Strings 2017
Tel Aviv, Israel

Genus Two Modular Bootstrap

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Harvard University

based on work with Minjae Cho and Scott Collier

see also related works in [Cardy, Maloney, Maxfield] [Keller, Mathys, Zadeh]

Our Goal

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To carve out the space of 2D (unitary) CFTs.

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What are the possible spectra of local operators and structure constants?

Conformal Bootstrap

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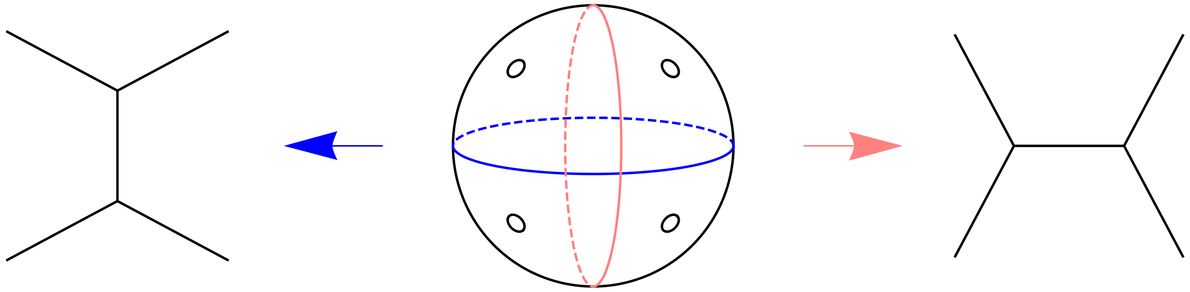
crossing invariance
(associativity of OPE)



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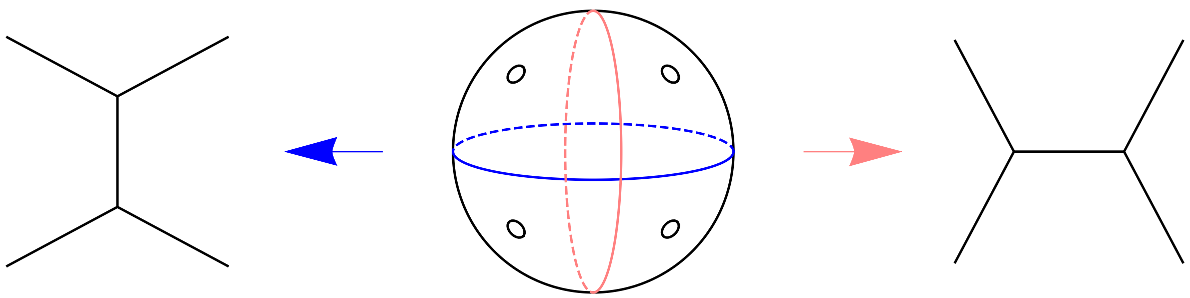
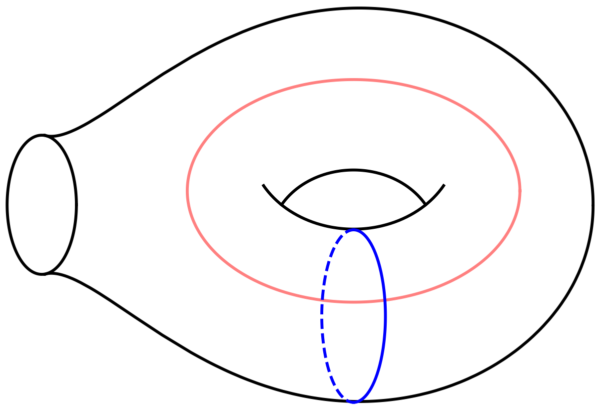


modular invariance



To illustrate with some examples…

Modular invariance of torus partition function

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[Hellerman '09, Friedan-Keller '13, Qualls-Shapere '13, Collier-Lin-XY '16]

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Bounding the gap in the spectrum (all-spin Virasoro primaries vs scalar Virasoro primaries) [Collier-Lin-XY '16]

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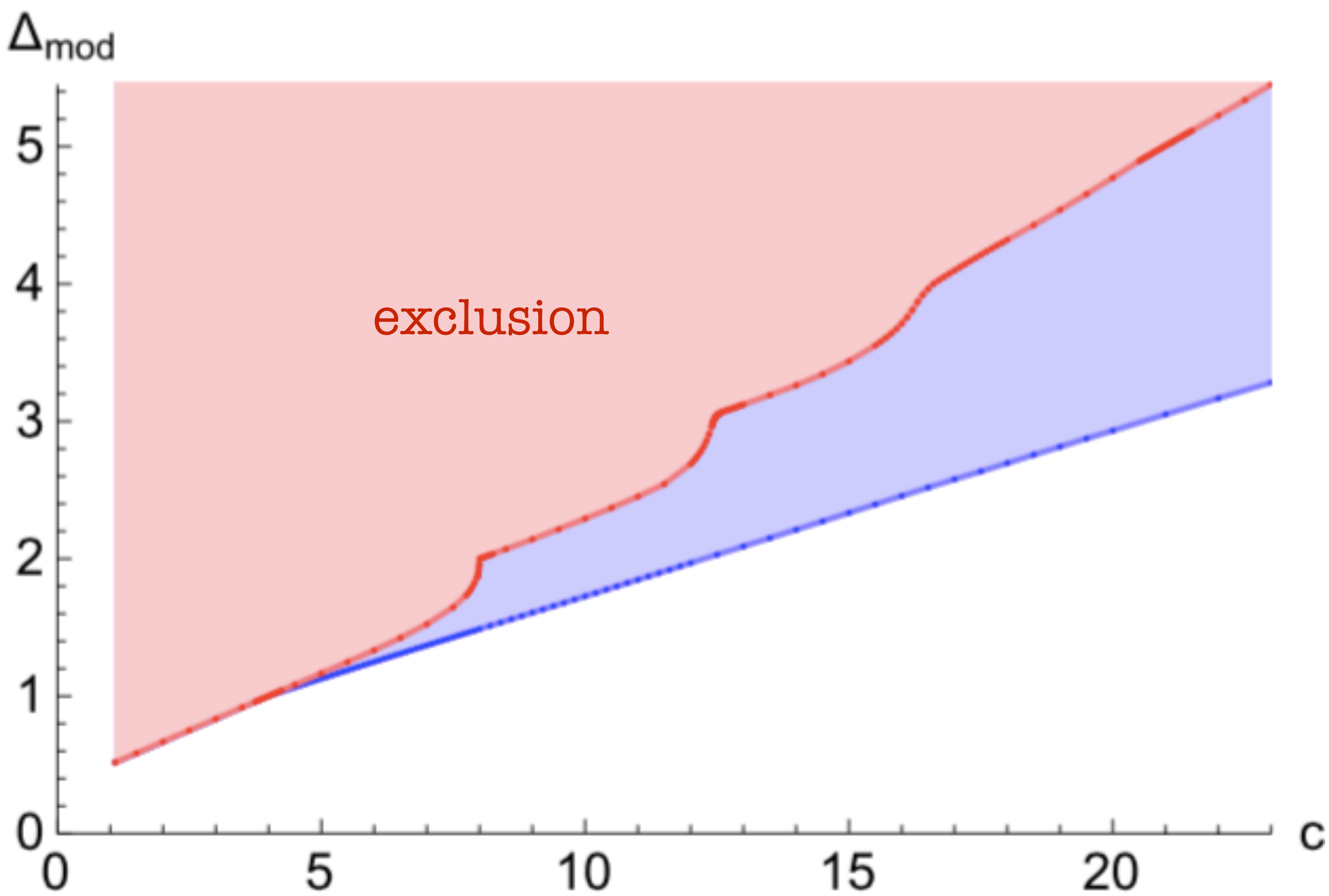

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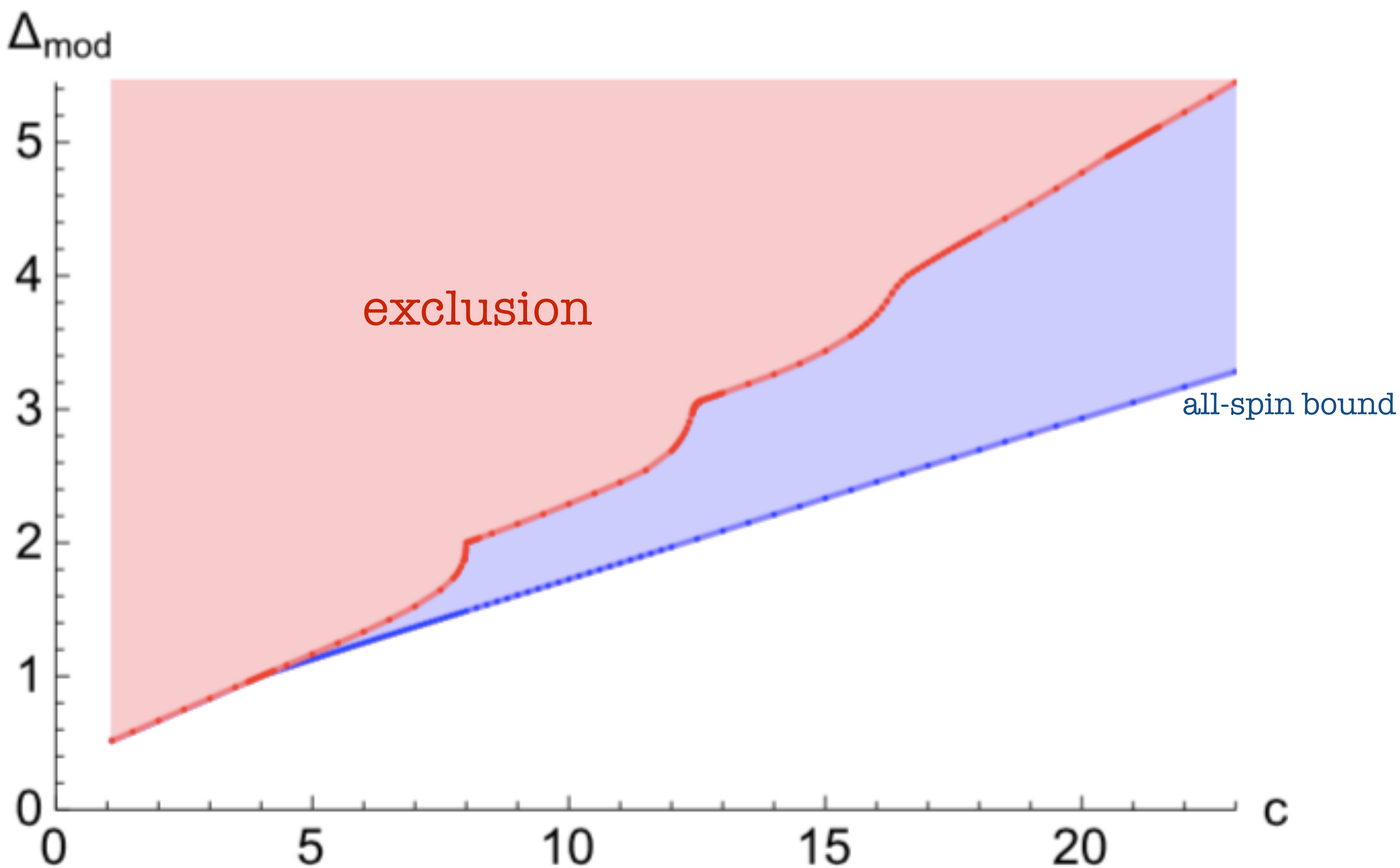

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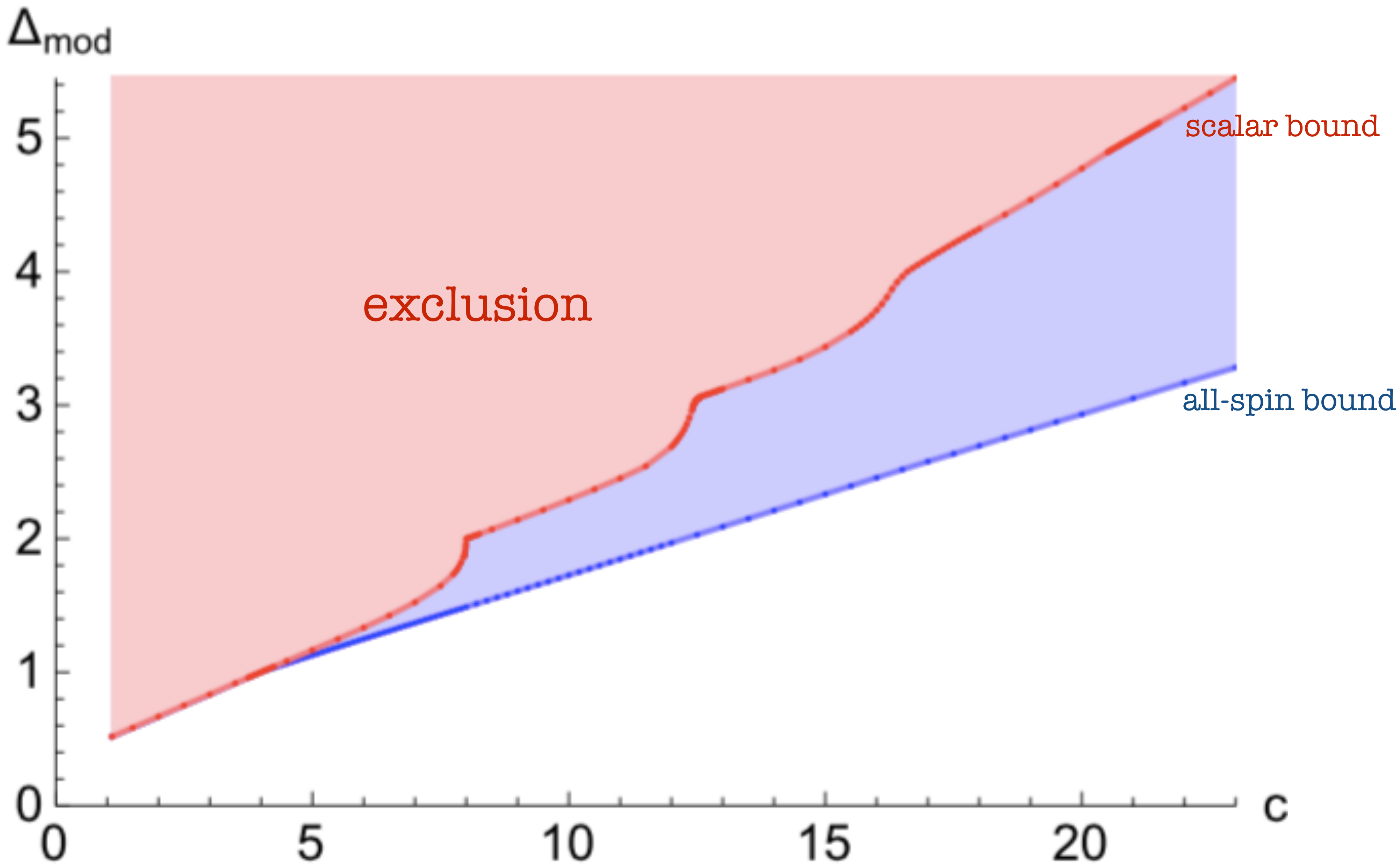

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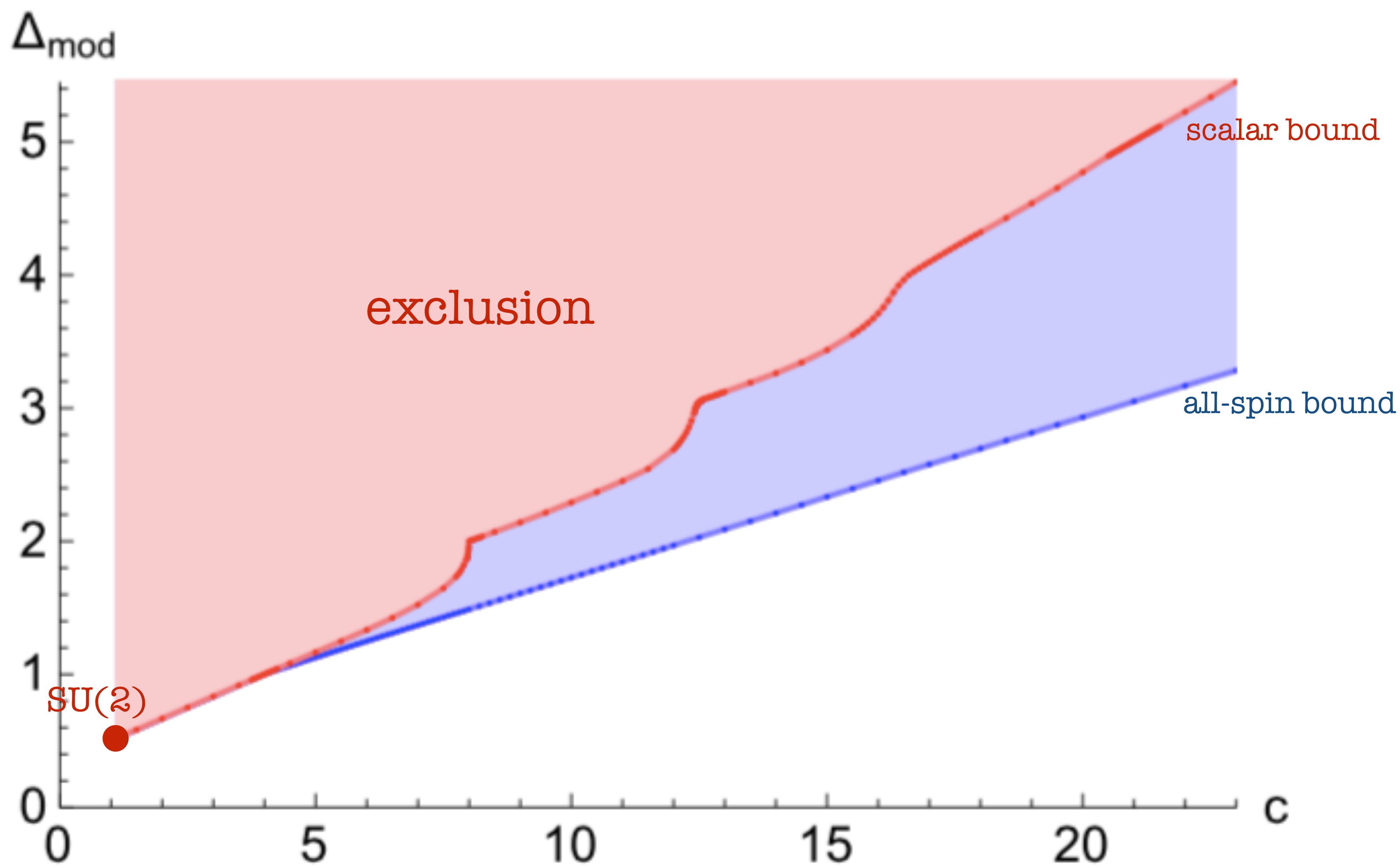

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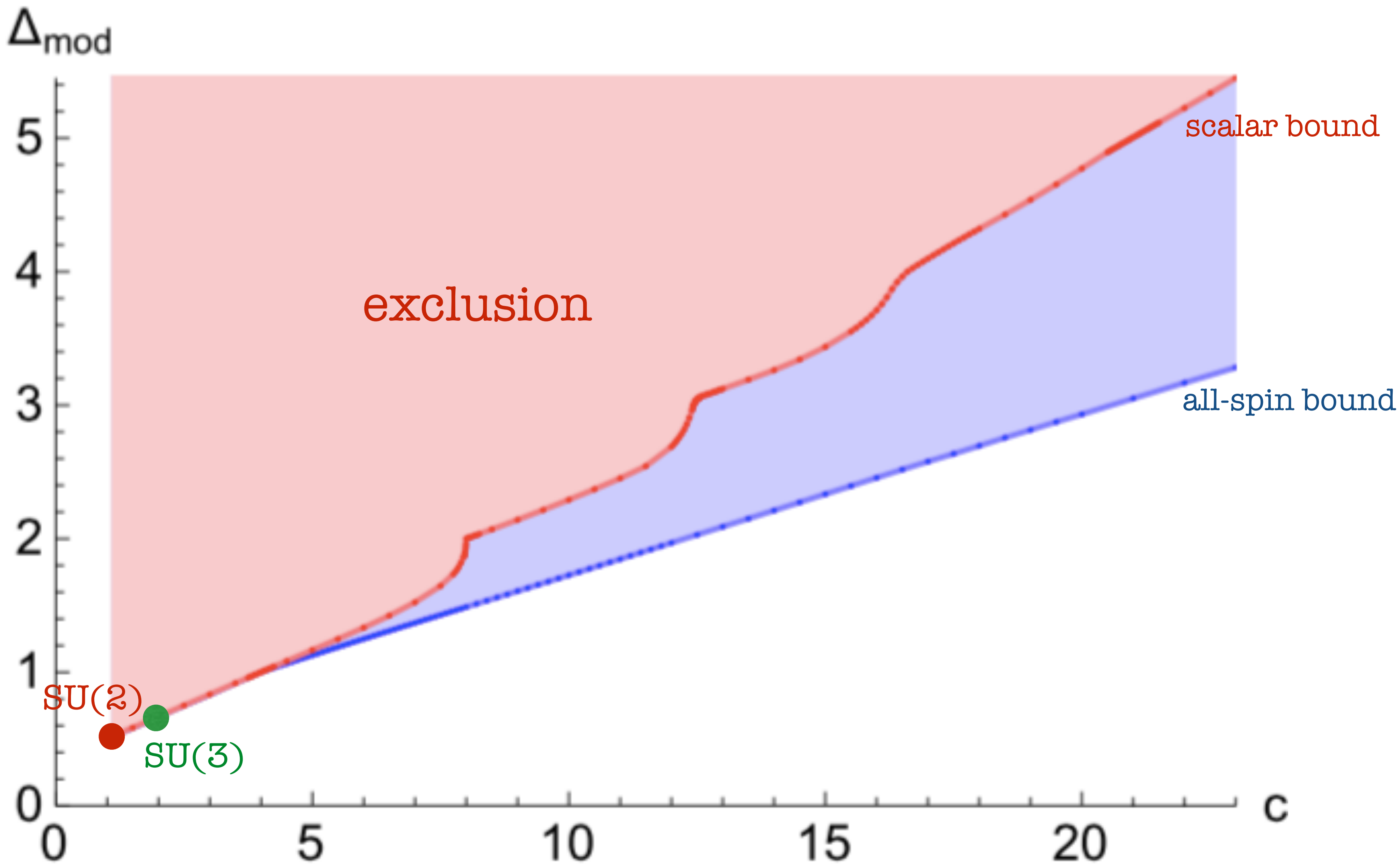

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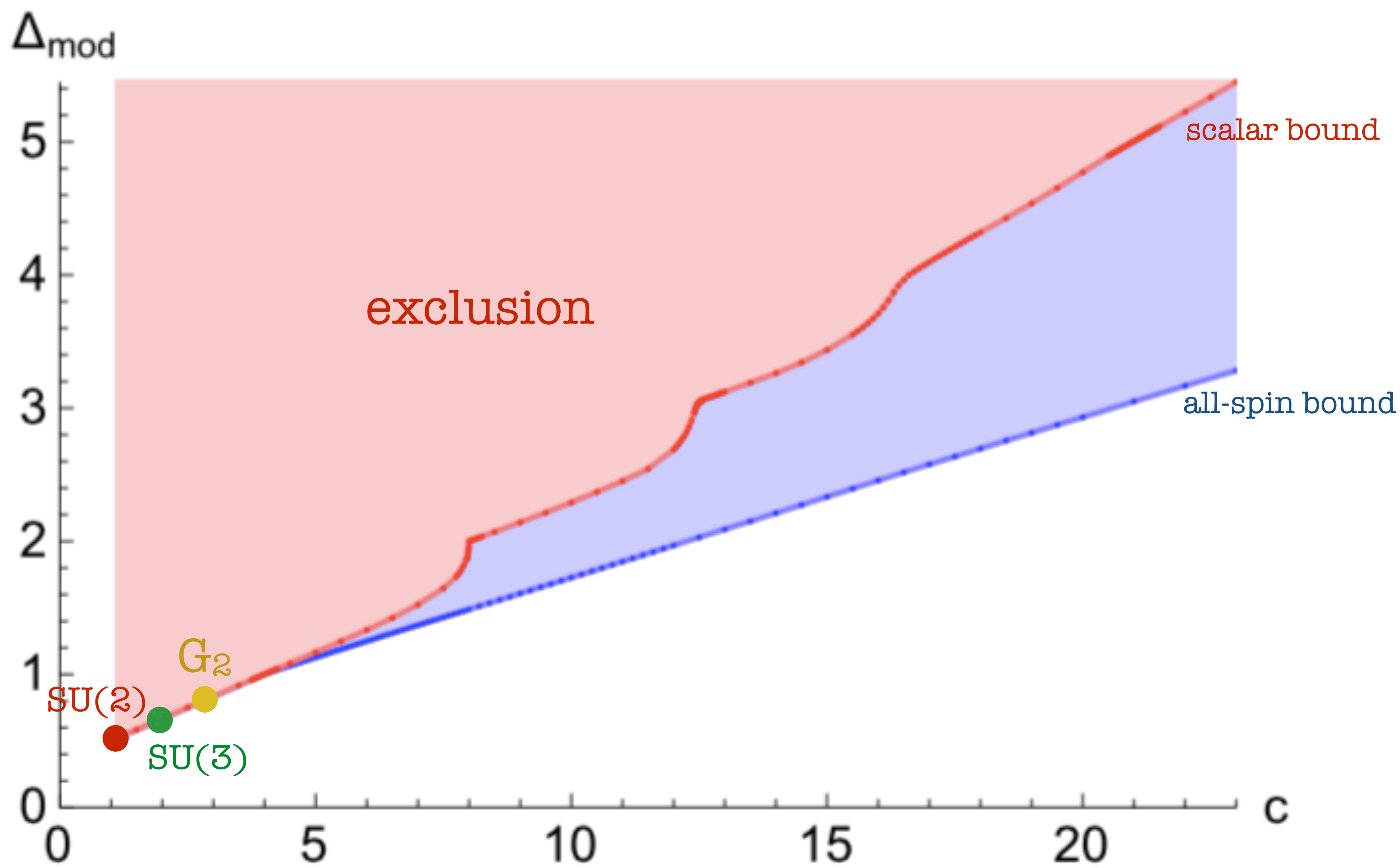

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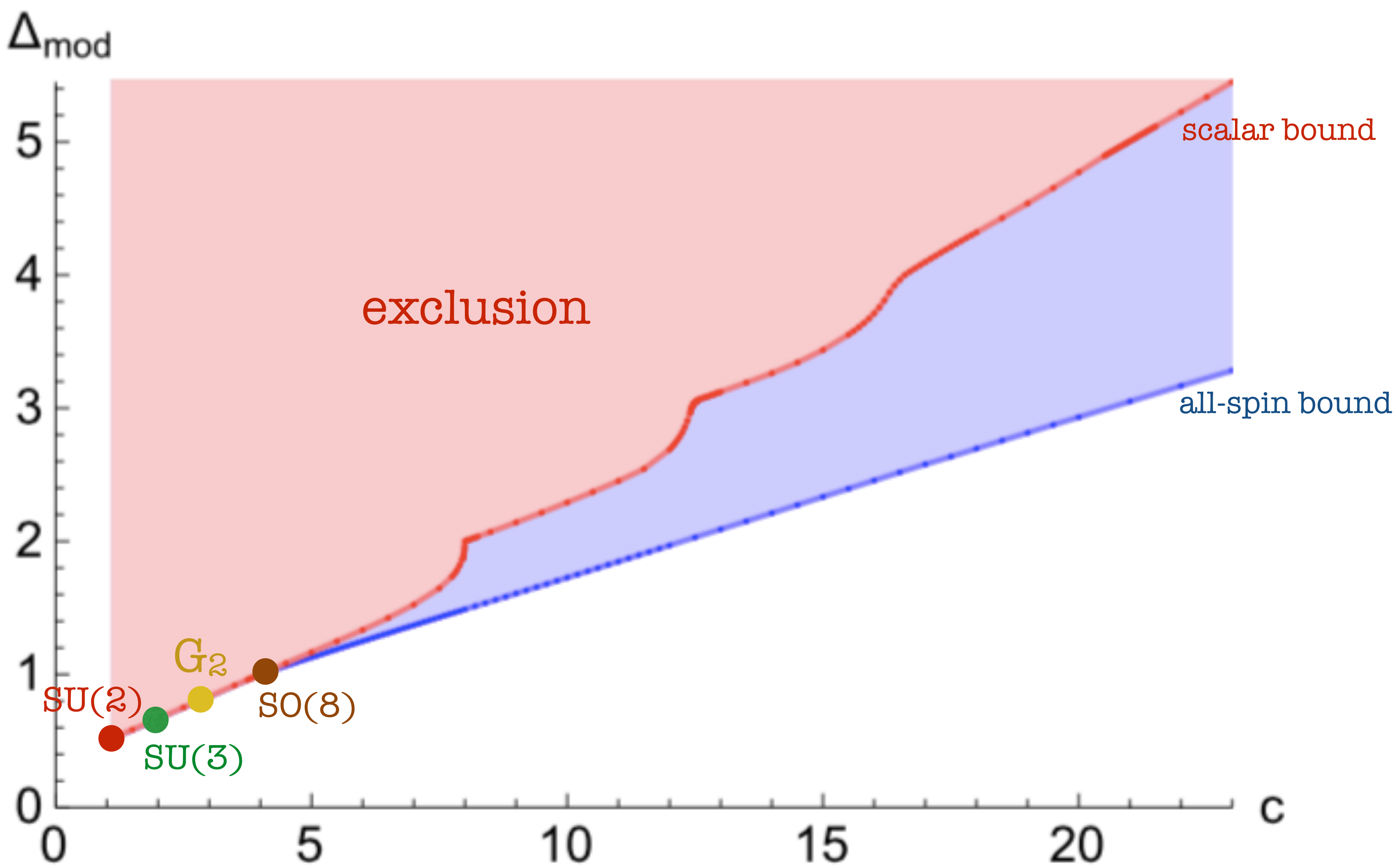

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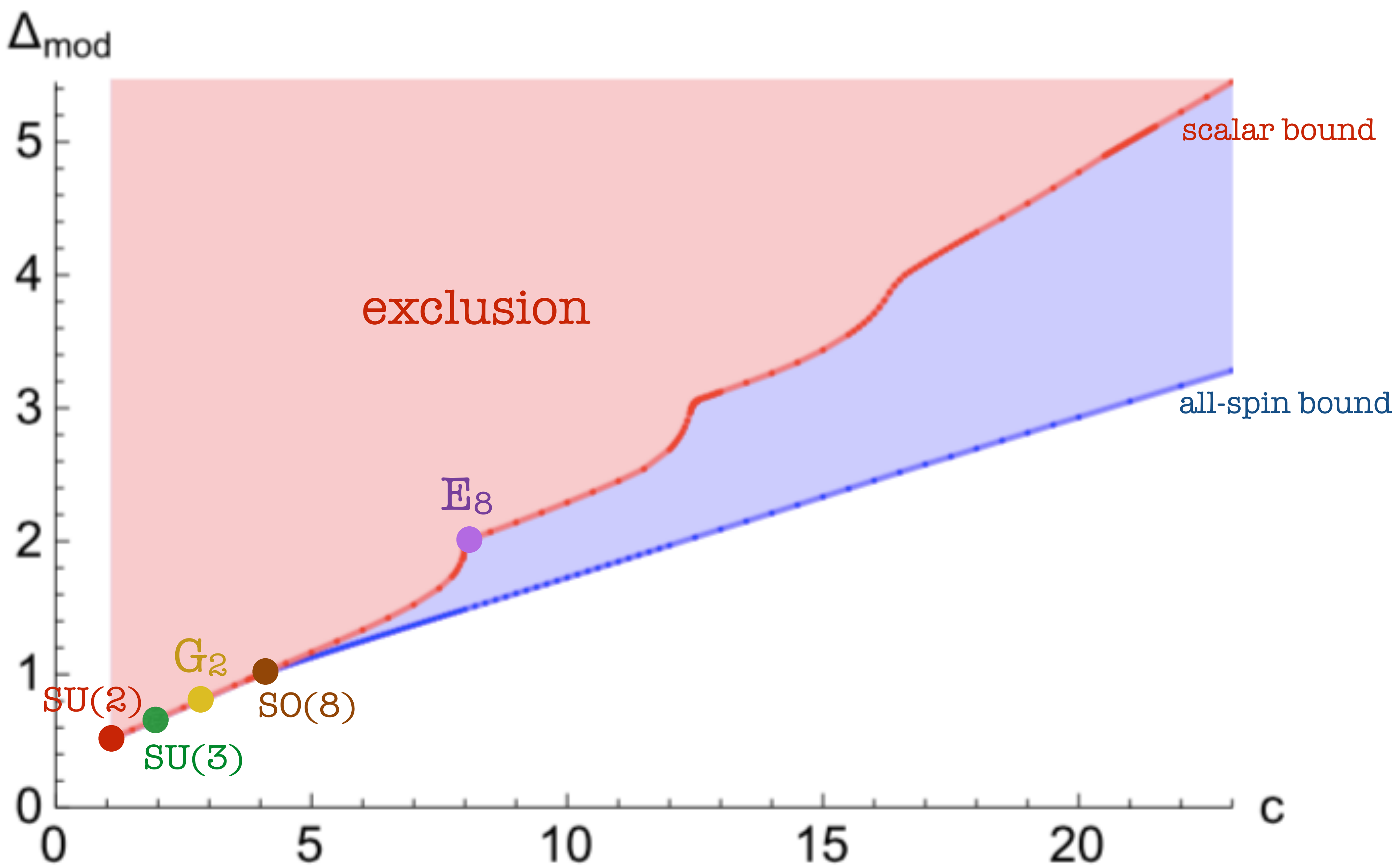

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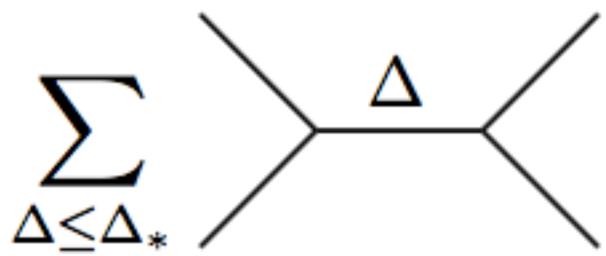

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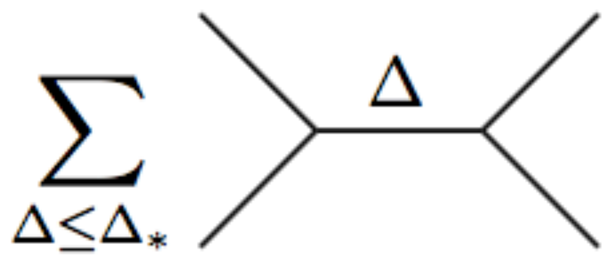

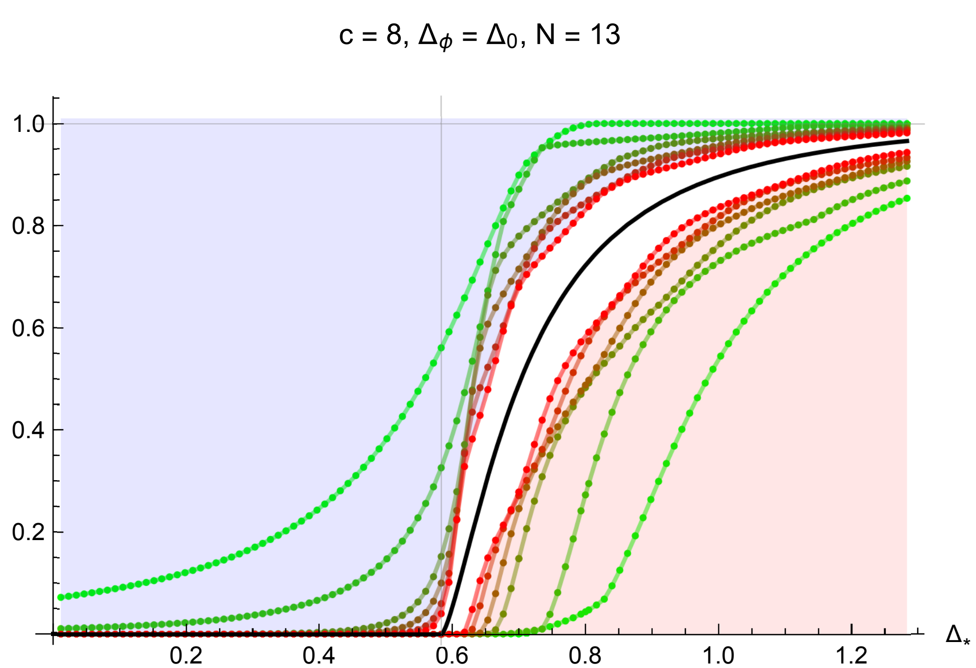

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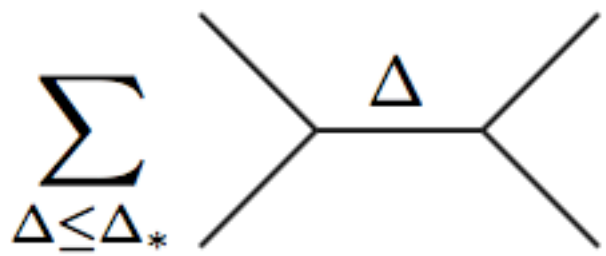

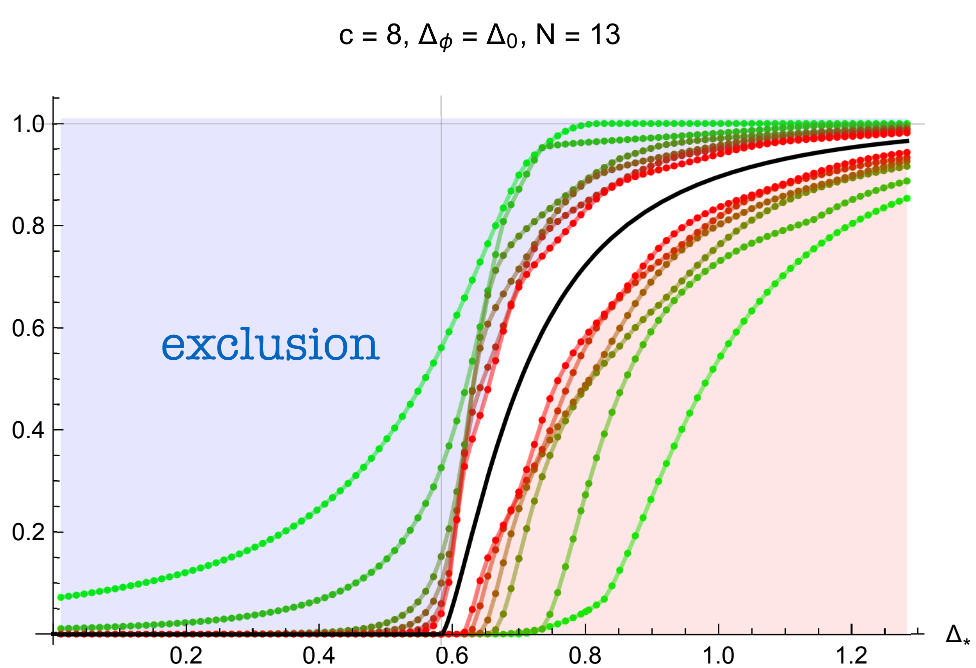

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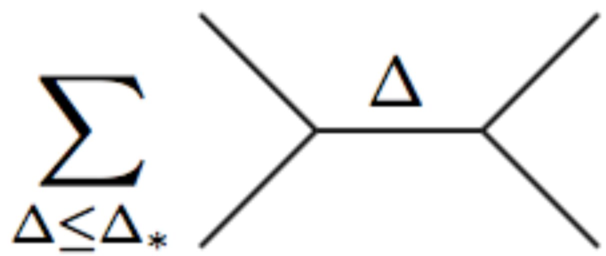

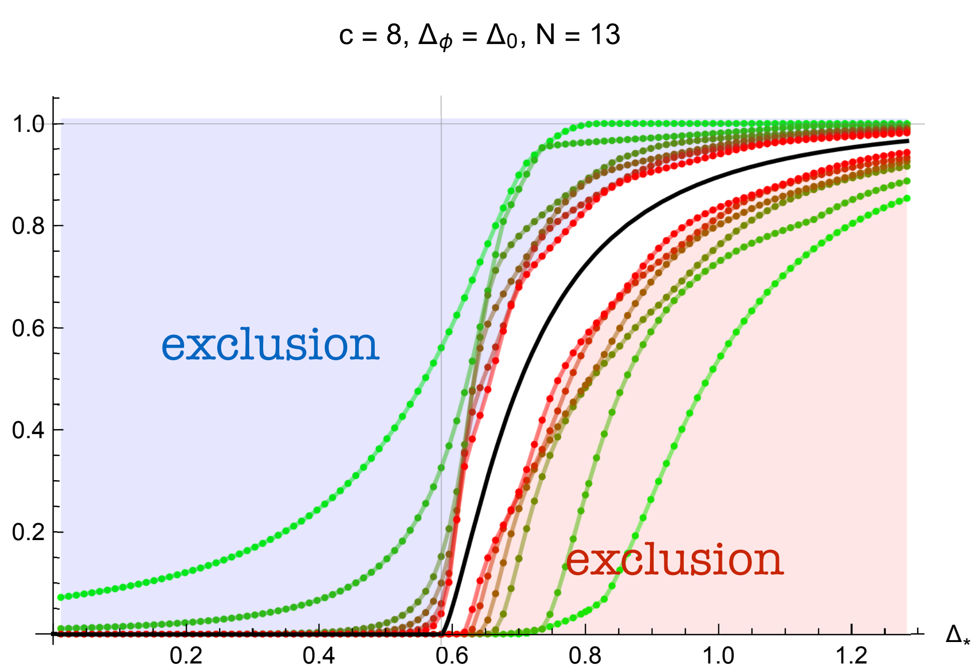

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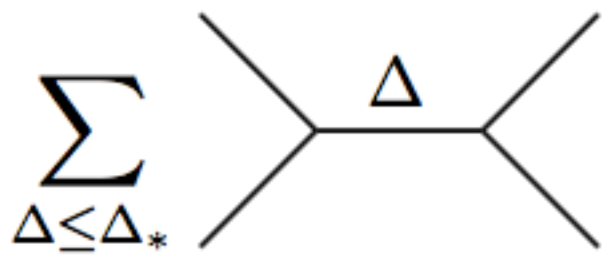

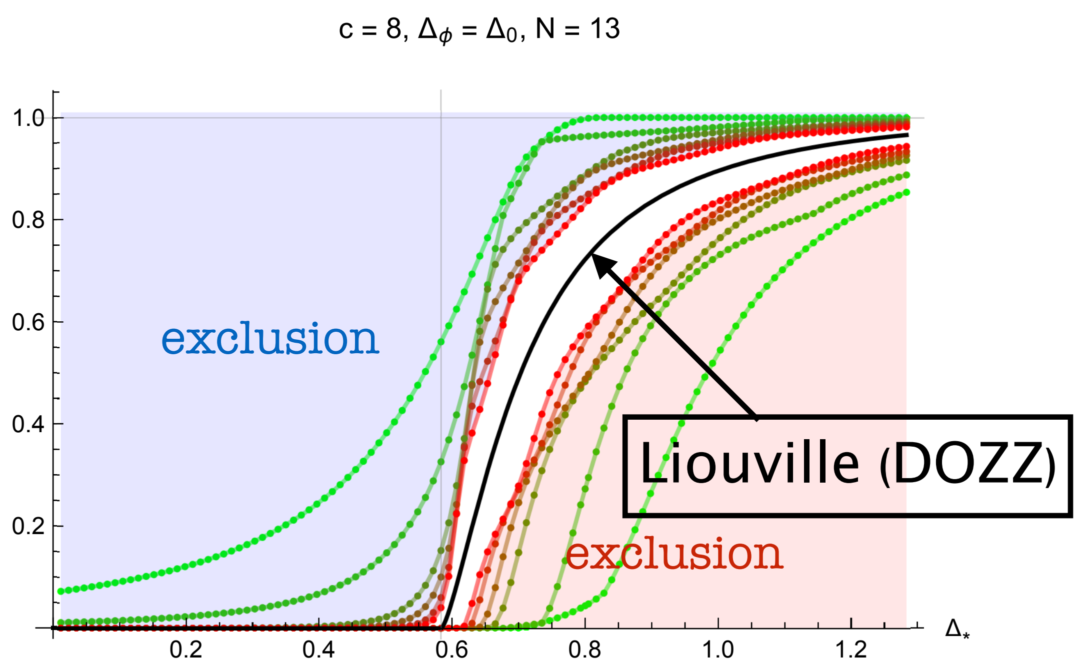

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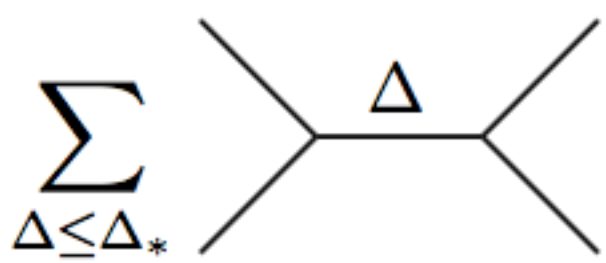

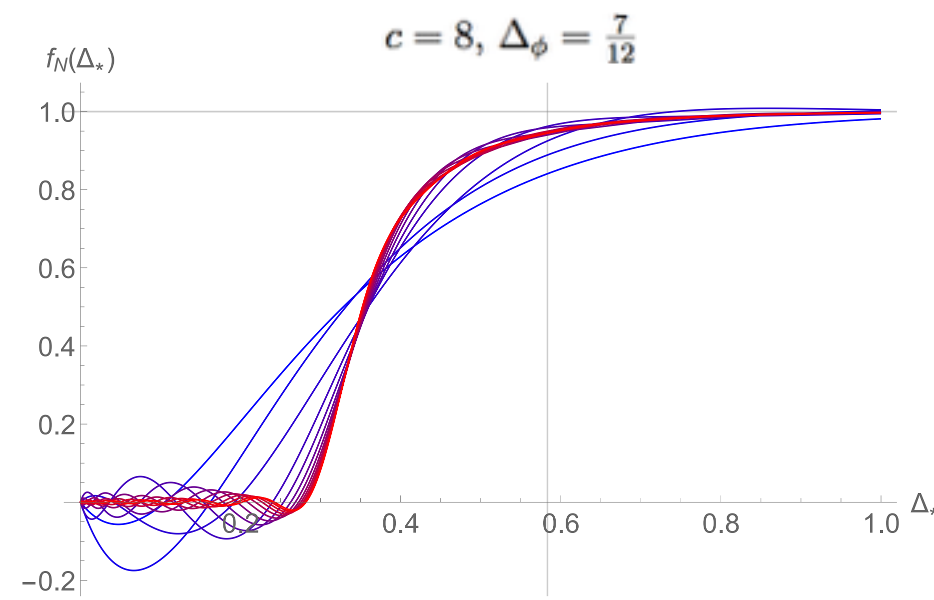

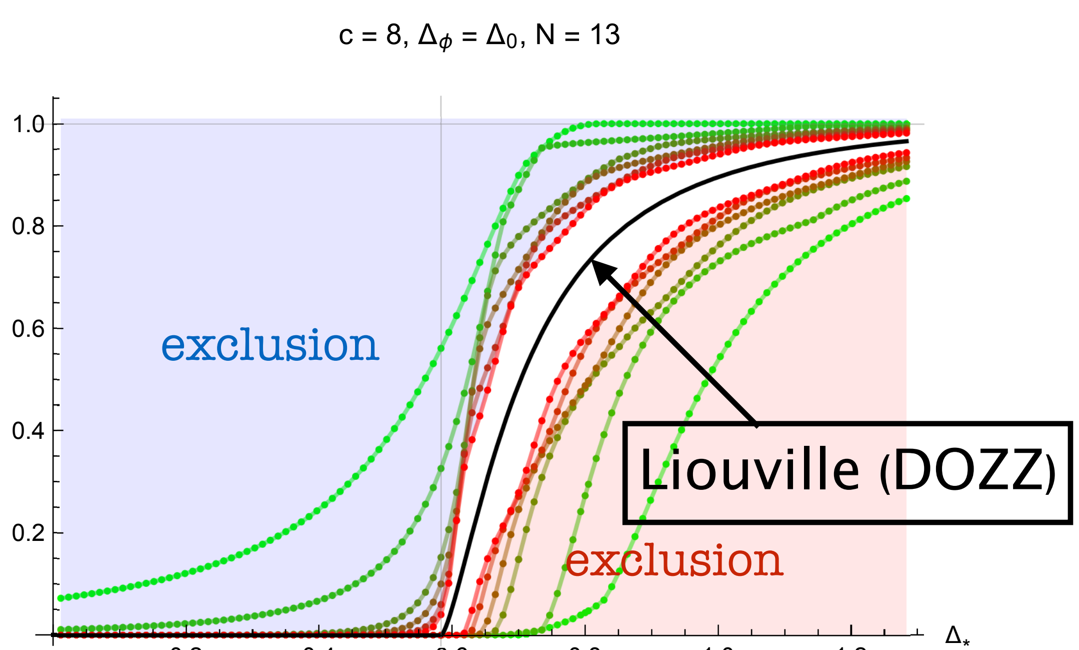

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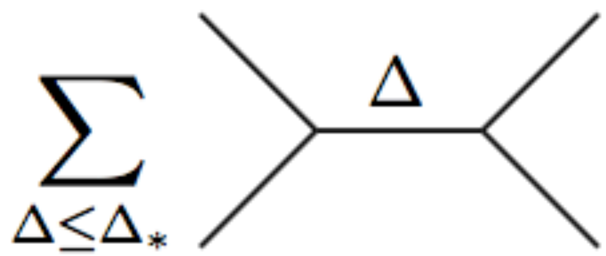

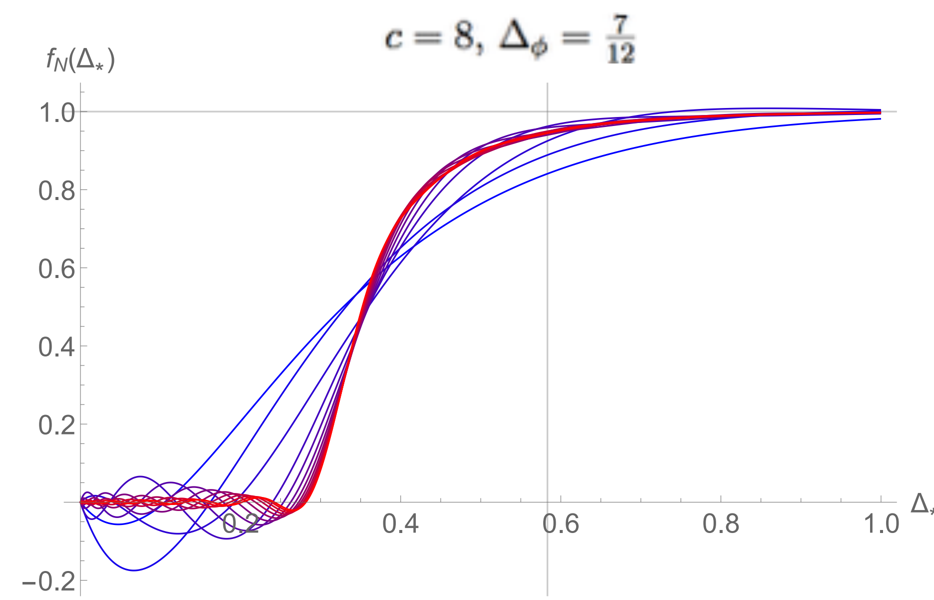

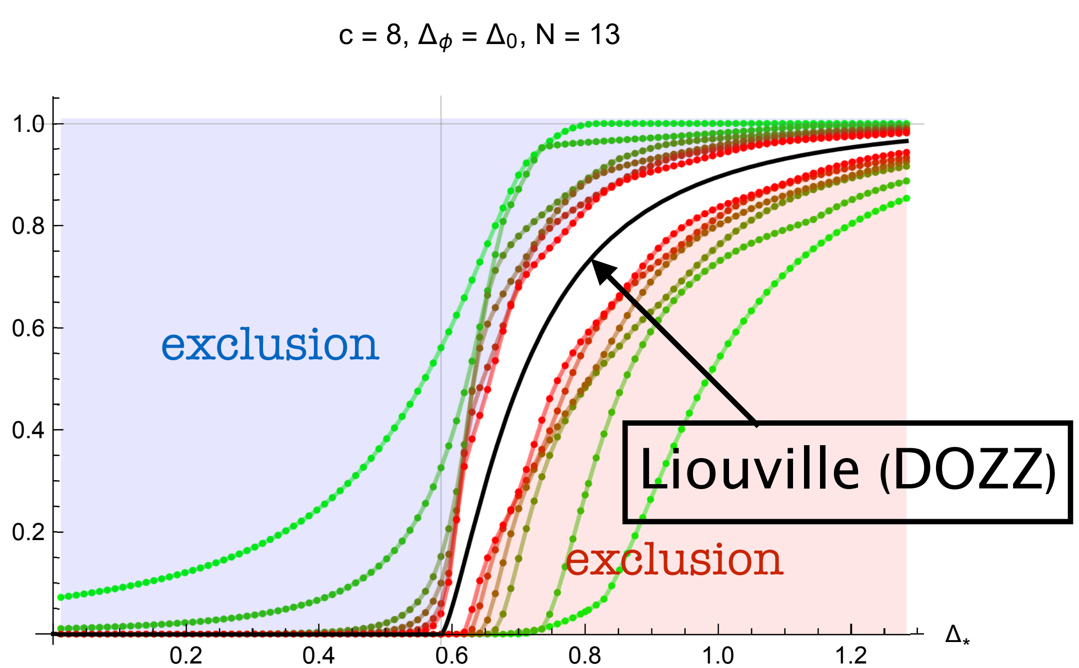

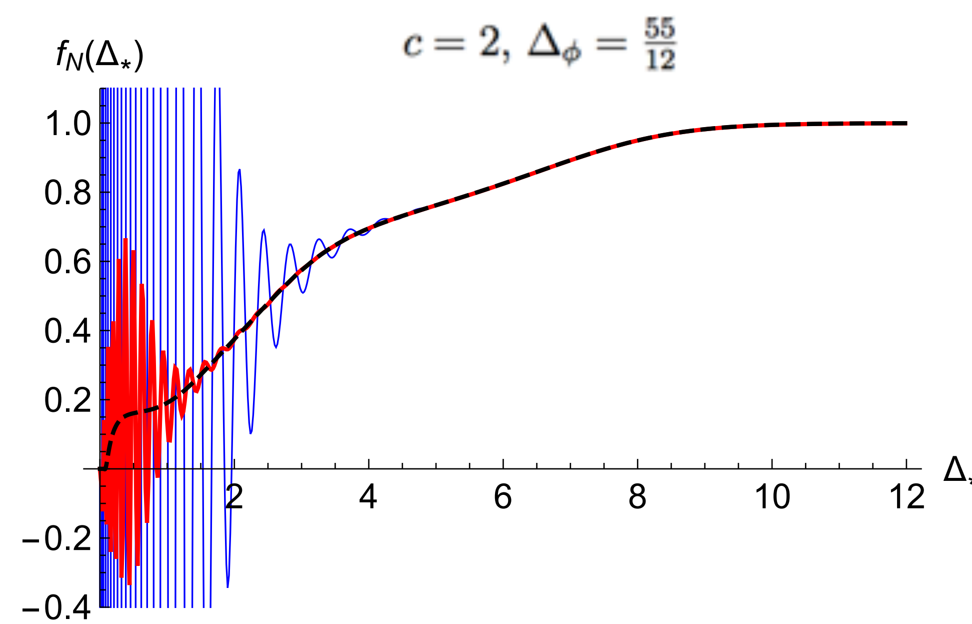

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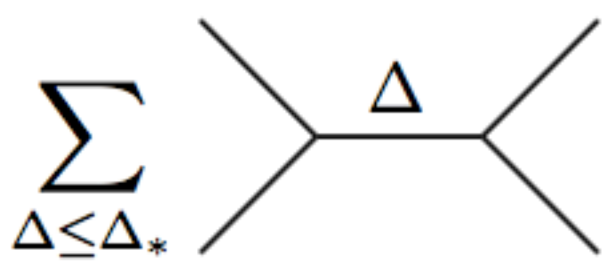

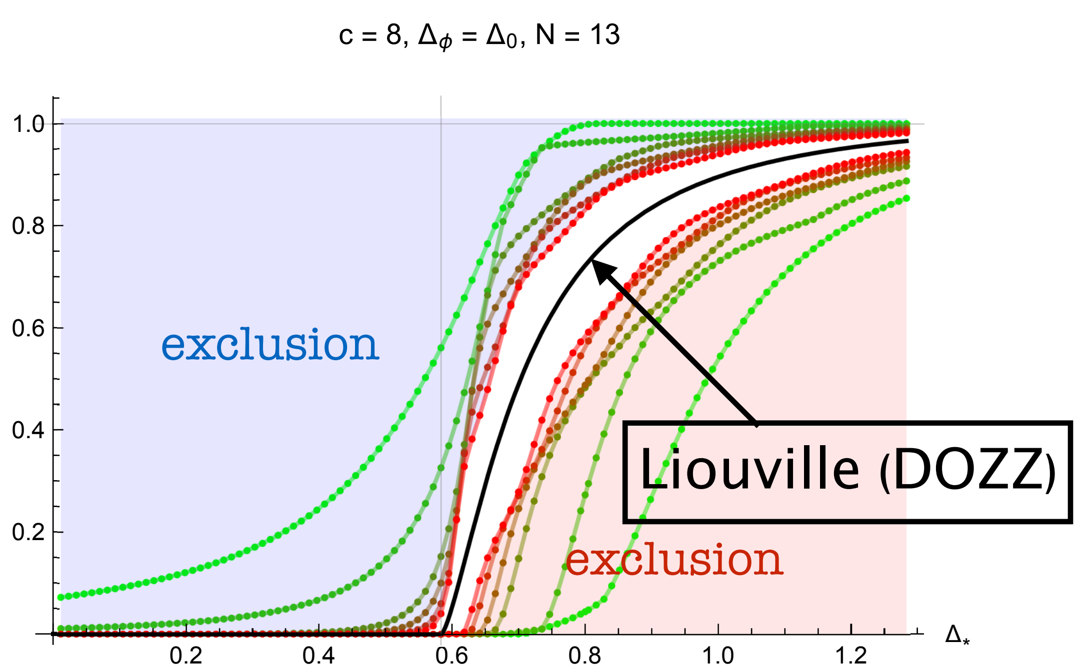

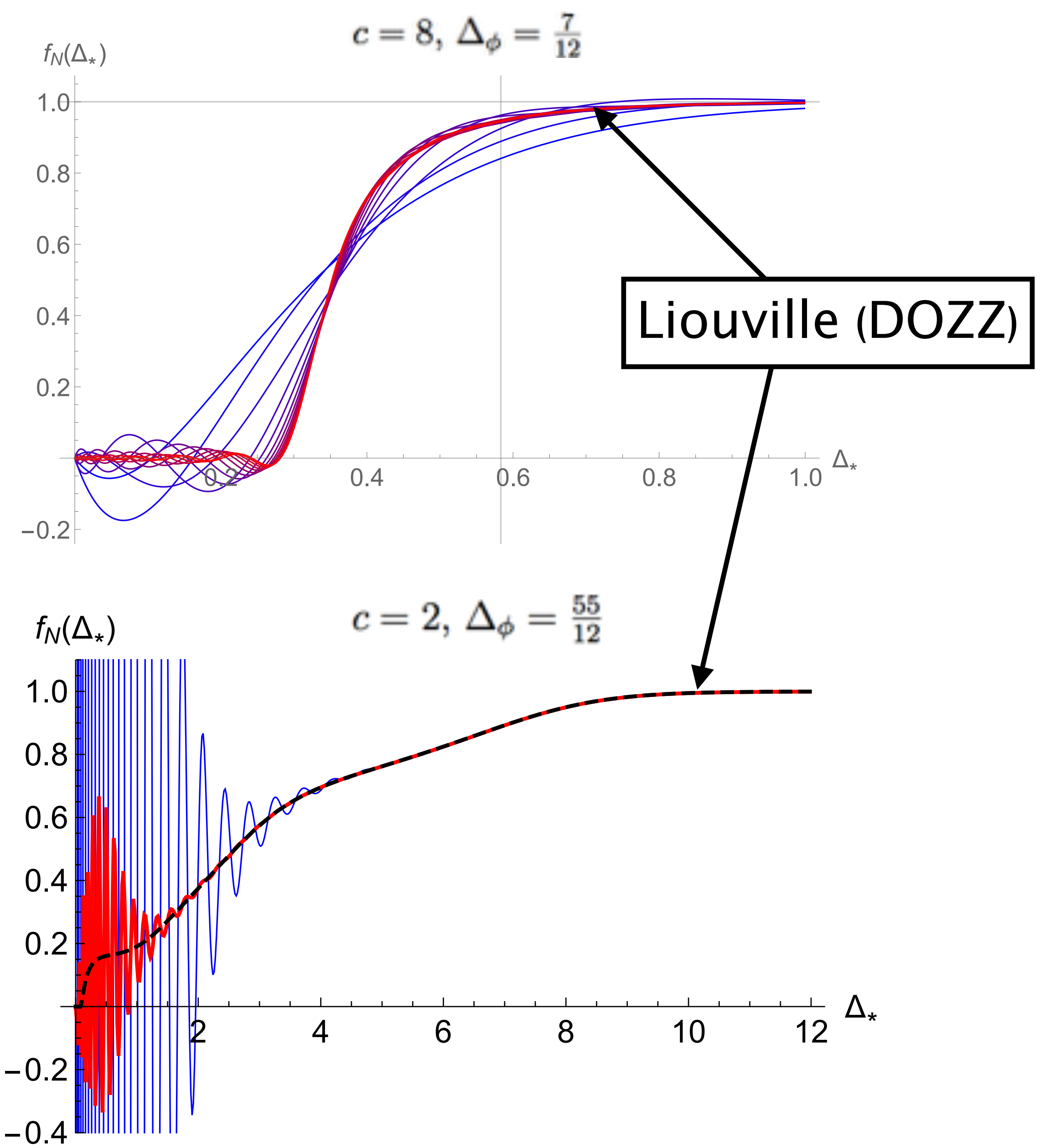

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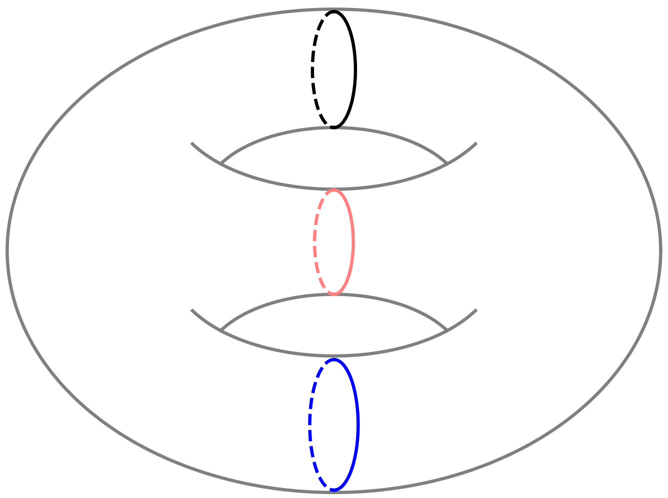

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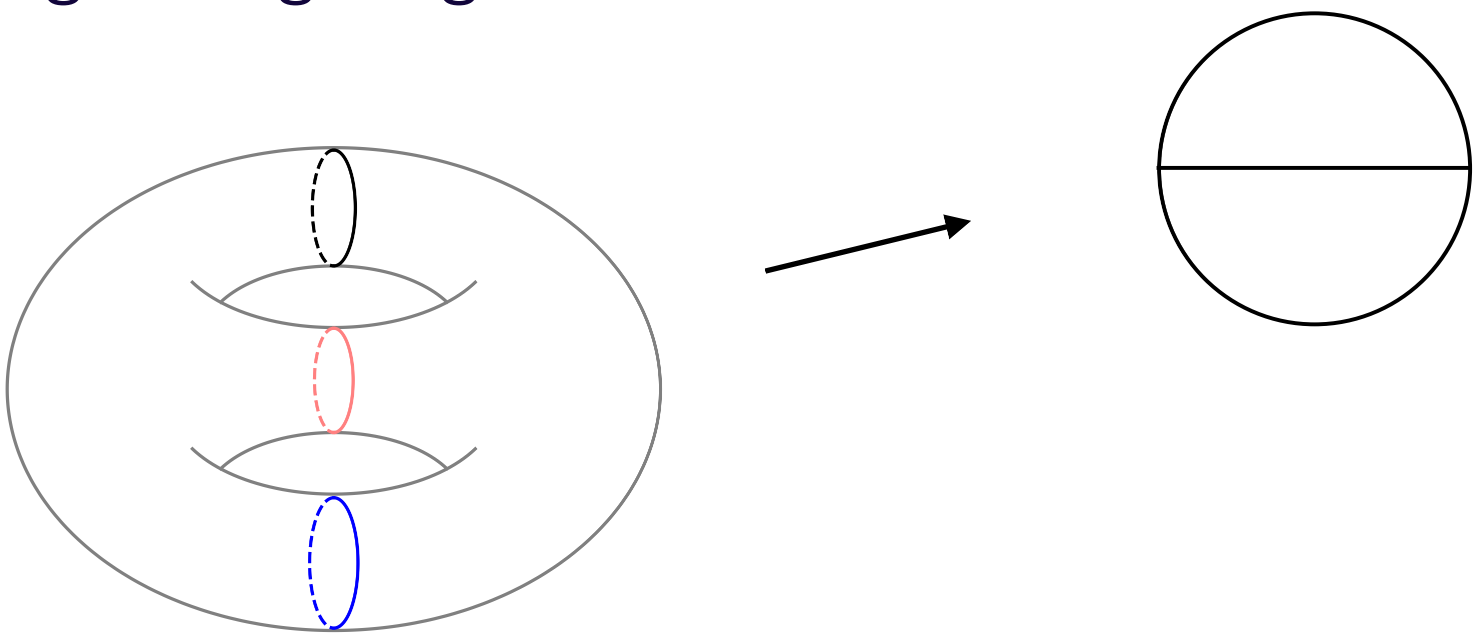

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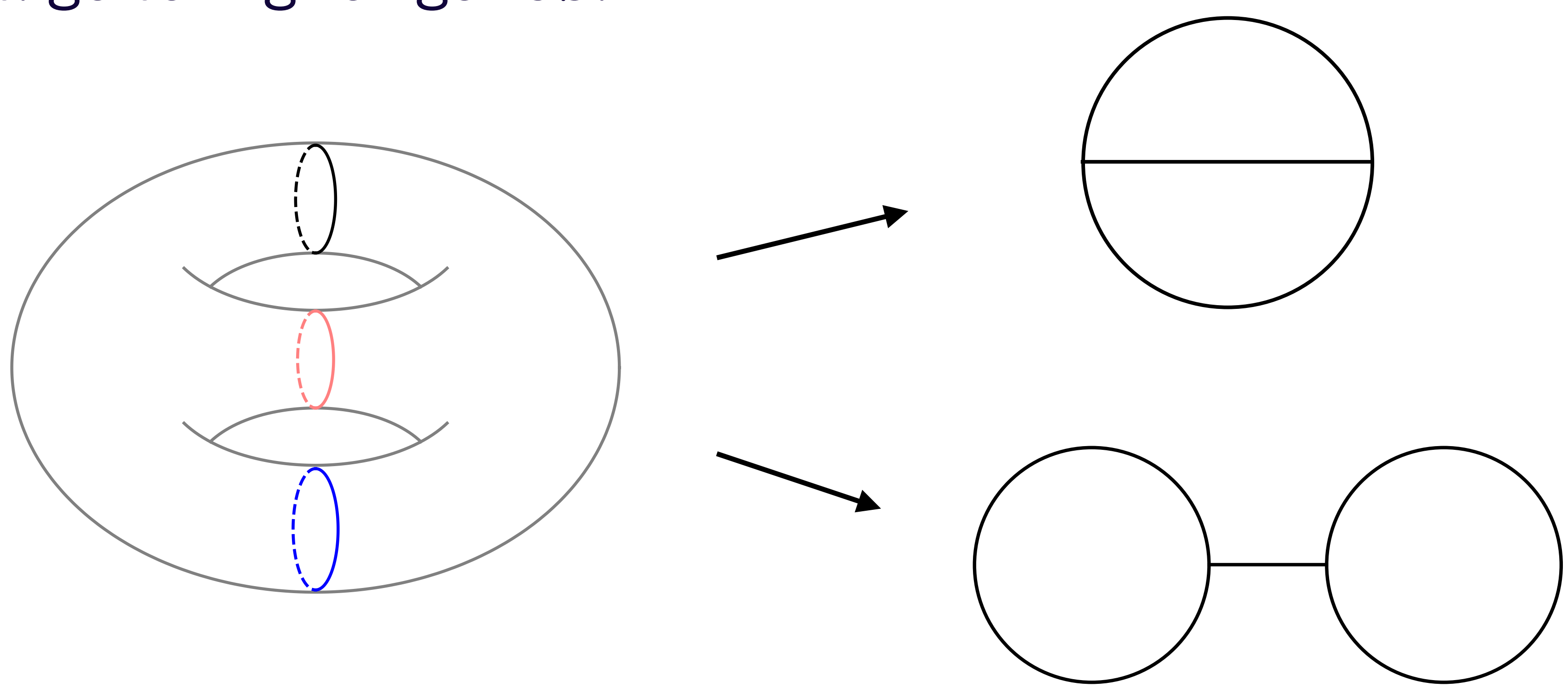

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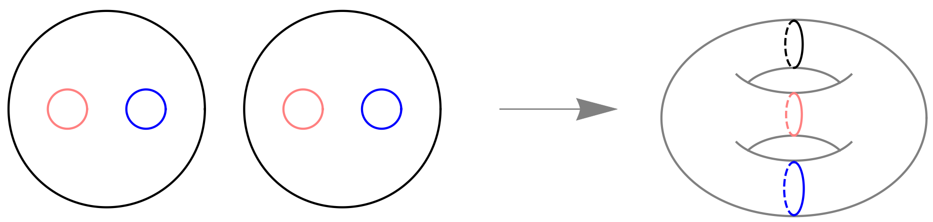

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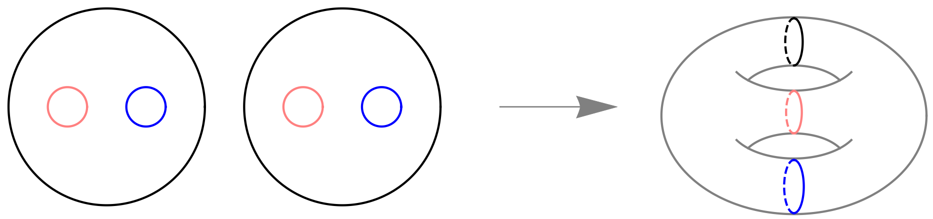

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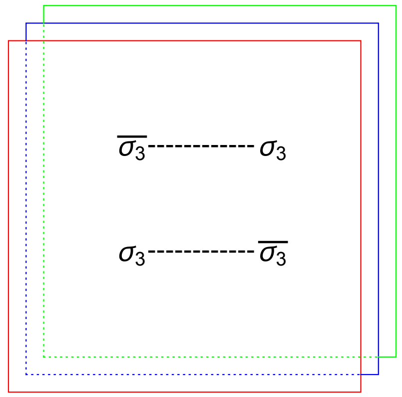

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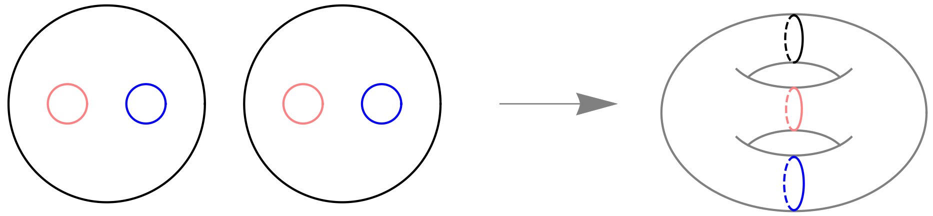

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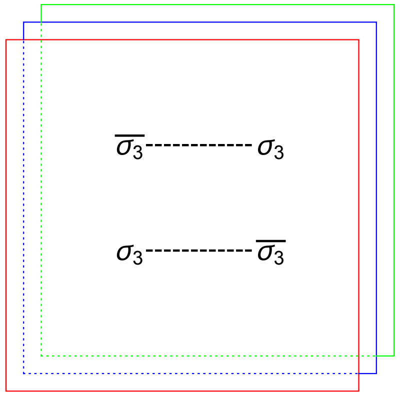

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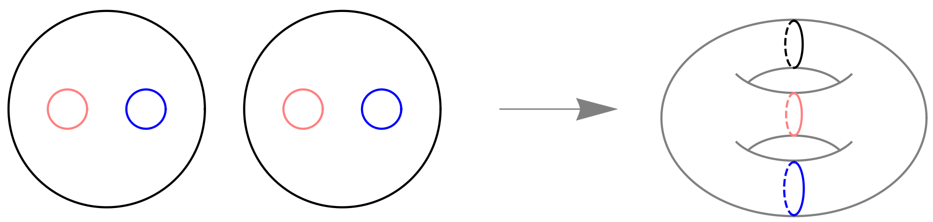

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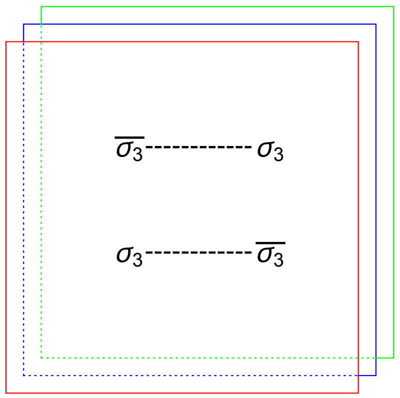

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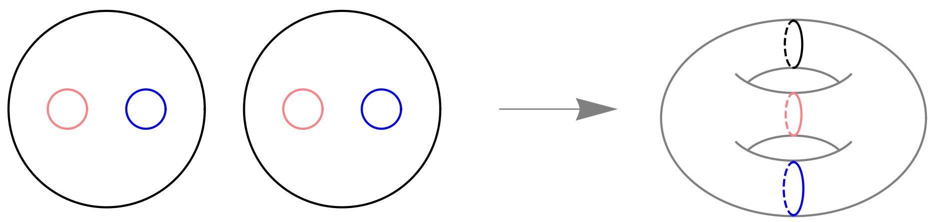

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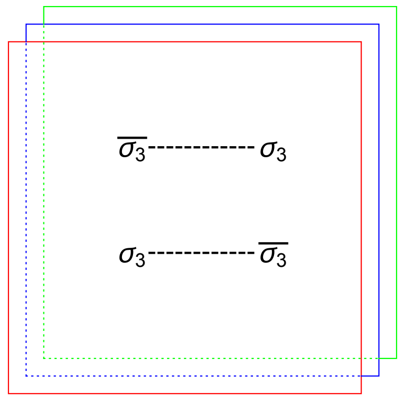

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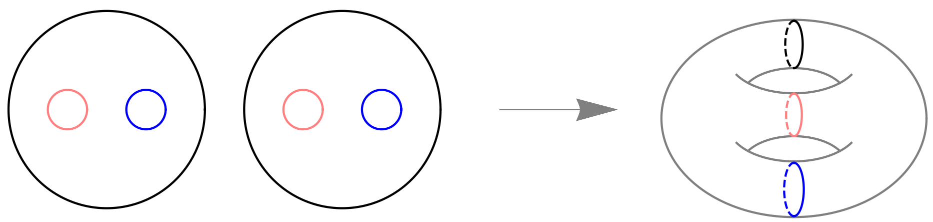

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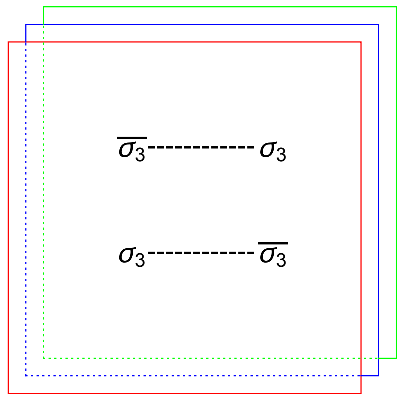

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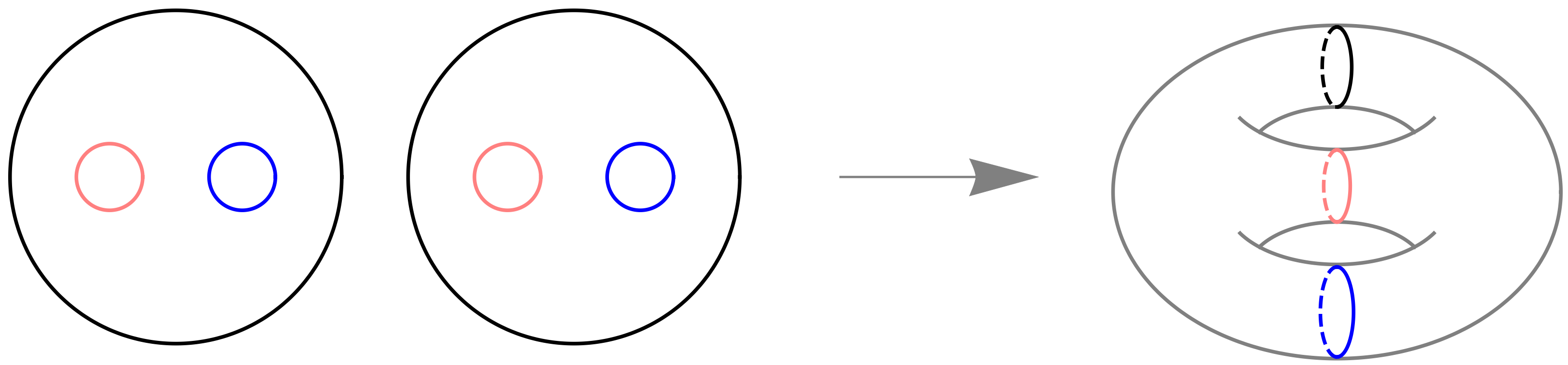

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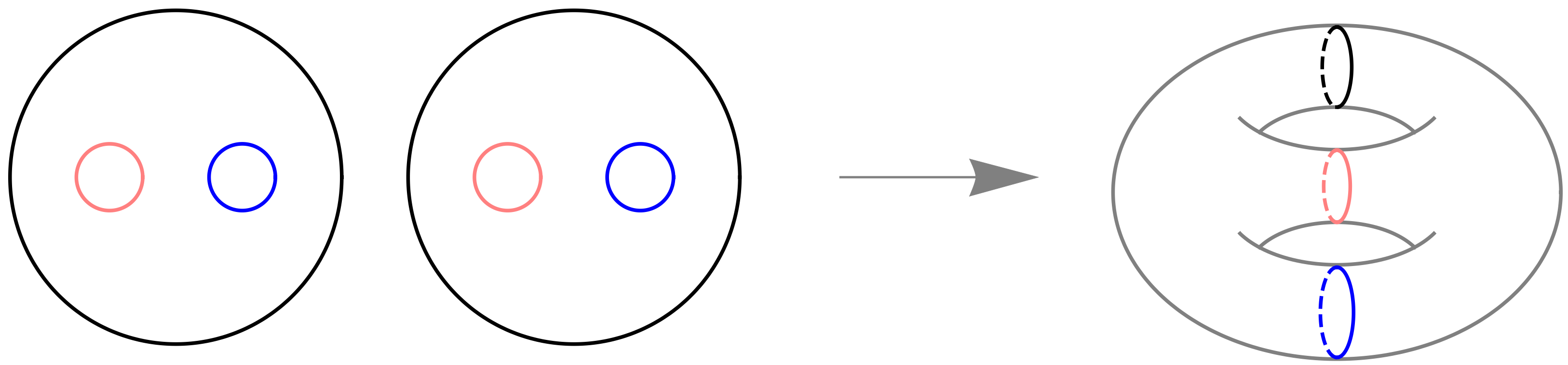

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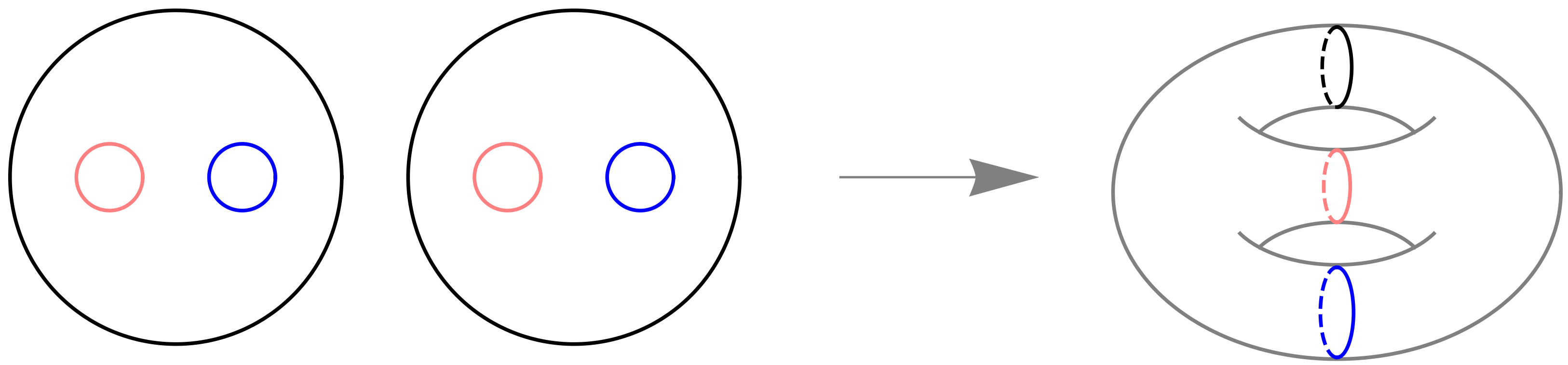

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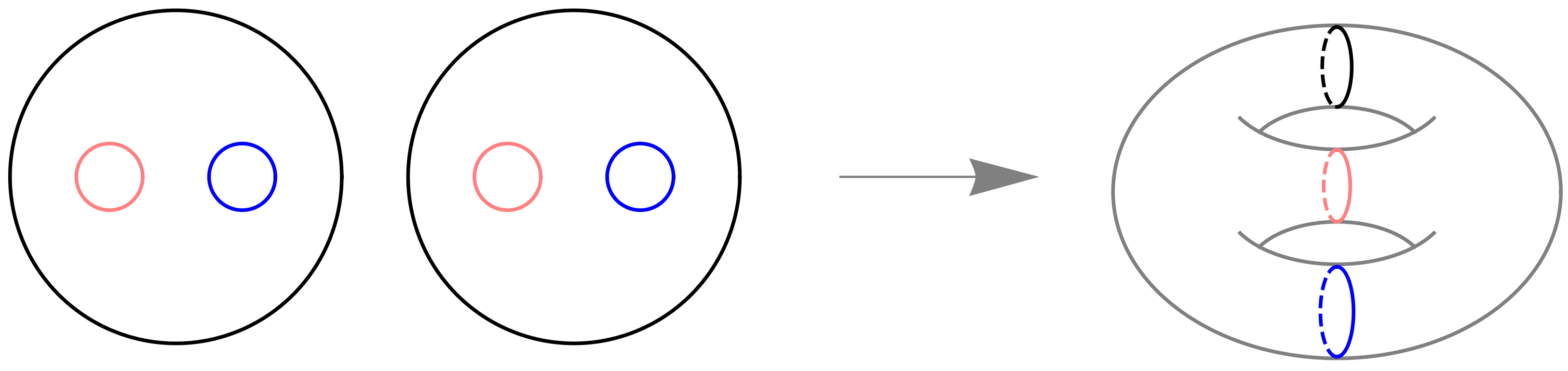

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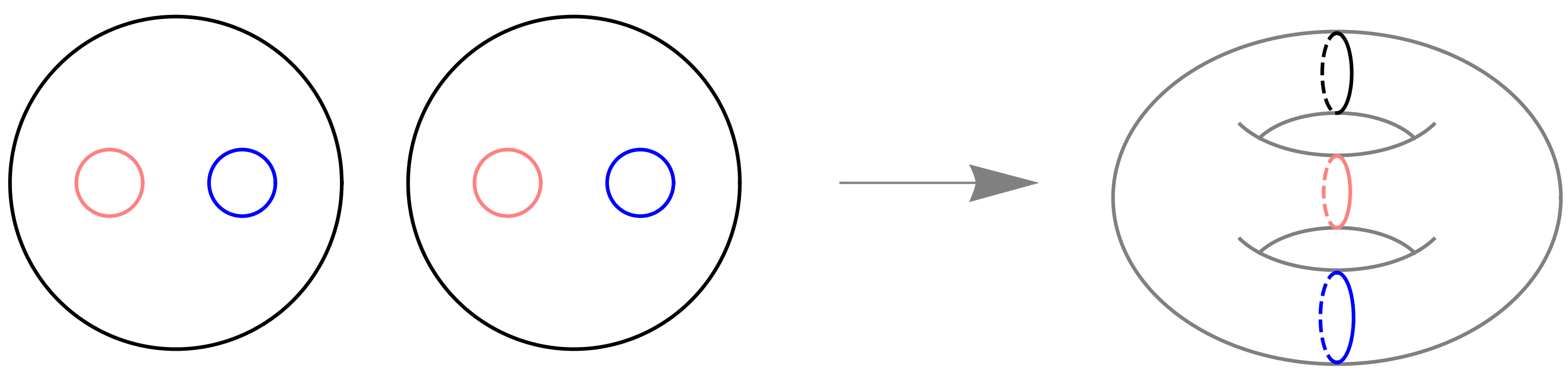

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[Zamolodchikov '84]

generalizations by [Hadasz, Jaskolski, Suchanek '09] [Cho, Collier, XY '17]

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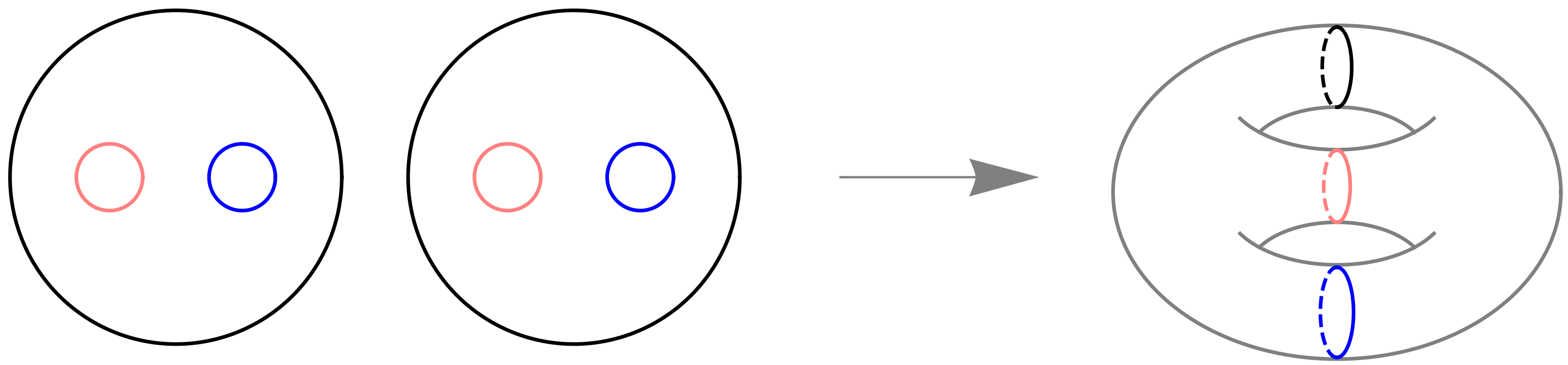

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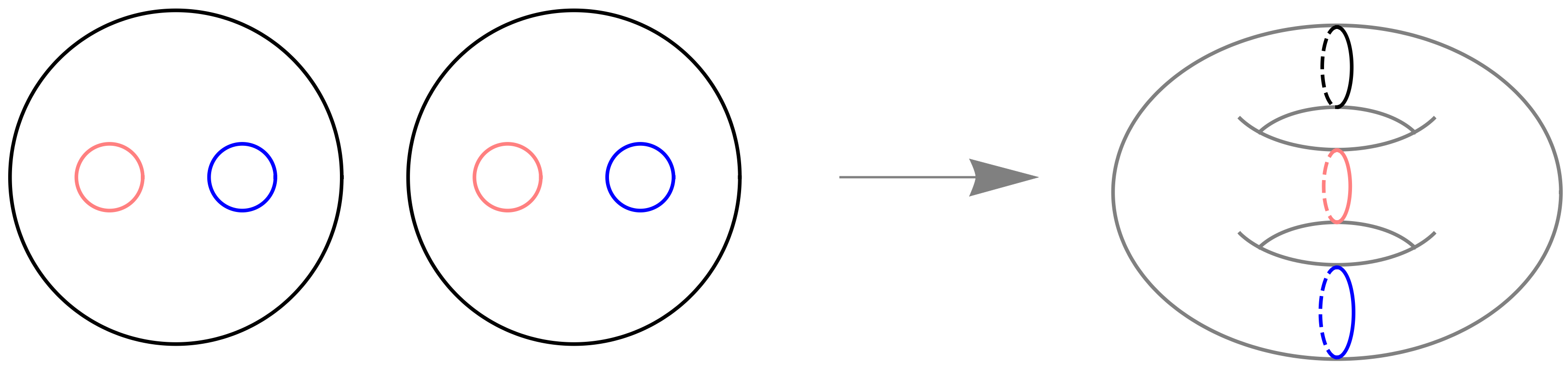

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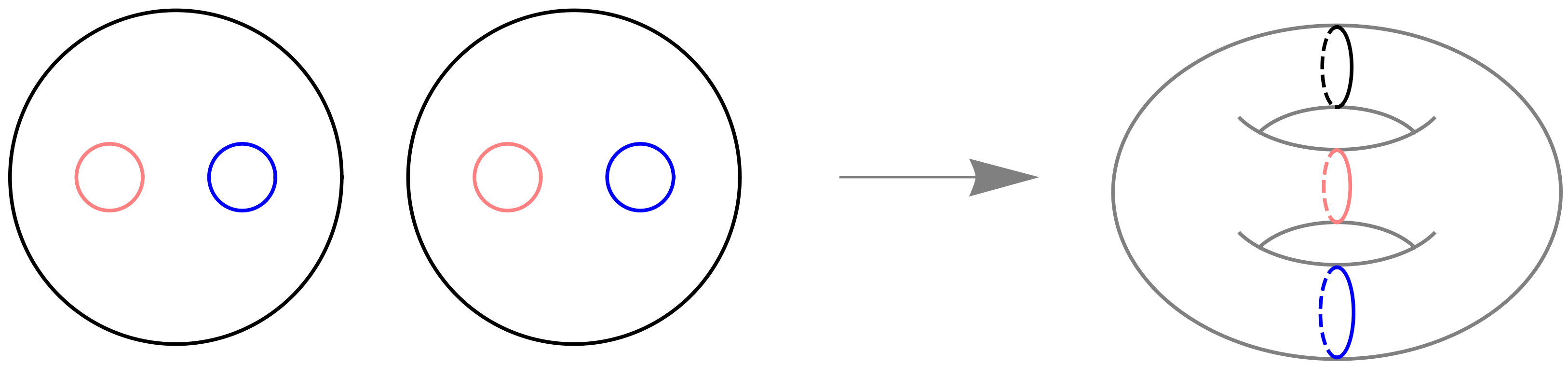

(captured by 1-loop partition function of 3D gravity on a handlebody) [Giombi-Maloney-XY '08]

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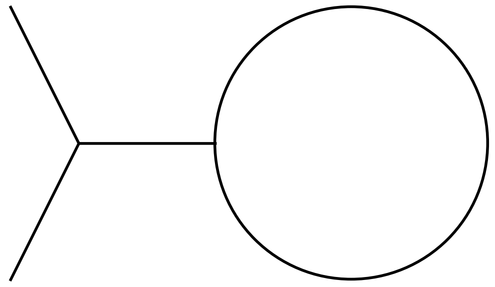

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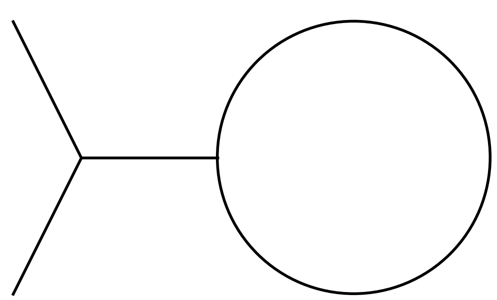

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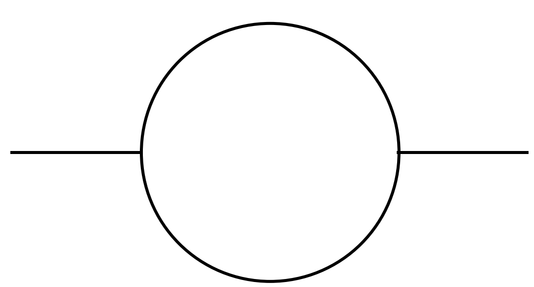

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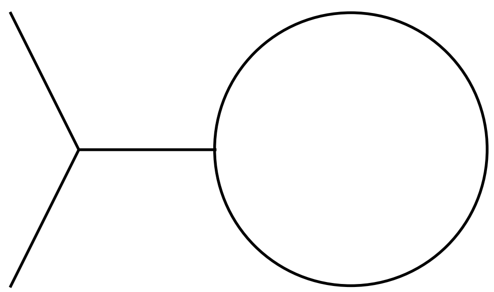

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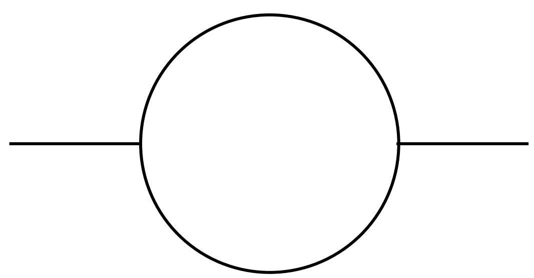

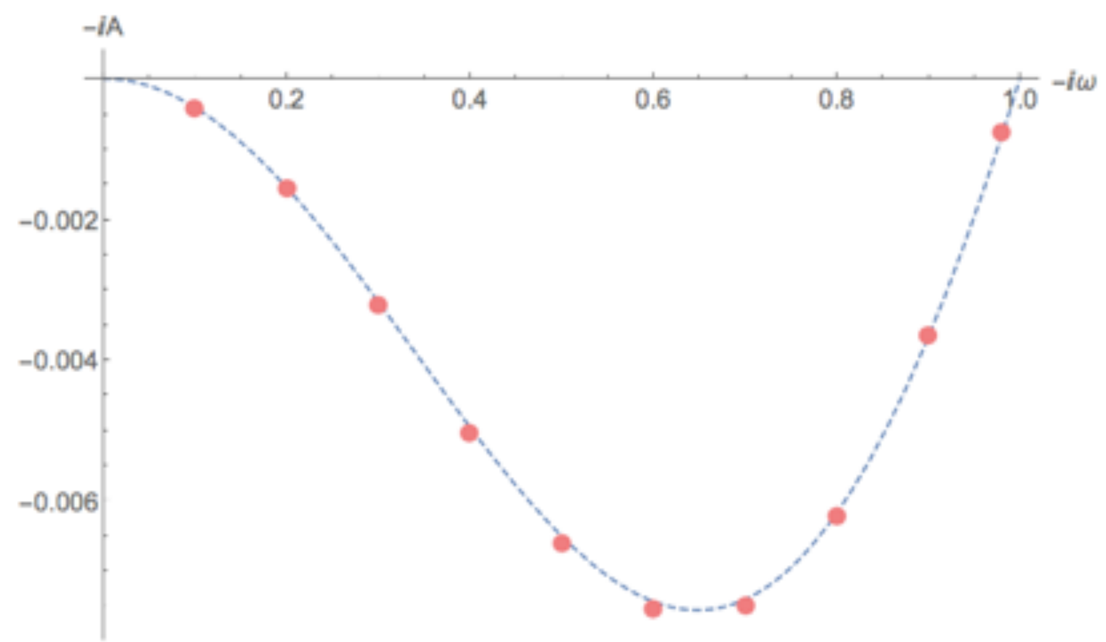

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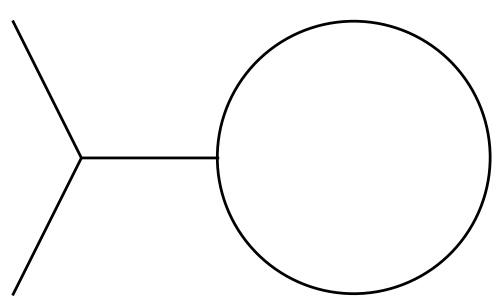

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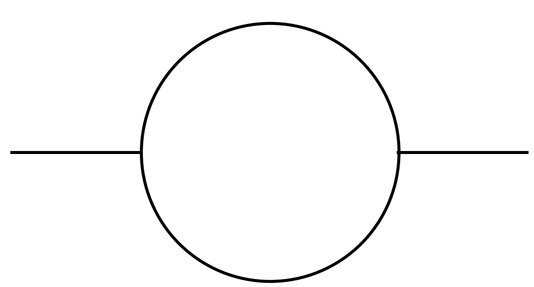

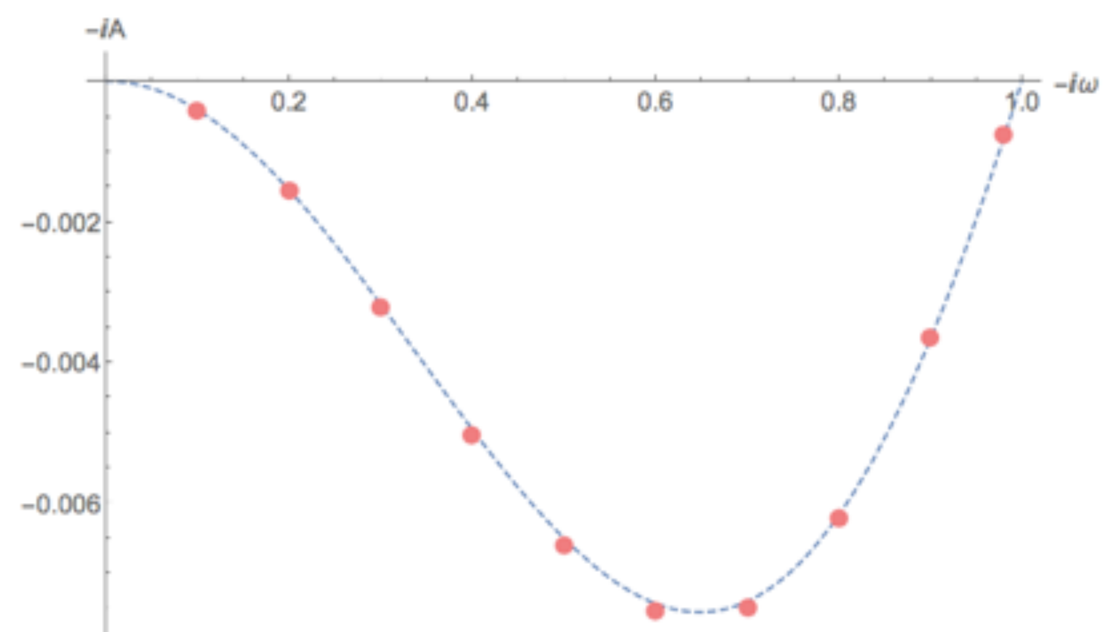

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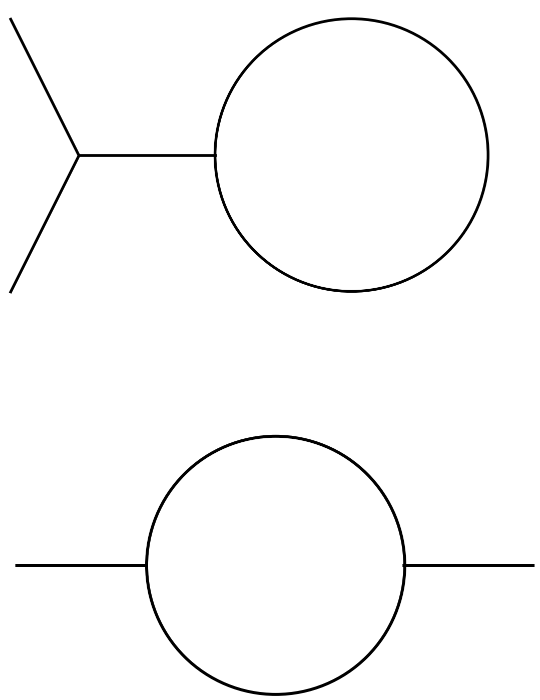

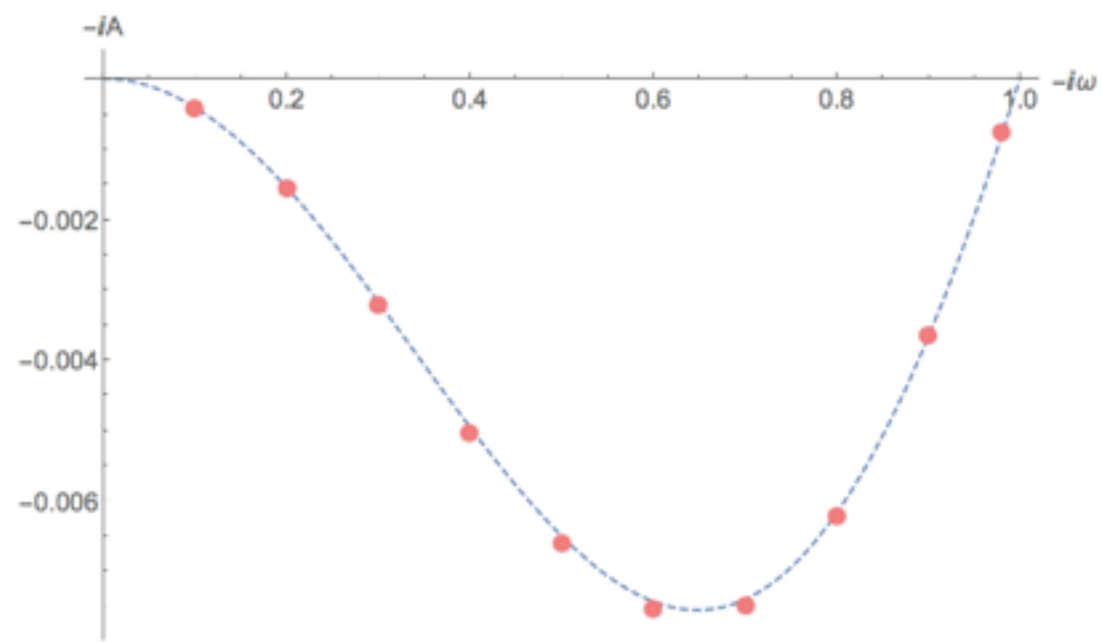

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Blue: matrix model result

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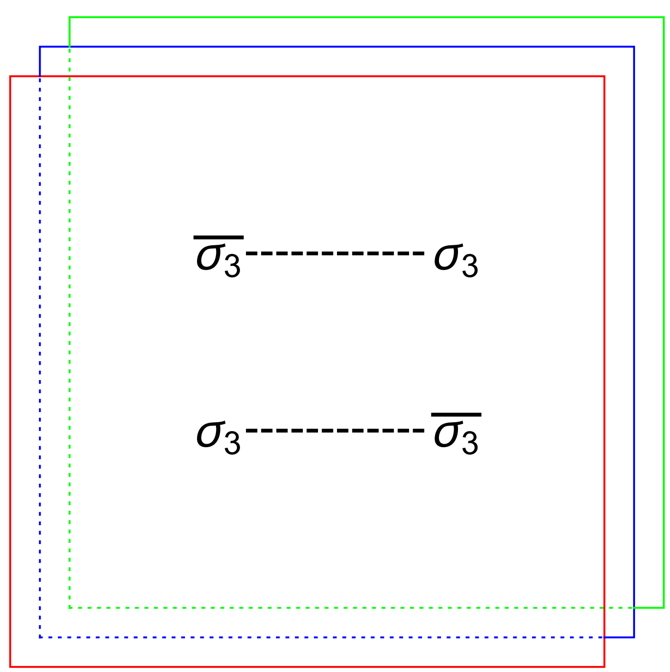

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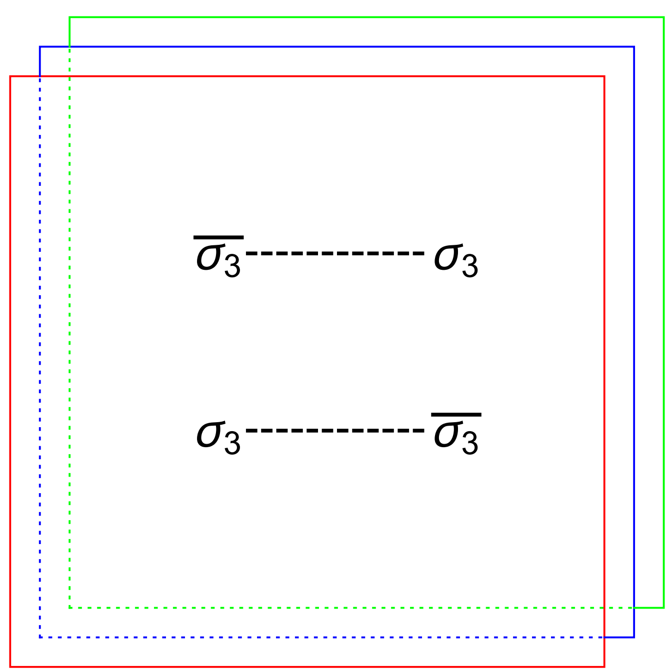

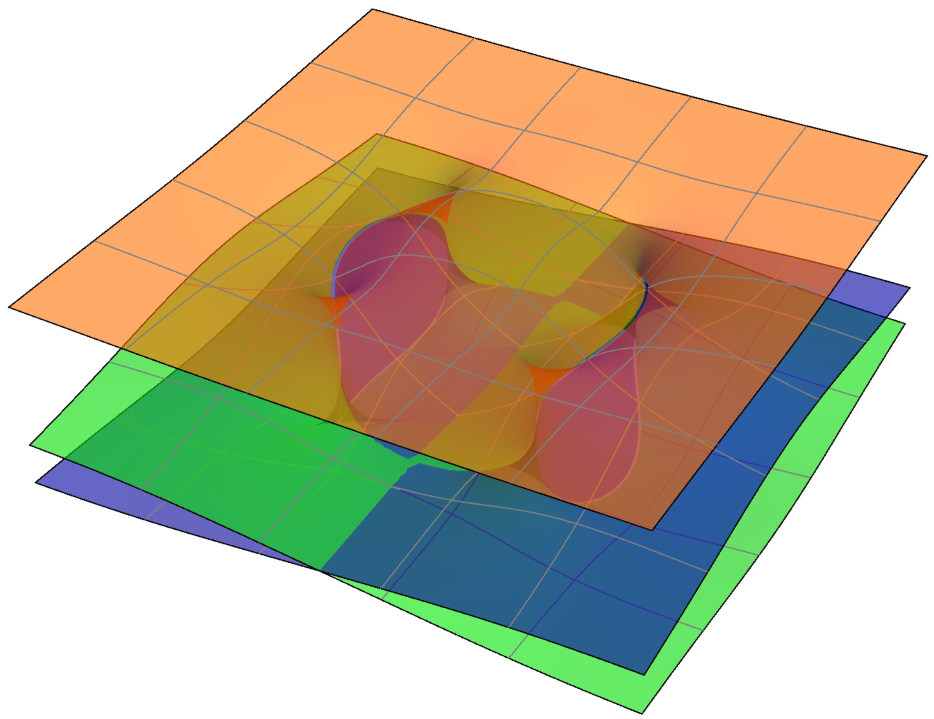

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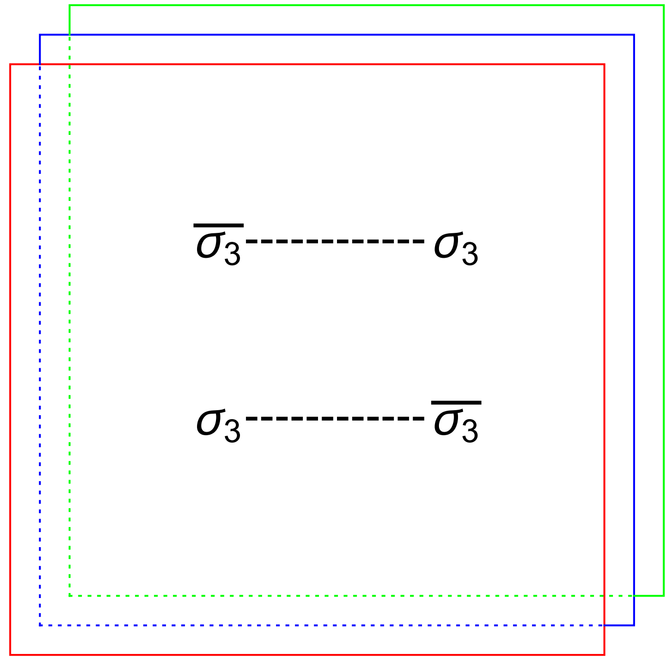

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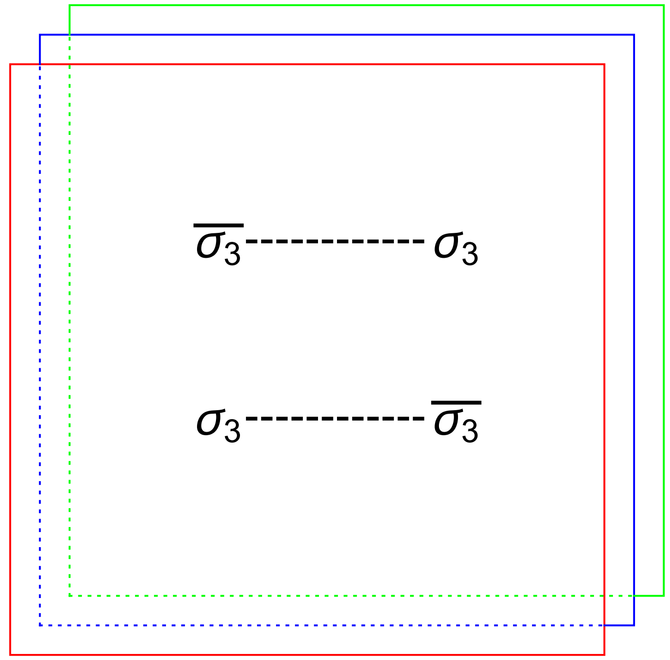

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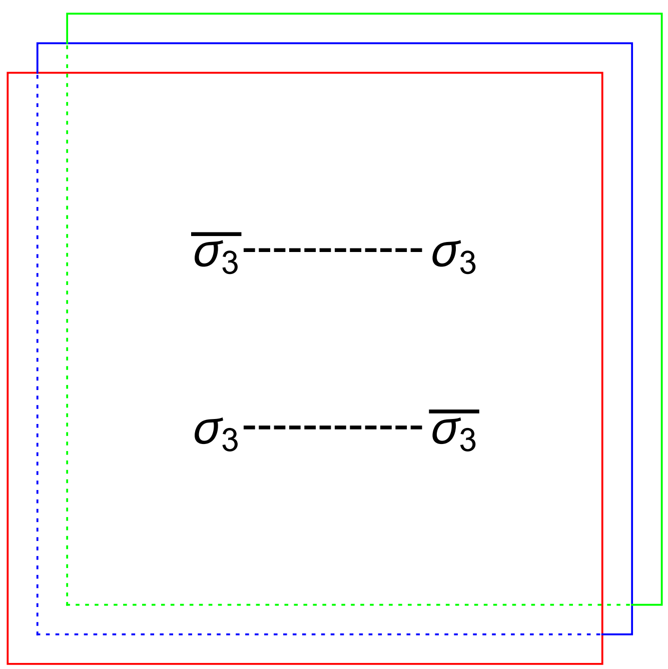

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The parameter **z** is the cross ratio of the four branch points on the sphere.

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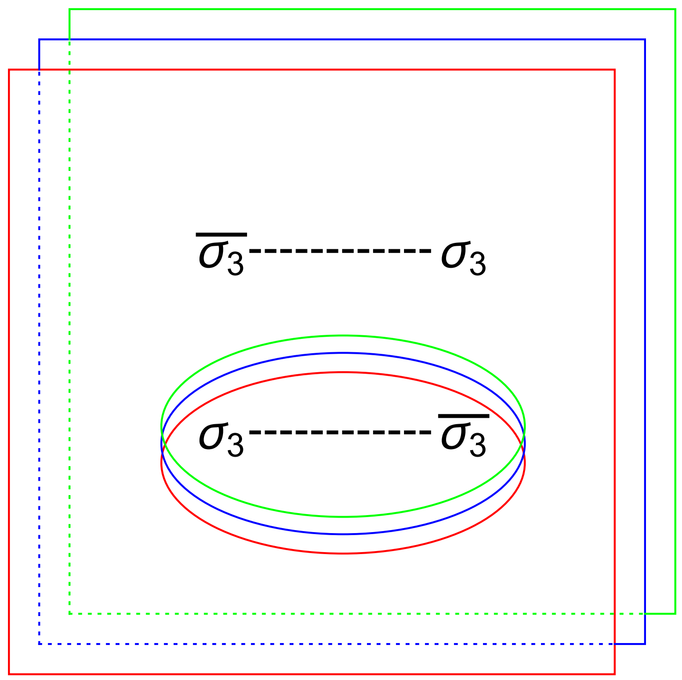

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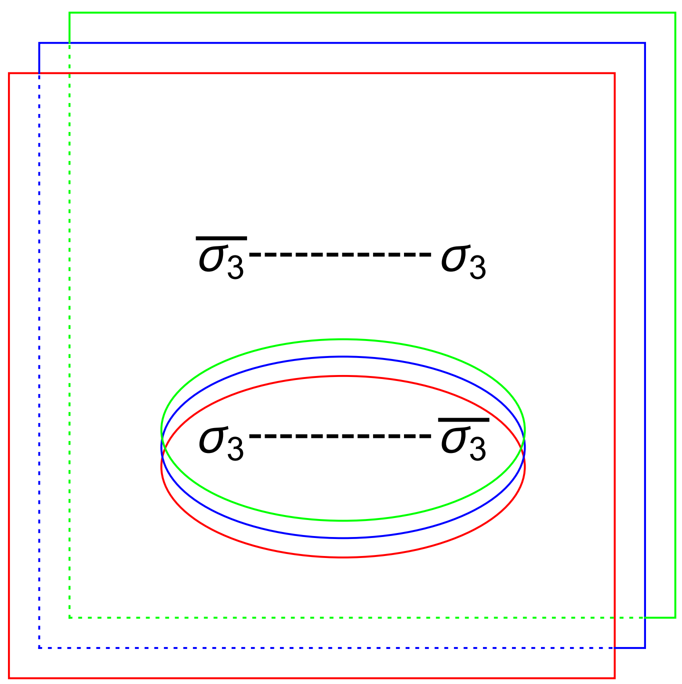

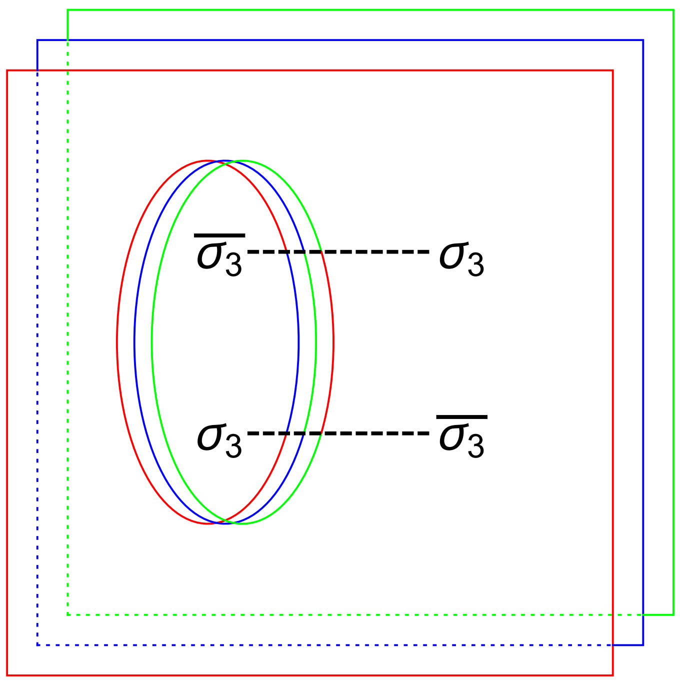

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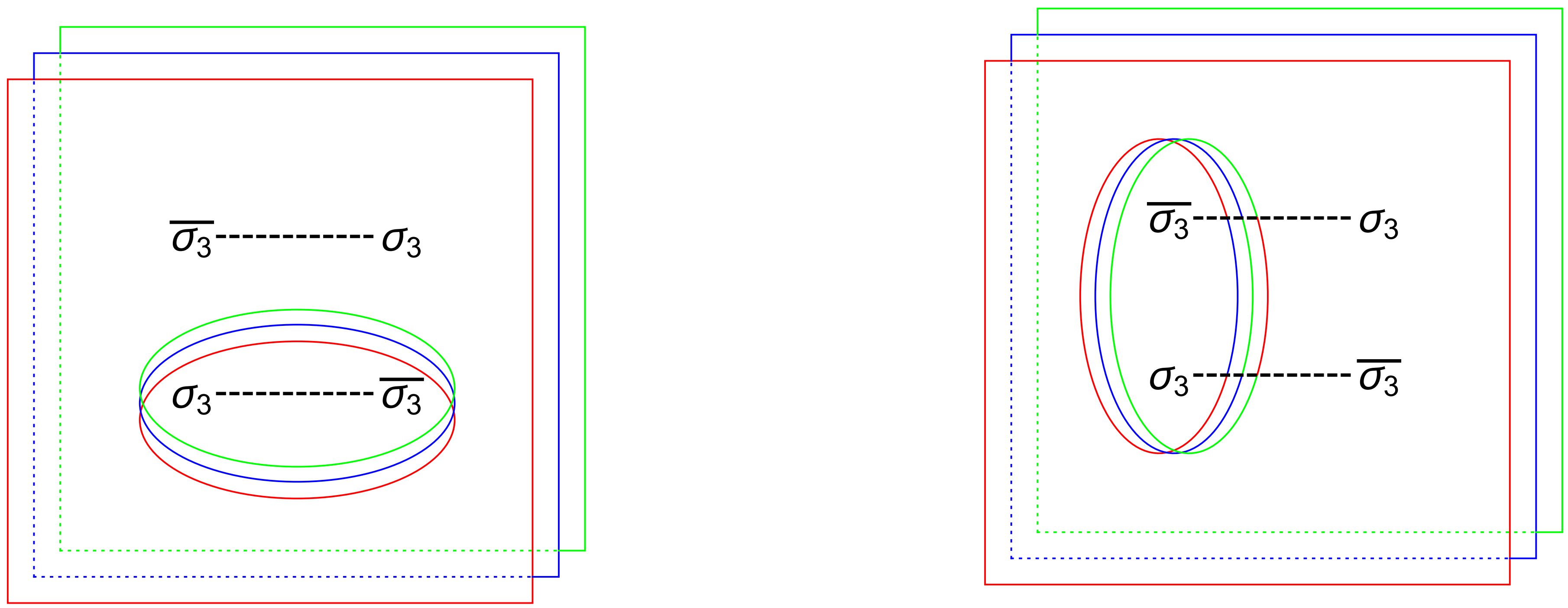

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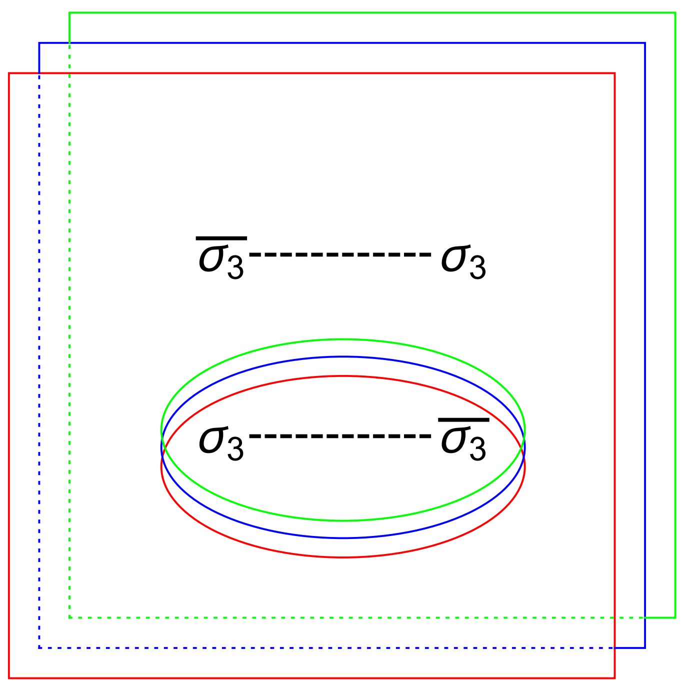

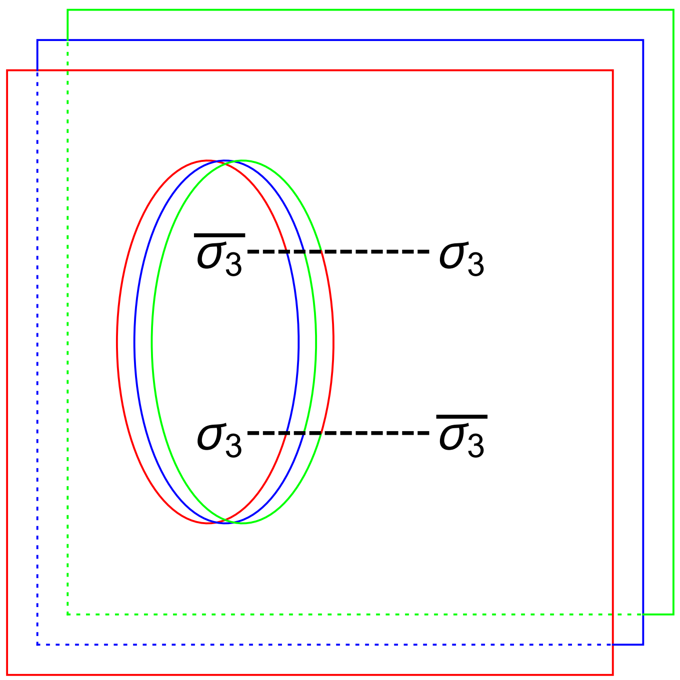

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We will focus on this crossing relation here.

Genus two conformal block decomposition

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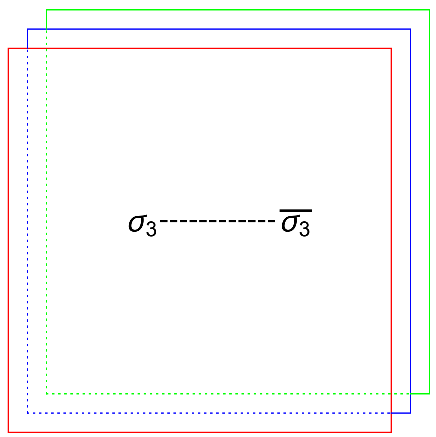

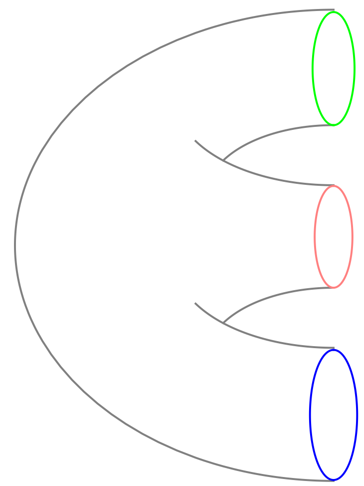

The OPE of a pair of Z_3 twist fields is a sum over tensor products of Virasoro descendants of three primaries.

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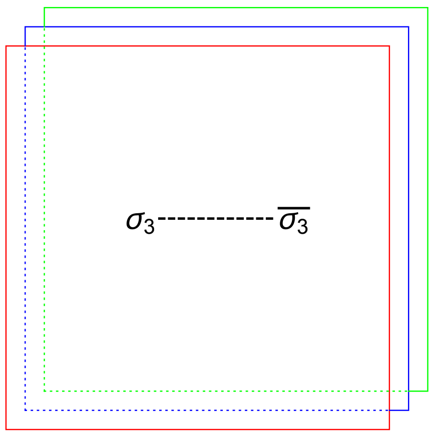

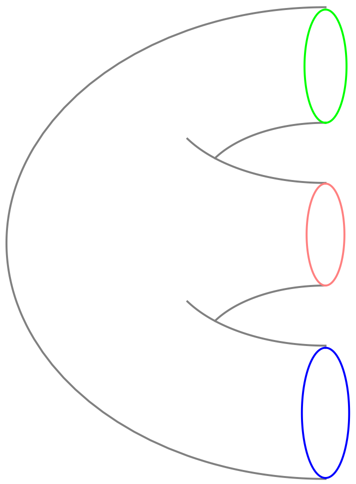

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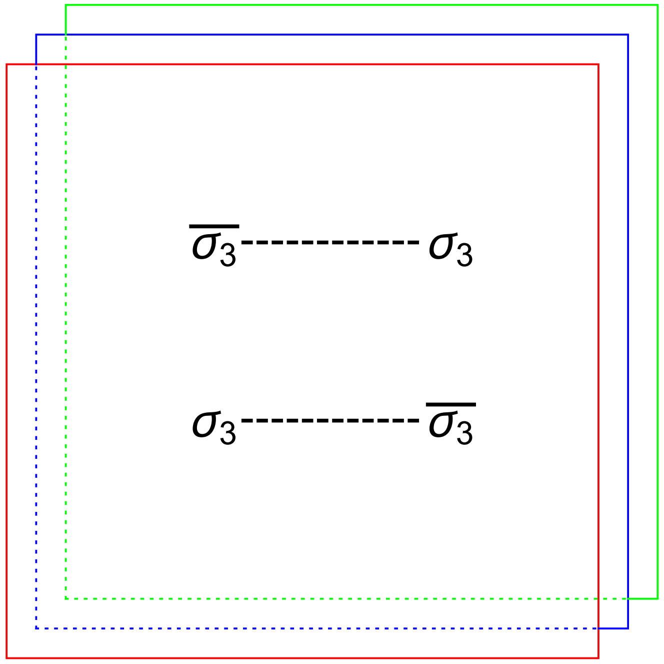

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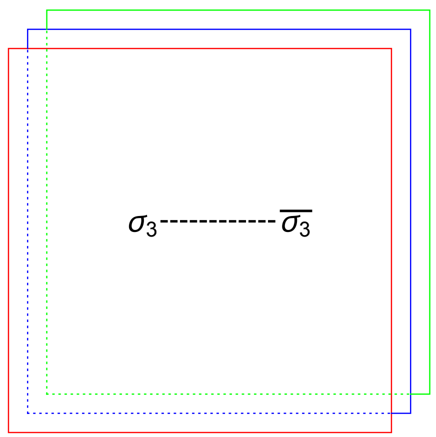

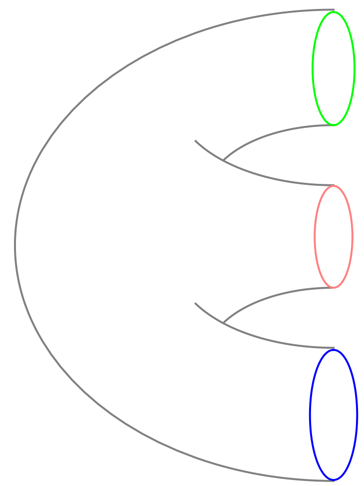

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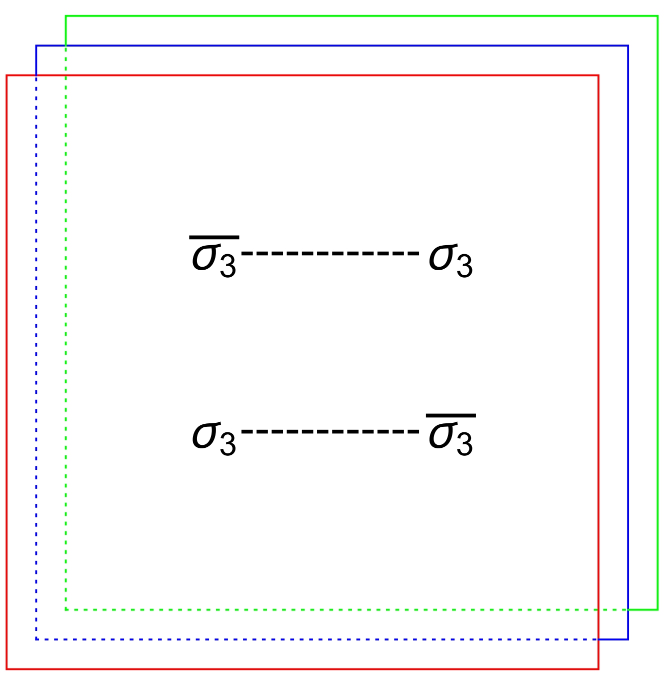

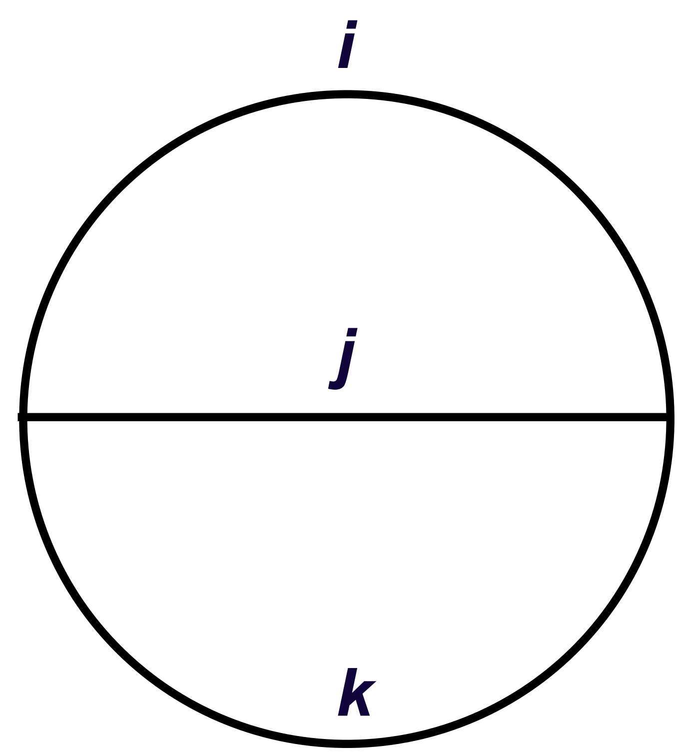

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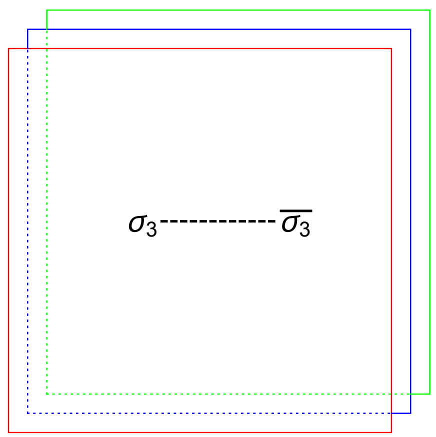

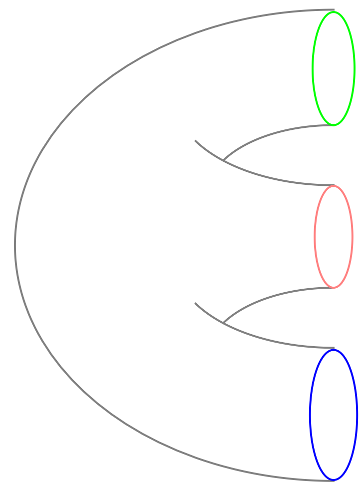

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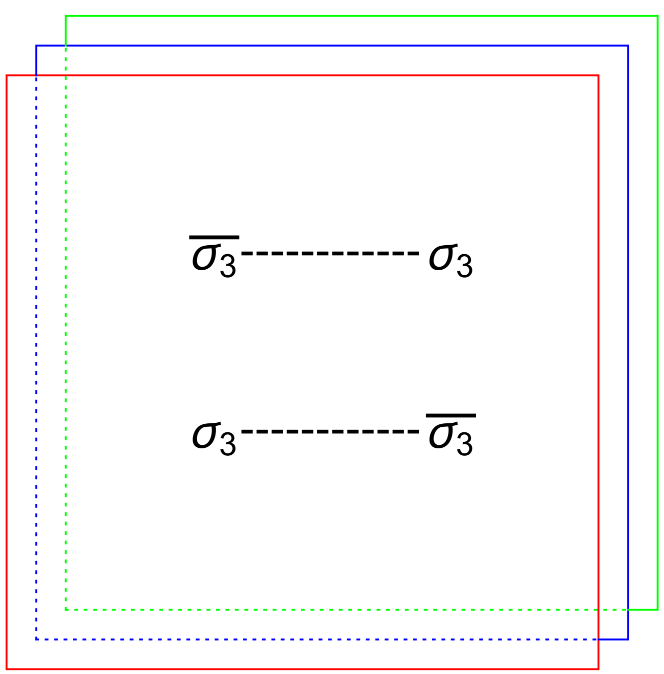

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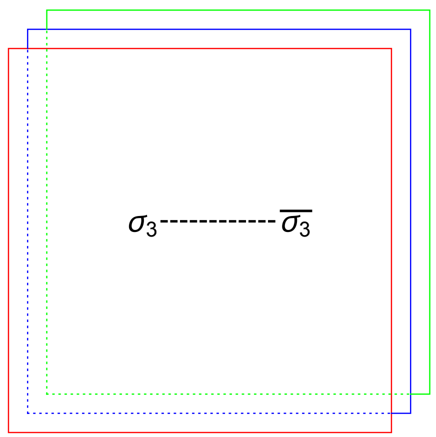

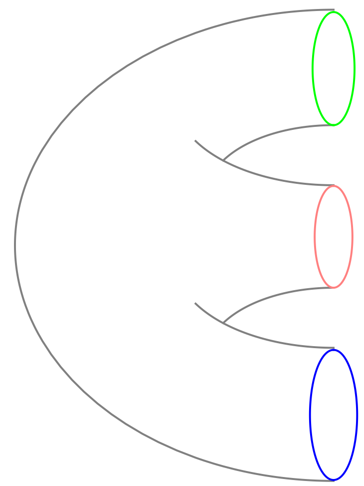

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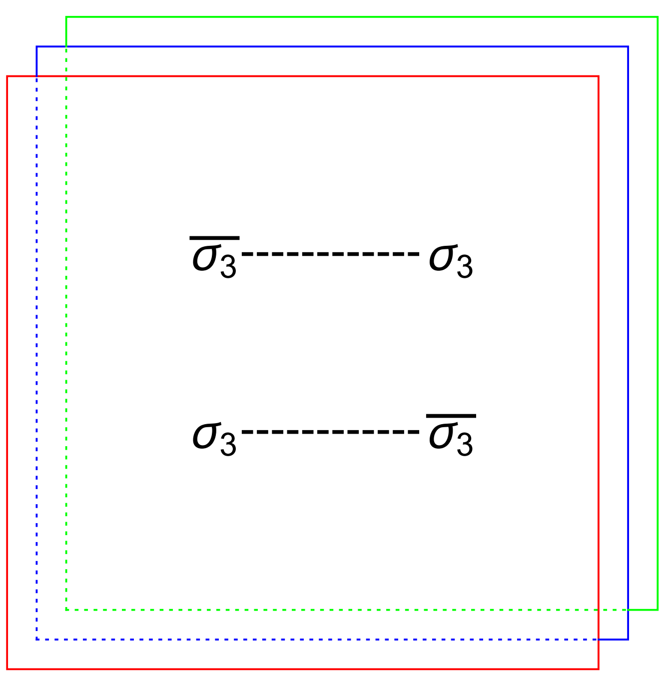

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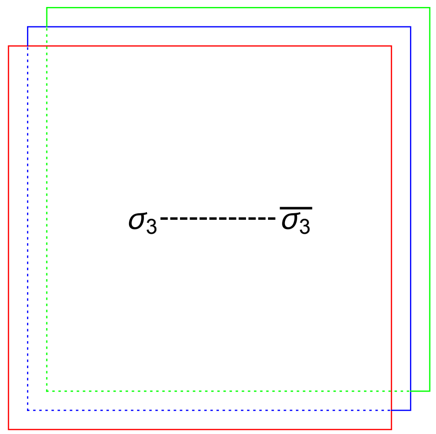
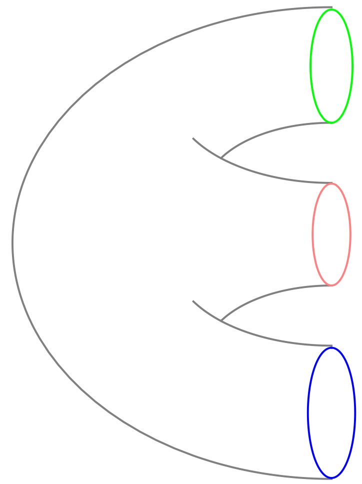
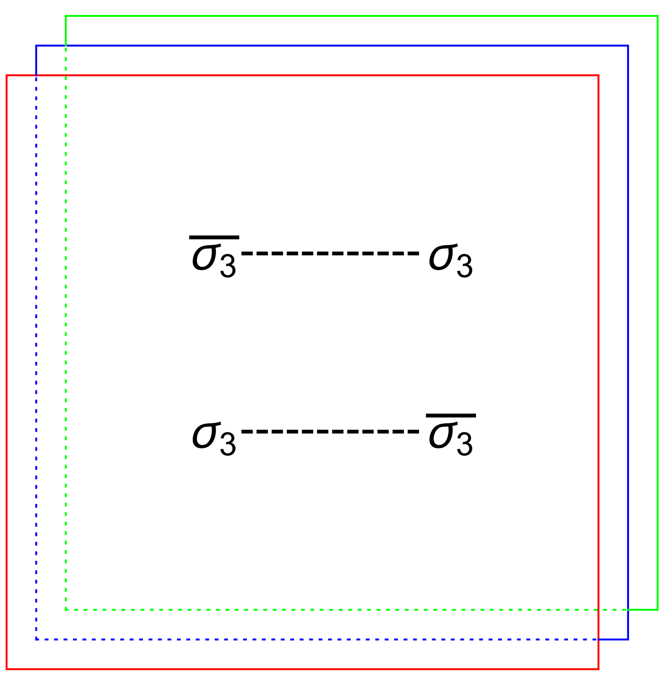
$$\mathcal{F}_c(h_1,h_2,h_3;z) = \exp\left[c\mathcal{F}^{cl}(z)\right]\mathcal{G}_c(h_1,h_2,h_3;z)$$

Genus two conformal block decomposition





The OPE of a pair of Z_3 twist fields is a sum over tensor products of Virasoro descendants of three primaries.

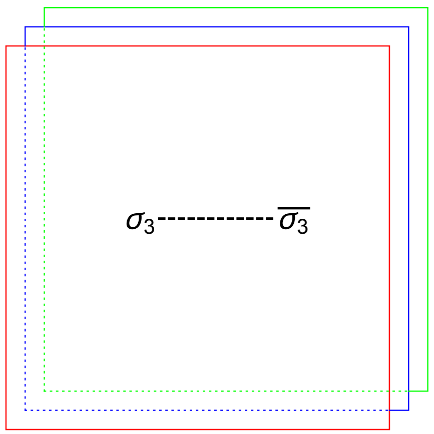


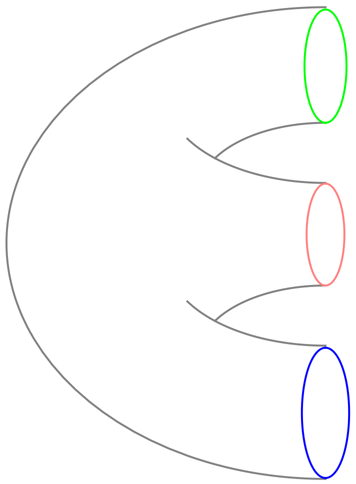
$$\langle \sigma_3(0)\overline{\sigma}_3(z,\bar{z})\sigma_3(1)\overline{\sigma}_3'(\infty)\rangle = \sum_{i,j,k} C_{ijk}^2 \mathcal{F}_c(h_i,h_j,h_k;z)\overline{\mathcal{F}}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k;\bar{z})$$

$$\mathcal{F}_c(h_1,h_2,h_3;z) = \exp\left[c\mathcal{F}^{cl}(z)\right]\mathcal{G}_c(h_1,h_2,h_3;z)$$

conformal anomaly

Genus two conformal block decomposition





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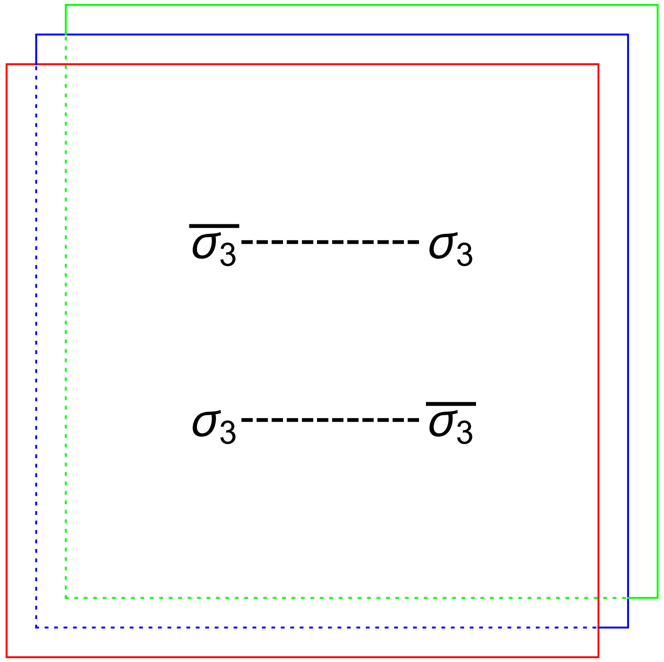

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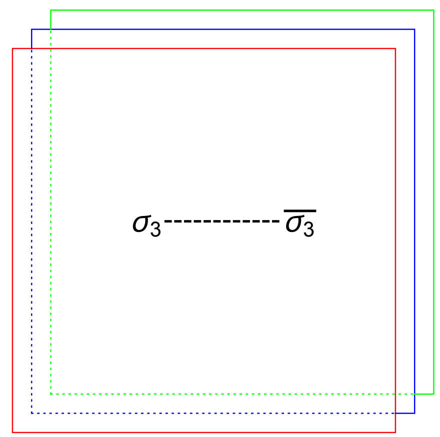
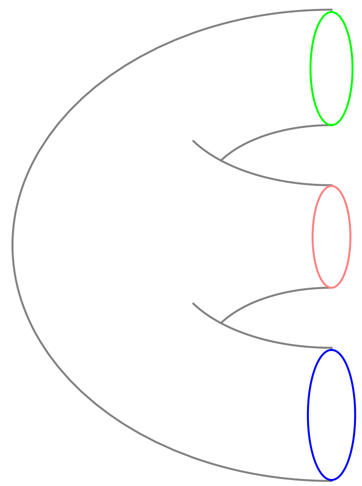
conformal anomaly

$$\mathcal{F}^{cl}(z) = -\frac{2}{9}\log(z) + 6\left(\frac{z}{27}\right)^2 + 162\left(\frac{z}{27}\right)^3 + 3975\left(\frac{z}{27}\right)^4 + 96552\left(\frac{z}{27}\right)^5 + 2356039\left(\frac{z}{27}\right)^6 + \cdots$$

Genus two conformal block decomposition





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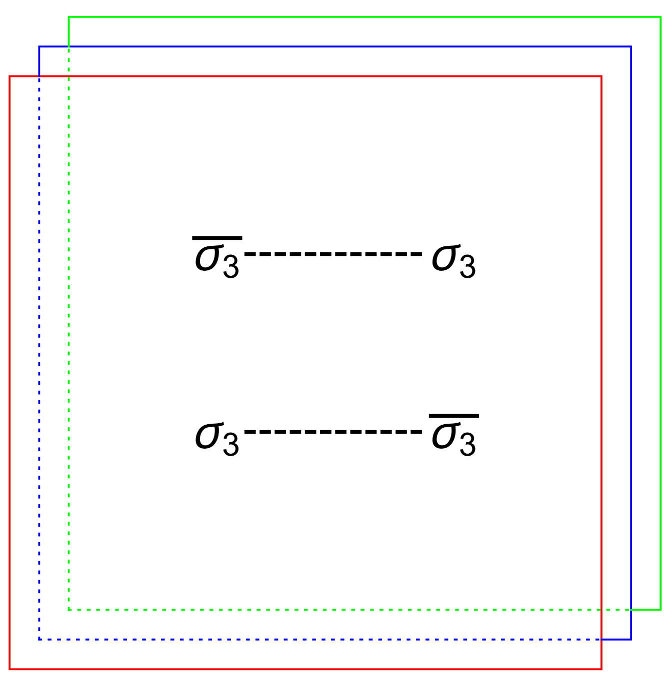

$$\langle \sigma_3(0)\overline{\sigma}_3(z,\bar{z})\sigma_3(1)\overline{\sigma}_3'(\infty)\rangle = \sum_{i,j,k} C^2_{ijk}\mathcal{F}_c(h_i,h_j,h_k;z)\overline{\mathcal{F}}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k;\bar{z})$$

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conformal anomaly

plumbing frame block

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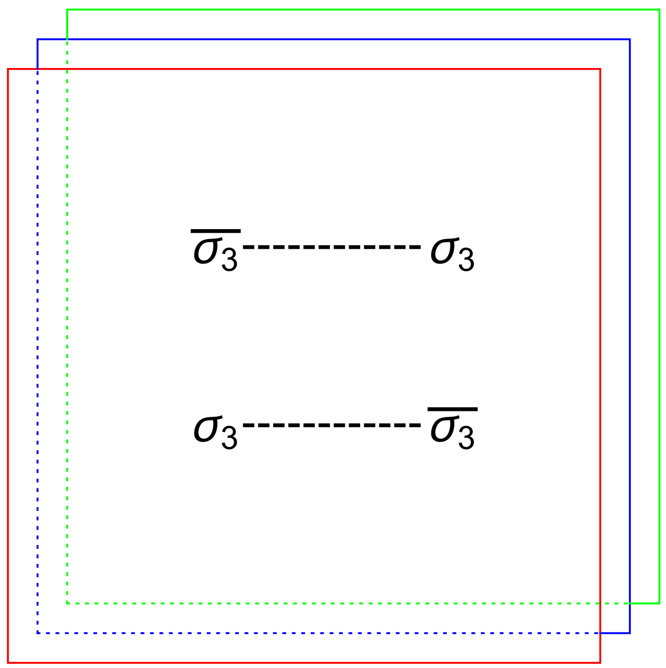
Genus two conformal block



$$\langle \sigma_3(0)\overline{\sigma}_3(z,\bar{z})\sigma_3(1)\overline{\sigma}'_3(\infty)\rangle = \sum_{i,j,k} C_{ijk}^2 \mathcal{F}_c(h_i,h_j,h_k;z)\overline{\mathcal{F}}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k;\bar{z})$$

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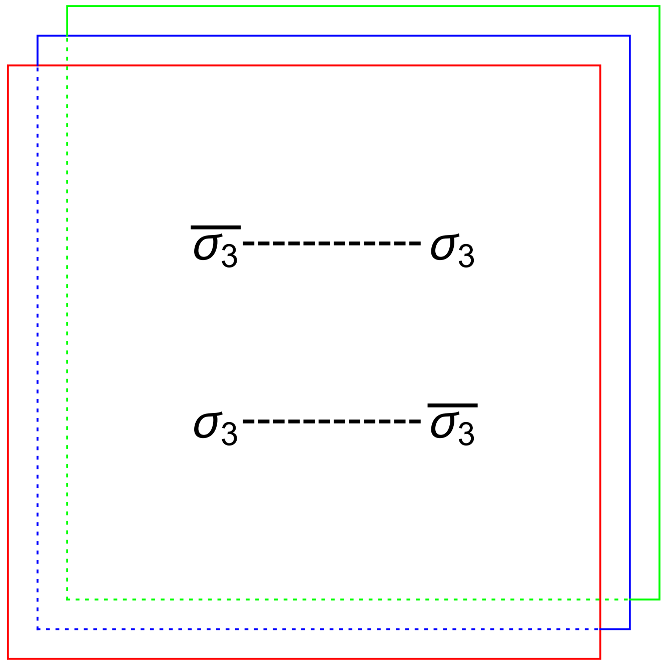
Genus two conformal block



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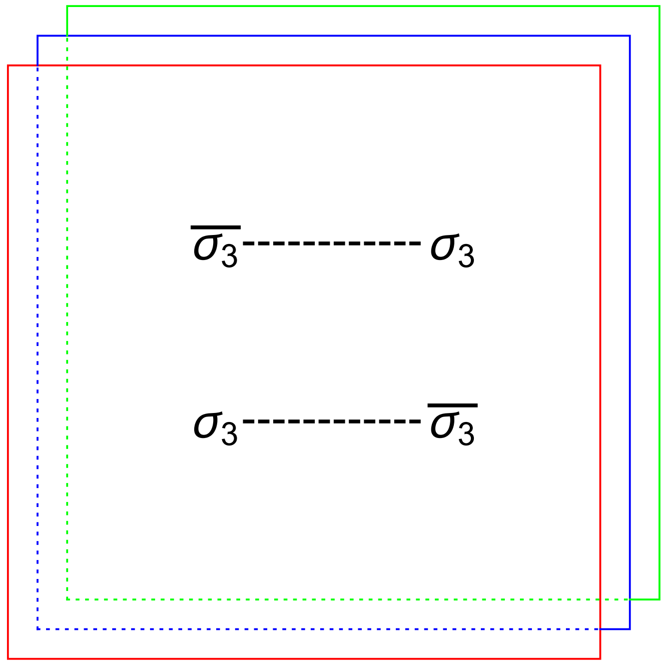
The infinite c limit of the plumbing frame block for the Renyi surface is

$$\mathcal{G}_\infty(h_1,h_2,h_3|z) = \left(\frac{z}{27}\right)^{h_1+h_2+h_3}\left\{1 + \left[\frac{h_1+h_2+h_3}{2} + \frac{(h_2-h_3)^2}{54h_1} + \frac{(h_3-h_1)^2}{54h_2} + \frac{(h_1-h_2)^2}{54h_3}\right]z\right.$$

[illegible]

$$\left. + \mathcal{O}(z^3)\right\}$$

Genus two conformal block



$$\langle \sigma_3(0)\overline{\sigma}_3(z,\bar{z})\sigma_3(1)\overline{\sigma}_3'(\infty)\rangle = \sum_{i,j,k} C_{ijk}^2 \mathcal{F}_c(h_i,h_j,h_k;z)\overline{\mathcal{F}}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k;\bar{z})$$

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conformal anomaly

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$$\left. + \text{[illegible]} + \mathcal{O}(z^3)\right\}$$

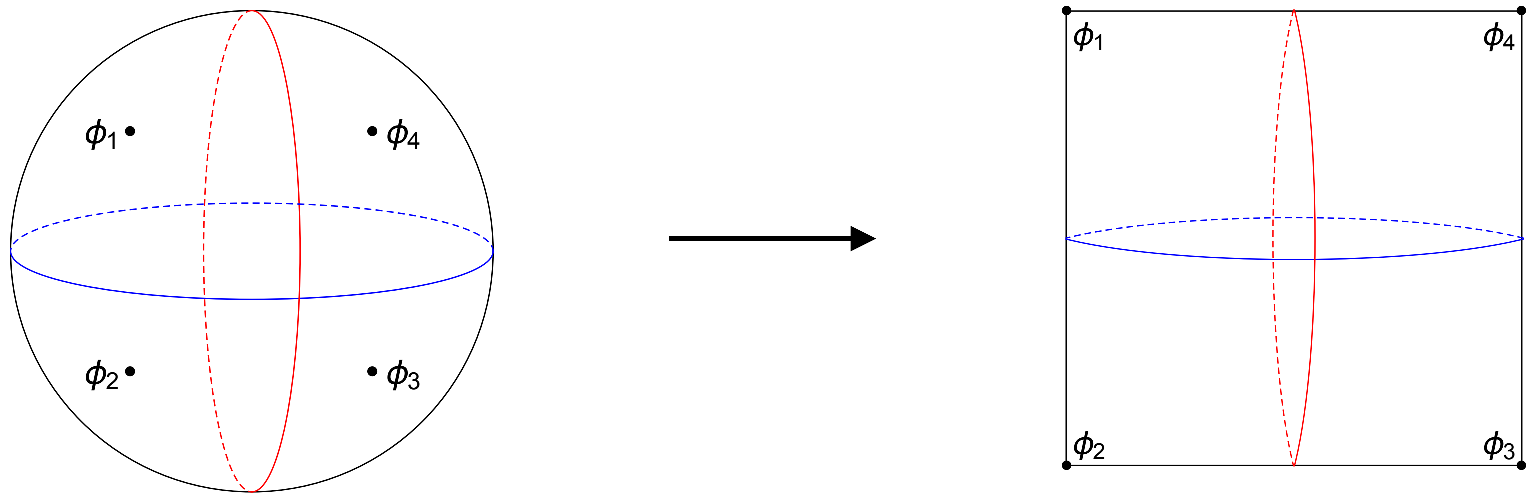
(Finite c result can be recovered by recursion formula.)

The pillow

[Maldacena-Simmons-Duffin-Zhiboedov ’15]

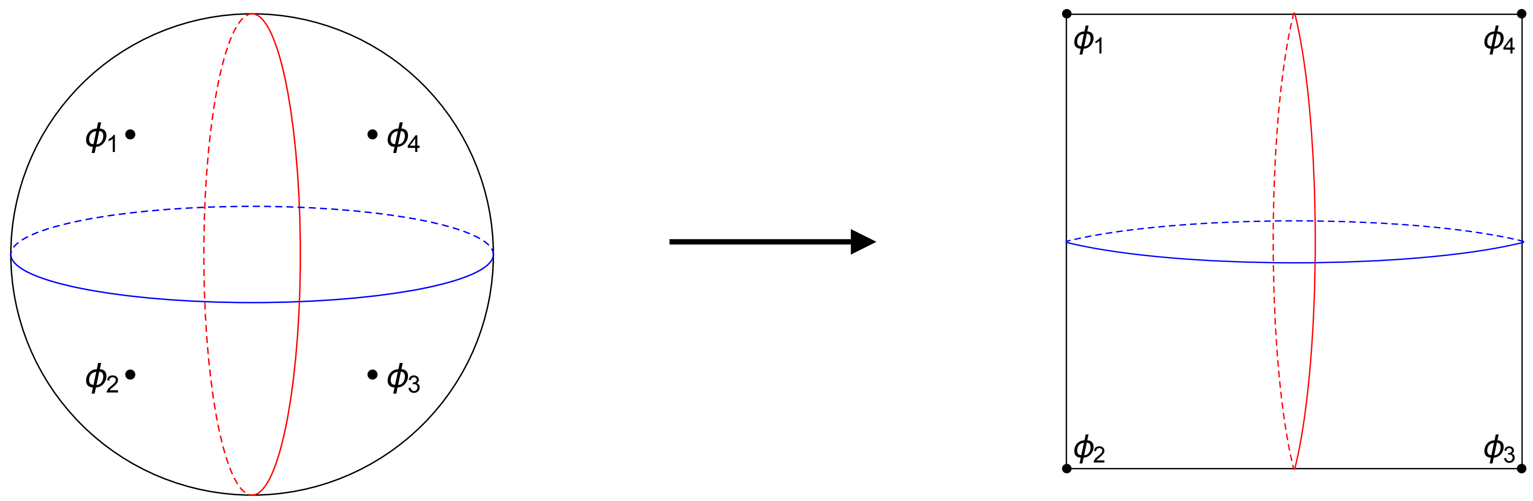
The pillow

[Maldacena-Simmons-Duffin-Zhiboedov ’15]



The pillow

[Maldacena-Simmons-Duffin-Zhiboedov ’15]

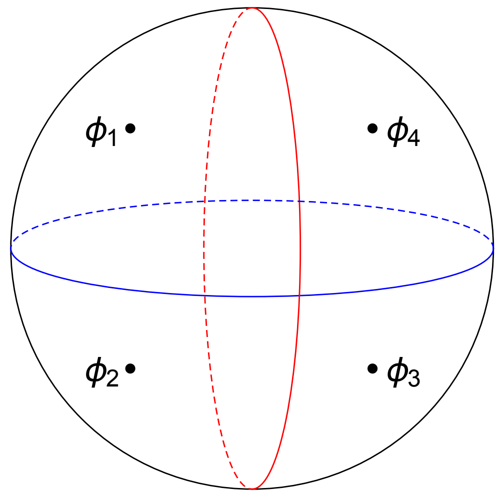


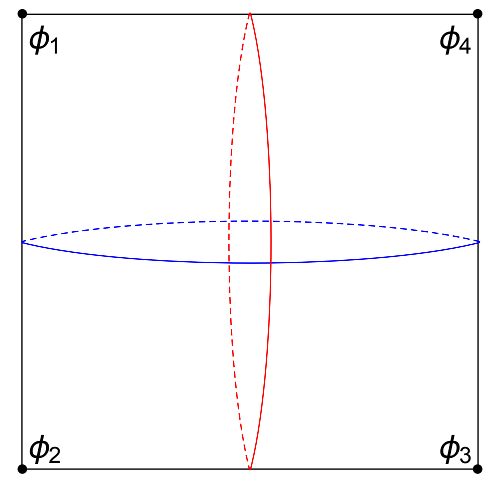
The 4-punctured sphere is conformally mapped to the pillow geometry (T^2/Z_2), with the identification of moduli

$$\tau = i\frac{K(1-z)}{K(z)}, \quad K(z) = {}_2F_1(\frac{1}{2}, \frac{1}{2}; 1; z) \qquad\qquad q = e^{\pi i \tau}$$

The pillow

[Maldacena-Simmons-Duffin-Zhiboedov ’15]

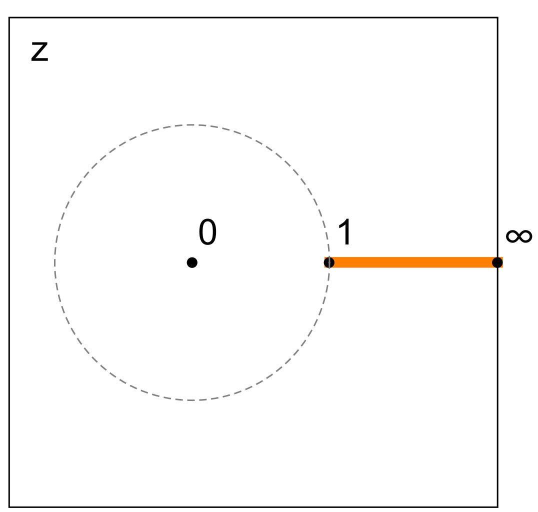




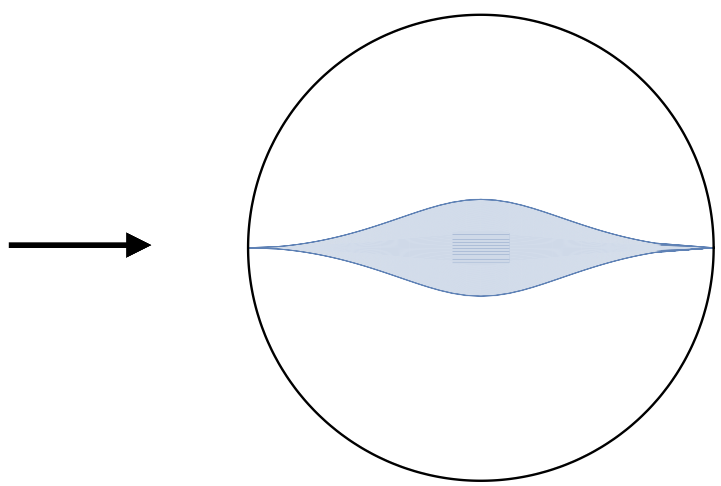
The 4-punctured sphere is conformally mapped to the pillow geometry (T^2/Z_2), with the identification of moduli

$$\tau = i\frac{K(1-z)}{K(z)}, \quad K(z) = {}_2F_1(\frac{1}{2},\frac{1}{2};1;z) \qquad q = e^{\pi i \tau}$$

z-plane

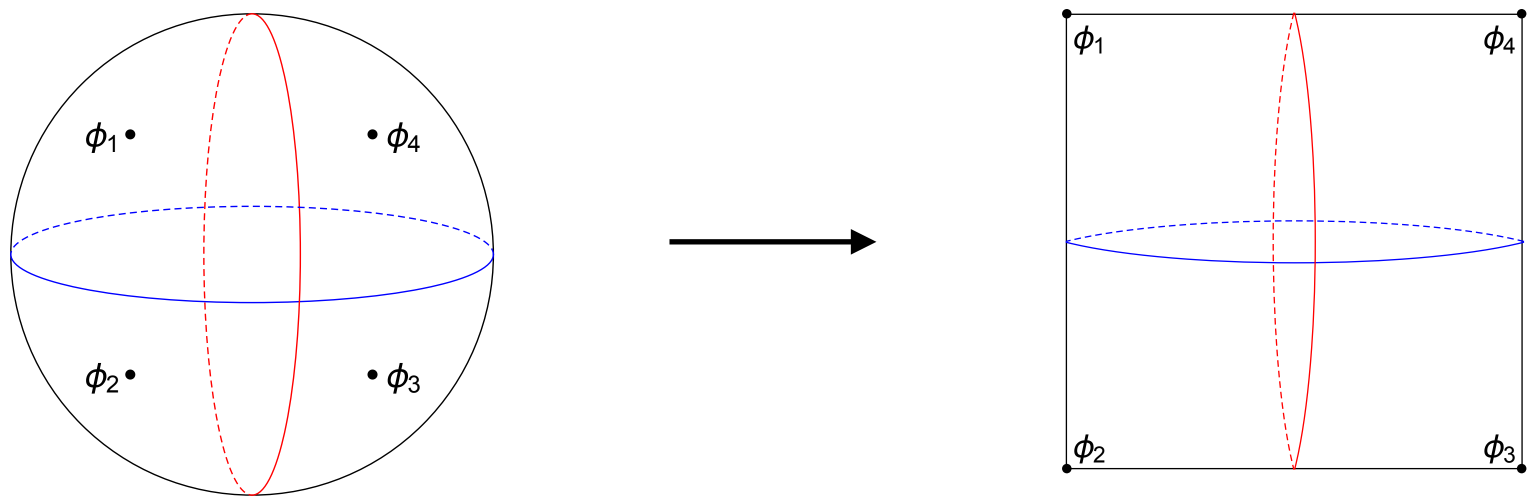


q-disc



The pillow

[Maldacena-Simmons-Duffin-Zhiboedov ’15]



The 4-punctured sphere is conformally mapped to the pillow geometry (T^2/Z_2), with the identification of moduli

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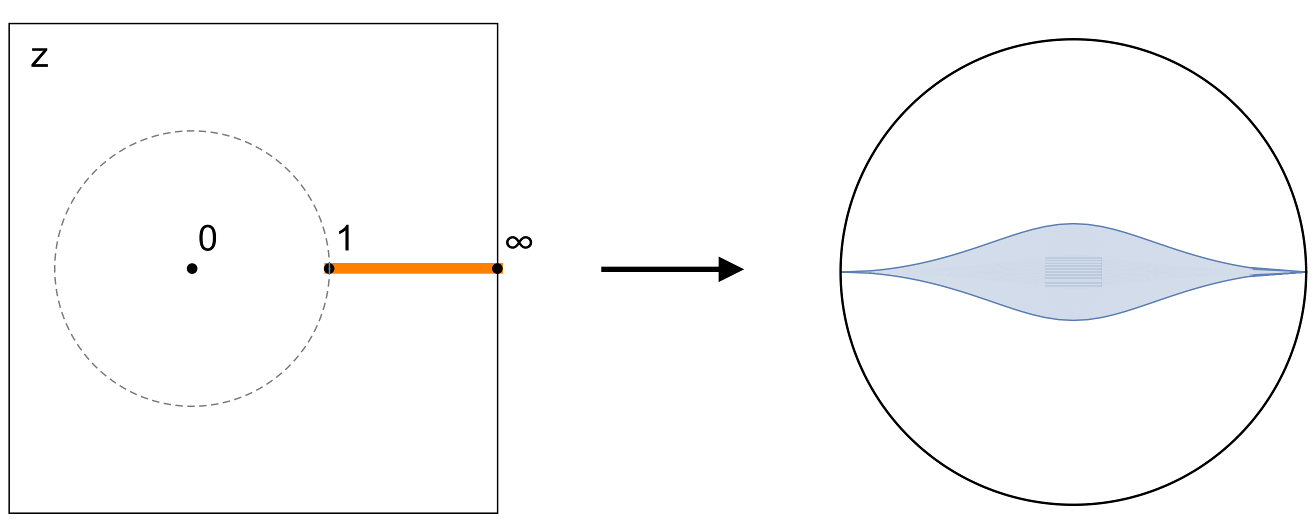
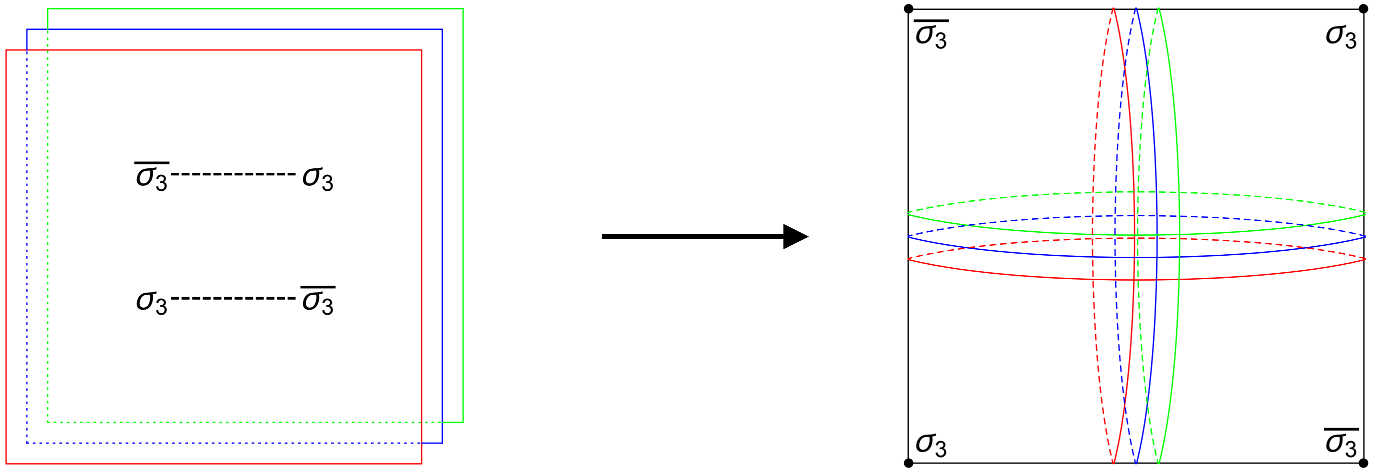
z-plane q-disc

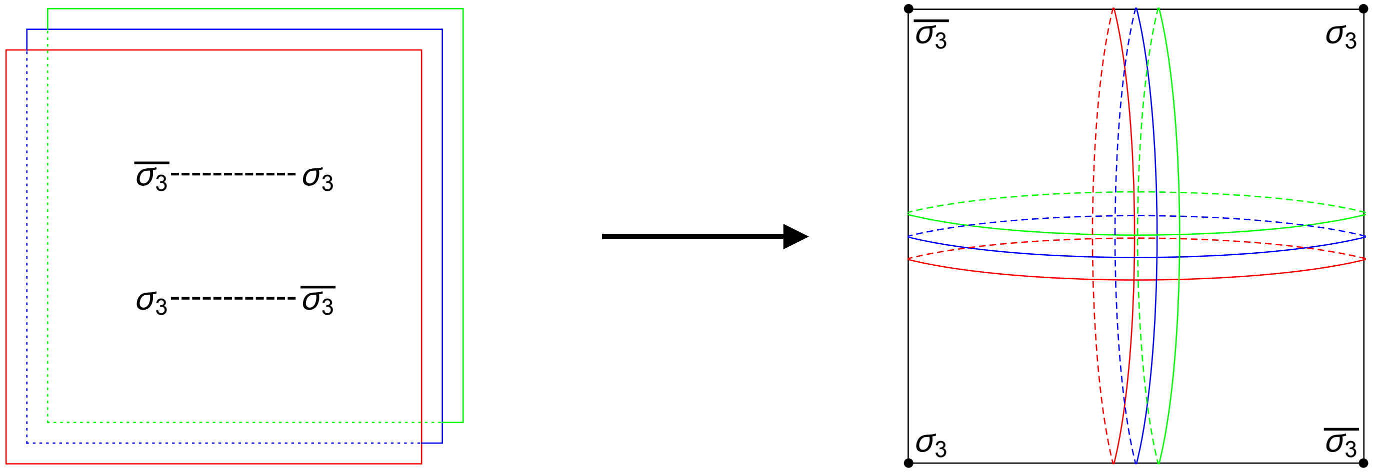


The q-expansion makes manifest certain analyticity and positivity properties of Virasoro conformal blocks.

The 3-fold pillow

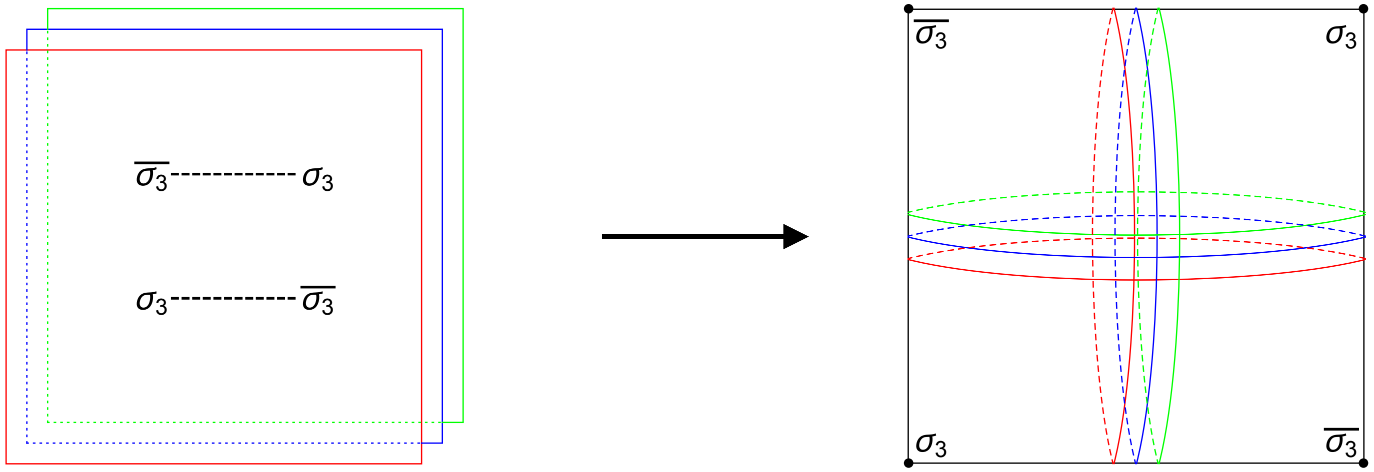


The 3-fold pillow



$$\mathcal{F}_c(h_1, h_2, h_3; z) = (z(1-z))^{-\frac{7c}{72}} (\theta_3(\tau))^{-\frac{5}{18}} q^{h_1+h_2+h_3-\frac{c}{8}} \sum_{n=0}^{\infty} A_n(h_1, h_2, h_3) q^n$$

The 3-fold pillow



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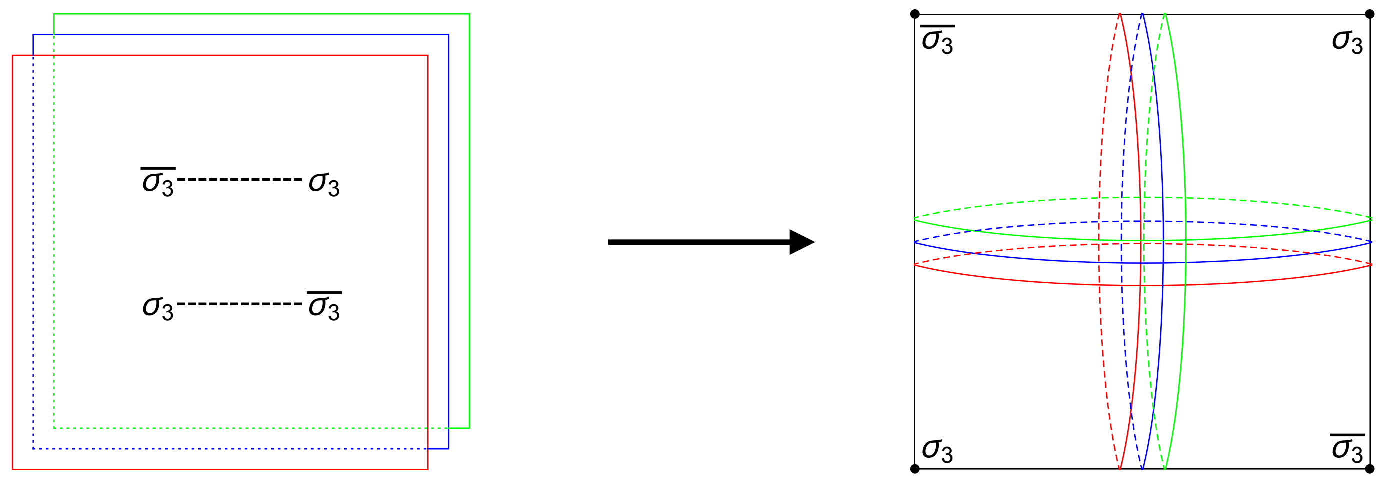
The coefficients A_n are non-negative.

The 3-fold pillow



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The coefficients A_n are non-negative.

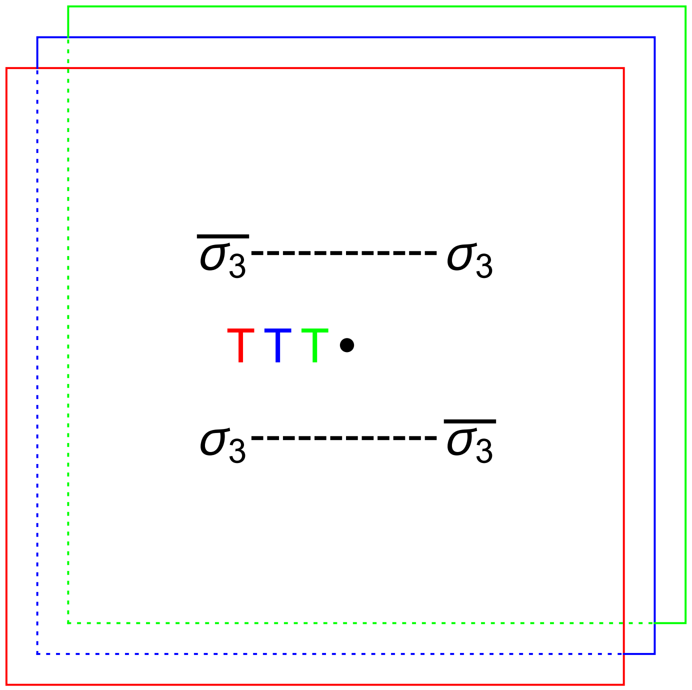
$$A_0 = 2^{-\frac{c}{2}} \left(\frac{16}{27}\right)^{h_1+h_2+h_3},$$

$$A_1 = 2^{-\frac{c}{2}-1} \left(\frac{16}{27}\right)^{h_1+h_2+h_3+1} \left[\frac{(h_1-h_2)^2}{h_3} + \frac{(h_2-h_3)^2}{h_1} + \frac{(h_3-h_1)^2}{h_2}\right],$$

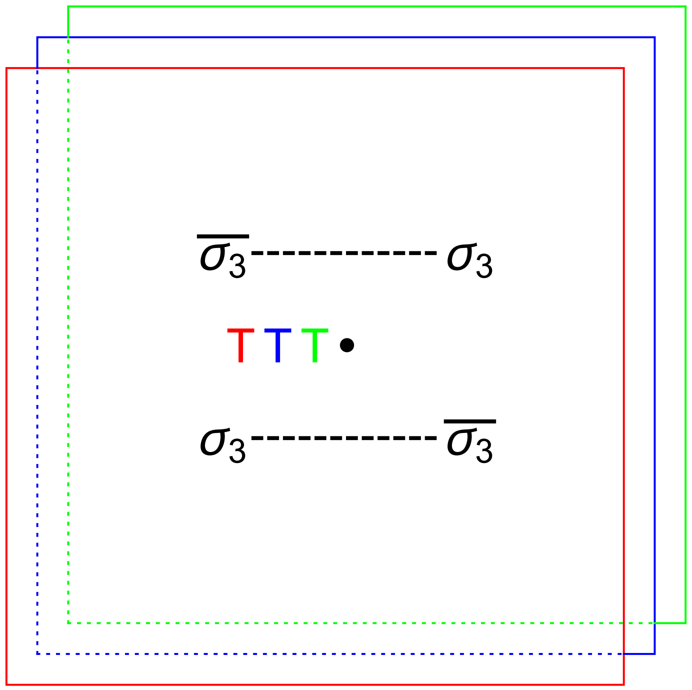
$$A_2 = \frac{2^{-\frac{c}{2}-9} \left(\frac{16}{27}\right)^{h_1+h_2+h_3+2}}{h_1(c+2h_1(c+8h_1-5))h_2(c+2h_2(c+8h_2-5))h_3(c+2h_3(c+8h_3-5))} \times$$

etc.

Genus two crossing equation beyond the Renyi surface



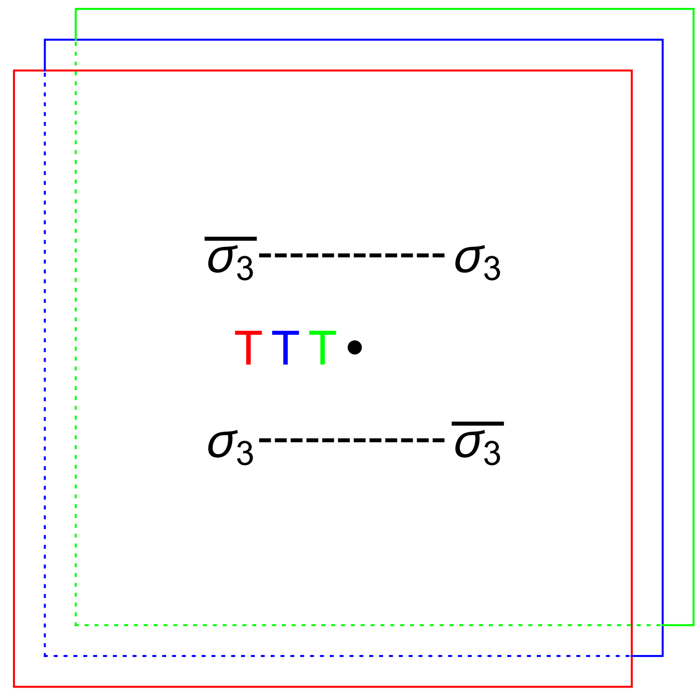
Genus two crossing equation beyond the Renyi surface



$$(-)^{\sum_{j=1}^{3}(|R_j|+|\tilde{R}_j|)}\sum_{(h_i,\tilde{h}_i)} C^2_{h_1,h_2,h_3;\tilde{h}_1,\tilde{h}_2,\tilde{h}_3}\mathbb{F}(h_1,h_2,h_3;R_1,R_2,R_3;w|z)\mathbb{F}(\tilde{h}_1,\tilde{h}_2,\tilde{h}_3;\tilde{R}_1,\tilde{R}_2,\tilde{R}_3;\bar{w}|\bar{z})$$

$$=\sum_{(h_i,\tilde{h}_i)} C^2_{h_1,h_2,h_3;\tilde{h}_1,\tilde{h}_2,\tilde{h}_3}\mathbb{F}(h_1,h_2,h_3;R_1,R_2,R_3;1-w|1-z)\mathbb{F}(\tilde{h}_1,\tilde{h}_2,\tilde{h}_3;\tilde{R}_1,\tilde{R}_2,\tilde{R}_3;1-\bar{w}|1-\bar{z}).$$

Genus two crossing equation beyond the Renyi surface

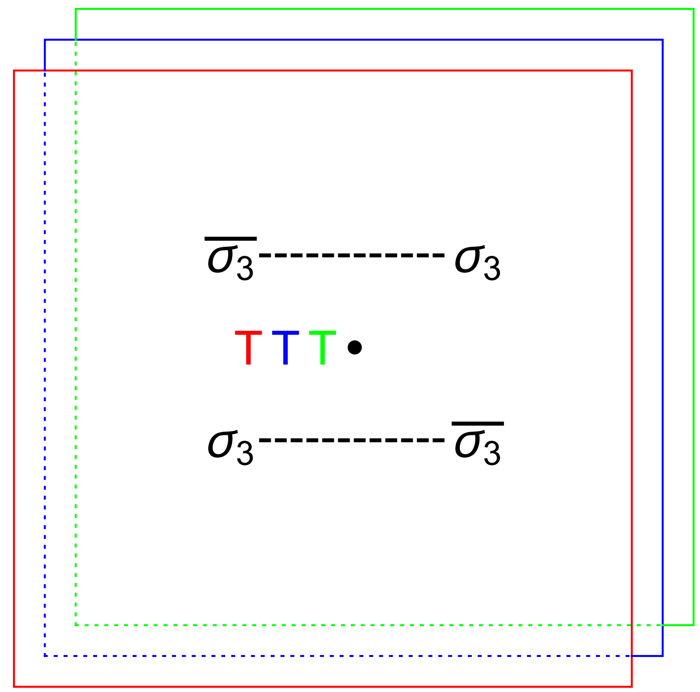


$$(-)^{\sum_{j=1}^{3}(|R_j|+|\tilde{R}_j|)} \sum_{(h_i,\tilde{h}_i)} C^2_{h_1,h_2,h_3;\tilde{h}_1,\tilde{h}_2,\tilde{h}_3} \mathbb{F}(h_1,h_2,h_3;R_1,R_2,R_3;w|z)\mathbb{F}(\tilde{h}_1,\tilde{h}_2,\tilde{h}_3;\tilde{R}_1,\tilde{R}_2,\tilde{R}_3;\bar{w}|\bar{z})$$
$$= \sum_{(h_i,\tilde{h}_i)} C^2_{h_1,h_2,h_3;\tilde{h}_1,\tilde{h}_2,\tilde{h}_3} \mathbb{F}(h_1,h_2,h_3;R_1,R_2,R_3;1-w|1-z)\mathbb{F}(\tilde{h}_1,\tilde{h}_2,\tilde{h}_3;\tilde{R}_1,\tilde{R}_2,\tilde{R}_3;1-\bar{w}|1-\bar{z}).$$

Modified genus two conformal blocks (with insertions of Virasoro descendants of id)

$$\mathbb{F}(h_1,h_2,h_3;R_1,R_2,R_3;w|z) = 3^{-3\sum_{i=1}^{3} h_i} \sum_{\{N_i\},\{M_i\}} z^{-2h_\sigma+\sum_{i=1}^{3}(h_i+|N_i|)} w^{\sum_{k=1}^{3}(|M_k|-|N_k|-|R_k|)}$$
$$\times \rho(\mathcal{L}^{\infty}_{-N_3}h_3, \mathcal{L}^{1}_{-N_2}h_2, \mathcal{L}^{0}_{-N_1}h_1)\rho(\mathcal{L}^{\infty *}_{-M_3}h_3, \mathcal{L}^{1*}_{-M_2}h_2, \mathcal{L}^{0*}_{-M_1}h_1)$$
$$\times \sum_{|P_i|=|N_i|,\ |Q_i|=|M_i|} \prod_{k=1}^{3} G^{N_k P_k}_{h_k} G^{M_k Q_k}_{h_k} \rho(L_{-Q_k}h_k, L_{-R_k}\mathrm{id}, L_{-P_k}h_k)$$

Genus two crossing equation beyond the Renyi surface



Triplet of Virasoro descendants of identity operator inserted on the three sheets

$$(-)^{\sum_{j=1}^{3}(|R_j|+|\tilde{R}_j|)} \sum_{(h_i,\tilde{h}_i)} C^2_{h_1,h_2,h_3;\tilde{h}_1,\tilde{h}_2,\tilde{h}_3} \mathbb{F}(h_1,h_2,h_3;R_1,R_2,R_3;w|z)\mathbb{F}(\tilde{h}_1,\tilde{h}_2,\tilde{h}_3;\tilde{R}_1,\tilde{R}_2,\tilde{R}_3;\bar{w}|\bar{z})$$

$$= \sum_{(h_i,\tilde{h}_i)} C^2_{h_1,h_2,h_3;\tilde{h}_1,\tilde{h}_2,\tilde{h}_3} \mathbb{F}(h_1,h_2,h_3;R_1,R_2,R_3;1-w|1-z)\mathbb{F}(\tilde{h}_1,\tilde{h}_2,\tilde{h}_3;\tilde{R}_1,\tilde{R}_2,\tilde{R}_3;1-\bar{w}|1-\bar{z}).$$

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$$\times \rho(\mathcal{L}^{\infty}_{-N_3}h_3, \mathcal{L}^{1}_{-N_2}h_2, \mathcal{L}^{0}_{-N_1}h_1)\rho(\mathcal{L}^{\infty *}_{-M_3}h_3, \mathcal{L}^{1*}_{-M_2}h_2, \mathcal{L}^{0*}_{-M_1}h_1)$$
$$\times \sum_{|P_i|=|N_i|,\ |Q_i|=|M_i|} \prod_{k=1}^{3} G^{N_k P_k}_{h_k} G^{M_k Q_k}_{h_k} \rho(L_{-Q_k}h_k, L_{-R_k}\mathrm{id}, L_{-P_k}h_k)$$

Constraints on structure constants from genus two crossing equation

Constraints on structure constants from genus two crossing equation

Restricting to Renyi surfaces for simplicity.

$$\sum_{i,j,k\in\mathcal{I}} C_{ijk}^2 \left[\mathcal{F}_c(h_i,h_j,h_k|z)\mathcal{F}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k|\bar{z}) - \mathcal{F}_c(h_i,h_j,h_k|1-z)\mathcal{F}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k|1-\bar{z})\right] = 0.$$

Constraints on structure constants from genus two crossing equation

Restricting to Renyi surfaces for simplicity.

$$\sum_{i,j,k\in\mathcal{I}} C^2_{ijk}\left[\mathcal{F}_c(h_i,h_j,h_k|z)\mathcal{F}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k|\bar{z}) - \mathcal{F}_c(h_i,h_j,h_k|1-z)\mathcal{F}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k|1-\bar{z})\right] = 0.$$

Applying linear functional

$$\alpha = \sum_{n+m=\text{odd}} a_{n,m}\partial_z^n\partial_{\bar{z}}^m\Big|_{z=\bar{z}=\frac{1}{2}}$$

Constraints on structure constants from genus two crossing equation

Restricting to Renyi surfaces for simplicity.

$$\sum_{i,j,k\in\mathcal{I}} C_{ijk}^2 \left[\mathcal{F}_c(h_i,h_j,h_k|z)\mathcal{F}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k|\bar{z}) - \mathcal{F}_c(h_i,h_j,h_k|1-z)\mathcal{F}_c(\tilde{h}_i,\tilde{h}_j,\tilde{h}_k|1-\bar{z})\right] = 0.$$

Applying linear functional

$$\alpha = \sum_{n+m=\text{odd}} a_{n,m}\partial_z^n\partial_{\bar{z}}^m\Big|_{z=\bar{z}=\frac{1}{2}}$$

Obtain constraints on structure constants of the form

$$\sum_{i,j,k\in\mathcal{I}} C_{ijk}^2 F_c^{\alpha}(h_i,h_j,h_k;\tilde{h}_i,\tilde{h}_j,\tilde{h}_k) = 0$$

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For instance, if $F_c^\alpha(h_1,h_2,h_3;\tilde{h}_1,\tilde{h}_2,\tilde{h}_3)$ takes negative value on some domain *D* and positive value on the interior of the complement of *D*, then the structure constants outside of *D* are bounded by those within *D*.

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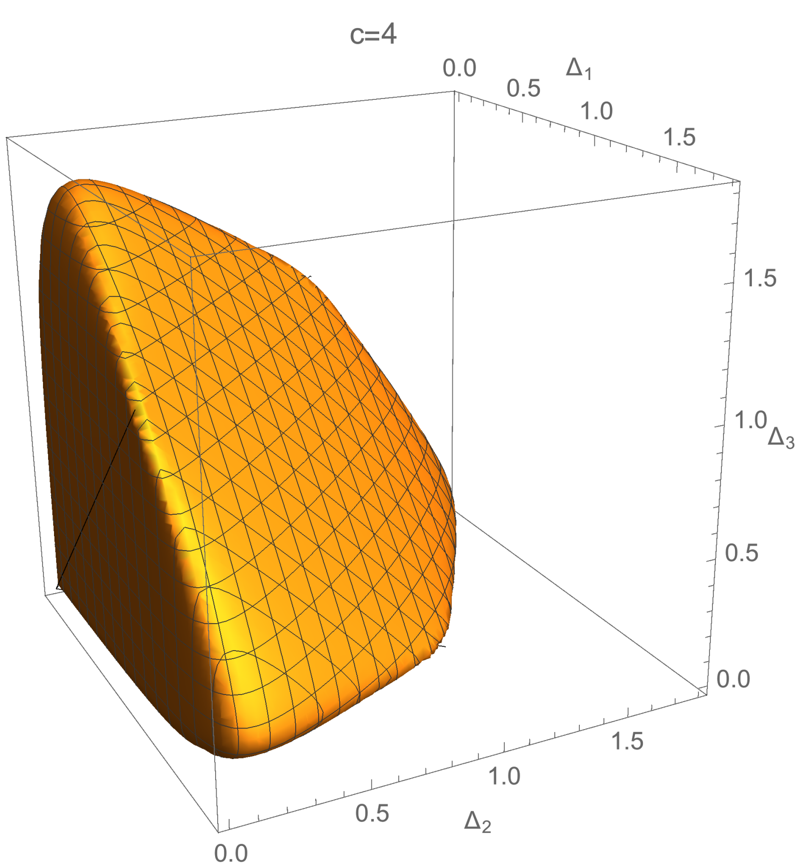

Constraints on structure constants from genus two crossing equation

$$\sum_{i,j,k\in\mathcal{I}} C^2_{ijk} F^\alpha_c(h_i,h_j,h_k;\tilde{h}_i,\tilde{h}_j,\tilde{h}_k) = 0$$

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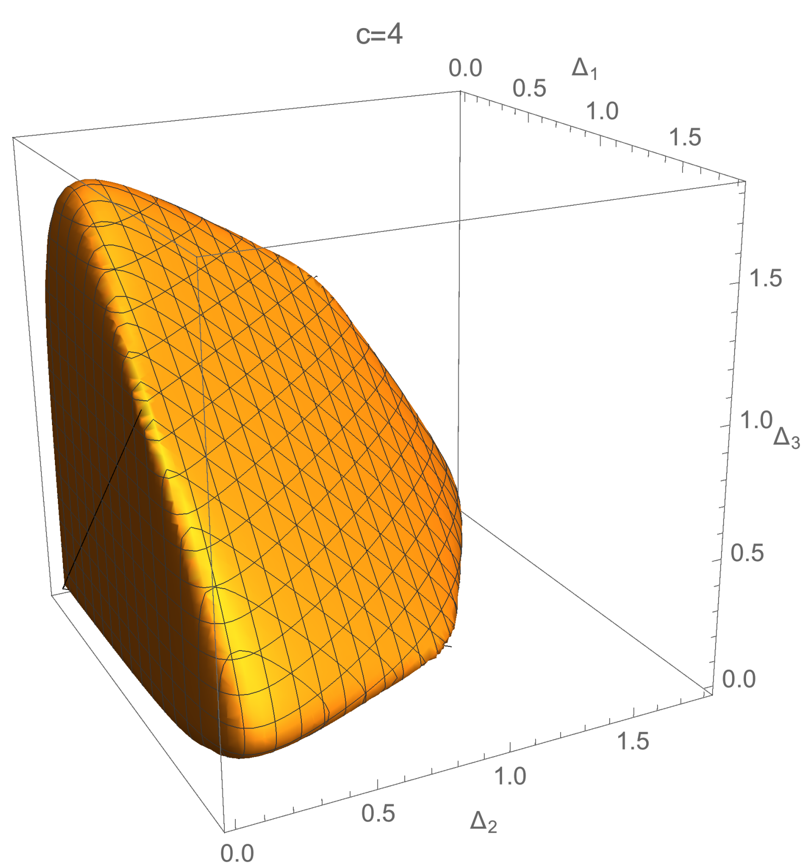

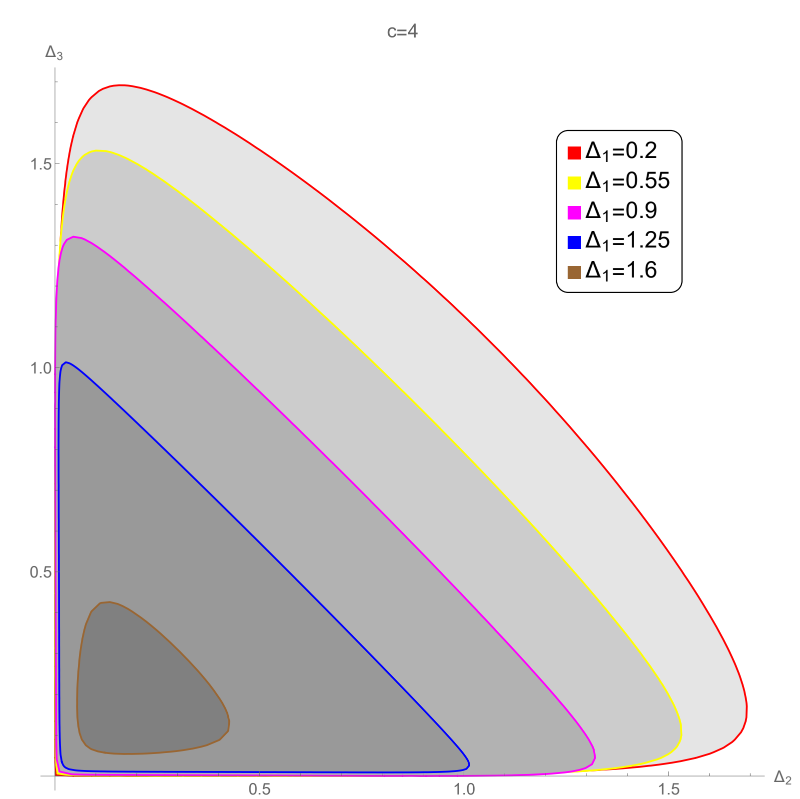

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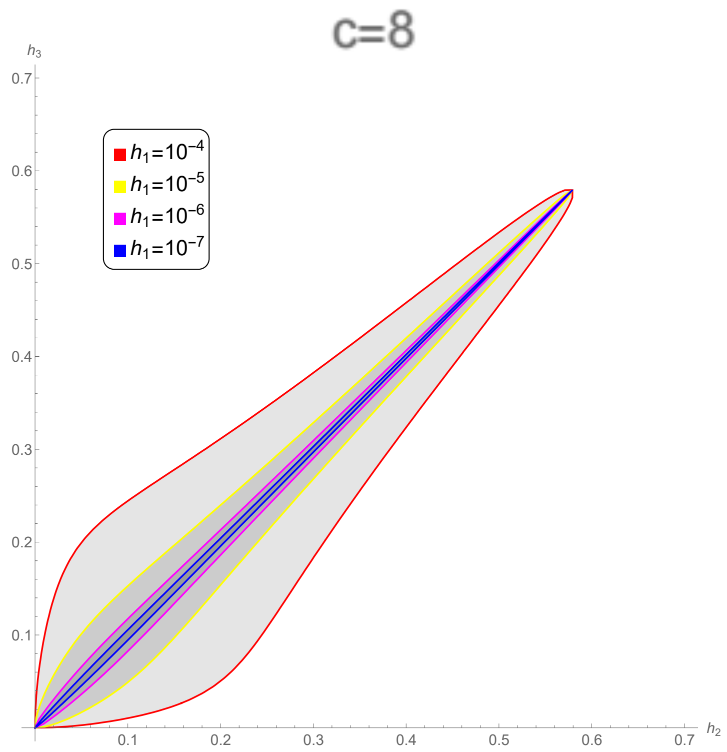

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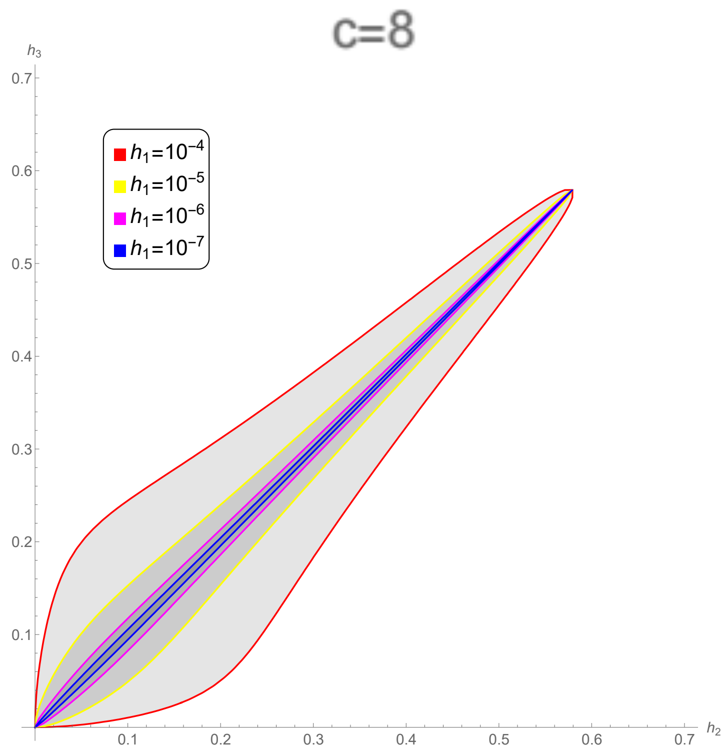

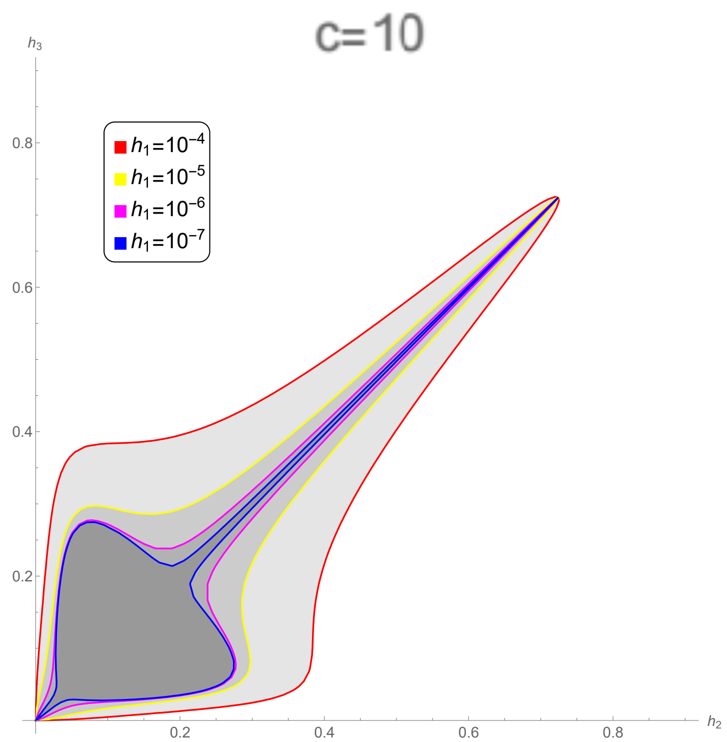

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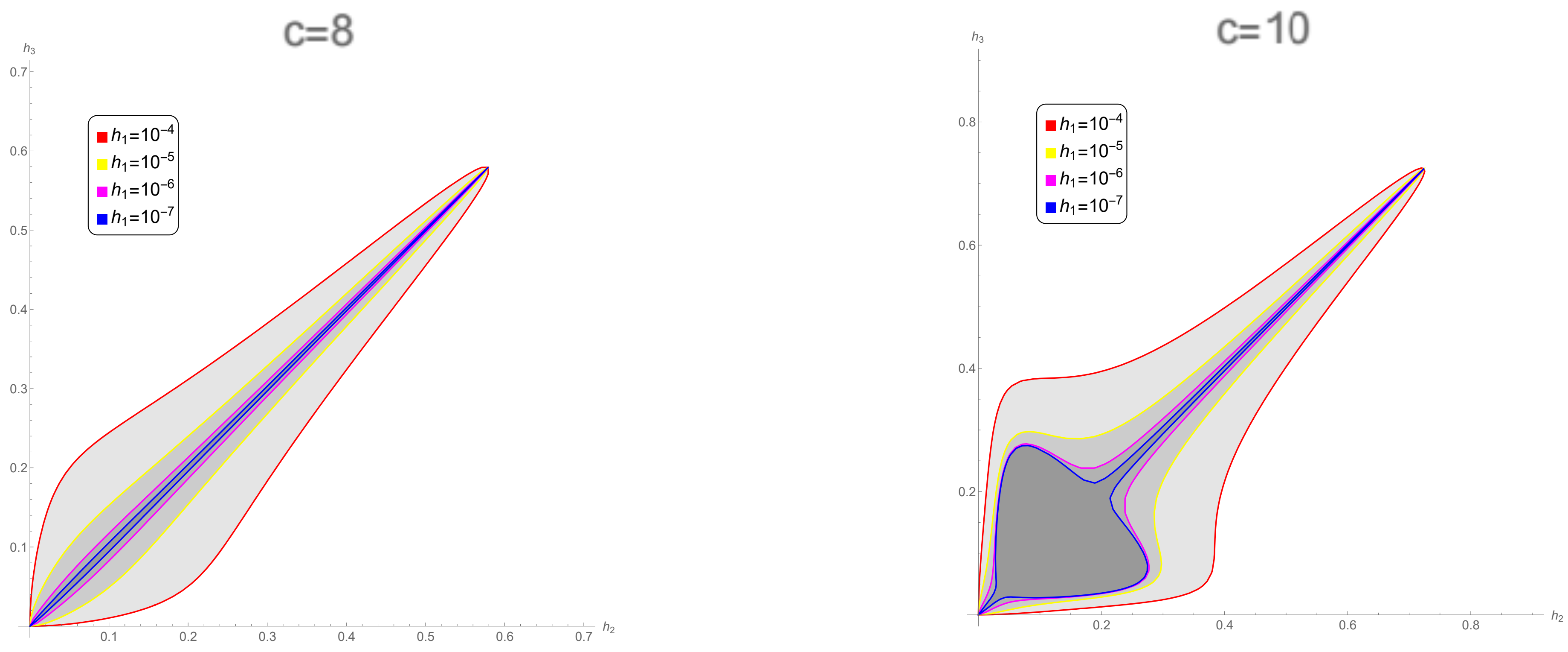

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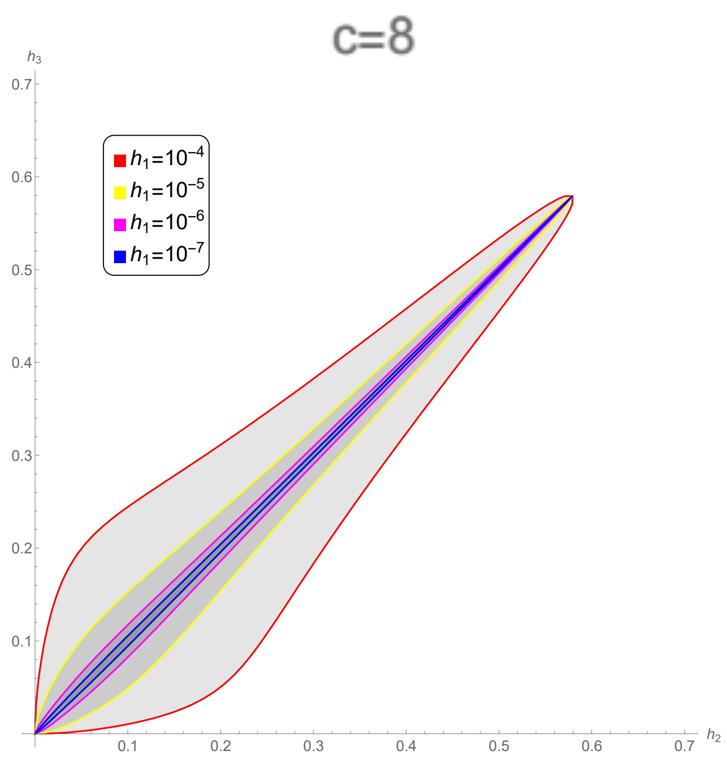

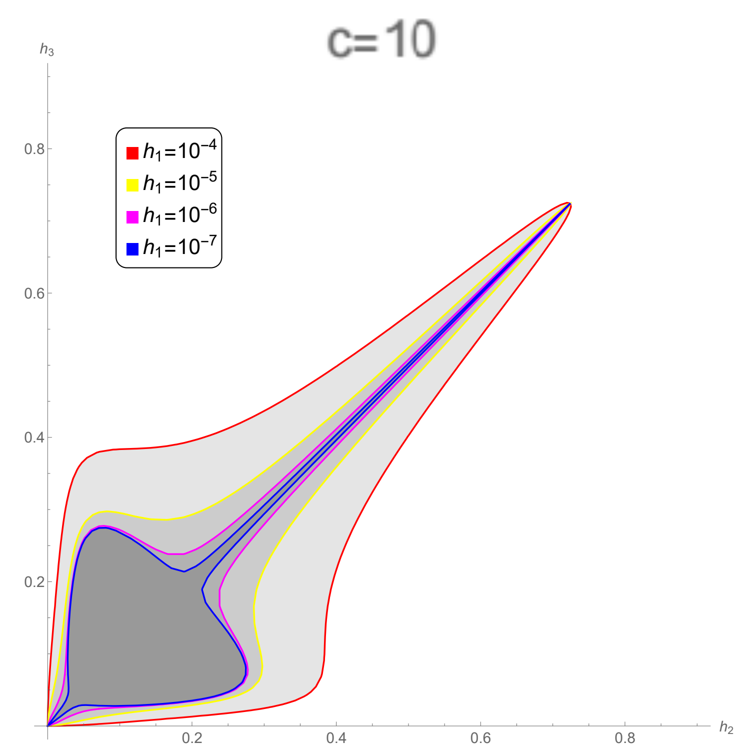

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In analyzing the critical domains so far, we only expanded the crossing equation to first order around z=1/2 along the Renyi locus. Full implications of the crossing equation remain to be uncovered.

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4. To what extent does the low lying operator spectrum of a CFT pin down the entire theory? (Existence and uniqueness of UV completion of gravity+matter in AdS?)

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