

Bulk reconstruction in the Hartle- Hawking formalism

Daniel L. Jafferis

Harvard University

Strings Conference

Tel Aviv

June 26, 2017

Outline

- Review of bulk reconstruction and long wavelength gravitational “paradoxes”
- Non-perturbative framework for the bulk effective theory – Hartle-Hawking in AdS
- Resolution of the paradoxes in the eternal black hole.

The basic idea

- Do the information paradox and other IR gravitational constraints obstruct the mapping of bulk operators to boundary ones? Is there a linear map from the entire bulk non-perturbative long distance effective theory Hilbert space into the exact CFT Hilbert space?
- The nonperturbative bulk effective theory described by Hartle-Hawking wavefunctions appears to be fully consistent with the exact CFT. The breakdown of the CFT description of bulk observables outside a perturbative subspace is paralleled by the lack of invariance of such observables under temporal diffeomorphisms beyond perturbation theory.

AdS/CFT lessons for quantum gravity

- Often use the duality by solving strongly interacting QFT problems with classical gravity.
- The idea of bulk reconstruction is to go the other way and learn about quantum gravity from the exactly defined dual. An aspect is that it has to be UV completed by string theory.
- This talk will be about non-perturbative IR “paradoxes” in gravity.

Bulk reconstruction

- What CFT operators correspond to bulk observables?
- The best answer would be if there was a general principle that selected the appropriate non-local CFT operators that reproduce the observations of local observers in the bulk. However in gravity there are no local operators, so we don't know what this principle is.
- Instead, find expressions that have the correct large N limit, around a fixed state (ie. background metric).

Causal reconstruction

- In a code subspace (bulk perturbation theory) around a given state, the bulk operators can be written in terms of CFT ones, by solving the bulk theory perturbatively and using the evolution equation to map them to the boundary. At higher orders, one must make choices for the diffeomorphism dressing.

$$\Phi(x, z) = \int dy G(x, z; y) \mathcal{O}(y) + \dots$$

[Banks Douglas Horowitz Martinec, Hamilton Kabat Lifschytz Lowe, ...]

Entanglement reconstruction

- Bulk reconstruction also works (in the code subspace) behind causal horizons (and the entire entanglement wedge for a boundary subregion).
[Dong Harlow Wall]
- Similarly, in perturbation theory around a given pure black hole microstates, [Papadodimas and Raju](#) showed that one can represent the perturbative bulk operator algebra using CFT operators.
- The basic idea is that by the bulk Reeh-Schlieder theorem any state may be obtained by acting with boundary operators. Therefore, given the projector onto the background state one can construct any perturbative operator.

IR paradoxes

- In recent years, contradictions between the gravitational effective theory and exact quantum descriptions have been sharpened in various situations.

[Mathur, Almheiri Marolf Polchinski Sully]

- Hawking's black hole information paradox is an incompatibility between unitarity and the existence of bulk operators (described by the expected bulk effective theory) behind the horizon.
- The paradoxes appear in configurations that are non-perturbatively different from the vacuum, but within the regime of validity of the bulk long distance theory. 8

Answered by AdS/CFT?

- In the context of AdS/CFT, these questions would be solved if one could find the CFT operators dual to the desired bulk observables.
- This mapping exists perturbatively around the AdS vacuum and some other states, but the whole question here is whether it works on all states that are expected to be described by the bulk EFT.

The main question

- Does the black hole information paradox and related puzzles really imply that there is not a linear map?

$$\mathcal{H}_{\text{bulk}} \rightarrow \mathcal{H}_{\text{CFT}}$$

- So the either the domain of validity of the bulk effective theory is smaller than expected (the firewalls of [Almheiri Marolf Polchinski Sully](#)), or the map is non-linear (as suggested by the state dependent operators of [Papadodimas Raju](#) or CFT entanglement encoding gravitational observables)?

RG flow of states

- The Hilbert space isn't a tensor product of long and short distance modes, so one can't obtain an RG flow on density matrices by tracing some degrees of freedom.
- Instead one has to project. For example, one can project onto an N dimensional subspace of the N^8 dimensional Hilbert space of a $2 \times 2 \times 2$ block (for example, in 3 spatial dimensions).

Linearity of the projection

- Traditionally, this was a projection onto a fixed subspace, for example of lowest energy density. This leads to a linear map. Note that it is not a low total energy subspace.
- A newer idea (MERA) is to take the projection on the maximally entangled subspace. This is useful numerically for finding the ground state wavefunction.
- Unclear whether the latter actually describes effective long distance observables in gravity (or laboratory systems). It is non-linear, and we certainly don't expect violations of quantum mechanics in collective observables in condensed matter systems.

The eternal black hole

- The eternal black hole, with the maximally extended Penrose diagram, has two asymptotically AdS regions.

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2, \text{ where } f(r) = r^2 + 1 - \frac{8G_N M \Gamma(d/2)}{(d-1)\pi^{(d-2)/2} r^{d-2}}$$

- Therefore the Hamiltonian, being a boundary term, is a sum $H = H_L + H_R$, and this spacetime is a state of two noninteracting copies of the CFT.

$$|\text{tfd}\rangle = \sum e^{-\beta E/2} |E_L\rangle |E_R\rangle$$

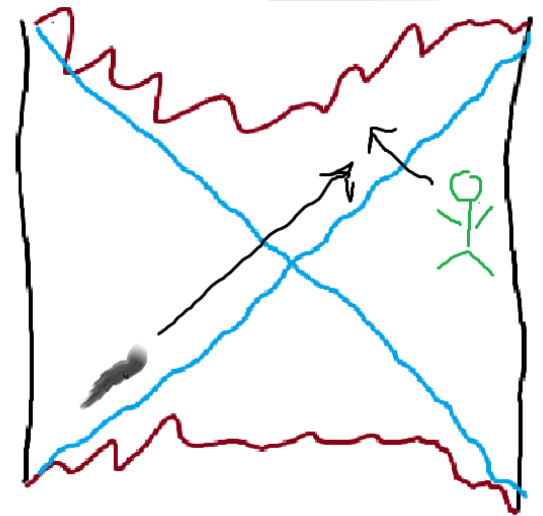
A paradox

- Consider a doubled CFT, and a measurement made by an apparatus from the right.

[Marolf Wall]

- Completeness of quantum mechanical description implies that it is a pure right operator in factorized states.
- Duality to the eternal black hole in the thermofield state implies it acts on the left.
- This contradicts linearity of the operator.

$$|\text{tfd}\rangle = \frac{1}{\sqrt{Z_\beta}} \sum_E e^{-\beta E/2} |E_L\rangle |E_R\rangle$$



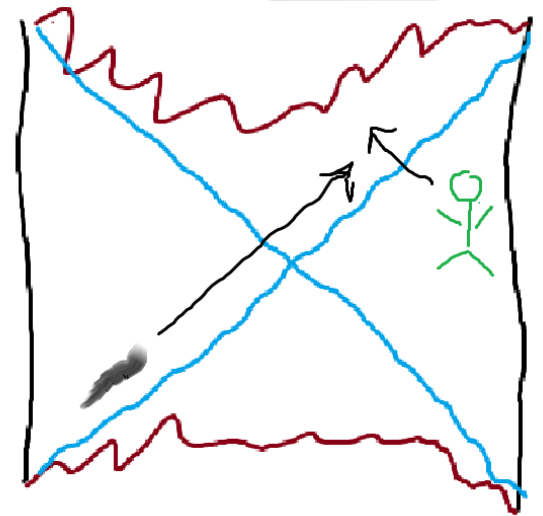
Small corrections don't help

- One may time evolve the tfd state to obtain many states with semiclassical descriptions as connected geometries – all that changes is the relative boundary time.
- By integrating over a long time, one can pick out contributions from individual energy eigenstates, and the results contradict the desired action on factorized states.

[Papdodimas Raju]
- Therefore, allowing exponentially small corrections to the operators in the G_N expansion doesn't help.

A simpler paradox

- The number of connected components, C , of space appears to be a well-defined observable in classical general relativity.
- It clearly equals 2 in any factorized state $|\psi_L\rangle |\psi_R\rangle$.
- However it is supposed to be 1 in the entangled thermofield state.
- This contradicts linearity of the operator.



Nonperturbative bulk effective theory

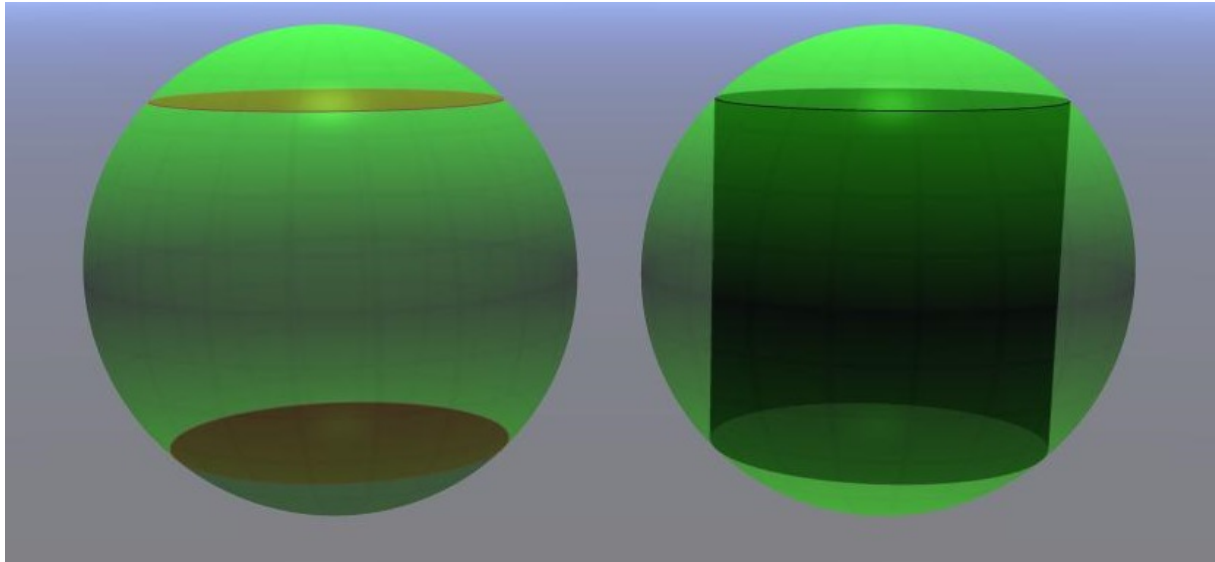
- In perturbation theory around a given state with a semiclassical geometric dual, C will be a c-number. We need a nonperturbative description of the bulk effective theory to even talk about this paradox.
- The effective field theory is valid for curvatures and energy densities that are much below the Planck scale. This is not a subspace of small total energy.
- Nonperturbative corrections to C can't resolve the problem, since too many states have the “wrong” value.

Linear dependence from the CFT

- This paradox is very similar to the information paradox. In both cases, the essential point is that the CFT states dual to different configurations have a small, but larger than expected, overlap.
- In the ordinary black hole, acting with behind the horizon creation operators gives too many states to agree with the CFT.
- In the eternal black hole, long time evolutions don't become independent as $e^{-a(\Delta t)^2}$

Gravity already knows

- The lack of linear independence between the eternal black hole and factorized states can already be seen in the euclidean path integral.



$$\langle 0, 0 | \text{tfd} \rangle = \frac{Z_{S^2}}{\sqrt{Z_{S^2}^2 Z_\beta}}$$

Hartle-Hawking formalism

- Analogous to writing wavefunctionals in quantum field theory in field basis, $\Psi(\varphi(x))$.
- In gravity, there is no canonical time slicing. Therefore the kets $|g(x)\rangle$ are not independent.
$$\Psi(g) = \langle g|\psi\rangle$$
- The resulting constraints on matrix elements of operators are the Hamiltonian constraints of diffeomorphism invariance.

Hartle-Hawking in AdS

- Given a state, for example specified by a euclidean path integral with sources at the AdS boundary, one can compute the path integral up to a slice with intrinsic metric h .

$$\Psi(h) = \int_{g|_{\partial M}=h, g|_{\infty}=\mathcal{J}} Dg e^{-S(g)}$$

- In AdS, h must obey the asymptotic conditions. One integrates over the lapse and the shift, but they also must obey the AdS asymptotics.

$$ds^2 = (N^2 - N_i N^i) dt^2 + 2N_i dx^i dt + h_{ij} dx^i dx^j$$

- Then it will be dominated by a saddle, and the integral is defined in perturbation theory around the saddle.

The Wheeler de Witt equation

- This defines a set of maps $h : \mathcal{H}_{\text{bulk}} \rightarrow \mathbf{C}$, and thus ket vectors $|h\rangle$
- They are not linearly independent – the overlap is given by the path integral between two h . Therefore the data $\Psi(h)$ is redundant. This is because there are many ways to slice the same spacetime.
- Infinitesimally, the content is that Ψ satisfies the Hamiltonian constraint equations. These are second order, so there is no way to fix the gauge using a condition only on h (unlike in gauge theory).

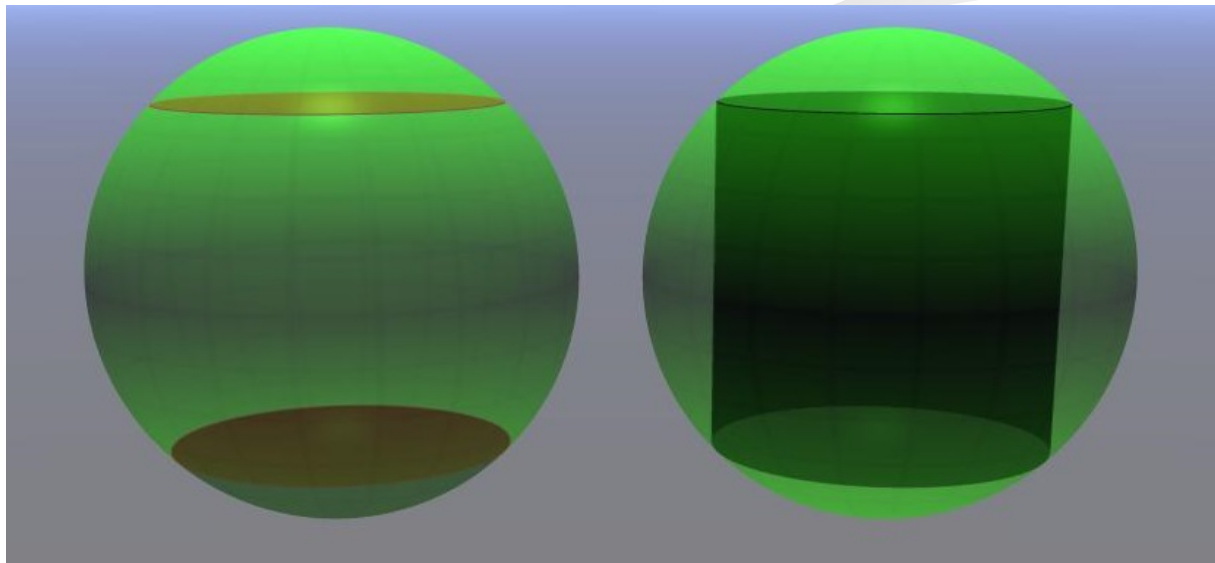
$$\left(-\frac{1}{2}h^{-1/2}(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl})\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - R(h)h^{1/2} + 2\Lambda h^{1/2} \right) \Psi(h) = 0$$

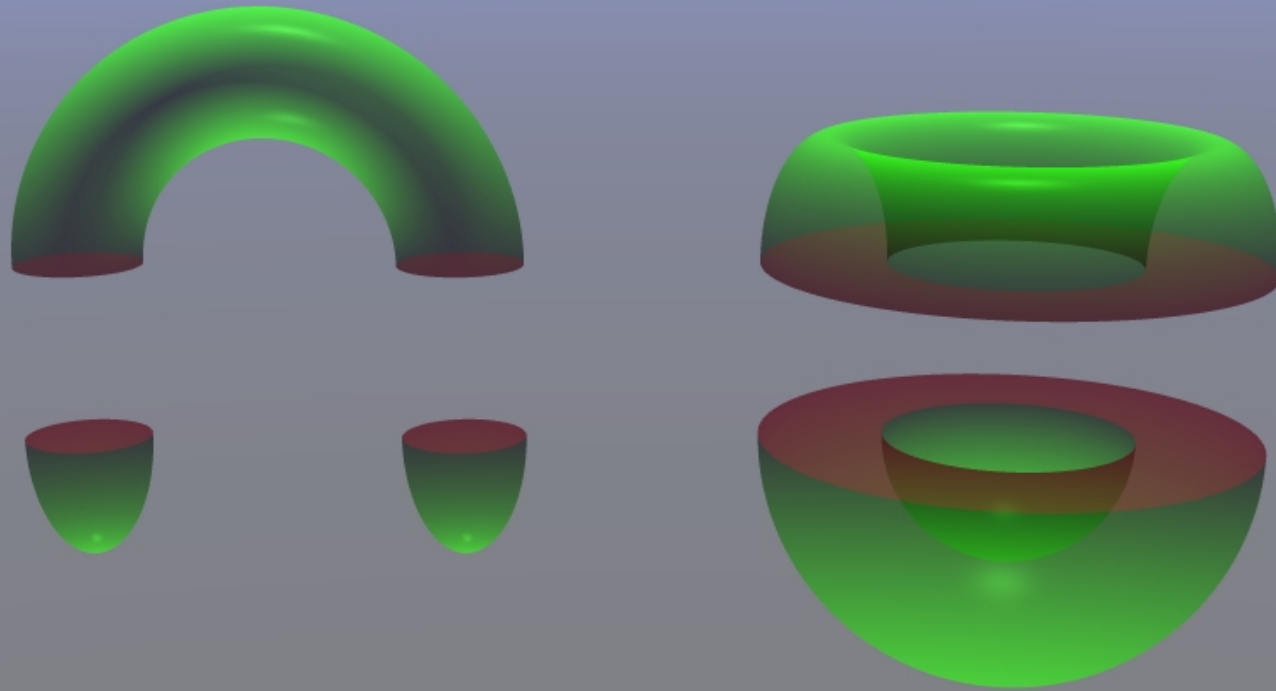
Nonperturbative constraints

- The overlap between kets of different topology is nonzero, taking the form $e^{-\frac{1}{G_N} S_{\text{saddle}}}$. Therefore nonperturbatively gauge invariant operators obey additional constraints.
- It would be hard to see these by integrating the WdW equation. This is because one would have to go through singular \hbar , where the effective theory breaks down.
- In a susy string theory example (Lin-Lunin-Maldacena geometries), [Berenstein-Miller](#) were able to expand one topology in terms of another by using the relation with free fermions.

Lack of topology independence

- One can interpret it as the amplitude that the tfd state is disconnected, or that the factorized vacuum is connected. The kets defining those topologies have a nonzero overlap.





The topology operator depends on the slice, so it is not invariant under temporal diffeomorphisms non-perturbatively.

The resolution

- The “operator” C is not well-defined in the bulk either. It has the same problem as an operator defined on the bulk wavefunctional that does not commute with the Hamiltonian constraint equations – it is not gauge invariant.
- The bulk effective theory perfectly agrees the CFT, on all bulk states after all!

Right framed operators

- Consider a bulk field at a position defined by a certain proper distance along a transverse geodesic from a point on the right boundary (this would be the field in Fefferman-Graham gauge if that were a good gauge).
- These are diffeomorphism invariant in classical gravity in AdS.
- Naively, one could insert this into the Hartle-Hawking path integral to obtain an action on states. But in general, the geodesic just crosses the slice and exits the manifold. So this definition does not make sense. The problem is that the geodesic doesn't have to lie in a slice.

Fixing the gauge nonperturbatively

- We can define a related operator that is nonperturbatively diffeomorphism invariant, by specifying its action on a partly gauge fixed set of kets.
- They are defined by doing the path integral only over metrics that satisfy, on the slice,

$$ds^2 = g_{ij} dx^i dx^j + N_i dt dx^i + S dt^2, \text{ where } g_{zz}(x_*, z) = \frac{L}{z^2}, g_{za} = 0, N_z = 0 \text{ for } 0 < z < z_*$$

- Now the value of a field at the endpoint of the geodesic is gauge invariant.

Implications

- However, the price is that this operator does not act exactly like the naïve one on all states.
- In particular, it projects on to configurations where the whole geodesic is on a spatial slice with spatial slice of the boundary. This implies that the geodesic is achronal.
- More importantly, it is not a pure right operator, by definition!

Resolution

- One can check that now there is no paradox of the Marolf-Wall type. Note that there are many ways of fixing the gauge non-perturbatively, and correspondingly many different operators. The main point is that the naïve definition, which doesn't make sense in the CFT, also doesn't make sense on the gravity side.

Observables vs. measurements

- No local diffeomorphism invariant observables. However, there are perfectly gauge invariant local Hamiltonians that describe the measuring process. They are relational.
- There is no canonical way to separate diffeomorphism dressing. Moreover, clearly measurements do not actually project onto states with different ADM energy (even by exponentially small amounts).

$$H_{\text{int}} = \mathcal{O}_{\text{app}} \mathcal{O}_{\text{sys}}$$

$$H_{\text{int}} \sim \psi_{\text{sys}} W_{xy} \psi_{\text{app}}^\dagger$$

Summary

- The information paradox and its cousins seemed to imply a breakdown of the bulk effective theory or a nonlinear relation between the bulk and boundary Hilbert spaces.
- However, the problematic operators also don't exist in the bulk – they are not diffeomorphism invariant.
- Important question for the future is what is the right framework to actually describe the outcomes of bulk measurements.