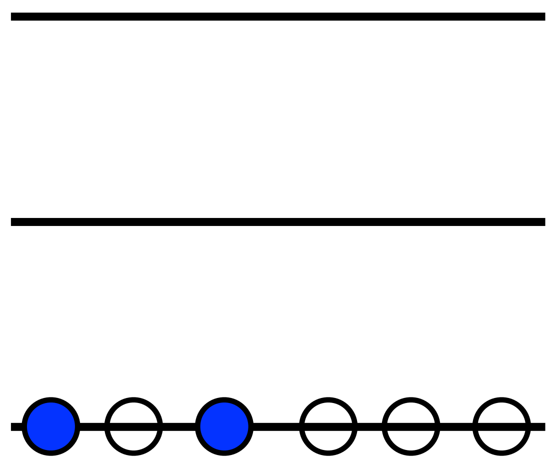

n=3

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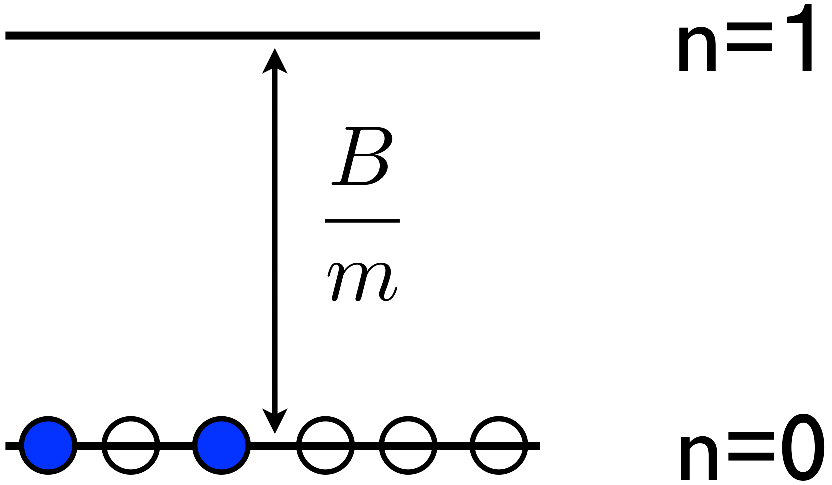
$$\nu = \frac{1}{3}$$

Filling fraction

$$\nu = \frac{n}{B/2\pi}$$

Lowest Landau level

$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$



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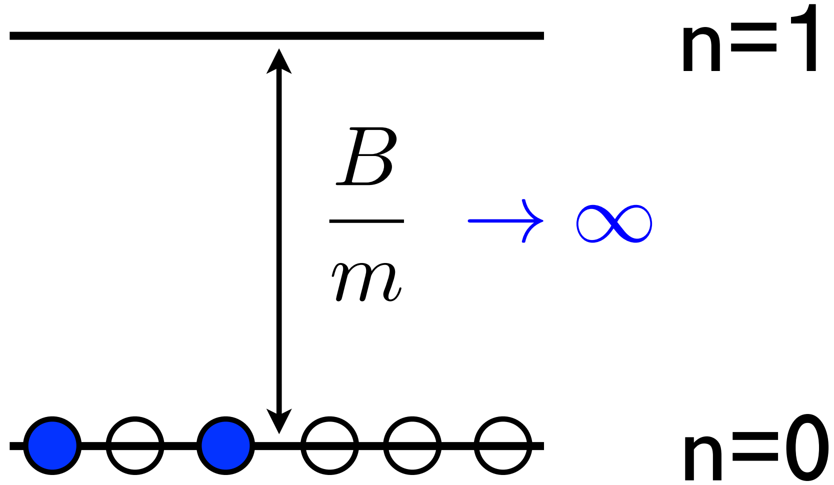
$m \to 0$



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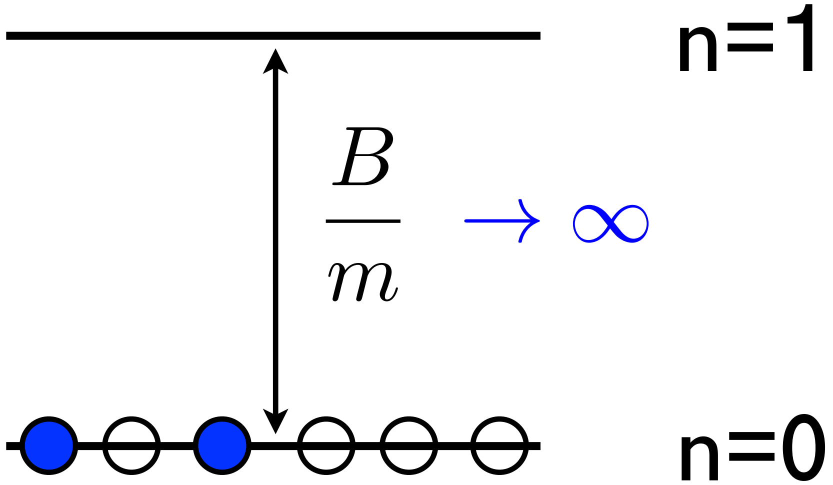
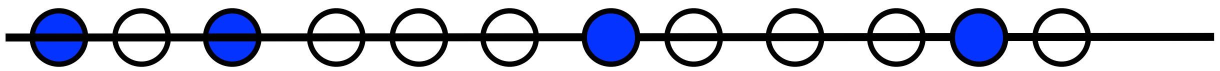


$$H = P_{\mathrm{LLL}} \sum_{a,b} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

Projection to
lowest Landau level

Why the FQH problem is hard



- Problem of degenerate perturbation theory
- Starting point: exponentially large number of degenerate states
- Any small perturbation lifts the degeneracy
- no small parameter

Flux attachment

- Experimental hint from late 80s: gapless state at $\nu=1/2$,
  - nontrivial low-energy effective theory
- late 80s - early 90s: idea of composite fermion
  - electron = CF with 2 attached flux quanta

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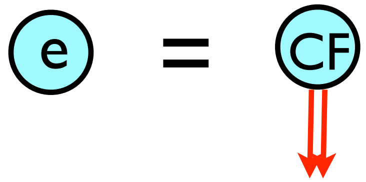
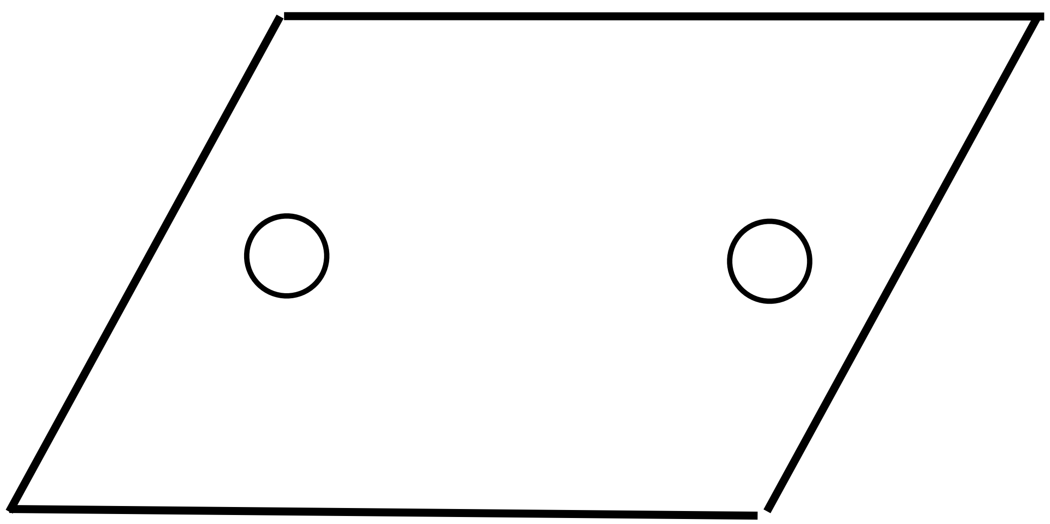


Flux attachment

(Wilczek 1982, Jain 1989)

- Flux attachment: statistics does not change by attaching an even number of flux quanta



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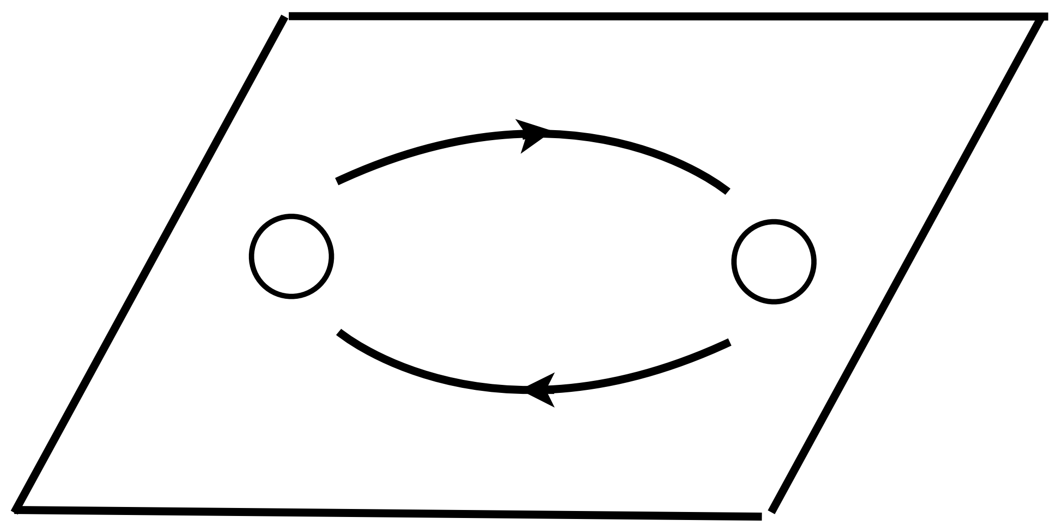

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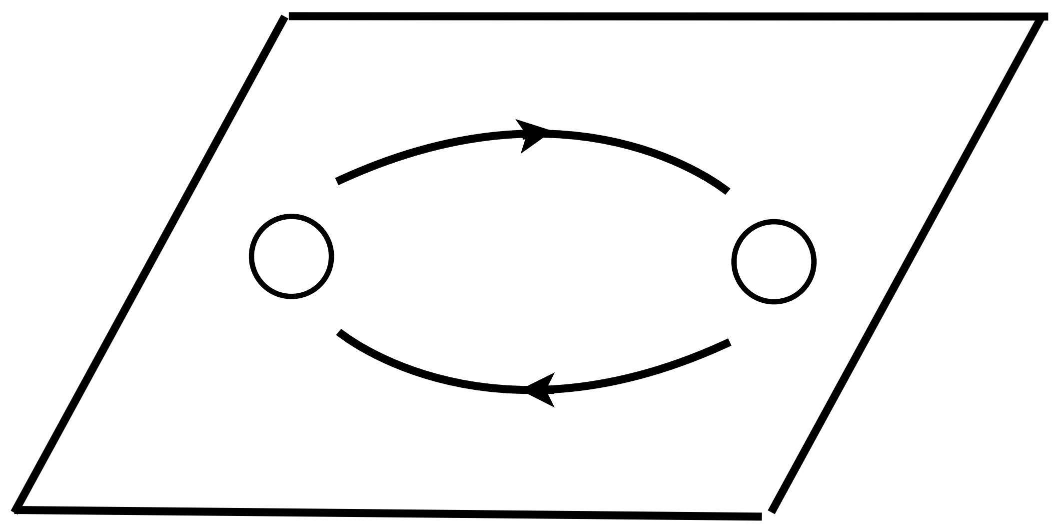

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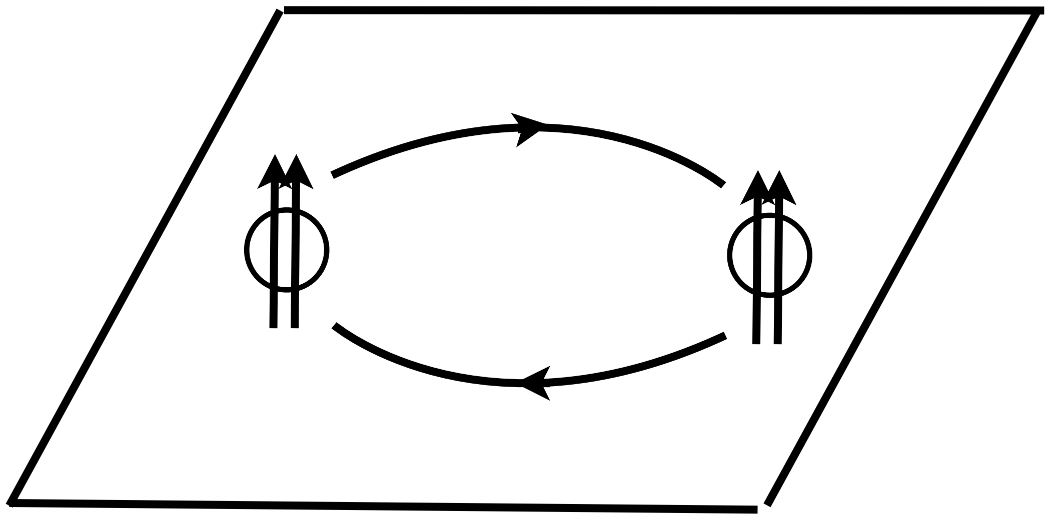

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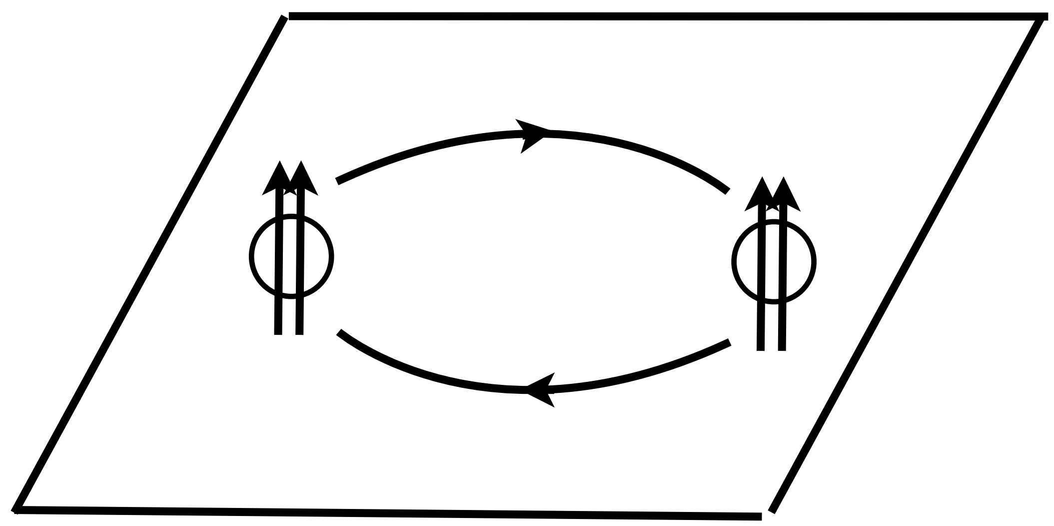

$$(-1)\exp(2i\pi) = (-1)$$

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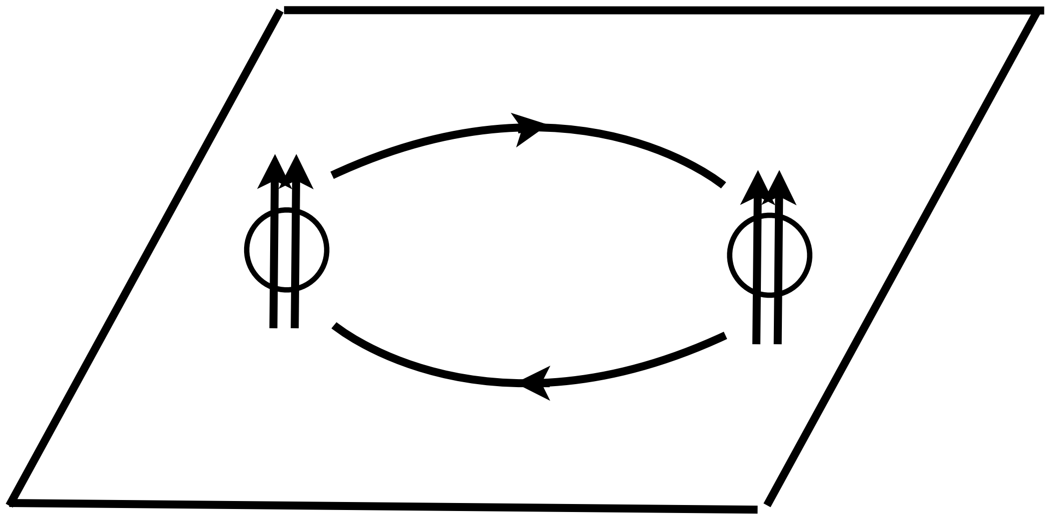

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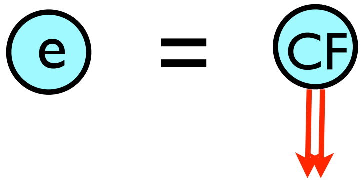
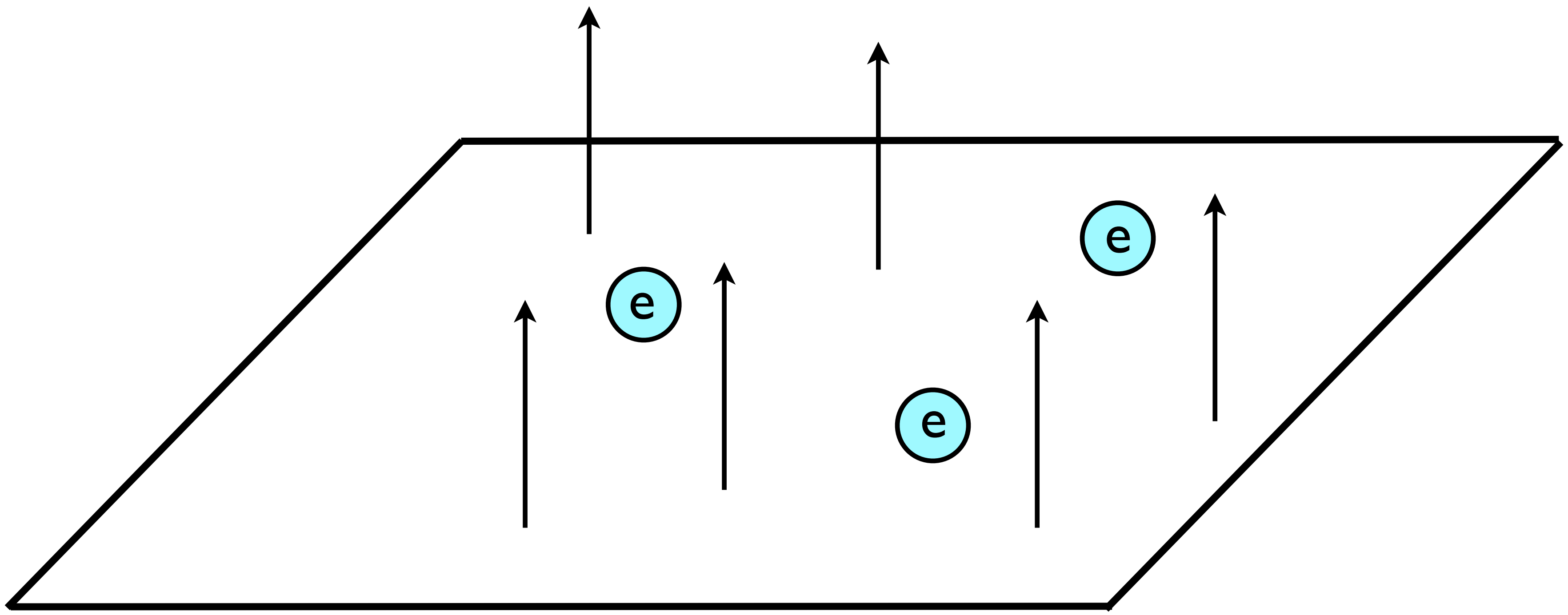
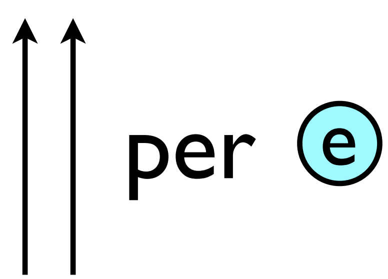


Fermi liquid at nu=1/2

Halperin Lee Read 1993





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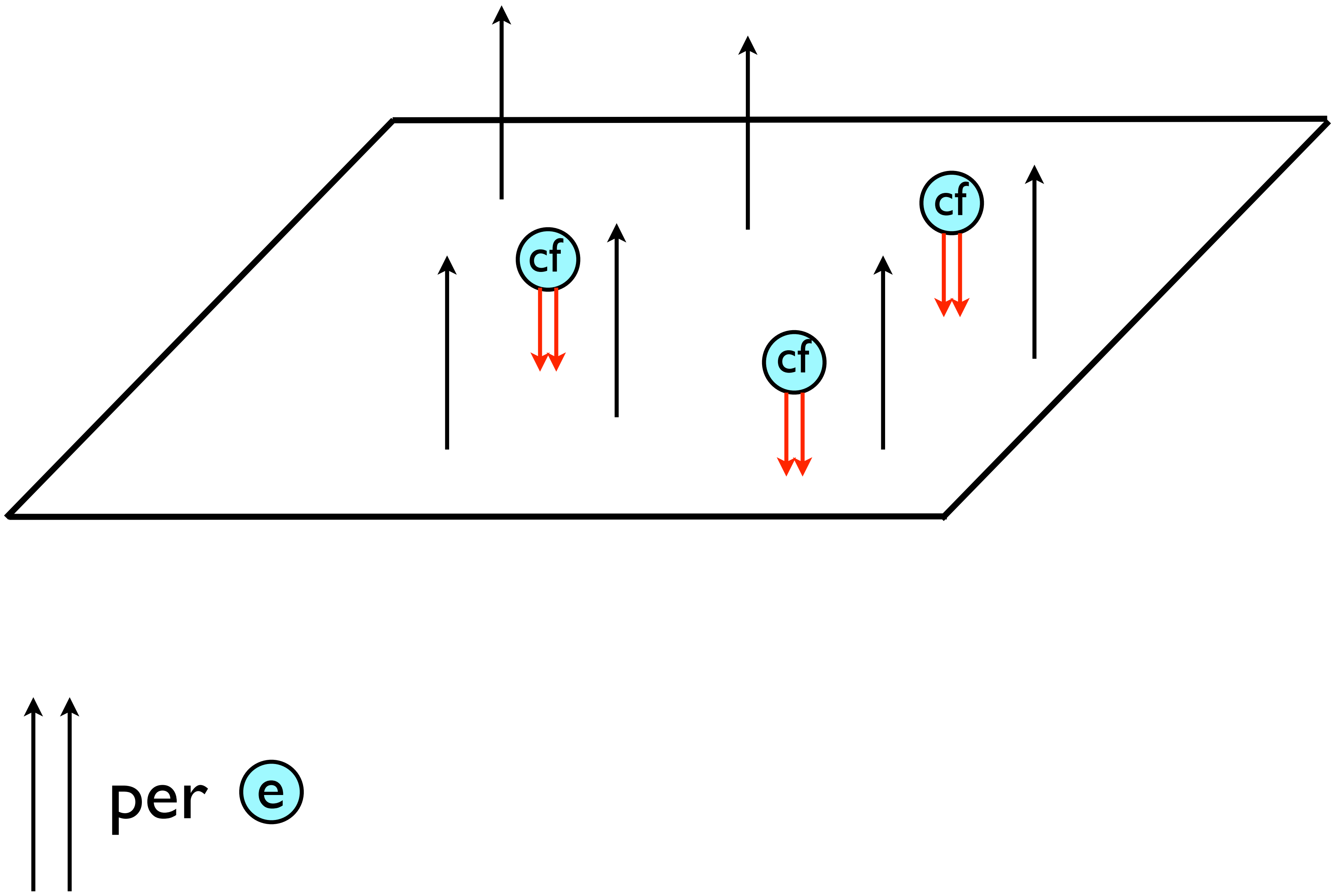

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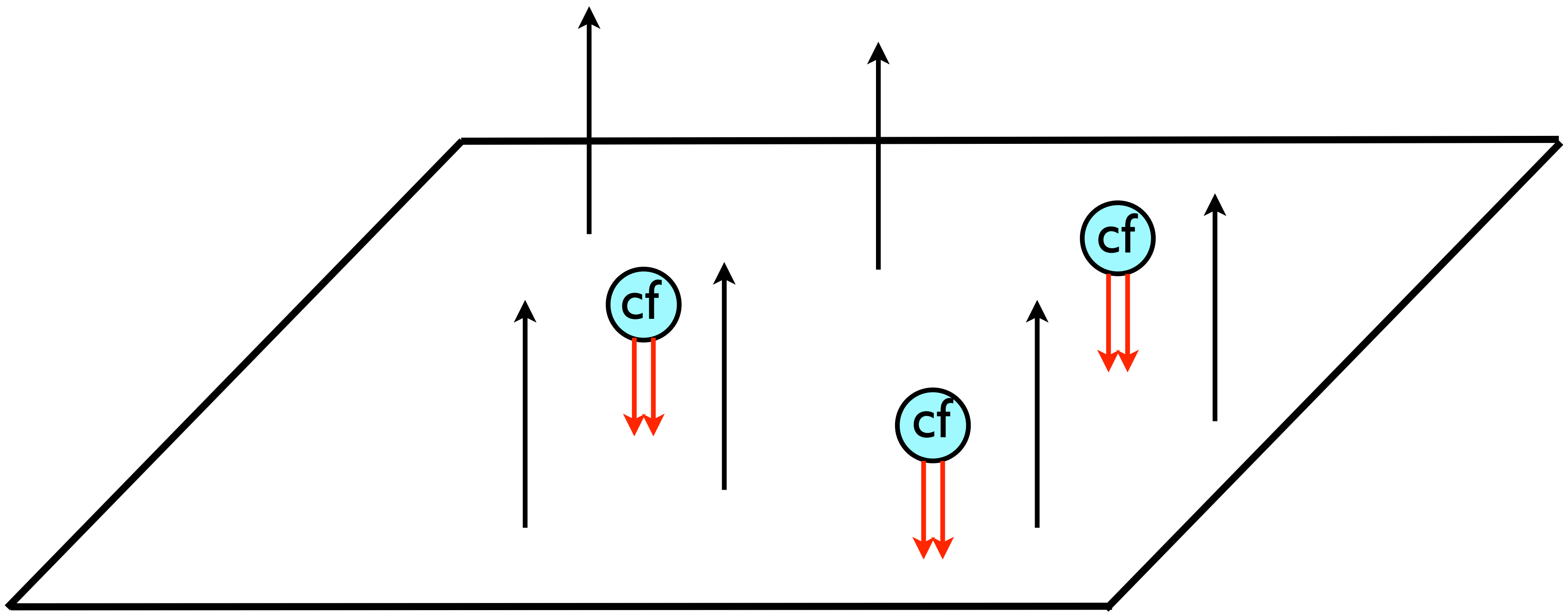

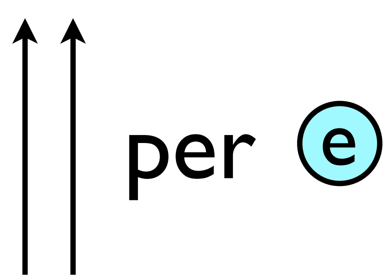

Zero B field for cf

Fermi liquid at nu=1/2

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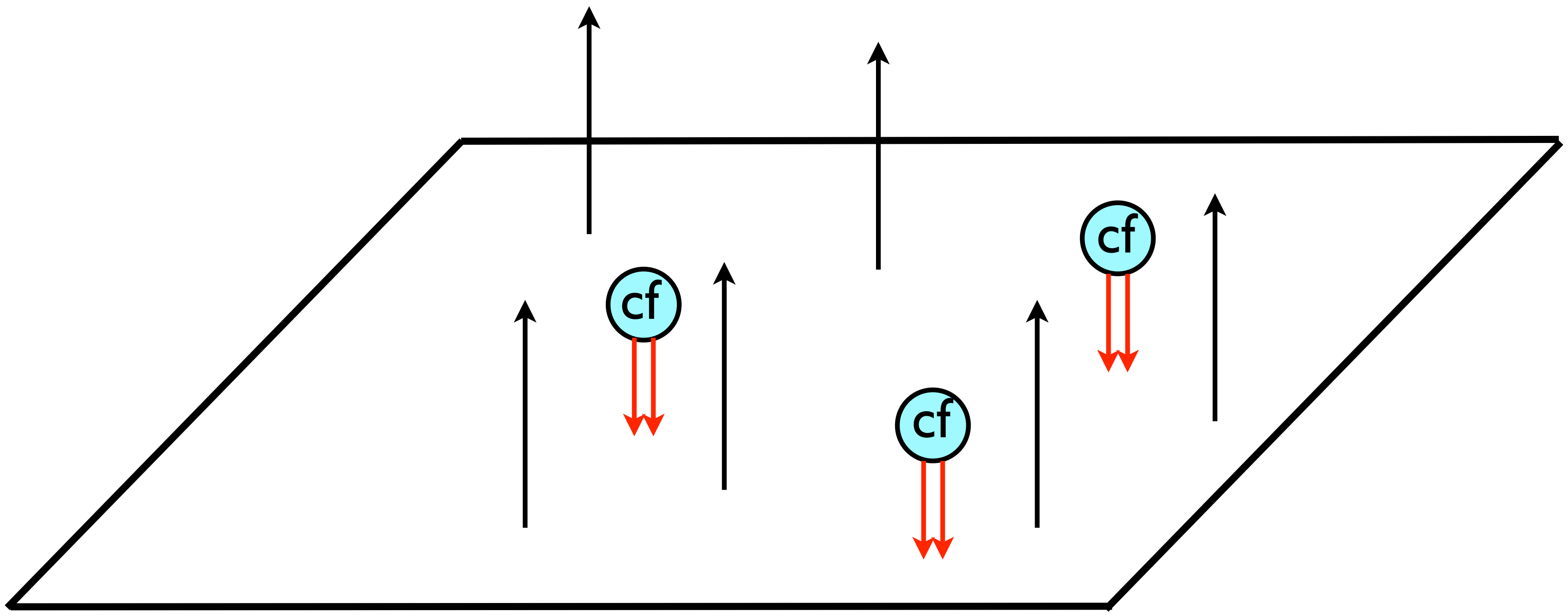
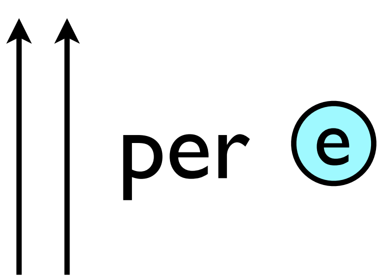




Zero B field for cf

CFs form a Fermi liquid; HLR theory

HLR field theory

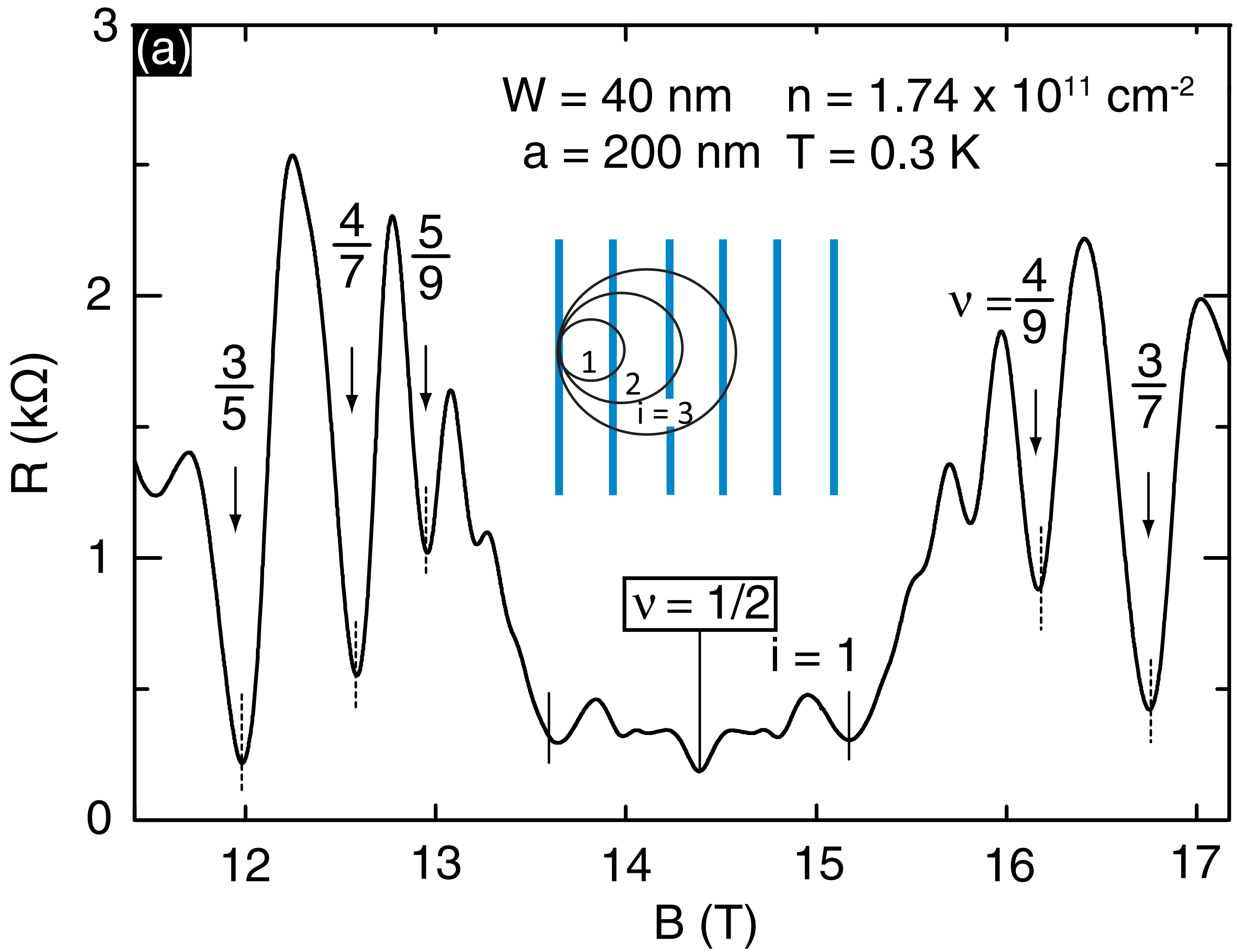
$$\mathcal{L} = i\psi^\dagger(\partial_0 - iA_0 + ia_0)\psi - \frac{1}{2m}|(\partial_i - iA_i + ia_i)\psi|^2 + \frac{1}{2}\frac{1}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$

$$b = \nabla \times a = 2 \times 2\pi\psi^\dagger\psi$$ "flux attachment"

mean field: $$B_{\text{eff}} = B - b = B - 4\pi n$$

$$\nu = \frac{1}{2} \qquad B_{\text{eff}} = 0$$

Reality of composite fermion confirmed in experiment (Kang et al, 1990)



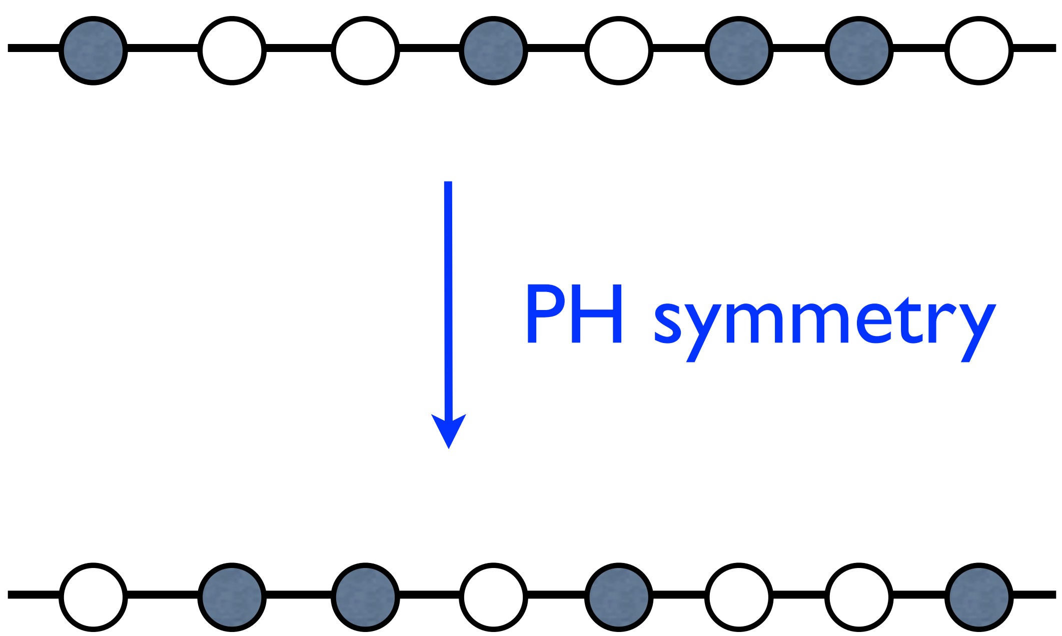
(Kamburov et al, 2014)

2 features of HLR theory

- Number of CFs = number of electrons (by construction)
- Chern-Simons term *ada*

- For a long time it was thought that the HLR theory (zoomed in the near Fermi surface region) gives the correct low-energy effective theory
- problems were known
- one problem turns out to be crucial

Particle-hole symmetry



$$\Theta|\text{empty}\rangle = |\text{full}\rangle$$

$$\Theta c_k^\dagger \Theta^{-1} = c_k$$

$$\Theta\, i\, \Theta^{-1} = -i$$

$$\nu \to 1 - \nu$$

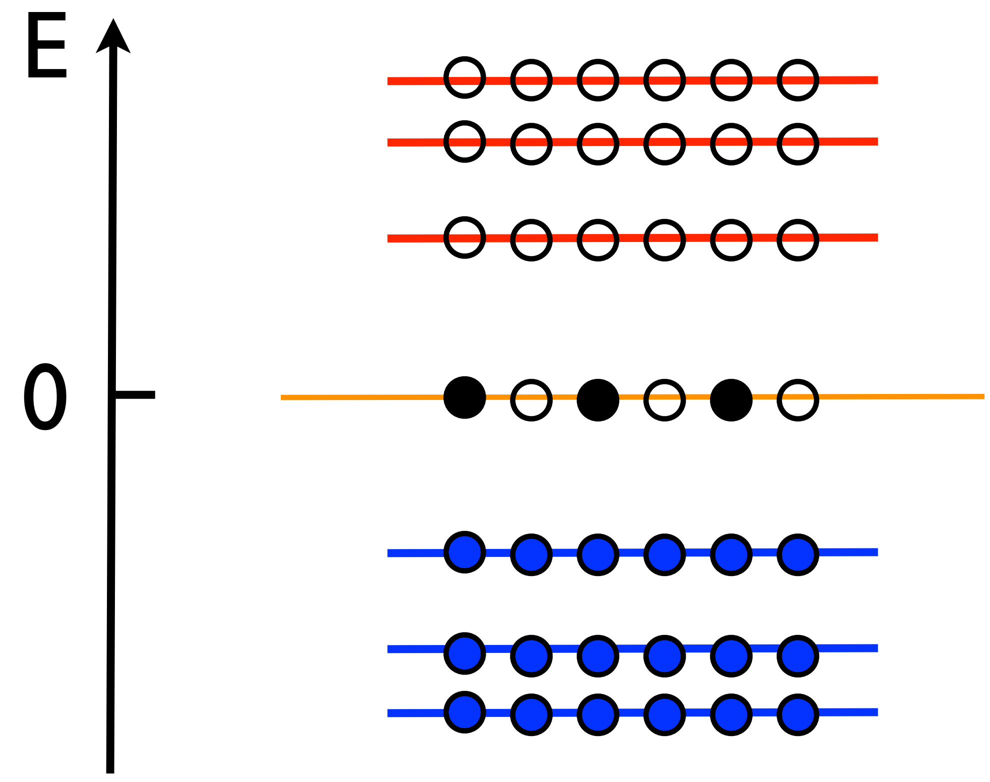
exact symmetry the Hamiltonian on the LLL, when mixing of higher LLs negligible

PH symmetry in HLR

- HLR Lagrangian does not have any symmetry that can be identified with PH symmetry ~1997
- The problem was considered "hard" as it requires projection to lowest Landau level
  - PH conjugation acts nonlocally

Sharpening the problem

- Consider a 2-component massless Dirac fermion
- Can realize fractional quantum Hall effect
- Natural particle-hole symmetry at zero density



FQHE for Dirac fermion

- FQHE for Dirac fermion sharpens the problem of particle-hole symmetry:
- Half filled Landau level at zero charge density
- ground state should be a Fermi liquid, volume of Fermi sphere ~ magnetic field
- Luttinger's theorem: Fermi volume = charge density
    - which charge density?

Dirac composite fermion

DTS 2015

electron theory $\mathcal{L} = i\bar{\psi}_e \gamma^\mu (\partial_\mu - iA_\mu)\psi_e$

CF theory $\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda$

Note: no *ada*

number of CFs $\neq$ number of electrons

consistent with a large number of exp. constraints

Particle-vortex duality

original fermion ψ	composite fermion ψ_e
magnetic field	density
density	magnetic field

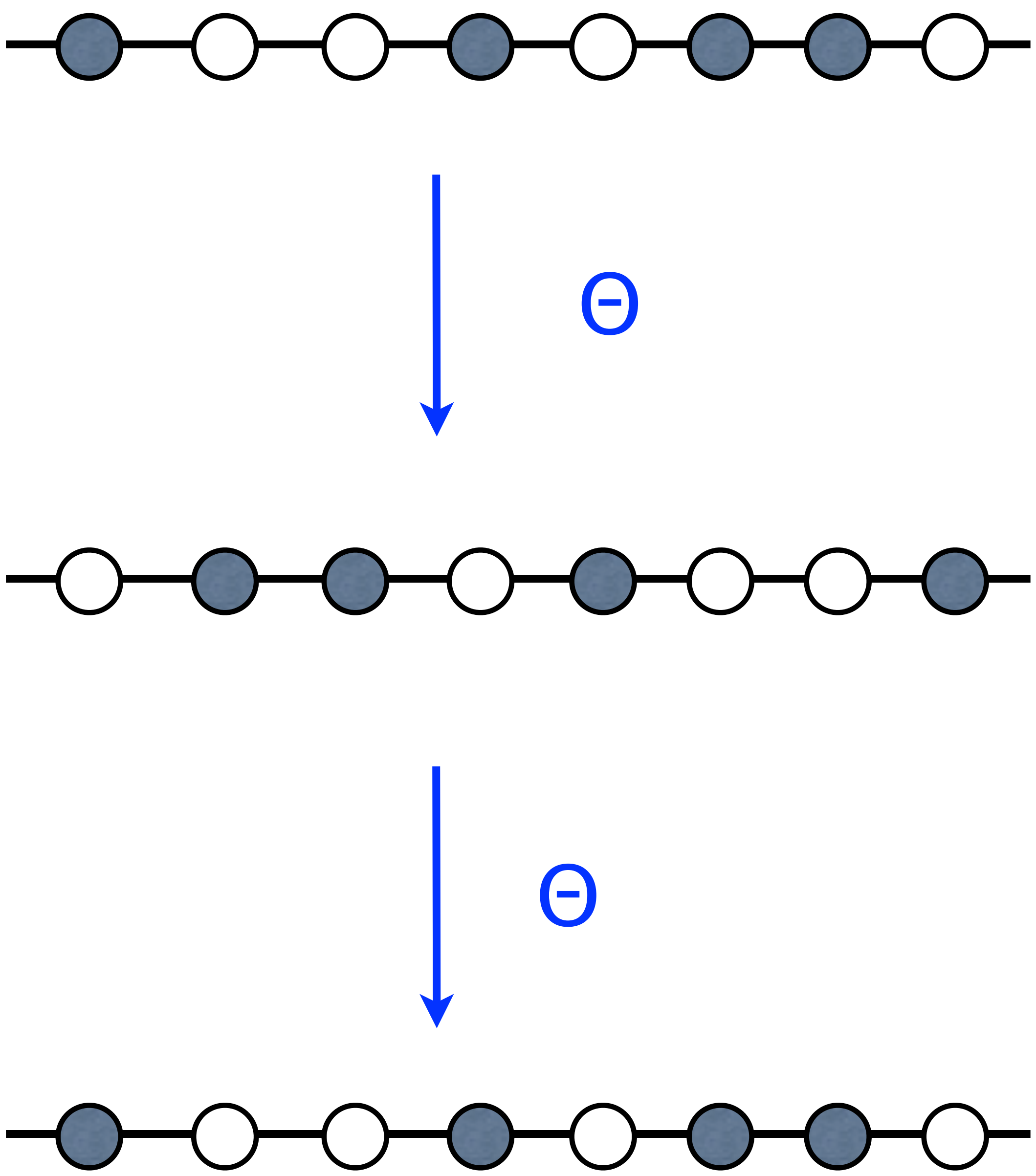
$$S = \int d^3x \left[i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{4\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda \right]$$

$$\rho = \frac{\delta S}{\delta A_0} = -\frac{b}{4\pi}$$

$$\frac{\delta S}{\delta a_0} = 0 \longrightarrow \langle \psi\bar{\gamma}^0\psi \rangle = \frac{B}{4\pi}$$

Fermi sphere from B

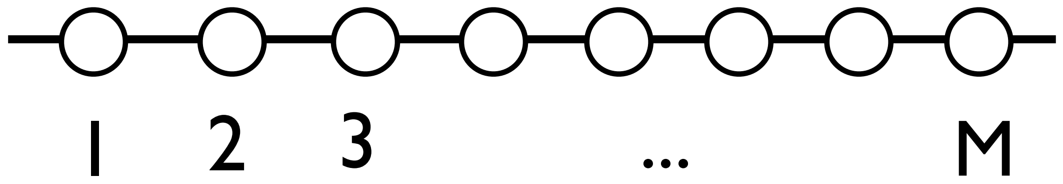
(Particle-hole)^2



$$\Theta^2 = \pm 1$$

Necessity of Dirac CF

On a single Landau level



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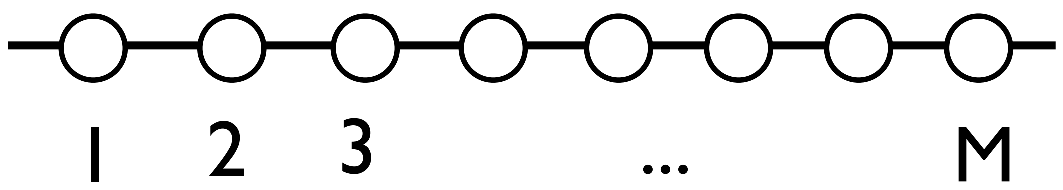

$$\Theta^2 = (-1)^{M(M-1)/2}$$

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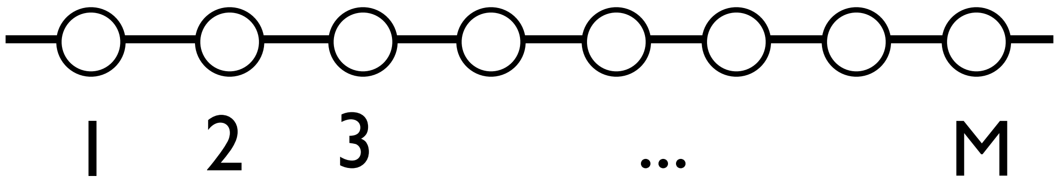

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$$M = 2N_{\rm CF}$$

$$\Theta^2 = (-1)^{N_{\rm CF}}$$

Necessity of Dirac CF

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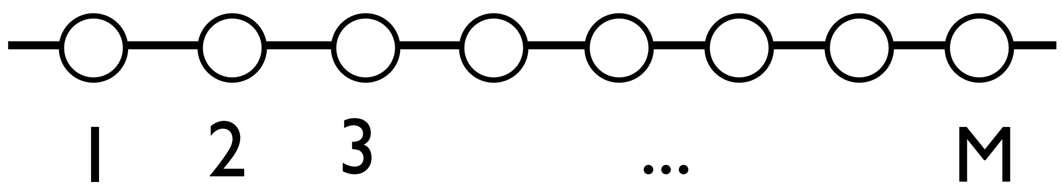

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$$\Theta^2 = (-1)^{N_{\rm CF}}$$

Natural for Dirac CF

Geraedts, Zaletel, Mong, Metlitski, Vishwanath, Motrunich; Levin, Son

More careful version of duality

$$\mathcal{L} = i\bar{\psi}_e \gamma^\mu (\partial_\mu - iA_\mu)\psi_e$$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ia_\mu)\psi - \frac{1}{2}\frac{1}{4\pi}ada + \frac{1}{2\pi}adb - \frac{2}{4\pi}bdb + \frac{1}{2\pi}Adb - \frac{1}{2}\frac{1}{4\pi}AdA$$

Naively integrating over b: $b = \frac{1}{2}(A + a)$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - a_\mu)\psi + \frac{1}{4\pi}Ada$$

Seiberg, Senthil, Wang, Witten, 1606.01989

Consequences of DCF

- Satisfies symmetry constraints on transport coefficient (conductivity, thermoelectric) at half filling
- A gapped particle-hole symmetric state: PH-Pfaffian
- Absence of Friedel oscillations in correlation of PH-symmetric operators Geraedts et al.

Consequences of PH symmetry

$$\mathbf{j} = \sigma_{xx}\mathbf{E} + \sigma_{xy}\mathbf{E}\times\hat{\mathbf{z}} + \alpha_{xx}\boldsymbol{\nabla}T + \alpha_{xy}\boldsymbol{\nabla}T\times\hat{\mathbf{z}}$$

conductivities

thermoelectric coefficients

- At exact half filling, in the presence of particle-hole symmetric disorders

$$\sigma_{xy} = \frac{e^2}{2h} \qquad \alpha_{xx} = 0$$

HLR $\rho_{xy} = \frac{2h}{e^2}$

Potter, Serbyn, Vishwanath 2015

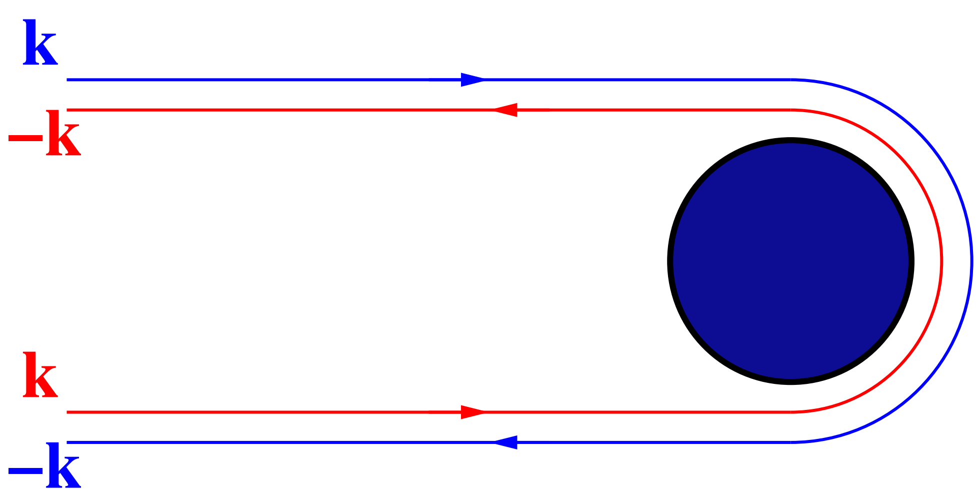
PH Pfaffian state

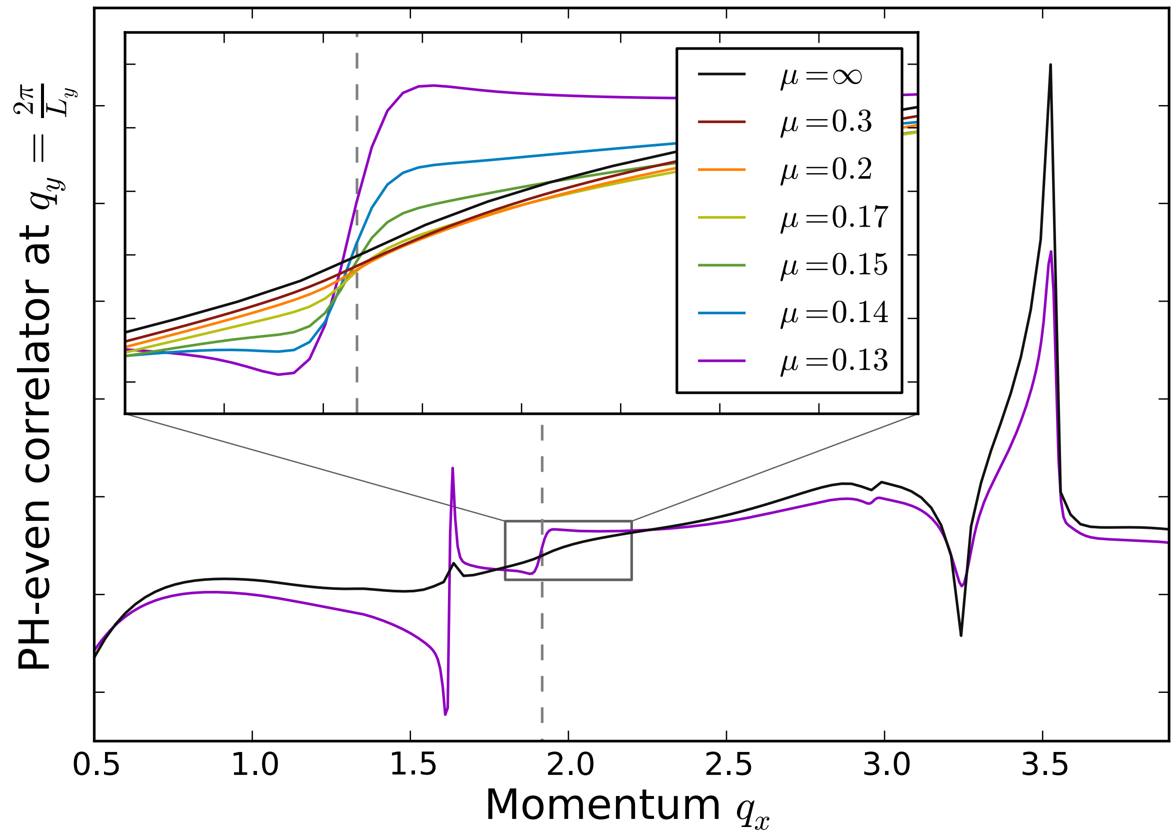
- The composite fermions can form Cooper pairs
- Simplest pairing does not break particle-hole symmetry

$$\langle \epsilon^{\alpha\beta} \psi_\alpha \psi_\beta \rangle \neq 0$$

Consequences of Dirac CF

Suppression of Friedel oscillations in correlations of particle-hole symmetric observables $\hat{O} = (\rho - \rho_0)\nabla^2\rho$





Geraedts, Zaletel, Mong, Metlitsky, Vishwanath, Montrunich, 2015

Direct proof of Berry phase π of the composite fermion

A window to duality

- Fermionic particle-vortex duality is a consequence of a more "elementary" fermion-boson duality Karch, Tong; Seiberg, Senthil, Wang, Witten
    - small N version of duality between CS theories, tested at large N
- New dualities can be obtained
    - Example: Nf=2 QED3 is self-dual Cenke Xu

The elementary duality

$$\mathcal{L} = L[\psi, A] - \frac{1}{2}\frac{1}{4\pi}AdA$$

$$\mathcal{L} = L[\phi, a] + \frac{1}{4\pi}ada + \frac{1}{2\pi}Ada$$

Conclusion and open questions

- Dirac CF solves the 20-year old problem of PH symmetry of half-filled Landau level
- Distinct predictions, numerically checked
- A experimentally accessible window to field-theoretical duality between (2+1) dimensional theories