Advances in 5d / 6d QFTs

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My task today:

1) Review recent developments in higher dimensional field theories (6d & 5d)
2) Should not assume any specific knowledge of the field
3) Not too much overlap with Vafa's review talk at Strings 2016

Recently, progress (among others) on

- Engineering new 5d/6d SCFTs
- New calculations of 5d/6d observables, especially w/ SUSY
- Implications of $d > 4$ to $d \leq 4$
- Abstract QFT methods (e.g. bootstrap)
- Formal developments w/ EFT and/or SUSY (anomalies, higher derivative terms, …)
- New AdS_6 and AdS_7 gravity duals

I'll focus on advances w/ exactly computable SUSY obervables & their lessons to 5d/6d.

To complement my talk, please refer to the past reviews

- Vafa, "6d SCFTs" (2016)
- Tachikawa, "Recent advances in SUSY" (2014)
- Moore, "The recent role of (2,0) theories in physical mathematics" (2011)

Plan

1) Some backgrounds

2) What can we compute in $d > 4$?

3) What physics do we learn?

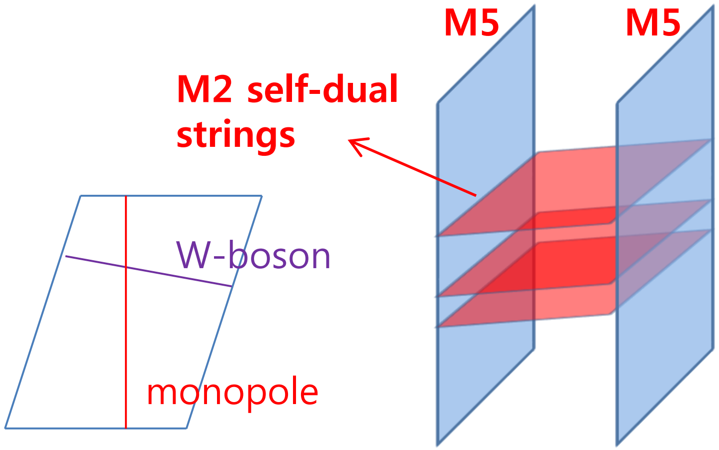
4) Other directions

   which I think should be explored more intensively in the future,
   or, which I keep brief just due to my lack of expertise

6d SCFTs

- Symmetry: $OSp(8^*|2N)$ (including $SO(6,2) \times SO(5)_R$) with $N = 1,2$.

- $N = (2,0)$: M5's, or IIB on C^2/Γ ($\Gamma = A_{n-1}, D_n, E_n$). Lagrangian unknown.
- Reduce on T^2: 4d N=4 SYM w/ S-duality. Hard.
- Gauge theories: E/M particles on unequal footing
- W-boson/monopole from 6d self-dual strings
- Many other mysteries: N^3, “non-Abelian tensors” …



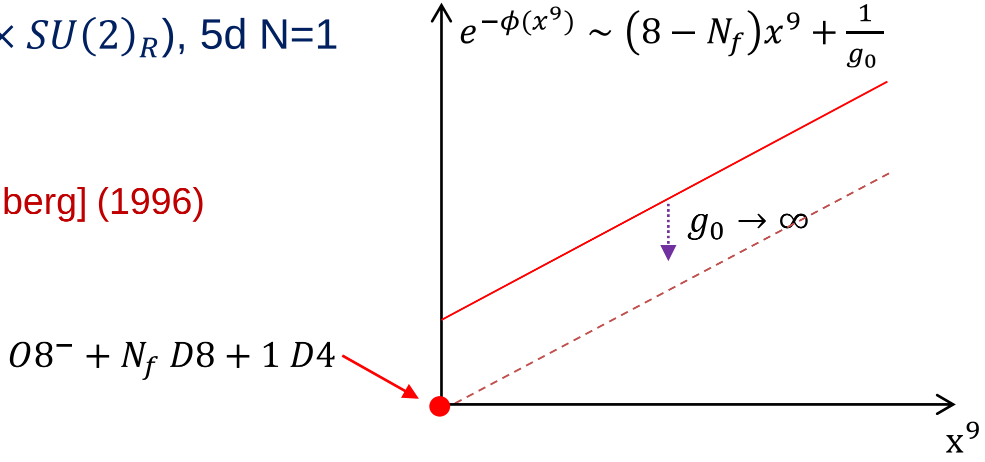
- $N = (1,0)$: various models from 5-branes (+ others); F-theory on singular CY3
- On T^2: 4d systems w/ both light E/M objects, e.g. Argyres-Douglas theories.
- Many 6d (1,0) theories are related to such 4d systems

- In a sense, these are QFTs for (tensionless) strings

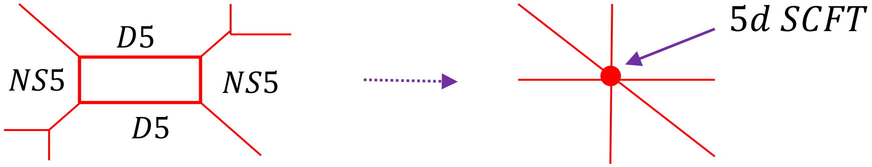
$B_{\mu\nu}$ with $H = dB = \star dB$, Ψ^A , Φ $\qquad dH_3 = d \star H_3 \sim \delta^{(4)}$

5d SCFTs

- Symmetry: $F(4)$ (including $SO(3,2) \times SU(2)_R$), 5d N=1
- Simple examples:
  - Strong-coupling limit of D4-D8-O8 [Seiberg] (1996)



  - Other examples:
    branes (esp. IIB 5-brane webs),
    M-theory on singular CY3



- on S^1: related to various 4d isolated CFTs (AD, Minahan-Nemeschansky, …)
- Global symmetry often enhances at ∞ coupling: $SO(2N_f) \times U(1) \to E_{N_f+1}$
  - due to 'nonperturbative' particles being massless: all ptl's to be treated on equal footing

How to study…?

- 5d/6d QFTs defy Lagrangian descriptions (note the constraints from $d \leq 4$)
- So how can we study them?

- Use string theory.
- Use effective QFTs (after deformations, going to branches, …)
- Use lower d QFTs (e.g. 1d, 2d) for 5d/6d "solitons/strings"
- ……

- Often, focus given to SUSY observables: important roles in $d > 4$

- Historical perspective:

- Active studies in mid/late 90's
- Gradual revival, I think (partly) triggered by…
  - better intuitions on QFT (dualities, AdS/CFT, …)
  - advances in strong coupling QFT (w/ SUSY, abstract methods, …)
  - progress in M2, ABJM, especially w/ SUSY (≥ 2009)
  - their implications to $d \leq 4$ (new dualities, AGT, …, ≥ 2009)

Deformations of SCFTs

- Helpful to deform SCFTs w/ massive parameters (preserving Poincare SUSY)

5d: massive deformations:

- Relevant deformations: Some yield Yang-Mills descriptions in IR. $[1/g_{YM}^2] \sim M^1$

$$\mathcal{L}_{\rm YM} \leftarrow -\frac{1}{4g_{YM}^2}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) + \cdots$$

$$\Delta\mathcal{L}_{\text{quark mass}} \leftarrow \bar{\psi}_i M^i{}_j \psi^j + \cdots$$

all associated w/ global symmetries
[Cordova, Dumitrescu, Intriligator] …

- Scalar VEVs. Higgs branch & Coulomb branch (← today) $A_\mu\ ,\ \lambda^A\ ,\ \boxed{\phi} \longrightarrow$ VEV

6d: Scalar VEVs. Today → tensor branch $B_{\mu\nu}$ with $H = dB = \star dB\ ,\ \Psi^A\ ,\ \boxed{\Phi} \longrightarrow$ VEV

- no vector multiplets: free Abelian tensor theory in IR
- ∃vector multiplets: 6d Yang-Mills-tensor(-matter) theory, w/ $\langle\Phi\rangle \sim 1/g_{YM}^2$

$$S_{\rm v+t}^{\rm bos} = \int \left[\frac{1}{2}d\Phi\wedge\star d\Phi + \frac{1}{2}H\wedge\star H\right] + \sqrt{c}\int\left[-\Phi\mathrm{tr}(F\wedge\star F) + B\wedge\mathrm{tr}(F\wedge F)\right]$$

$$H \equiv dB + \sqrt{c}\ \mathrm{tr}\left(AdA - \frac{2i}{3}A^3\right)$$

- Easier to study various observables

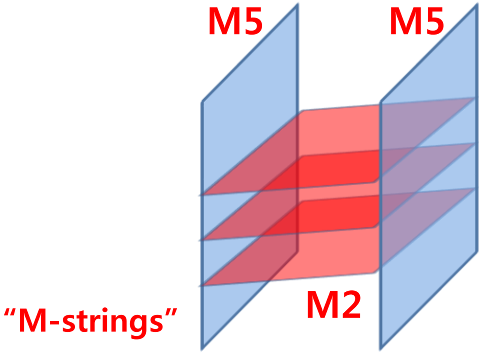
- Observables asymptotically encode CFT info: e.g. go to “high T”
- Also related to 5d/6d CFT observables (often combined with some guessworks)

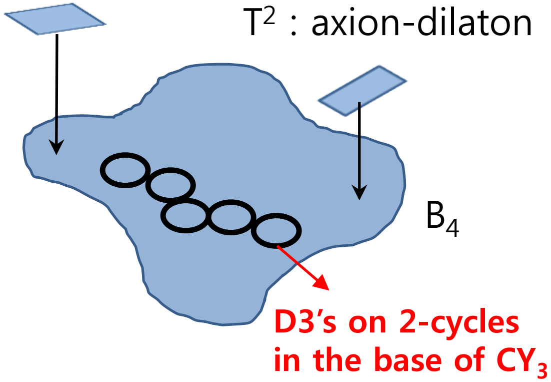
Objects in tensor/Coulomb branches

- 6d self-dual strings: tension $\tau \sim v = \langle\Phi\rangle$ in the tensor branch

- couples to 2-form tensor: “electric = magnetic charges” $dH_3 = d\star H_3 \sim \delta^{(4)}$

- Examples





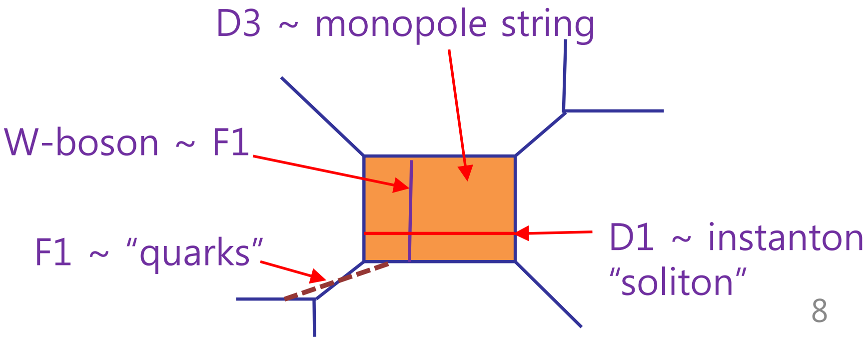
- w/ 6d gauge group: Yang-Mills instanton strings

$$dH_3 = d\star H_3 = \sqrt{c}\,\mathrm{tr}(F\wedge F) \longrightarrow F = \pm\star_4 F \qquad H = \mp\star_4 d\Phi. \quad \text{on the transverse } \mathbb{R}^4$$

BPS & solves e.o.m. at leading order in $v^{-1} \sim g_{YM}^2$

- 5d W-bosons, quarks, solitons, strings

- Example: “SU(2) w/ $N_f = 2$ quarks”



Compactifications of 6d CFTs

- M-theory on S^1_R : IIA description at $E \ll 1/R$. At momentum k, D0 QM at $\Delta E \ll 1/R$

- N M5 on S^1 w/ momemtum k: D0-D4 QM at $\Delta E \ll 1/R$

“ADHM” quantum mechanics
For Yang-Mills instantons

$$q_{\dot\alpha} = (q, \tilde{q}^\dagger) : \ k \times N$$

$$a_m \& \text{ and other fields} : k \times k$$

$$\begin{aligned} L_{QM} \ &= \ \frac{1}{g_{QM}^2}\mathrm{tr}_{k,N}\Big[\frac{1}{2}(D_t\varphi^I)^2 + \frac{1}{2}(D_t a_m)^2 + \frac{1}{4}[\varphi^I, \varphi^J]^2 + \frac{1}{2}[a_m, \varphi^I]^2 \\ &\quad + D_t q_{\dot\alpha} D_t \bar{q}^{\dot\alpha} - (\varphi^I \bar{q}^{\dot\alpha} - \bar{q}^{\dot\alpha} v^I)(q_{\dot\alpha}\varphi^I - v^I q_{\dot\alpha}) - D^{\dot\alpha}{}_{\dot\beta} D^{\dot\beta}{}_{\dot\alpha} + \cdots \Big] \end{aligned}$$

$$D^{\dot\alpha}{}_{\dot\beta} = \bar{q}^{\dot\alpha} q_{\dot\beta} - \frac{1}{2}\zeta^A(\tau^A)^{\dot\alpha}{}_{\dot\beta} + \frac{1}{4}(\bar\sigma^{mn})^{\dot\alpha}{}_{\dot\beta}[a_m, a_n] - \frac{1}{2}\delta^{\dot\alpha}_{\dot\beta}(\mathrm{trace})$$

- “E-string theory” M5-M9 on S^1 : O8 + 8 D8 w/ D4 + k D0

- Subtler descriptions recently:

- S.-S. Kim, Hayashi, K. Lee, Taki, Yagi, … : deformations of 5-brane webs, etc.

- connections of 6d SCFTs on S^1 & 5d SCFTs [Hayashi,Ohmori] [Del Zotto,Heckman,Morrison]

- Often comes w/ calculus from the topological vertex method

- Can study a sector of 6d QFT (at small R): E.g. by QM (GLSM) at given k (like D0-D4)

  But for BPS observables, often can continue to other parameter regimes

SUSY observables

- SUSY partition functions: BPS objects transverse to R^4
  - $j_1, j_2 \in SO(4)$; $J_R \in SU(2)_R$; q_i → #(W-boson) or #(string-winding); F_a → other charges
  - 5d SUSY index $Z[R^4 \times S^1]$

$$Z_{\mathbb{R}^4\times S^1}(\epsilon_{1,2}, v_i, z_a) = \mathrm{Tr}_{5\mathrm{d}}\left[(-1)^F e^{-tQ^2} e^{2\pi i\epsilon_1(j_1+J_R)} e^{2\pi i\epsilon_2(j_2+J_R)} \prod_i e^{-v_i q_i} \prod_{a\in F} e^{2\pi i z_a F_a}\right] \qquad Q \equiv Q^{J_R=+\frac{1}{2}}_{j_1=j_2=-\frac{1}{2}} + \text{h.c.}$$

$$\longrightarrow \text{“}Z_{\text{pert}}\text{”} \sum_{k=0} q^k Z_k(\epsilon_{1,2}, v_i, z_{\text{rest}})$$

$q = e^{2\pi i z_1} \ll 1$, e.g. for “instanton number” k in 5d: $\Delta E \ll 1/g_{YM}^2$

  - 6d index $Z[R^4 \times T^2]$ on $R^{4,1} \times S^1$: BPS strings winding S^1

$$Z_{\mathbb{R}^4\times T^2}(\epsilon_{1,2}, \tau, v_i, z_a) = \mathrm{Tr}_{6\mathrm{d}}\left[(-1)^F e^{-2\pi i\bar{\tau}Q^2} e^{2\pi i\tau\frac{H+P}{2}} e^{2\pi i\epsilon_1(j_1+J_R)} e^{2\pi i\epsilon_2(j_2+J_R)} \prod_i e^{-v_i q_i} \prod_{a\in F} e^{2\pi i z_a F_a}\right]$$

$$\longrightarrow \text{“}Z_{\text{tensor}}\text{”} \sum_{n_1,\cdots,n_r=0}^{\infty} Z_{(n_i)}(\tau, v_i, z_a) \prod_{i=1}^{r} w_i^{n_i}$$

$w_i = e^{-v_i} \ll 1$ for tensor VEV: $\Delta E \ll v_i^{1/2}$

$$\longrightarrow \text{“}Z_{\text{pert}}\text{”} \sum_{k=0}^{\infty} q^k Z_k(\epsilon_{1,2}, v_i, z_a)$$

$q = e^{2\pi i\tau} \ll 1$ for KK momentum: $\Delta E \ll 1/R$

In all expansions, coefficients are 1d/2d indices.

  - All “Nekrasov partition function” in a broad sense [Nekrasov] (2002)

- Dual roles of $\epsilon_{1,2}$: “chemical potentials” & “IR regulator”
  - lifts c.o.m. 0-modes on R^4. "Ω-background”

$$Z = \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} \frac{z(n\epsilon_{1,2}, nv_i, nz_a, \cdots)}{2\sin(\pi n\epsilon_1)\cdot 2\sin(\pi n\epsilon_2)}\right] \xrightarrow{\epsilon_1,\epsilon_2 \to 0} \exp\left[-\frac{f(v_i, z_a, \cdots)}{\epsilon_1\epsilon_2}\right]$$

From 1d/2d gauge theories

- 2d elliptic genera [Benini,Eager,Hori,Tachikawa] (2013)

\- Subtleties w/ fermion 0-modes on T^2 → contour prescription "Jeffrey-Kirwan residues"

$$Z_{T^2}(\tau,\{z\}) = \oint [du] Z_{\text{1-loop}}(\tau,\{u,z\}) \qquad u_I \sim u_I + 1 \sim u_I + \tau$$

flat connections on T^2

- 1d index: In our context, [Nekrasov] (2002) pioneered it. Recent works w/ contour & other subtleties [Hwang,Kim²,Park] [Hori,Kim,Yi] [Cordova,Shao] (2014)

$$Z_{S^1}(\{z\}) = \oint [d\phi] Z_{\text{1-loop}}(\{\phi,z\}) \qquad \phi_I \sim \phi_I + i$$

S^1 flat connection & scalar vector multiplet

- Moduli space σ-models of 5d/6d solitons are incomplete: completion by GLSM

\- 4d: Singular saddle points of QFTs which are good at short distances

\- 5d/6d: part of UV completing sick YM in $d > 4$

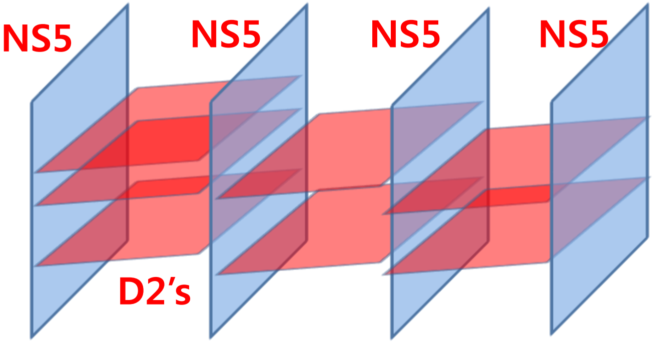
single SU(N) instanton:

$$ds^2 = g_{MN}(X)dX^M dX^N = \underbrace{\boxed{ds^2(\mathbb{R}^4)}}_{\text{c.o.m.}} + \overbrace{\underbrace{d\lambda^2}_{\text{instanton ``size''}} + \lambda^2\left[\underbrace{\boxed{ds^2(S^3/\mathbb{Z}_2)}}_{SU(2)\text{ orientation}} + \underbrace{\boxed{ds^2(\mathcal{M}_{4N-8})}}_{\frac{SU(N)}{SU(2)\times U(N-2)}}\right]}^{\text{small instanton singularity}}$$

\- Troubles w/ wrong UV uplift in 'early days': E.g. 5d SU(2) instantons at $N_f = 7$. It is an example which does not have 4d analogue.

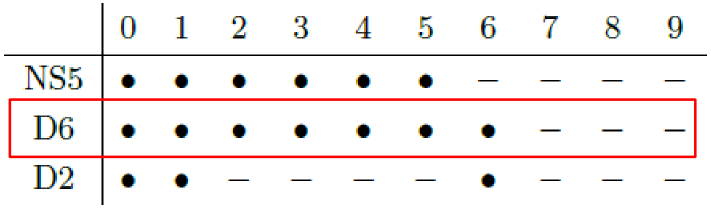
New examples (mostly 2d)

- Strings of 6d $N=(2{,}0)$: Uplift to IIA, M2-M5 → D2-NS5

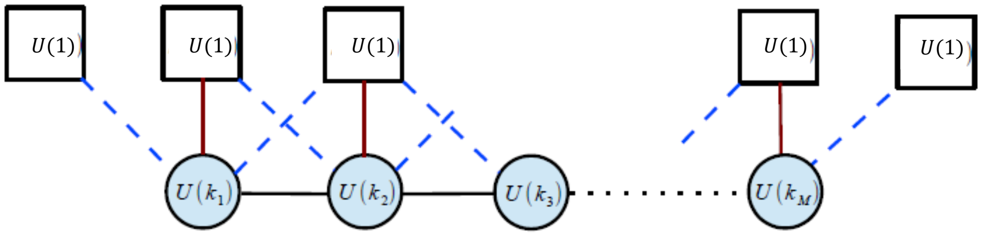


yields 2d N=(4,4) quiver

tricky UV theory use: only sees $SO(3)_R \subset SO(4)_R$
(similar to 3d $N=8$ SYM vs. M2: $SO(7) \subset SO(8)$)



	0	1	2	3	4	5	6	7	8	9
NS5	•	•	•	•	•	•	–	–	–	–
D6	•	•	•	•	•	•	•	–	–	–
D2	•	•	–	–	–	–	•	–	–	–

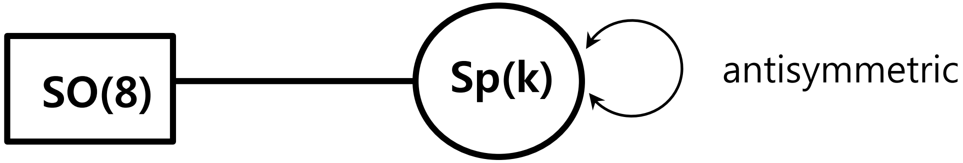


Sees $SU(2) \times U(1) \subset SO(4)_R$: similar to "mirror dual" of 3d $N=8$ SYM $U(1)^2 \times SU(2)^2 \subset SO(8)$ [Haghighat,Iqbal,Kozcaz,Lockhart,Vafa]

- $N=(1{,}0)$ M5-M9: M2-M5-M9 → D2-NS5-O8-D8 [Kim^2,Lee,Park,Vafa] (2014)
- With 6d gauge symmetry: Often 2d GLSM given by "ADHM instanton strings."

\- E.g. 6d $SO(8)$ YM coupled to a tensor [Haghighat, Klemm, Lockhart, Vafa] (2014)

\- For k strings, 2d $N=(0{,}4)$ quiver given by ADHM construction



\- Also, progress on "ADHM-like" 1d/2d gauge theory descriptions of certain exceptional instantons (!!!) [Kim^2,Park] (2016) [Kim^3,Park] (to appear)

CFT physics from Coulomb/tensor branches

- Can we learn about CFTs?
- “High T” limit may overcome all massive deformations & compactifications.

- E.g. 6d $N = (2{,}0)$ SCFT physics from $Z[R^4 \times T^2]$ …?

- S^1 w/ radius R; VEVs v_i; mass m (previously z) for $SU(2)_l \subset SO(5)_{R-symmetry}$
- Ω-background: effectively provides a finite “volume” of R^4: will turn off
- “The temperature” T, to be sent large, is conjugate to KK momentum on S^1
- D0-D4 description: Only know series expansion at $q \ll 1$ (Note, $q = e^{2\pi i\tau} = e^{-\frac{1}{TR}}$)

- S-duality on T^2…? (S-duality of $N = 4$ SYM for small T^2)

- Flips $\tau \to i0^+$ to $-1/\tau \to i\infty$, so will be helpful to study high T from easier low T
- But exact S-duality can’t relate low T ($\log Z \sim N^2$) & high T ($\log Z \sim N^3$) regions
- Answer: [SK, Nahmgoong] (2017)

“A simple S-duality anomaly $\propto N^3$” balances the mismatch of high/low T.

- derivation: Many ways. E.g. expand in tensor VEV, coefficients are exact functions of τ

S-duality & its anomaly

- "M-string expansion"

$$Z(\tau,v,m,\epsilon_{1,2}) = e^{-\varepsilon_0} Z_{U(1)}(\tau,m,\epsilon_{1,2})^N \sum_{n_1,\cdots,n_{N-1}=0}^{\infty} e^{-\sum_{i=1}^{N-1} n_i \alpha_i(v)} Z_{(n_i)}(\tau,m,\epsilon_{1,2})$$

simple roots of A_{N-1}

[Haghighat,Iqbal,Kozcaz,Lockhart,Vafa]

$$Z_{(n_i)} = \sum_{Y_1,\cdots,Y_{N-1};|Y_i|=n_i} \prod_{i=1}^{N} \prod_{s\in Y_i} \frac{\theta_1(\tau|\frac{E_{i,i+1}(s)-m+\epsilon_-}{2\pi i})\theta_1(\tau|\frac{E_{i,i-1}(s)+m+\epsilon_-}{2\pi i})}{\theta_1(\tau|\frac{E_{i,i}(s)+\epsilon_1}{2\pi i})\theta_1(\tau|\frac{E_{i,i}(s)-\epsilon_2}{2\pi i})}$$

$$Z_{(n_i)}\left(-\frac{1}{\tau},\frac{m}{\tau},\frac{\epsilon_{1,2}}{\tau}\right) = \exp\left[\frac{1}{4\pi i\tau}\left(\epsilon_1\epsilon_2\Omega^{ij}n_in_j - \Omega^{ij}(2m^2-2\epsilon_+^2)\rho_in_j\right)\right] Z_{n_i}(\tau,m,\epsilon_{1,2})$$

Fixed by 6d & 2d chiral anomalies [Ohmori,Shimizu,Tachikawa,Yonekura] [Intriligator] ; [Kim^2,Park] [Shimizu,Tachikawa]. So many results below apply to general 6d $N = (2,0), (1,0)$

- $\epsilon_{1,2} \to 0$ (for technical convenience): induces S-modular anomaly of $-\log Z \to f(v,m,\tau)/\epsilon_1\epsilon_2$

- $F \equiv \pi i\tau v^2 + f$. ("classical prepotential" in SW)
- $F = N\, f_{U(1)}(m,\tau) + F_{S-dual}(v,m,\tau) + \frac{N^3-N}{288} m^4 E_2(\tau)$ (definition of F_{S-dual})
- "Standard" anomaly for F_{S-dual} & "anomaly" of the "standard" one

$$\tau^2 F_{\text{S-dual}}\left(\tau_D = -\frac{1}{\tau}, v_D = v + \frac{1}{2\pi i\tau}\frac{\partial f}{\partial v}, \frac{m}{\tau}\right) = F_{\text{S-dual}}(\tau,v,m) - v\frac{\partial F_{\text{S-dual}}}{\partial v}(\tau,v,m)$$

4d limit (small τ^2)

$$\sim F_{\text{S-dual}}\left(-\frac{1}{\tau}, v_D \equiv \tau v + \frac{1}{2\pi i}\frac{\partial f}{\partial v}, m\right)$$

"Standard" S-duality in 4d Seiberg-Witten:
"magnetic dual prepotential ~ S-dual prepotential"

[4d limit of these, for $N = 2^*$ SYM, are studied in [Billo,Frau,Fucito,Lerda,Morales] (2015)]

Asymptotic free energy

- Use “dual low T setting”: anomalous part + 5d perturbative part
- Results:

$$-\log Z \sim \frac{f(\tau\to 0, v, m)}{\epsilon_1\epsilon_2} \sim \frac{i}{\epsilon_1\epsilon_2\tau}\left[\boxed{\frac{N^3m^4}{48\pi} - \frac{\pi N m^2}{12}} \boxed{\mp i\frac{N^2m^3}{12}}\right] \qquad \text{for } 0 < \pm\mathrm{Im}(m) < \frac{2\pi}{N}$$

from anomalous parts

from “low T” dual perturbative part

- At $m = 0$, SUSY enhances, #(boson) – #(fermion) = 0. Obstruct full cancelation by $m \neq 0$
- Mechanism, w/ light D0’s…? Interpretation?

- A check: imaginary part of O(m^4) can be computed directly from 6d anomaly

[Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma] (2012) [Di Pietro, Komargodski] (2014)

- 6d background gauge fields in $U(1) \subset SU(2)_L \subset SO(5)_R$: $(2\pi)^4 I_8 \to \frac{N^3}{24}F^4$
- Small temporal S^1: 5d CS terms determined by this anomaly are

$$S^{(2)}_{\rm CS} = -\frac{iN^3r_1}{96\pi^2}\int\left(A_6^4 a\wedge da\wedge da + 4A_6^3\mathcal{A}\wedge da\wedge da + 6A_6^2\mathcal{A}\wedge d\mathcal{A}\wedge da + 4A_6\mathcal{A}\wedge d\mathcal{A}\wedge\mathcal{A}\right)$$

- background: $ds^2(\mathbb{R}^4\times T^2) = \sum_{a=1,2}\left|dz_a - \frac{2i\epsilon_a}{\beta}z_a dy\right|^2 + (dx-\mu dy)^2 + dy^2 = e^{2\phi}(dy+a)^2 + h_{ij}dx^i dx^j$

$$a = \frac{1}{1+\mu^2+\frac{4\epsilon_a^2|z_a|^2}{\beta^2}}\left(-\mu dx - \frac{2\epsilon_a|z_a|^2}{\beta}d\phi_a\right) \qquad A_6 = \frac{2m}{\beta} \qquad \mathcal{A} = -A_6 a \qquad \tau = \frac{\beta}{4\pi}(\mu+i)$$

Other CFT observables?

- SUSY partition functions on curved space(time): $S^4 \times S^1$, S^5, $S^5 \times S^1$, …
- I don't know how to use string theory to compute.
- "SUSY path integral w/ 5d SYM", as deformed 5d CFT, or S^1 compactified 6d CFT (w/ instantons ~ "D0-branes")

- SUSY path integrals certainly look better than generic ones. May look finite, unambiguous, computable. But they (mostly) have ambiguities
- Singular saddle points: We need guesses to write down concrete formulae.
- Also issues on whether the series of irrelevant operators correcting them matter or not.

- $N = (2,0)$ index on $S^5 \times S^1$: $Z(S^5 \times S^1) = \mathrm{Tr}\left[(-1)^F q^{\Delta - R_1} y^{R_1 - R_2} y_1^{j_1 - j_3} y_2^{j_2 - j_3}\right]$

$$Z(S^5 \times S^1) = \frac{1}{N!} \sum_{s_1, \cdots, s_N = -\infty}^{\infty} \oint \left[\frac{d\lambda_i}{2\pi}\right] q^{-\frac{1}{2}\sum_i s_i^2} e^{-i\sum_i s_i \lambda_i} Z^{(1)}_{\mathbb{R}^4 \times T^2}(\lambda, q, y, y_{1,2}) Z^{(2)}_{\mathbb{R}^4 \times T^2} Z^{(3)}_{\mathbb{R}^4 \times T^2}$$

[e.g. "from" 5d SYM on $CP^2 \times S^1$ [Kim³,Lee]: see also [Lockhart,Vafa] [Kim³] [Qiu,Zabzine]…]

- Some consistency tests (some low orders at large N, etc. See next page for more)

6d index

- Simplest setting: turn off most of the fugacities, $y = y_1 = y_2 = 1$,
- $Tr[(-1)^F q^{\Delta - R_1}]$ commutes w/ 16 SUSY.
- I had a naïve (& completely wrong) expectation before calculation, which drove me.

"*I'll be counting ½-BPS operators, w/ vacuum Casimir energy factor on* $S^5 \times R$."

$$Z(q) \overset{?}{=} q^{\epsilon_0} Z_{\frac{1}{2}\text{-BPS}}(q) = q^{\epsilon_0} \prod_{n=1}^{N} \frac{1}{1-q^n} \qquad \epsilon_0 = -\frac{5N^3}{24} + (\text{subleading in } N)$$

computed from AdS_7 dual [Awad,Johnson] (2000)

- But we found…
  [Hee-Cheol Kim, SK] (2012)

$$q^{(\epsilon_0)_{\text{SUSY}}} \prod_{n=1}^{N} \prod_{s=0}^{\infty} \frac{1}{1-q^{n+s}} \qquad (\epsilon_0)_{\text{SUSY}} = -\frac{N^3 - N}{6} - \frac{N}{24}$$

- Whenever I got wrong, we learned something new.
- $Tr\left[(-1)^F q^{\Delta - R_1}\right] \neq Z_{1/2-BPS}$: counts local operators whose operator products yield W_{N-1} algebra [Beem, Rastelli, van Rees] (2014)
- "Supersymmetric Casimir energy": a BPS cousin [H.-C. Kim, SK] [Cassani, Martelli] [Lorenzen, Martelli] [Assel, Cassani, Di Pietro, Komargodski, Lorenzen, Martelli] [Bobev, Bullimore, H.-C. Kim] …
- $(\epsilon_0)_{SUSY}$ from AdS_7?: SUSY holographic renormalization [Genolini,Cassani,Martelli,Sparks]

5d partition functions

- 5d superconformal index: [Hee-Cheol Kim, Sung-Soo Kim, Kimyeong Lee] (2012)

$$Z(S^4 \times S^1) = \oint [d\lambda_i]\, Z^{N}_{\mathbb{R}^4 \times S^1}(\lambda, \epsilon_{1,2}, z_a) Z^{S}_{\mathbb{R}^4 \times S^1}(\cdots)$$

- Technically, a natural extension of Pestun's calculus on S^4
- Tests of UV physics, e.g. UV symmetry enhancements [H.-C.Kim, S.-S.Kim, K. Lee], [Rodriguez-Gomez, Zafrir], [Bergman, Zafrir], [Hayashi, H.-C.Kim, Nishinaka], [Taki], [Hwang, J.Kim, SK, Park], [Zafrir], [Hayashi, Zoccarato], [Bergman, Zafrir], ……

- $N^{5/2}$ scaling of $Z[S^5]$ (only needs '5d perturbative' part at large N) [Jafferis, Pufu]
- 5d Sp(N) theory w/ $N_f \leq 7$ fundamental & 1 antisymmetric hypers: N D4 + N_f D8 + O8

$$F = -\log Z_{S^5} \xrightarrow{N\to\infty} -\frac{9\sqrt{2}\pi N^{\frac{5}{2}}}{5\sqrt{8-N_f}}$$

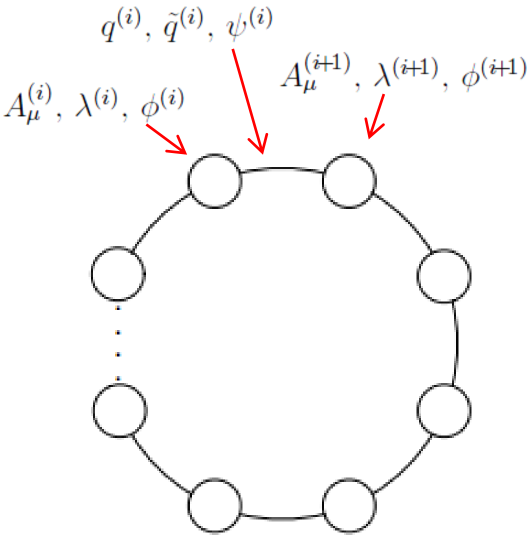
Agrees w/ AdS_6 gravity dual calculus

- Concrete answers often better define questions.
- Could they be like "Veneziano amplitude" for string theory…?

Other directions

- Many ideas in $d > 4$: Some offered by string theory. Some not intensively studied yet.

- DLCQ M5: [Aharony,Berkooz,Kachru,Seiberg,Silverstein] [Aharony,Berkooz,Seiberg] (1997)
- Compactify 6d (2,0) on light-like S^1 (vanishingly small spatial S^1 [Sen] [Seiberg])
- momentum k sector: nonrelativistic superconformal QM. UV uplift to QM for D0-D4
- Non-relativistic SC index [Nakayama]: agrees at large N w/ DLCQ SUGRA index on $AdS_7 \times S^4$ [Kim^2,Koh,Lee^2] (2011), [SK, Nahmgoong] (unpublished)



- Deconstruction: [Arkani-Hamed,Cohen,Kaplan,Karch,Motl] (2001)
- Classically, "latticize" 5d MSYM on S^1 to K sites
- 4d $N = 2$ SCFT w/ circular $U(N)^K$ quiver
- Quantum mechanically, proposes that a suitable large K limit in the Higgs branch constructs 6d $N = (2{,}0)$ theory on T^2
- Recent works [Hayling,Papageorgakis,Pomoni,Rodriguez-Gomez] (2017)

Other directions *(continued)*

- M5's from BMN matrix model…?
  - large N of BMN: non-Abelian spherical M2 or M5 [Maldacena,Sheikh-Jabbari,Raamsdonk] (2002)
  - Recent works [Asano,Ishiki,Okada,Shimasaki] (2014), [Asano,Ishiki,Shimasaki,Terashima] (2017)

- Constraining $Z[T^2]$ for 6d strings from modularity, etc. [Huang,Katz,Klemm] (2015) [Del Zotto,Lockhart] (2016), [Gu,Huang,Kashani-Poor,Klemm] (2017) …

- Studies on "instanton operators" in 5d or 6d on S^1: [Lambert,Papageorgakis,Schmidt-Sommerfeld] [Tachikawa] [Zafrir] [Yonekura] [Bergman, Rodriguez-Gomez] [Cremonesi, Ferlito, Hanany, Mekareeya] ……

- I think further interesting works can be done by fully exploring these settings: esp. w/ new SUSY skills, by "designing" good observables (computable & interesting).

Concluding remarks

- Non-Lagrangian / non-perturbative QFTs highlight the limitations of our current formulation of QFTs, or of our technical controls over them.
- QFTs in d>4 encode such limitations in essential manners (e.g. E/M particles & T^2)

- Despite many constraints, we can explore some of their quantum questions.

- 6d/5d string/soliton physics, and their implications to 6d/5d QFTs
- Certain proposals for 5d/6d CFT observables
- Summary of other approaches (may be much more interesting in the future)

- Further questions:

- Microscopic description? Note: 4d Argyres-Douglas theories [Maruyoshi,Song,Agarwal] (2016))Perhaps without manifest Lorentz symmetry…?
- DLCQ, deconstruction, …
- More about 5d: e.g. more examples, “classification” [Jefferson, H.-C. Kim, Vafa, Zafrir] (2017)
- Better understanding on 5d magnetic monopole strings [Haghighat] [J,K,V,Z]