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Quantum Chaos of pure states in Random Matrices and in the SYK model

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Based on work arXiv:1901.02025
+ work in progress with Tomoki Nosaka(KIAS)

Introduction/Motivation

- In this talk, we consider the **time evolution of pure states**

- Study black hole microstate dynamics (non-perturbatively)
  [cf: Hartman-Maldacena, Cooper-Rozali-Swingle-Raamsdonk-Waddell-Wakeham]

- To study the time evolution after projection measurement
  [Shiba-TN-Takayanagi-Watanabe, 16] [Maldacena-Stanford-Yang, 17]

- To understand state dependence of BH interiors
  (by deforming the Hamiltonian in a state dependent way)
  [cf: Kourkoulou-Maldacena, Almheiri-Mousatov-Shyani]

Pure State dynamics in RMT and in the SYK

- Generically, a time evolved state $|\psi(t)\rangle$ is a complicated superposition of vectors:

$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle = \sum_i c_i(t)|\psi_i\rangle$$

- What we consider in this talk is the (square of)amplitude

$$\langle |c_i(t)|^2 \rangle_{\text{ensemble}}$$

in **Random Matrix theory** (analytically) and

in the (mass deformed) **SYK model** (numerically).

It is related to the spectral form factor, which is diagnostic of **quantum chaos** [Berry] and brought to BH physics by [CGHPSSSST]

[Papadodimas-Raju]

Return(Evolution) Amplitude

- The overlap between time evolved states and the initial states

$$g_R(t) = |\langle\psi_0| e^{-iHt} |\psi_0\rangle|^2$$

We call this **return amplitude** according to [Cardy, 14]

- We can also consider the amplitude to evolve to initially orthogonal states:

$$g_{ev}(t) = |\langle\psi_1|e^{-iHt}|\psi_0\rangle|^2 \qquad \text{where} \quad \langle\psi_1|\psi_0\rangle = 0$$

We call this **evolution amplitude**.

Return/Evolution Amplitude in Random Matrices

The results are : [TN, 19]

$$\langle g_R(t)\rangle_{\mathrm{GUE}} = \frac{1}{L(L+1)}(\langle g(t)\rangle_{\mathrm{GUE}} + L)$$

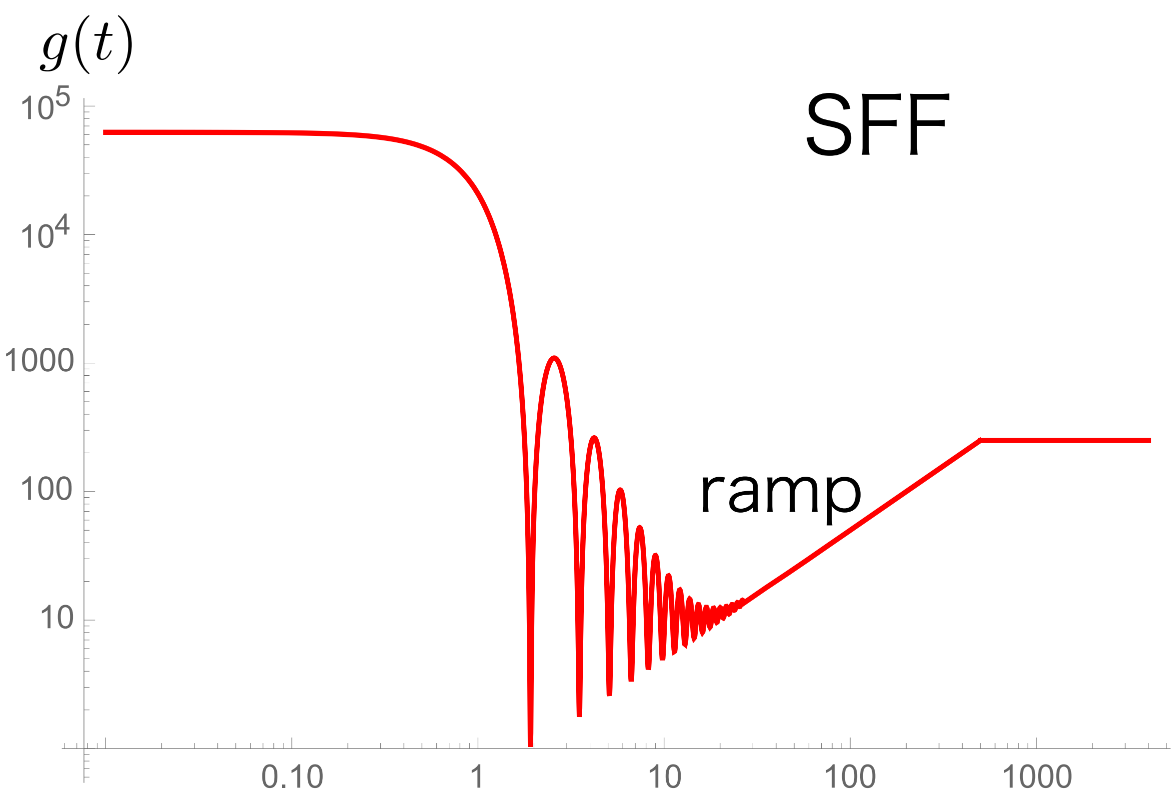
$$\langle g_{ev}(t)\rangle_{\mathrm{GUE}} = \frac{1}{L^2-1}(L - \frac{1}{L}\langle g(t)\rangle_{\mathrm{GUE}})$$

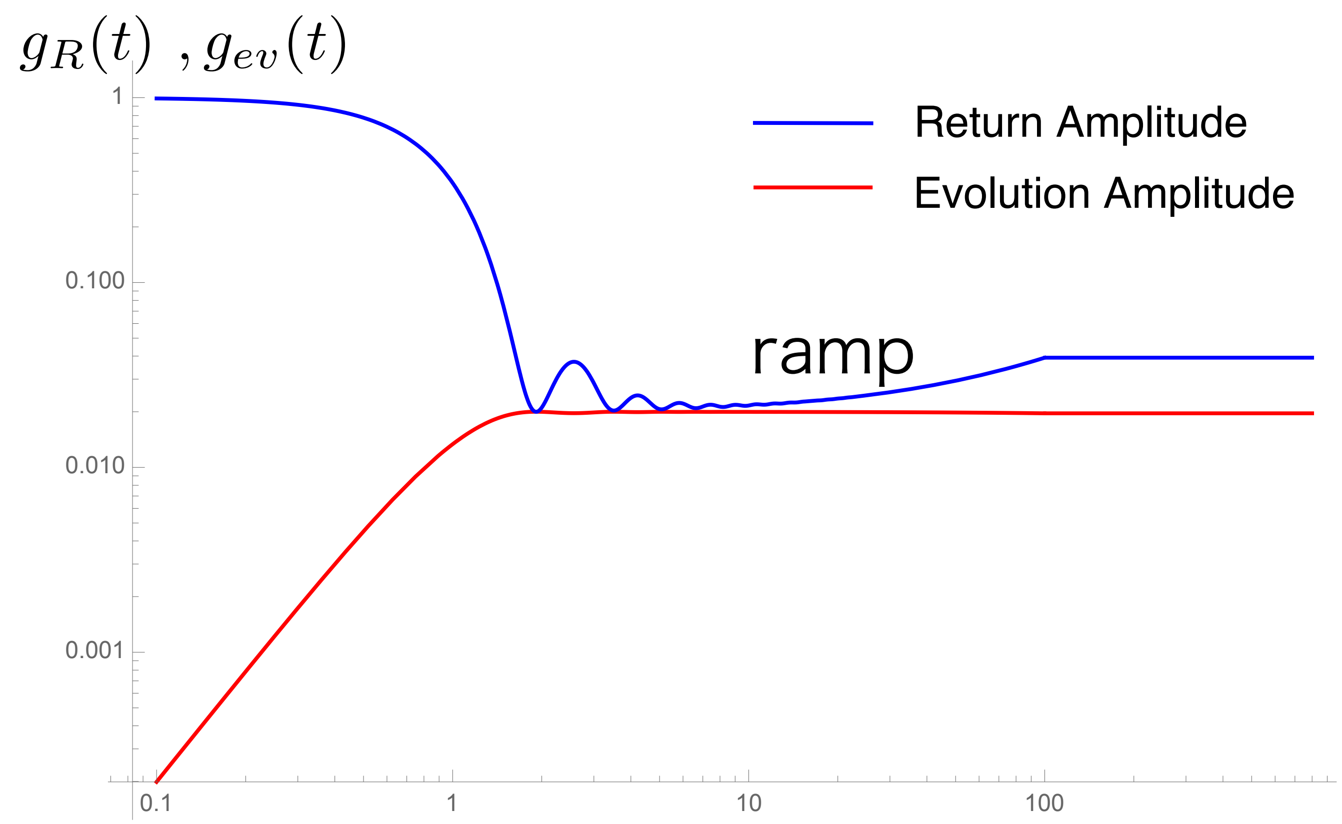
(We can also evaluate in GOE/GSE ensembles)

where $g(t) = \mathrm{Tr}(e^{-iHt})$ is the **spectral form factor** (SFF), which is

$$\langle g(t)\rangle_{GUE} = L^2\frac{J_1(2t)^2}{t^2} - L(1-\frac{t}{2L})\theta(2L-t) + L$$

[Cotler - Hunter Jones - Liu-Yoshida, 16]





ramp: Fourier transform of the long range **level repulsion**

Pure states in the SYK model

The model :

[Sachdev-Ye 93] [Kitaev 14,15]

N Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$

$$H_{SYK} = \sum_{i<j<k<l} J_{ijkl}\psi_i\psi_j\psi_k\psi_l$$, with $\langle J_{ijkl}\rangle_J = 0$ and $\langle J_{ijkl}^2\rangle_J = \frac{3!J^2}{N^3}$

Spin operators:

$$S_k = -2i\psi_{2k-1}\psi_{2k} \qquad (k = 1, \cdots, N/2)$$

Pure states:

[Kourkoulou-Maldacena 17]

$|B_s\rangle$: simultaneous eigenstates of S_k $(k = 1, \cdots, N/2)$

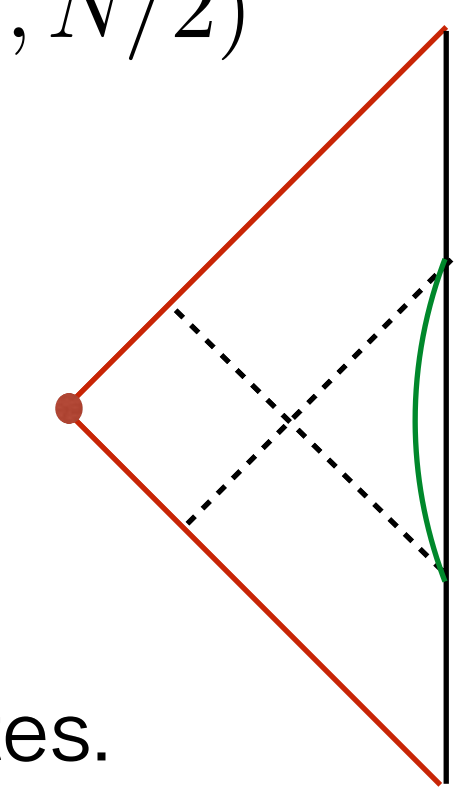
$S_k |B_s\rangle = s_k |B_s\rangle$ ($2^{\frac{N}{2}}$ states, form a basis)

- Lower energy states by Euclidean evolution

$$|B_s(\beta)\rangle = e^{-\frac{\beta}{2}H_{SYK}} |B_s\rangle$$

- Have a NAdS2 gravity interpretation

Projection measurement of the Left CFT in TFD states.



[Shiba-TN-Takayanagi-Watanabe, 16] [Maldacena-Stanford-Yang, 17]

[Goel-Lam-Turiaci-Verlinde, 18]

Return Amplitudes in the SYK

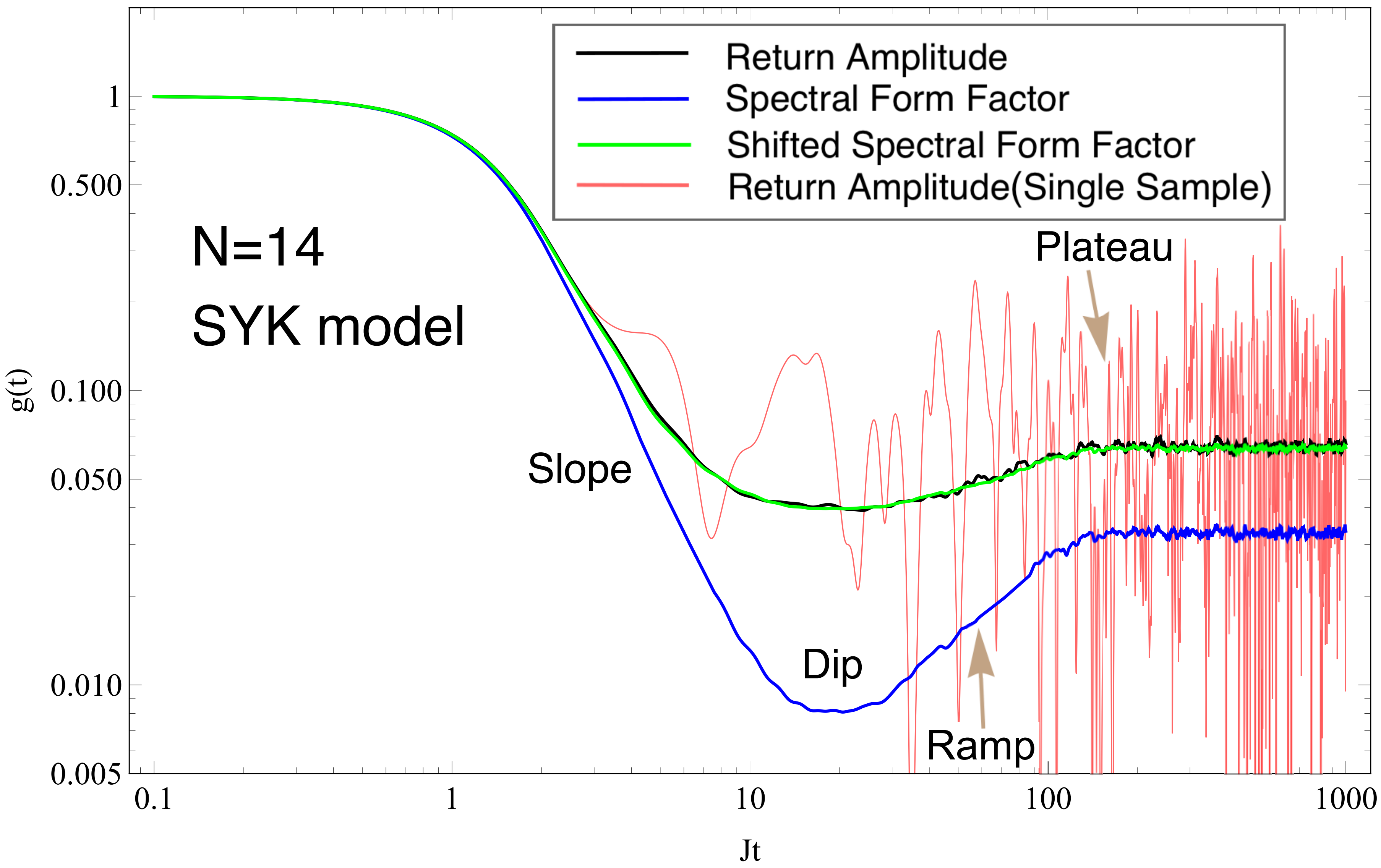
[TN, 19]

- finite N, Exactly diagonalize. $\beta = 1.5$, average over 1500 samples

$$g_p(t;\beta) = |\langle B_{\uparrow\uparrow\uparrow\cdots}|e^{-iH_{SYK}t}e^{-\beta H_{SYK}}|B_{\uparrow\uparrow\uparrow\cdots}\rangle|^2$$

return amplitude: $\langle g_p(t;\beta)\rangle_J$

shifted SFF: $\dfrac{\langle g(t;\beta)\rangle_J + \langle \mathrm{Tr}(e^{-2\beta H_{SYK}})\rangle_J}{\langle g(0;\beta)\rangle_J + \langle \mathrm{Tr}(e^{-2\beta H_{SYK}})\rangle_J}$

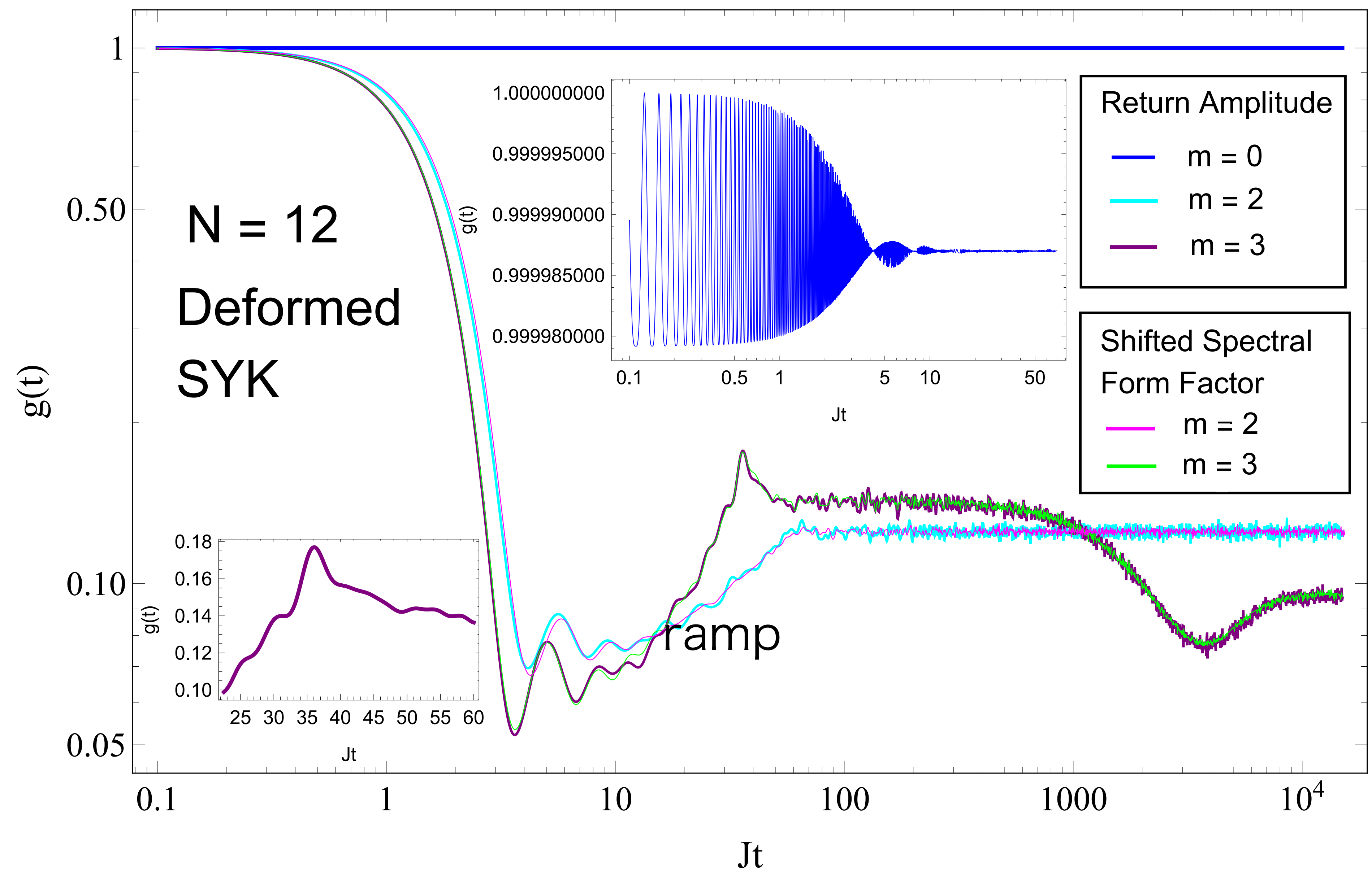


Return amplitude in mass deformed SYK [TN, 19]

- State dependent defamation introduced in [Kourkoulou-Maldacena]

$$H_{def} = H_{SYK} + \mu H_M \qquad H_M = -\frac{1}{2}\sum_{k=1}^{N/2} s_k S_k = \sum_{k=1}^{N/2} i s_k \psi_{2k-1}\psi_{2k}$$

- $\beta = 0 \quad \mu = 50$, average over 2000 samples
- $m = 0 : |B_{\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow}\rangle \qquad m = 2 : \ |B_{\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow}\rangle \qquad m = 3 : |B_{\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow}\rangle$



Conclusion

- We derive the analytic expression that relates Return Amplitude to the Spectral Form Factor
- A simple relation between the spectral form factor and the return amplitude
- The SYK model also obeys the same relation.
- State dependent deformation keeps the value of RA, but if there is an error they show random matrix behaviors.

Future problem

- Understanding results analytically from collective fields G, Σ (that exist even in infinite temperature)
- Study Finite temperature in the deformed SYK further
- Relation to the generation of entanglement.
- Relation between Hawking-Page like transition and integrable/chaotic transition

[cf:Maldacena-Qi 18 ,Garcia Garcia-Nosaka-Rosa-Verbaarschot 19]

Thank you !