FIELD THEORY ACTIONS FOR AMBITWISTOR STRINGS*

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*1903.05792(H.Flores, M.L)

MOTIVATION

- CHY developed a string-like formulae for calculating tree-level amplitudes for **n** massless particles. (integral over moduli space of **n**-punctured Riemann sphere)

- Mason & Skinner introduced the ambitwistor string as a way to derive CHY formulas from a worldsheet perspective, then was extended to loops and pure spinor version

- Siegel introduced an approach closely related to standard string theory, by considering a singular gauge limit and a change in the worldsheet boundary conditions

- Toy model to study string field theory / New ways to evaluate Feynman diagrams in field theory

AMBITWISTOR CRASH-COURSE

Gauge fixed action:

(1,0) (2,-1)

$$S = \frac{1}{2\pi}\int d^2z\,(\underbrace{P_m\bar{\partial}X^m + b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c}}_{Bosonic} + \overbrace{\psi_1\bar{\partial}\psi_1 + \beta_1\bar{\partial}\gamma_1 + \underbrace{\psi_2\bar{\partial}\psi_2 + \beta_2\bar{\partial}\gamma_2}_{Heterotic})}^{TypeII} + \underbrace{S_J}_{Heterotic}$$

(1/2) (3/2,-1/2)

XX trivial OPE → e^{ikX} No anomalous dimension

null constraint

BRST operator:

$$Q = \oint \frac{dz}{2\pi i}(cT^M + cT_{\tilde{b}\tilde{c}} + cT_{\beta_1\gamma_1} + cT_{\beta_2\gamma_2} + bc\partial c + \frac{1}{2}\tilde{c}P^2 + \gamma_1 P\cdot\psi_1 + \gamma_2 P\cdot\psi_2 - \gamma_1^2\tilde{b} - \gamma_2^2\tilde{b})$$

$$Q^2 = 0 \Rightarrow dim = \begin{cases} 26, & \text{Bosonic} \\ 10, & \text{Type II} \\ \frac{2}{5}(41 - c_j), & \text{Heterotic} \end{cases}$$

→ SO(32) : 16

Does not describe the massless string spectrum

$$\bar{\delta}\,(k\cdot P)$$

BOSONIC SPECTRUM

Vertex operator with ghost number two:

$$V(z) = \boxed{c\tilde{c}P_m P_n \mathbf{g}^{mn}} + \boxed{c\tilde{c}\partial X_m \partial X_n \mathbf{f}^{mn} + c\tilde{c}P_{(m}\partial X_{n)}\mathbf{r}^{mn} + c\tilde{c}P_{[m}\partial X_{n]}\mathbf{b}^{mn}}$$

EOM: $\boxed{\Box\, \mathbf{g}_{mn} = \partial^m \mathbf{g}_{mn} = 0}$ → $\mathcal{A}_3(k_1, k_2, k_3) \propto k^6$

The amplitude does not describe gravity

The amplitude suggests a kinetic term proportional to k^6

If you allow the vertex operator depend on ∂X

$$Q_{brst} V(z) = 0$$

Sketch of the cohomology

EOM: $\mathbf{r}_{mn} \propto \Box \mathbf{g}_{mn}$, $\mathbf{f}_{mn} \propto \Box \mathbf{r}_{mn}$ and $\Box \mathbf{f}_{mn} = 0$

auxiliary fields

$\boxed{\Box^3 \mathbf{g}_{mn} = 0}$

Kalb-Ramond field

$$\boxed{\begin{aligned} \Box\, \mathbf{b}_{mn} &= 0 \\ \partial^m \mathbf{b}_{mn} &= 0 \end{aligned}}$$

KINETIC TERM: BOSONIC

$$S[\Psi] = \frac{1}{2}\langle\Psi|c_0 Q\Psi\rangle = \frac{1}{2}\langle I\circ V(0)|\partial c Q V(0)\rangle$$

Similar to closed string field theory

$\frac{1}{2}\langle\Psi|(c_0-\bar{c}_0)Q\Psi\rangle$

BPZ conjugate

$$\langle\Psi| = \langle 0|I\circ V(0)$$

$$I(z) = 1/z.$$

Constraints

$$L_0|\Psi\rangle = b_0|\Psi\rangle = 0$$

Using normalization: $\langle c_{-1}c_0c_1\tilde{c}_{-1}\tilde{c}_0\tilde{c}_1\rangle = 1$

Kalb-Ramond Field strength:

Action after a field redefinition:

$$S = -\int d^{26}x\left[\frac{1}{2}h^{mn}\Box^2\left(R_{mn}-\frac{1}{2}\eta_{mn}R\right) - \phi R + (H^{mnp}H_{mnp})\right]$$

$$\delta h_{mn} = \partial_{(m}\lambda_{n)}, \quad \delta\phi = 0, \quad \delta b_{mn} = \partial_{[m}\omega_{n]}$$

almost gravity

TYPE II-NS

GSO(+): gravity + dilation + Kalb-Ramond

$$(e^{-\phi_1}, \psi_1^m) \to (-e^{-\phi_1}, -\psi_1^m)$$

$$V^{-1}_{GSO+} = e^{-\phi_1} e^{-\phi_2} c\tilde{c}(h_{mn}\psi_1^{(m}\psi_2^{n)} + b_{mn}\psi_1^{[m}\psi_2^{n]}) + \eta_1 \partial\xi_2 e^{-2\phi_2} c\tilde{c}s + \ldots$$

Action:

$$S_+ = -\int d^{10}x \left[\frac{1}{2} h^{mn}\left(R_{mn} - \frac{1}{2}\eta_{mn}R\right) - \phi R + (H^{mnp}H_{mnp})\right]$$

GSO(-): gauge field + Kalb-Ramond + …

$$V^{-1}_{GSO-} = e^{-\phi_1} e^{-\phi_2} c\tilde{c}(c_{mn}\psi_1^m\psi_1^n + d_{mn}\psi_2^m\psi_2^n + \partial X^m A_m) + \ldots$$

$$b^{\pm}_{mn} = c_{mn} \pm d_{mn}$$

Action:

$$S_- = -\frac{1}{2}\int d^{10}x [H^{-\,mnp}H^-_{mnp} + H^{+\,mnp}H^+_{mnp} + b^{+\,mn}\partial_{[m}A_{n]}]$$

Gauge trans: $\delta b^-_{mn} = \partial_{[m}\sigma_{n]}, \quad \delta A_m = \partial_m \rho, \quad \delta b^+_{mn} = 0$

HETEROTIC - RNS

The theory contain higher derivative terms and has N=1 supersymmetry

Neveu- Schwarz sector:

$$V_{NS}^{-1} = e^{-\phi} c\tilde{c}\Big[g_{mn} P^{(m}\psi^{n)} + + b_{[mn]}\partial x^m \psi^n + s\, \partial x^m \psi_m + C_{mnp}\psi^m\psi^n\psi^p + J^a \psi^m A^a_m \Big] + \dots$$

Super Yang-Mills

$$S = -\int d^{10}x \left[\frac{1}{2} h^{mn} \Box \left(R_{mn} - \frac{1}{2}\eta_{mn} R \right) - \phi R + \frac{1}{4}\mathrm{Tr}(F^{mn}F_{mn}) + -C^{mnp}H_{mnp} + \frac{1}{2}C^{mnp}\left(\Box C_{mnp} - \frac{1}{2}\partial_{[p}\partial^r C_{mn]r} \right) \right]$$

$$\delta h_{mn} = \partial_{(m}\lambda_{n)}, \quad \delta\phi = 0, \quad \delta b_{mn} = \partial_{[m}\omega_{n]}, \quad \delta C_{mnp} = 0, \quad \delta A_m = \partial_m \rho$$

Not gravity /Conf. Gravity

Ramond sector:

$$V_R^{-1/2} = c\tilde{c} S_\alpha e^{-\phi/2}\partial x^m \mathbf{C}^\alpha_m + c\tilde{c} S_\alpha e^{-\phi/2} P_m \mathbf{D}^{m\alpha} + c\tilde{c}\partial\tilde{c}\partial\xi S^\alpha e^{-3\phi/2}\mathbf{I}_\alpha + c\tilde{c} S_\beta e^{-\phi/2} J^a \chi^{a\beta}$$

$$S_R = -\int d^{10}x \left[\frac{1}{2}\mathbf{d}^{m\alpha}\Box\left((\gamma^n \mathbf{F}_{mn})_\alpha - \frac{1}{2}(\gamma_m)_{\alpha\beta}(\gamma^{rs}\mathbf{F}_{rs})^\beta \right) + \frac{1}{2}(\gamma_m)_{\alpha\beta}(\gamma^{rs}\mathbf{F}_{rs})^\beta \mathbf{i}_\alpha - \frac{i}{2}\mathrm{Tr}\left(\chi \not{\partial} \chi \right) \right]$$

$$\delta \mathbf{d}^\alpha_m = \partial_m \lambda^\alpha, \quad \delta \mathbf{i}_\alpha = 0$$

$$\mathbf{F}^\alpha_{mn} = \partial_{[m}\mathbf{d}^\alpha_{n]}$$

CONCLUSIONS / OPEN PROBLEMS

- Type II GSO(+) and Heterotic Yang-Mills agrees with string theory. Bosonic / Heterotic (gravity) does not, but have a similar and interesting structure (higher derivative)
- Still missing an interpretation for these theories
- Why does not agree with string theory* / How can it be derived from usual closed string theory** ?
- How to construct ambitwistor like models in curved spacetime (adS) ?

*Null origin of the ambitwistor (E.Cassali, P.Tourkine) :1606.05636

**Left-handed string (W.Siegel): 1512.02569

HETEROTIC - SUSY

$$S = -\int d^{10}x \left[\frac{1}{2}h^{mn}\Box\left(R_{mn} - \frac{1}{2}\eta_{mn}R\right) - \phi R + \frac{1}{4}\mathrm{Tr}(F^{mn}F_{mn}) + \right.$$
$$\left. -C^{mnp}H_{mnp} + \frac{1}{2}C^{mnp}\left(\Box C_{mnp} - \frac{1}{2}\partial_{[p}\partial^{r}C_{mn]r}\right)\right]$$

$$S_R = -\int d^{10}x \left[\frac{1}{2}\mathbf{d}^{m\alpha}\Box\left((\gamma^{n}\mathbf{F}_{mn})_{\alpha} - \frac{1}{2}(\gamma_m)_{\alpha\beta}(\gamma^{rs}\mathbf{F}_{rs})^{\beta}\right) + \frac{1}{2}(\gamma_m)_{\alpha\beta}(\gamma^{rs}\mathbf{F}_{rs})^{\beta}\mathbf{i}_{\alpha} - \frac{i}{2}\mathrm{Tr}\left(\chi\partial\!\!\!/\chi\right)\right]$$

$$\mathbf{Q}_{\alpha}^{-1/2} = \frac{1}{2\pi i}\oint \mathrm{d}z\, S_{\alpha}e^{-\phi_6/2}$$

Susy transformations:

$$\delta_\zeta h_{mn} = 2\zeta\gamma_{(m}\mathbf{d}_{n)}$$
$$\delta_\zeta \phi = \zeta\mathbf{i}$$
$$\delta_\zeta C_{mnp} = -3(\zeta\gamma_{t[mn}\mathbf{F}^{t}{}_{p]}) - 3(\zeta\gamma_{[m}\mathbf{F}_{np]})$$
$$\delta_\zeta b_{mn} = -2\Box(\zeta\gamma_{[m}\mathbf{d}_{n]}) - (\zeta\gamma_{mn}\mathbf{i}) + \frac{1}{6}(\zeta\gamma_{mn}\partial_p\mathbf{F}^{p})$$
$$\delta_\zeta A^{a}_{m} = \frac{i}{2}(\zeta\gamma_m\chi^{a}).$$

$$\delta_\zeta \mathbf{d}^{\alpha}_{m} = +(\gamma^{rs}\zeta)^{\alpha}\partial_s h_{mr} - 2(\gamma^{np}\zeta)^{\alpha}C_{mnp} + \frac{1}{3}(\gamma_{mnps}\zeta)C^{nps}$$
$$\delta_\zeta \mathbf{i}_{\alpha} = 2(\partial\!\!\!/\zeta)_{\alpha}t - (\gamma^{mnp}\zeta)_{\alpha}H_{mnp} + \frac{1}{3}(\gamma^{mnp}\zeta)_{\alpha}\Box C_{mnp}$$
$$\delta_\zeta \chi^{a\beta} = -\frac{1}{4}F_{mn}(\gamma^{mn}\zeta)^{\beta}$$

**Pure spinor: Max Guillen