Irrelevant current-current deformations and holography

Monica Guica

IphT, CEA Saclay & Stockholm U. & Nordita

based on 1710.08415

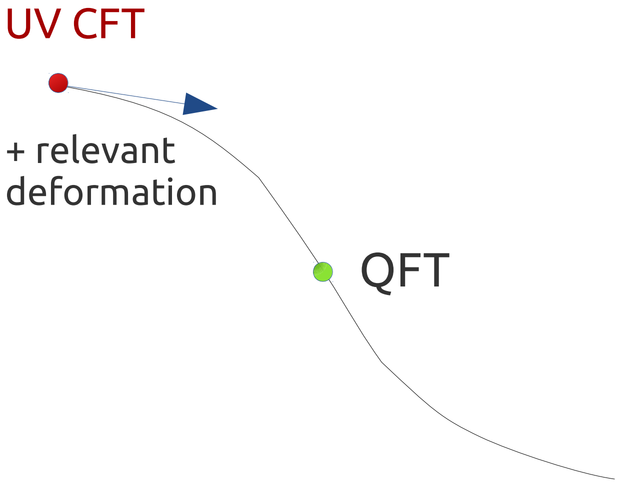
1902.01434

1803.09753 : with A. Bzowski

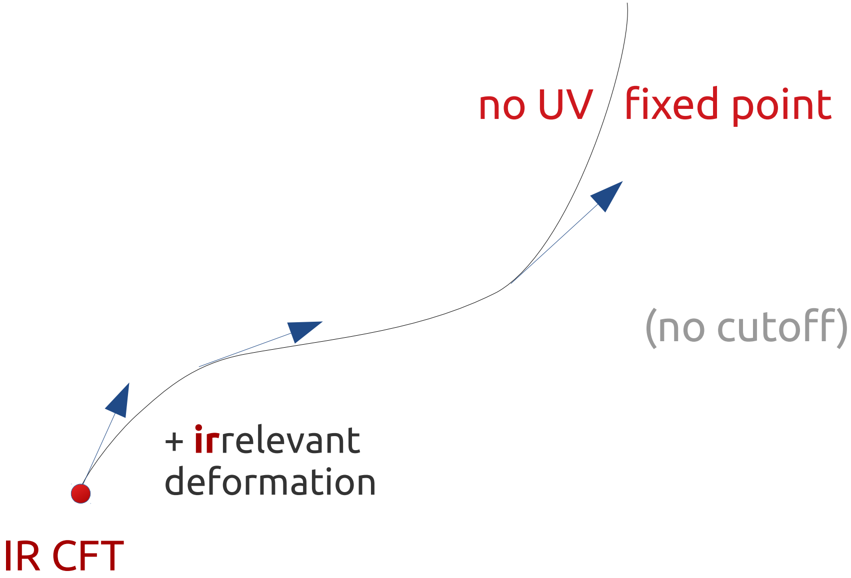
1906.11251: with R. Monten

Motivation

Usual framework: local, UV complete QFTs



Examples of non-local, UV complete QFTs ?



- Quantum gravity ?
- Holography in non-asymptotically AdS spacetimes

TT̄ - deformed QFTs

- universal deformation of 2d QFTs (see Mark Mezei's talk)

$$\frac{\partial S}{\partial \mu} = \int d^2z\, \underbrace{(T_{zz}T_{\bar{z}\bar{z}} - T_{z\bar{z}}^2)_\mu}_{\text{``}T\bar{T}\text{''}}$$

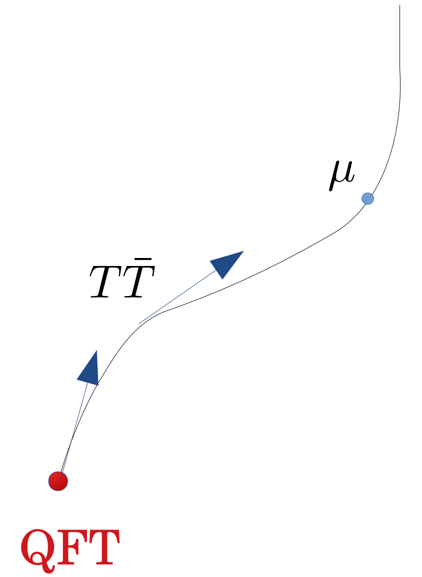
- deformation irrelevant (dim = (2,2)) but integrable

  finite size spectrum, partition function, thermodynamics

  Smirnov & Zamolodchikov, Cavaglia et al, Cardy

- deformed theory non-local (scale μ) but argued UV complete

  S-matrix (2→ 2): $\mathcal{S}_\mu = e^{\frac{i\mu s}{4}}\mathcal{S}_0$ Dubovsky et al.



- special case of more general Smirnov-Zamolodchikov (current-current) deformations

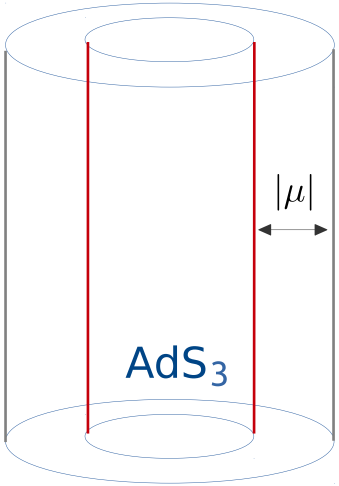
$$\frac{\partial S}{\partial \mu} = \int d^2z\, (J^A_{z\ldots}J^B_{\bar{z}\ldots} - J^B_{z\ldots}J^A_{\bar{z}\ldots})_\mu$$

$J^A,\, J^B$: (higher spin) conserved currents

Holography: why interesting

Double-trace TT̄ deformation

- universal , ∀ large c CFT



AdS_3 with mixed bnd. conditions at ∞

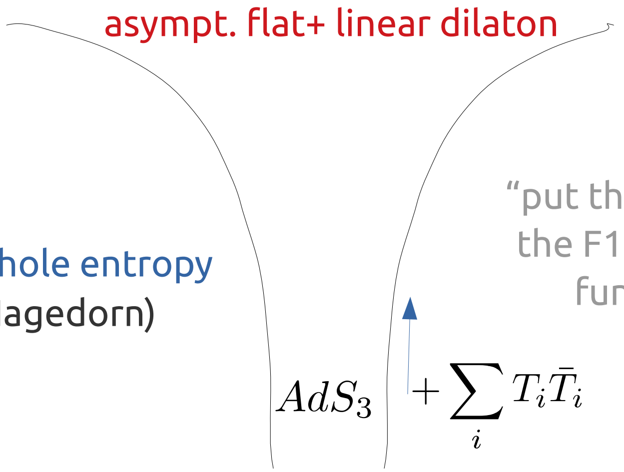
$\mu < 0$ $\approx$ Dirichlet at finite radius

McGough, Mezei, Verlinde

Single-trace TT̄ deformation $\sum_{i=1}^{p} T_i \bar{T}_i$

- near horizon NS5-F1 → $\mathcal{M}^p / S_p$

✓ black hole entropy
(Hagedorn)

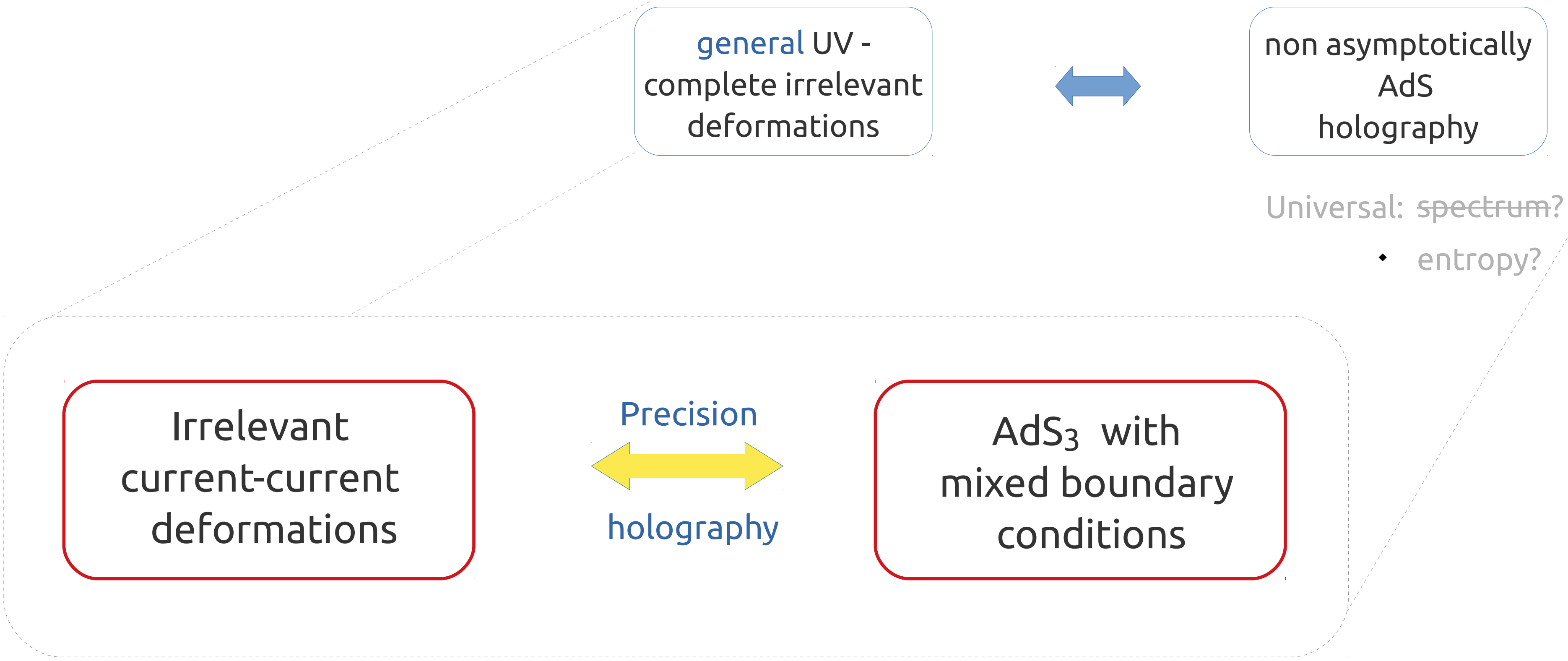


“put the 1 back in the F1 harmonic function”

Giveon, Itzhaki, Kutasov

Generalisations?

- tractable single-trace irrelevant flows with no UV fixed point?



general UV - complete irrelevant deformations

non asymptotically AdS holography

Universal: ~~spectrum~~?

- entropy?

Irrelevant current-current deformations

Precision holography

AdS$_3$ with mixed boundary conditions

- observables and precise holographic dictionary?
- (new kinds of) symmetries?
- constraints on the non-local structure (e.g. star product)?

Plan

- $J\bar{T}$ deformed CFTs
    - spectrum
    - correlation functions

field theory

- holography for $J\bar{T}$ deformed CFTs
- derivation & generalization of the holographic dictionary for $T\bar{T}$ - deformed CFTs

(precision)
holography

- conclusions

JT̄ deformed CFTs

- universal deformation of 2d QFTs with a $U(1)$ current

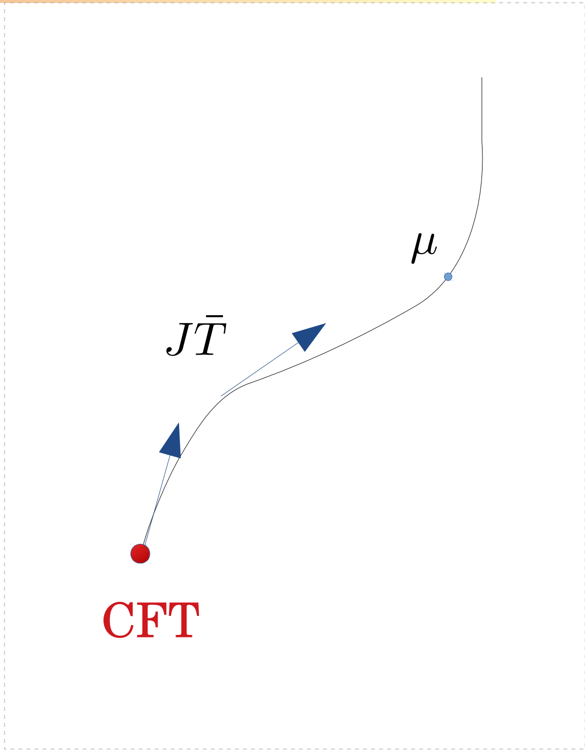
$$\frac{\partial S_{J\bar{T}}}{\partial \mu} = \int d^2 z \, \underset{\text{“}J\bar{T}\text{”}}{(J_z T_{\bar{z}\bar{z}} - J_{\bar{z}} T_{z\bar{z}})}_{\underset{(1,2)}{\mu}}$$

- breaks Lorentz invariance $T_{z\bar{z}} \neq T_{\bar{z}z} (= 0)$

- preserves $\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local!}}$

$$\boxed{CFT_1!}$$

- deformation irrelevant but integrable (UV complete ?)



★ off-shell observables

★ holographic duals to extremal black holes

Holography for $J\bar{T}$ deformed CFTs

Double-trace $J\bar{T}$ deformation

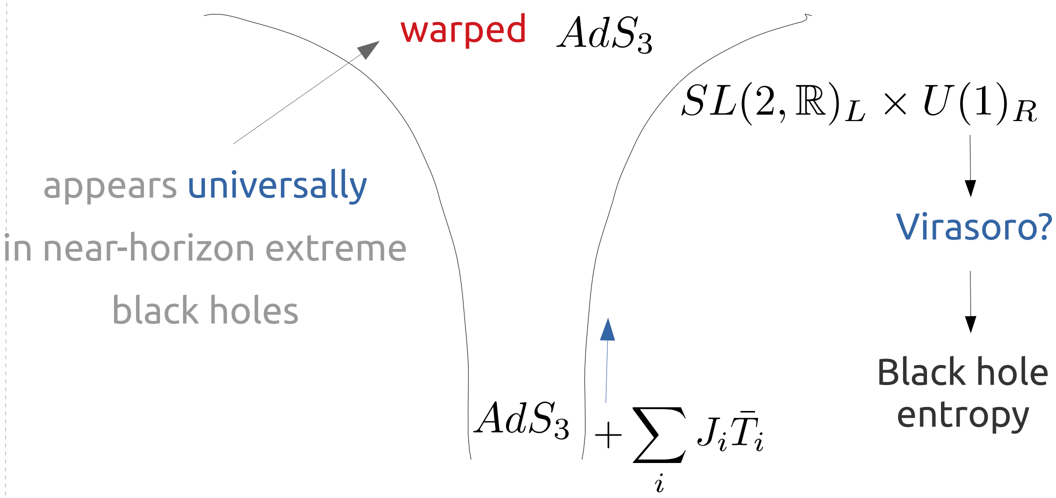
- universal , ∀ large c CFT

mixed boundary conditions on AdS$_3$

metric sector ≈ Compere – Song - Strominger b.c.

Single-trace $J\bar{T}$ deformation $\sum_{i=1}^{p} J_i\bar{T}_i$

- near horizon NS5-F1 → $\mathcal{M}^p/S_p$



warped AdS_3

appears universally in near-horizon extreme black holes

$SL(2,\mathbb{R})_L \times U(1)_R$

Virasoro?

Black hole entropy

AdS_3 $+\sum_i J_i\bar{T}_i$

Apolo, Song
Chakraborty, Giveon, Kutasov

Generalisations?

Field theory analysis of $J\bar{T}$

The finite-size spectrum

- JT̄ factorizes in translationally invariant states

$$\langle J\bar{T}\rangle = \langle J_z T_{\bar{z}\bar{z}} - J_{\bar{z}} T_{z\bar{z}}\rangle = \langle J_z\rangle\langle T_{\bar{z}\bar{z}}\rangle - \langle J_{\bar{z}}\rangle\langle T_{z\bar{z}}\rangle$$

- cylinder $z = \varphi + i\tau \ , \qquad \varphi \sim \varphi + R$

- eigenstates $|n\rangle$ of energy E_n , momentum P_n and charge Q_n

- dependence on μ, R

chiral anomaly

$$\frac{\partial E_n}{\partial \mu} = R\langle n|J\bar{T}|n\rangle \ , \qquad P_n R \in \mathbb{Z} \ , \qquad \frac{\partial Q_n}{\partial \mu} = \frac{k}{4\pi} R\langle n|\bar{T}|n\rangle$$

- replace $T_{\tau\tau} \sim \frac{E_n}{R} \ , \ T_{\tau\varphi} \sim \frac{P_n}{R} \ , \ J_\tau \sim \frac{Q_n}{R}$ etc. $\boxed{J_\varphi = ?}$ → make J chiral $J_{\bar{z}} = 0$

- equations determine spectrum universally in terms of initial $E, P, Q \to h, \bar{h}, q$

The finite-size spectrum

- solution for $E_{L,R} = \frac{1}{2}(E \pm P)$

$$E_R = \frac{4\pi}{\mu^2 k}\left(R - \mu q + \sqrt{(R-\mu q)^2 - \mu^2 k \left(\bar{h} - \frac{c}{24}\right)}\right)$$

$$E_L = E_R + \frac{2\pi(h-\bar{h})}{R} \qquad\qquad Q = q + \frac{\mu k}{4\pi} E_R$$

exact spectrum

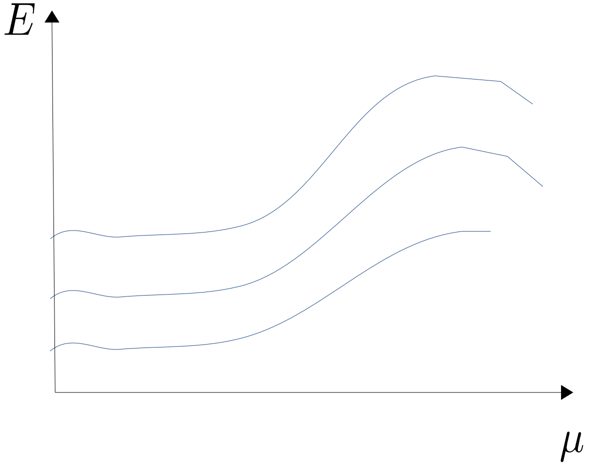
- breaks down for $\bar{h} - \frac{c}{24} > \frac{1}{\mu^2 k}(R-\mu q)^2$ $\quad (\sim \mu < 0 \;\; T\bar{T})$

- superluminal propagation → CTCs on a compact space

Cooper, Dubovsky, Moshen

- thermodynamics: smoothly deformed levels → unchanged density of states

$$S_{Cardy}(h, \bar{h}) \to S_{Cardy}(h(E,P,\mu), \bar{h}(E,P,\mu))$$



Correlation functions

- general structure: $SL(2,\mathbb{R})_L \times U(1)_R$ → CFT$_1$ structure with local operators $\mathcal{O}_{\bar{p}}(z)$

  local non-local → Fourier transform

MG, Skenderis, Taylor, van Rees

- in original CFT $\mathcal{O}_{\bar{p}}(z) = \int d\bar{z}\, e^{-i\bar{p}\bar{z}}\mathcal{O}(z,\bar{z})$

- two-point functions: $$\langle \mathcal{O}_{i,\bar{p}_1,q_1}(z_1)\mathcal{O}_{j,\bar{p}_2,q_2}(z_2)\rangle = \frac{\overbrace{\mathcal{N}_{ij}(\bar{p}_1)}^{\text{normalization}\ \propto\ \bar{p}_1^{2\bar{h}-1}\,\delta_{ij}} \cdot \delta(\bar{p}_1+\bar{p}_2)\delta(q_1+q_2)}{z_{12}^{2h(\mu\bar{p}_1)}}$$

Fourier transform antiholomorphic part CFT 3pf

- three-point functions: $$\langle \mathcal{O}_{i,\bar{p}_1,q_1}(z_1)\mathcal{O}_{j,\bar{p}_2,q_2}(z_2)\mathcal{O}_{k,\bar{p}_3,q_3}(z_3)\rangle = \frac{\mathcal{C}_{ijk}(\mu\bar{p}_l)\mathcal{K}(\bar{p}_l)\cdot\delta(\sum\bar{p}_l)\delta(\sum q_l)}{z_{12}^{2h_{ij;k}(\mu\bar{p}_l)}\,z_{23}^{2h_{jk;i}(\mu\bar{p}_l)}\,z_{13}^{2h_{ik;j}(\mu\bar{p}_l)}}$$

- higher-point functions can be constructed via OPE

- $h(\mu\bar{p}), C_{ijk}(\mu\bar{p}_l)$ → can be in principle computed to all orders using conformal perturbation theory

  → completely specifies the theory

Correlation functions

- general structure: $SL(2,\mathbb{R})_L \times U(1)_R$ → CFT$_1$ structure with local operators $\mathcal{O}_{\bar p}(z)$

  local non-local → Fourier transform

MG, Skenderis, Taylor, van Rees

- in original CFT $\mathcal{O}_{\bar p}(z) = \int d\bar z \, e^{-i\bar p \bar z}\mathcal{O}(z,\bar z)$

- two-point functions: $$\langle \mathcal{O}_{i,\bar p_1,q_1}(z_1)\mathcal{O}_{j,\bar p_2,q_2}(z_2)\rangle = \frac{\overbrace{\mathcal{N}_{ij}(\bar p_1)}^{\text{normalization } \propto \bar p^{2\bar h-1}} \cdot \delta(\bar p_1+\bar p_2)\delta(q_1+q_2)}{z_{12}^{2h(\mu \bar p_1)}}$$

conformal data $f(\mu\bar p)$

- three-point functions: $$\langle \mathcal{O}_{i,\bar p_1,q_1}(z_1)\mathcal{O}_{j,\bar p_2,q_2}(z_2)\mathcal{O}_{k,\bar p_3,q_3}(z_3)\rangle = \frac{\mathcal{C}_{ijk}(\mu\bar p_l)\mathcal{K}(\bar p_l)\cdot\delta(\sum \bar p_l)\delta(\sum q_l)}{z_{12}^{2h_{ij;k}(\mu\bar p_l)}\, z_{23}^{2h_{jk;i}(\mu\bar p_l)}\, z_{13}^{2h_{ik;j}(\mu\bar p_l)}}$$

- higher-point functions can be constructed via OPE

- $h(\mu\bar p), C_{ijk}(\mu\bar p_l)$ → can be in principle computed to all orders using conformal perturbation theory

  → completely specifies the theory

The spectrum of conformal dimensions

- CFT: energies on cylinder $\xrightarrow{\text{exp map}}$ conformal dimensions on the plane
- $SL(2,\mathbb{R})_L \times U(1)_R$, but the cylinder identification $\varphi \sim \varphi + R$ breaks $SL(2,\mathbb{R})_L$ down to $U(1)$
- to restore it, perform an infinite boost $x^\pm \to e^{\pm\gamma} x^\pm$, where $x^\pm = \varphi \pm t$

finite

$$\begin{cases} x^+ \sim x^+ + R\, e^{-\gamma} \\ x^- \sim x^- + R\, e^{\gamma} \end{cases}$$

infinite

- map to the plane and rewrite E_L in terms of $E_R \equiv \bar{p}$

$$\boxed{h(\mu) = h + \frac{q}{2\pi}\,\mu\bar{p} + \frac{k}{16\pi^2}\,\mu^2\bar{p}^2} \qquad \boxed{q(\mu) = q + \frac{k}{4\pi}\,\mu\bar{p}}$$

exact dimensions and charges

- no obvious pathology (JT̄ on the plane ✓)
- $h(\mu) - \dfrac{q^2(\mu)}{k} = const$ → spectral flow with momentum-dependent parameter
- matches CPT up to second order

Operator mixing

- degenerate operators mix: usually, hard problem → here, tractable
- CPT $\langle \mathcal{O}_1 \mathcal{O}_2 \ldots \rangle_\mu = \langle \mathcal{O}_1 \mathcal{O}_2 \ldots e^{\mu \int J\bar{T}} \rangle_0$ ← determined by Ward identities
- $(z, \bar{p})$ basis → large degeneracy (momentum space mixes only $SL(2,\mathbb{R})$ descendants)

original CFT

decompose into $SL(2,\mathbb{R})_R$ reps labeled by n : "$\bar{T}^n \mathcal{O}$"

$$\mathcal{O} \in Vir_L(h) \times Vir_R(\bar{h}) \quad \rightarrow \quad \mathcal{O}_{\bar{p}} \quad (\bar{T}\mathcal{O})_{\bar{p}} \quad (\bar{T}^2\mathcal{O})_{\bar{p}} \quad \ldots \quad \Big\} \text{ same } h, \forall \bar{p}, n$$

- for generic operators, mixing only between $\bar{T}^n \mathcal{O}$ and $\bar{T}^m \mathcal{O}$ with $m < n$ → tractable!
- after deformation: degeneracy in $\bar{p}$ lifted, but not in n → modified Virasoro-like symmetry?
- need also $C_{\mathcal{O}_1 \mathcal{O}_2 \bar{T}^n \mathcal{O}}$ ← scheme – dependence in CPT?
- $\Delta C_{\mathcal{O}_1 \mathcal{O}_2 \bar{T}^n \mathcal{O}} = 0$ is a consistent choice → determines all correlation functions

w.i.p. with A. Bombini, A Galliani

Holography

Double-trace deformations in AdS/CFT

- $J\bar{T}$ is a double-trace deformation → mixed boundary conditions for dual bulk fields
- e.g. scalar

$$\Phi = \mathcal{J}\; z^{d-\Delta} + \ldots + \langle \mathcal{O} \rangle\; z^{\Delta} + \ldots$$

source (fixed) vev (fluctuates)

- $S_\mu = S_{CFT} + \mu \int \mathcal{O}^2$
- variational principle (equivalent to Hubbard-Stratonovich)

$$\delta S_\mu = \delta S_{CFT} - \delta\left(\mu \int \mathcal{O}^2\right) = \int \mathcal{O}\delta\mathcal{J} - \mu \int \delta\mathcal{O}^2 = \int \overset{\text{new vev}}{\mathcal{O}}\,\delta(\overbrace{\mathcal{J} - 2\mu\mathcal{O}}^{\text{new source }\tilde{\mathcal{J}}})$$

large N

$J\bar{T}$

- introduce sources: $J^\alpha \leftrightarrow \mathrm{a}_\alpha \qquad T^a{}_\alpha \leftrightarrow e^a{}_\alpha$
- covariantize: $\mu \int d^2z\, J\bar{T} = \mu_a \int d^2x\, e\, T^a{}_\alpha J^\alpha$
- variational principle

$$\delta S_\mu = \delta S_{CFT} - \delta S_{J\bar{T}} = \int d^2x\, [\overbrace{eT^a{}_\alpha \delta e^\alpha_a + eJ^\alpha \delta \mathrm{a}_\alpha}^{\text{CFT}} - \delta(\overbrace{\mu_a T^a{}_\alpha J^\alpha e}^{\text{deformation}})] = \int d^2x\, \tilde{e}(\overbrace{\tilde{T}^a{}_\alpha \delta \tilde{e}^\alpha_a + \tilde{J}^\alpha \delta \mathrm{a}_\alpha}^{\text{new sources \& vevs}})$$

The J$\bar{T}$ holographic dictionary

- new sources $\quad \tilde{e}^{\alpha}_{a} = e^{\alpha}_{a} - \mu_a \langle J^{\alpha} \rangle \; , \qquad \tilde{\mathrm{a}}_{\alpha} = \mathrm{a}_{\alpha} - \mu_a \langle T^a_{\alpha} \rangle$
- new vevs $\quad \tilde{T}^a{}_{\alpha} = T^a{}_{\alpha} + (e^a_{\alpha} + \mu_{\alpha} J^a)\, \mu_b T^b{}_{\beta} J^{\beta} \; , \qquad \tilde{J}^{\alpha} = J^{\alpha}$

large N
field theory

Holography:

$(T^a{}_{\alpha}, e^a{}_{\alpha})$ modelled by 3d Einstein gravity

$(J^{\alpha}, \mathrm{a}_{\alpha})$ $U(1)$ Chern-Simons gauge field

non-dynamical

- AdS_3 gravity with mixed boundary conditions Compere-Song-Strominger-like
- use usual AdS/CFT dictionary to compute $J^{\alpha},\, T^a_{\alpha}$ → plug into above for $\tilde{J}^{\alpha},\, \tilde{T}^a_{\alpha}$
- perfect match between energies of black holes and the defomed CFT spectrum ✓

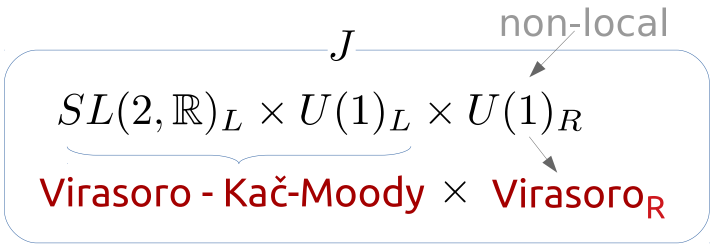
Asymptotic symmetry group

- algebra of diffeos + gauge transformations that preserve $\tilde{e}_a^\alpha = e_a^\alpha - \mu_a \langle J^\alpha \rangle \ , \qquad \tilde{\mathrm{a}}_\alpha = \mathrm{a}_\alpha - \mu_a \langle T_\alpha^a \rangle$

- background dependent → use modified Lie bracket

Barnich, Troessaert

- asymptotic symmetry group



non-local

$$\overset{J}{\overbrace{\underbrace{SL(2,\mathbb{R})_L \times U(1)_L}_{\text{Virasoro - Kač-Moody}} \times U(1)_R}}$$

Virasoro - Kač-Moody × Virasoro_R

- Virasoro_R → implemented by $g(\bar{z} - \mu \int^z J(z')dz')$: non-local, “state-dependent’’ deformation

of original Virasoro

- field theory interpretation ?

- different structure than seen in field theory!

$$\bar{\mathbf{T}} \ : \quad q_{\bar{T}} = \frac{k\mu\bar{p}}{4\pi} \ , \qquad h_{\bar{T}} = \frac{k\mu^2\bar{p}^2}{16\pi^2}$$

TT̄

Holographic dictionary for T$\bar{\text{T}}$ - deformed CFTs

- variational principle approach:

$$\delta S_{CFT} - \Delta\mu\,\delta S_{T\bar{T}} = \int d^2x\,\overbrace{\sqrt{\gamma}\,T_{\alpha\beta}\,\delta\gamma^{\alpha\beta}}^{\text{CFT}} - \Delta\mu\int d^2x\,\delta(\overbrace{\sqrt{\gamma}\,\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}T_{\alpha\beta}T_{\gamma\delta}}^{\text{deformation}}) = \int d^2x\,\overbrace{\sqrt{\tilde{\gamma}}\,\tilde{T}^{\alpha\beta}\delta\tilde{\gamma}^{\alpha\beta}}^{\text{new sources \& vevs}}$$

- flow equations

$$\partial_\mu\gamma_{\alpha\beta} = -2(T_{\alpha\beta} - \gamma_{\alpha\beta}\,T) \equiv -2\,\hat{T}_{\alpha\beta} \qquad \partial_\mu\hat{T}_{\alpha\beta} = -\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta} \qquad \partial_\mu(\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta}) = 0$$

- exact solution

$$\boxed{\begin{gathered}\gamma_{\alpha\beta}(\mu) = \gamma_{\alpha\beta}(0) - \mu\hat{T}_{\alpha\beta}(0) + \mu^2\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta}(0)\\ \hat{T}_{\alpha\beta}(\mu) = \hat{T}_{\alpha\beta}(0) - \mu\,\hat{T}_\alpha{}^\gamma\hat{T}_{\gamma\beta}(0)\end{gathered}}$$

- both signs of μ
- other (matter) vevs can be on
- large N field theory

Holography → Fefferman Graham expansion $g^{(0)}_{\alpha\beta} \leftrightarrow \gamma_{\alpha\beta}(0)\,,\qquad g^{(2)}_{\alpha\beta} \leftrightarrow 8\pi G\ell\,\hat{T}_{\alpha\beta}(0)$

$$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \left(\frac{g^{(0)}_{\alpha\beta}}{\rho} + g^{(2)}_{\alpha\beta} + \ldots\right)dx^\alpha dx^\beta$$

- mixed non-linear boundary conditions
- only depend on asymptotics

- linearized matter fields → sources ($\Rightarrow$ boundary conditions) unaffected $\sqrt{\gamma}\,\mathcal{O}\,\delta\mathcal{J} = \sqrt{\tilde{\gamma}}\,\tilde{\mathcal{O}}\,\delta\tilde{\mathcal{J}}$

Pure gravity

- pure 3d gravity → Fefferman-Graham expansion truncates $ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \frac{g^{(0)}_{\alpha\beta} + \rho\, g^{(2)}_{\alpha\beta} + \rho^2\, g^{(2)}_{\alpha}{}^{\gamma} g^{(2)}_{\gamma\beta}}{\rho} dx^\alpha dx^\beta$

- mixed boundary conditions at ∞ → coincide precisely with Dirichlet at $\boxed{\rho_c = -\frac{\mu}{4\pi G\ell}}$ $\mu < 0$

coincides with

McGough, Mezei, Verlinde

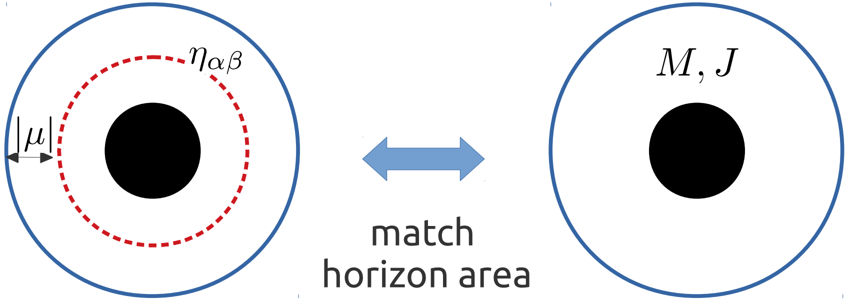
$$\gamma_{\alpha\beta}(\mu) = \gamma_{\alpha\beta}(0) - \mu\,\hat{T}_{\alpha\beta}(0) + \mu^2 \hat{T}_{\alpha}{}^{\gamma}\hat{T}_{\gamma\beta}(0) = fixed = \eta_{\alpha\beta}\ (\phi, T)$$

- deformed stress tensor → coincides precisely with Brown-York + counterterm at ρ_c

$$\hat{T}_{\alpha\beta}(\mu) = \hat{T}_{\alpha\beta}(0) - \mu\,\hat{T}_{\alpha}{}^{\gamma}\hat{T}_{\gamma\beta}(0)$$

- fixed by variational principle → no ambiguity!

- energy $E(\mu) = \int d\phi\, T_{TT}(\mu)$ match to field theory : $E(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu M}{R} + \frac{4\mu^2 J^2}{R^2}}\right)$



deformed state

undeformed state

- McGough et al computed energy on

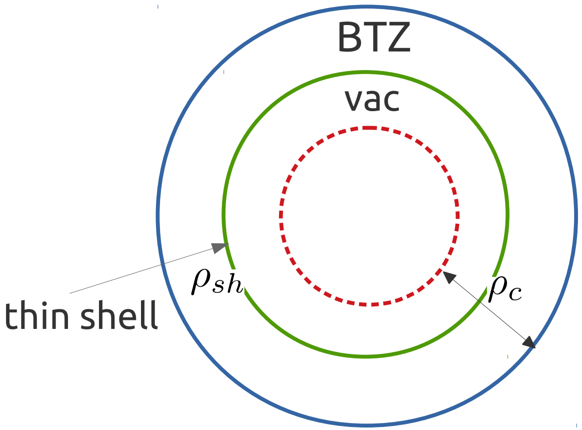
undeformed BTZ at Schwarzschild $r_c \sim |\mu|^{-\frac{1}{2}}$

- same as mixed phase space + FG coordinate

$\rho_c \sim |\mu|$

Adding matter

- difference between mixed at infinity and Dirichlet at finite radial distance for $\mu < 0$



- shell outside ρ_c → only mixed bnd. cond. give correct energy

  → configurations outside this surface ✓

  → 2d $T\overline{T}$ describes entire spacetime : UV completeness

  integrability

- imaginary energies → breakdown of coordinate transformation
- used to make $\gamma_{\alpha\beta}(\mu) = \eta_{\alpha\beta}$, which only depends on the asymptotic value of the metric

Take-home: universal formula for energy ↔ universal asymptotic behaviour

- McGough et al picture still holds in typical high energy states

Asymptotic symmetries

- the $T\bar{T}$ deformation breaks conformal symmetries to $U(1)_L \times U(1)_R$ and makes theory non-local
- phase space → parametrized by two arbitrary functions $\mathcal{L}(u)\,,\;\; \bar{\mathcal{L}}(v)$ state-dependent coordinates

$$x^+ = u - \mu \int^v dv' \bar{\mathcal{L}}(v')\,, \qquad x^- = v - \mu \int^u du' \mathcal{L}(u')$$

- asymptotic diffeomorphisms: depend on arbitrary $f(u)\,,\; g(v)$ and strongly background dependent
- asymptotic symmetry group: $Virasoro(u) \times Virasoro(v)$ with same **c** as in CFT
- identify in fied-theory → highly constraining!
- Note: on a purely gravitational background and for $\mu < 0$ → asymptotic symmetries of a finite box

→ make sense of ASG near e.g. BTZ horizon?

Conclusion

Summary and future directions

- exactly solvable irrelevant current-current deformations of 2d CFTs: $T\bar{T}$, $J\bar{T}$

  → spectrum (cylinder, plane: no pathology)

  → correlation functions, operator mixing

- large N holographic dictionary → variational principle: precision holography

  → non-local & state-dependent generalization of Virasoro: both $T\bar{T}$, $J\bar{T}$

  → $J\bar{T}$: different organisation of data in field theory vs. gravity

Future directions:

- precision match between all observables (e.g. correlation functions)?
- field theory interpretation of the Virasoro symmetries → constraints on the theory?
- 1/N corrections?
- generic single trace generalisations of these UV-complete irrelevant deformations?

Thank you !