Emergent entropy

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Strings 2017, Tel Aviv

$$\Delta S \geq 0$$

1824: Carnot

Clausius, Kelvin, Planck, Boltzmann, …….

Arguably one of the most widely applicable laws in nature

Implies an arrow of time, whose origin is still debated

At a heuristic level, the 2nd law says a system should become increasingly disordered, which is often considered self-evident.

Derivation from first principle: few and far between, no general derivation

Boltzmann H-theorem (dilute gases)

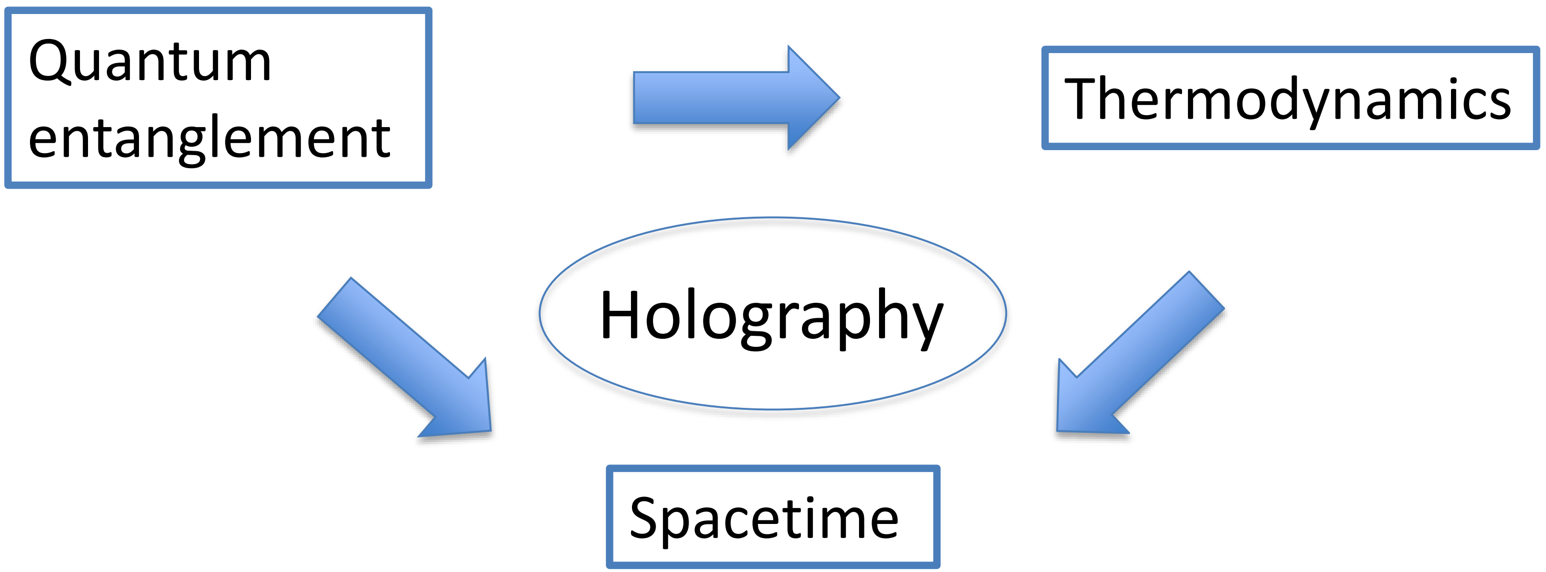
How does the 2^{nd} law emerge from the fundamental laws ?

What are the minimal inputs for it to hold?

These foundational questions also have modern bearings:

Black holes, generalized 2^{nd} law, information loss, …….

More generally, for an isolated quantum system



Systems in local equilibrium

For most macroscopic systems we observe in nature (including far-from-equilibrium situations) there is in addition a local version of 2nd law.

$$L \gg l_{\rm relax}$$ L: variation scales of physical quantities

At distance scales d: $L \gg d \gg l_{\rm relax}$ local equilibrium

Crucial phenomenological constraint in the description of dissipative systems: (in fluid approximation)

S^μ : local entropy current $\qquad \partial_\mu S^\mu \geq 0$

perturbatively at the level of derivative expansion.

Recently with Paolo Glorioso:

arXiv:1612.07705

With assumptions of unitarity and a Z_2 symmetry (characterizing time reversal and local equilibrium):

1. A general proof of the 2nd law for any local equilibrium systems

   Liquids, critical systems, quantum liquids such as superfluids and strongly correlated systems.

2. A proof of the local 2nd law perturbatively to all orders in derivative expansion

   An explicit algorithm to construct S^μ

- The second law exists only after coarse graining.

  the fine-grained entropy Von Neumann entropy remains constant under unitary time evolution.

- Coarse graining by itself does not introduce an arrow of time.

- The second law operates at classical level, in the thermodynamic limit.

  Both classical statistical fluctuations and quantum fluctuations are expected to violate it.

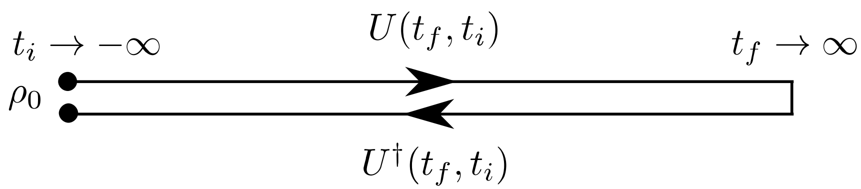
It turns out best approach is to start with

A quantum many-body system → coarse graining

Classical limit → thermodynamical limit

Coarse graining: effective action

Start with a general many-body system in some state ρ_0 which describes a macroscopic medium (well above vacuum)



$$\mathrm{Tr}\left(\rho_0 \cdots\right) = \int_{\rho_0} D\psi_1 D\psi_2 \, e^{iS[\psi_1] - iS[\psi_2]} \ldots$$

Integrate out all "fast" modes:

slow modes (two sets)

$$\mathrm{Tr}\left(\rho_0 \cdots\right) = \int D\chi_1 D\chi_2 \, e^{iS_{\mathrm{EFT}}[\chi_1, \chi_2; \rho_0]} \ldots$$

Slow modes: long-lived gapless excitations

- conserved quantities: universal

  Hydrodynamic modes S_{EFT} : hydrodynamics

- System-specific

  order parameters near a critical point, Fermi surfaces ,

  Goldstone bosons

  ....

there is a separation of scales: Λ

Non-equilibrium EFT

Microscopic description

Λ

Bare theory: $\int D\chi_1 D\chi_2\, e^{iS_{\mathrm{EFT}}[\chi_1,\chi_2;\rho_0]} \ldots$

Renormalization group, universality

Macroscopic phenomena

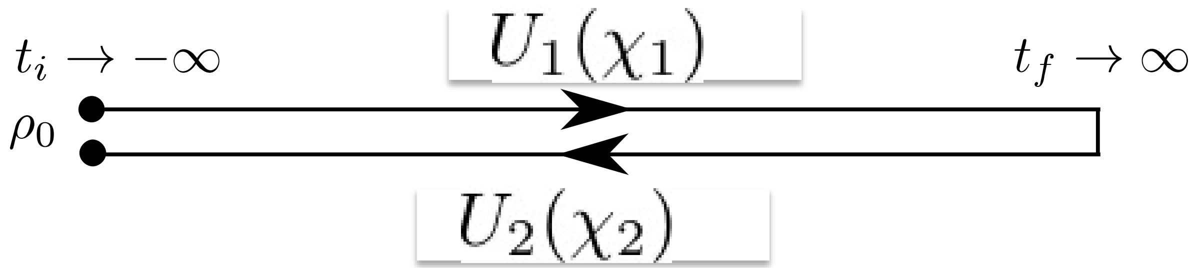
$S_{\mathrm{EFT}}[\chi_1,\chi_2;\rho_0]$: Local, real-time effective action , including dissipative, retardation, fluctuation effects.

Classical limit: path integrals survive

Thermodynamic limit: equations of motion

Constraints from unitarity time evolution

$$e^{iS_{\mathrm{EFT}}} =$$



1. $S^*_{\mathrm{EFT}}[\chi_1, \chi_2] = -S_{\mathrm{EFT}}[\chi_2, \chi_1]$ $\quad \mathrm{Tr}^*(U_1 \rho_0 U_2^\dagger) = \mathrm{Tr}(U_2 \rho_0 U_1^\dagger)$

   Terms symmetric in 1 <-->2 must be pure imaginary

2. $\mathrm{Im}\, S_{\mathrm{EFT}} \geq 0$ $\quad \left| \langle \psi | U_2^\dagger U_1 | \psi \rangle \right|^2 \leq 1$

3. $S_{\mathrm{EFT}}[\chi_1 = \chi, \chi_2 = \chi] = 0$ $\quad \mathrm{Tr}(U_1 \rho_0 U_1^\dagger) = 1$

Not visible in Euclidean EFT

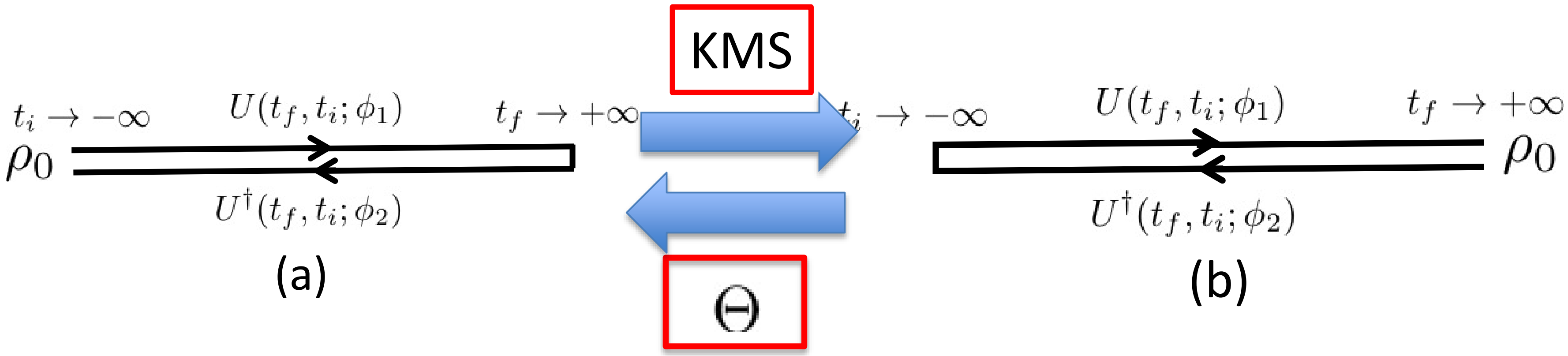
These constraints survive in the classical limit

Local equilibrium

How to impose local equilibrium?

Hint: Correlation functions in a thermal state satisfy a set of constraints: Kubo-Martin-Schwinger (KMS) conditions

Example: Euclidean 2-point functions periodic along the time circle



Suppose the underlying system is also invariant under:

Θ : any discrete symmetry containing time reversal

Θ and KMS: Z_2 operations

Θ and KMS have to be imposed together

Define a local equilibrium state as that satisfying a Z_2 symmetry

$$S_{\rm EFT}[\chi_1, \chi_2] = S_{\rm EFT}[\tilde{\chi}_1, \tilde{\chi}_2]$$

Example: a scalar order parameter in the classical limit $(\Theta = \mathcal{PT})$

$$\tilde{\chi}_r(-x) = \chi_r(x), \qquad \tilde{\chi}_a(-x) = \chi_a(x) + i\beta(x)\partial_0\phi_r(x)$$

$$\chi_r = \frac{\chi_1 + \chi_2}{2}, \quad \chi_a = \chi_1 - \chi_2 \qquad \beta(x):$$ Local inverse temperature

Specifies a class of non-equilibrium states.

→ Local first law, Onsager relations,

local fluctuation-dissipation relations

Theorem

For any action satisfying:

1. $S^*_{\rm EFT}[\chi_1, \chi_2] = -S_{\rm EFT}[\chi_2, \chi_1]$
2. $\mathrm{Im}\, S_{\rm EFT} \geq 0$
3. $S_{\rm EFT}[\chi_1 = \chi, \chi_2 = \chi] = 0$

and dynamical KMS symmetry (Z_2)

$$S_{\rm EFT}[\chi_1, \chi_2] = S_{\rm EFT}[\tilde{\chi}_1, \tilde{\chi}_2]$$

there exists a current S^μ satisfying

$$\Delta \int d^{d-1}x\, S^0 \geq 0 \qquad\qquad \partial_\mu S^\mu \geq 0$$

(perturbatively
in derivative expansion)

The Z_2 symmetry: $\tilde{\mathcal{L}} - \mathcal{L} = \partial_\mu V^\mu$

Equations of motion: $\partial_\mu S^\mu = W$

Z_2 symmetry: $W = \text{Integral transform of } \mathrm{Im}\, S_{\mathrm{EFT}}$

$$\mathrm{Im}\, S_{\mathrm{EFT}} \geq 0$$

$$\Rightarrow \quad \Delta \int d^{d-1}x\, S^0 \geq 0 \qquad \partial_\mu S^\mu \geq 0$$

In the equilibrium limit, the current recovers the standard expression of the thermodynamic entropy.

Ideal fluid limit:

$$\tilde{\chi}_r(\cancel{-}x) = \chi_r(x), \qquad \tilde{\chi}_a(\cancel{-}x) = \chi_a(x) + i\beta(x)\partial_0\phi_r(x)$$

enhances to a U(1) symmetry $\Rightarrow$ $\partial_\mu S^\mu = 0$

This is essentially the $U(1)_T$ symmetry of Haehl, Loganayagam, Rangamani.

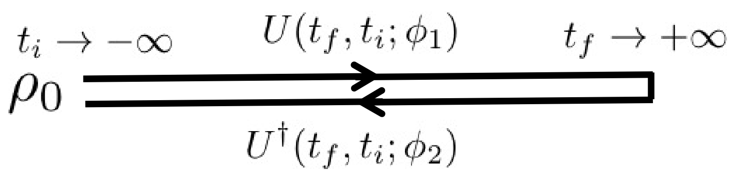
Arrow of time

$$\tilde{\chi}_r(-x) = \chi_r(x), \qquad \tilde{\chi}_a(-x) = \chi_a(x) + i\beta(x)\partial_0\phi_r(x)$$

$$+ \to - : \qquad \partial_\mu S^\mu \le 0$$

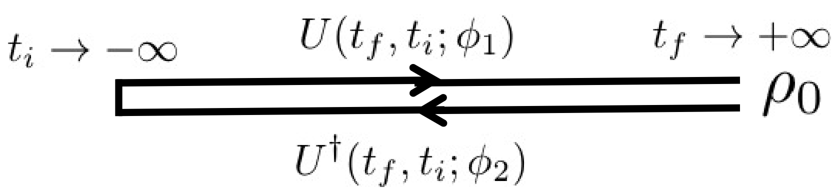
Causal arrow of time is also reversed.

+ :



(a)

- :



(b)

Entropy always increases evolving away from the state

Features

1. Thermodynamic entropy is emergent, consequence of Z_2

   In contrast to Boltzmann, who started with coarse grained

$$-\mathrm{Tr}\rho \log \rho$$

2. Monotonicity from classical remnants of quantum unitarity

3. Arrow of time comes from ”boundary” condition in time

4. The system need to have an underlying discrete symmetry involving time reversal.

Future questions

1. There must be a quantum information origin of it

$$\partial_\mu S^\mu = \text{Integral transform of Im}\, S_{\text{EFT}}$$

2. Include classical statistical and quantum fluctuations

Generalizations of Jarzynski, Crooks work relations?

3. Gravity counterpart of the Z_2 symmetry may give hint on how to formulate 2^{nd} law of BH in general higher derivative gravities

4. Relax/modify the Z_2 symmetry to include more general non-equilibrium states?

Thank You !