Universal Correction To The Veneziano Amplitude

Alexander Zhiboedov, Harvard U

Strings 2017, Tel Aviv, Israel

with S. Caron-Huot, Z. Komargodski, A. Sever, 1607’

(talk by Zohar at Strings2016)

with A. Sever, (to appear)

Homework from Strings2014

Problem 72 (Juan):

What is the general theory of weakly coupled, interacting, higher spin particles?

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(related homework from Nima about weakly coupled completion of gravity amplitudes)

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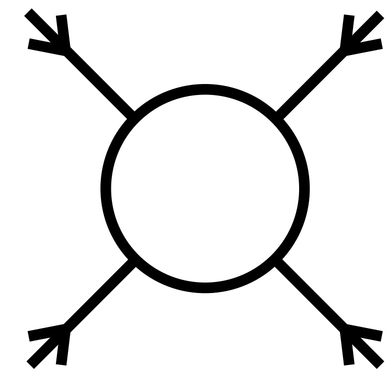
Two-to-two scattering amplitude $A(s,t) =$ 

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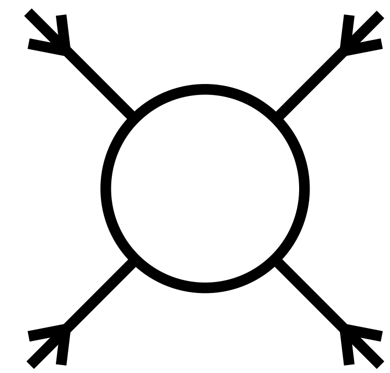
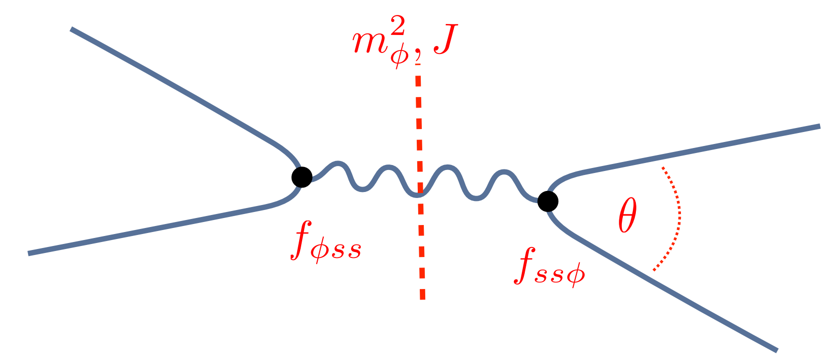
Two-to-two scattering amplitude $A(s,t) =$ 

- weakly coupled $\equiv$ meromorphic



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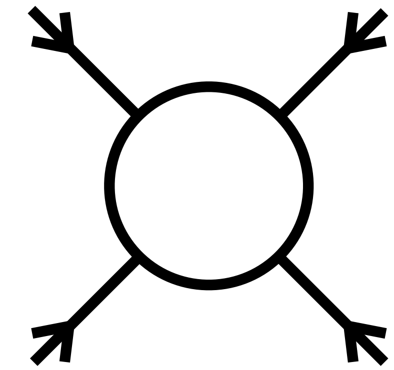
Two-to-two scattering amplitude $A(s,t) =$ 

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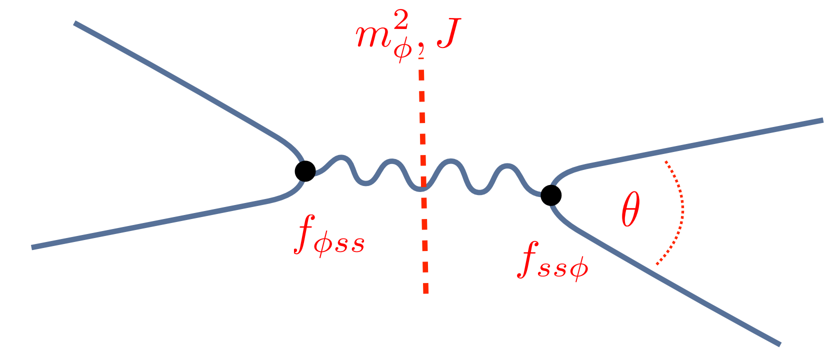

- unitarity

$$A(s,t)|_{s\simeq m_\phi^2} \simeq f_{ss\phi}^2 \frac{P_J(\overbrace{1+\frac{2t}{m_\phi^2-4m_s^2}}^{\cos\theta})}{s-m_\phi^2}$$

positive

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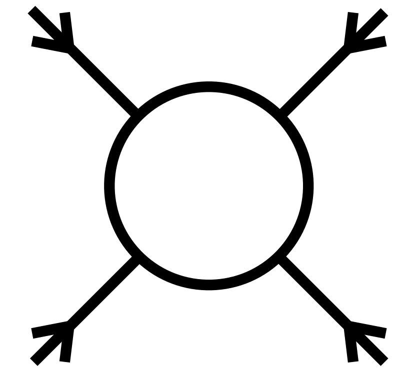
Two-to-two scattering amplitude $A(s,t) =$ 

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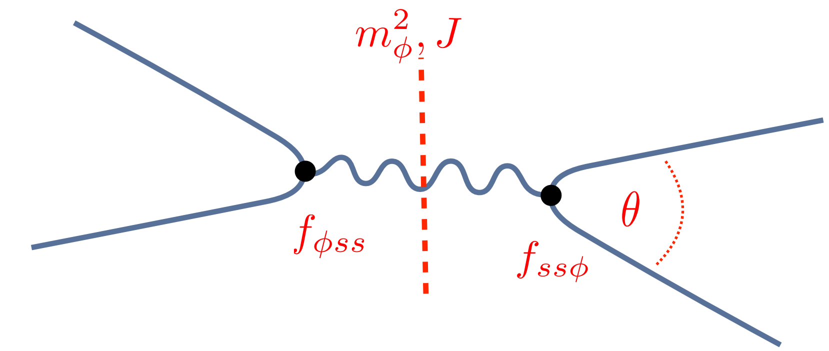

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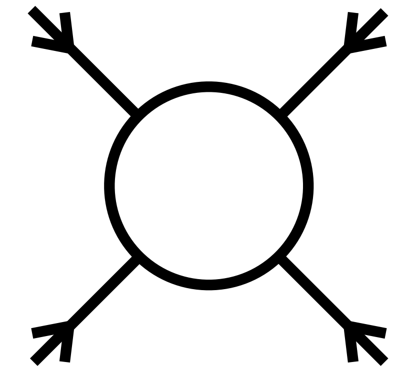
Two-to-two scattering amplitude $A(s,t) =$ 

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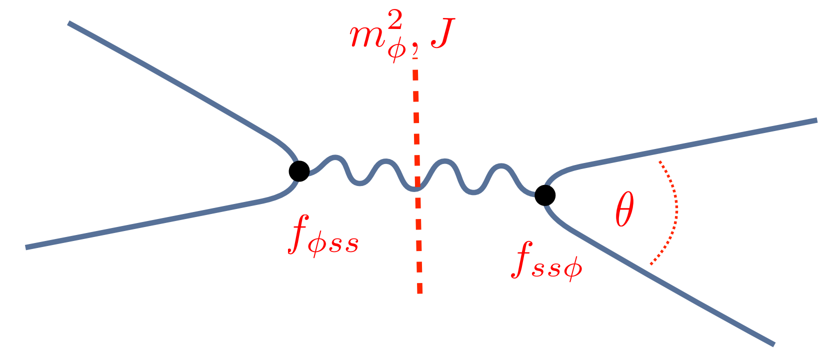

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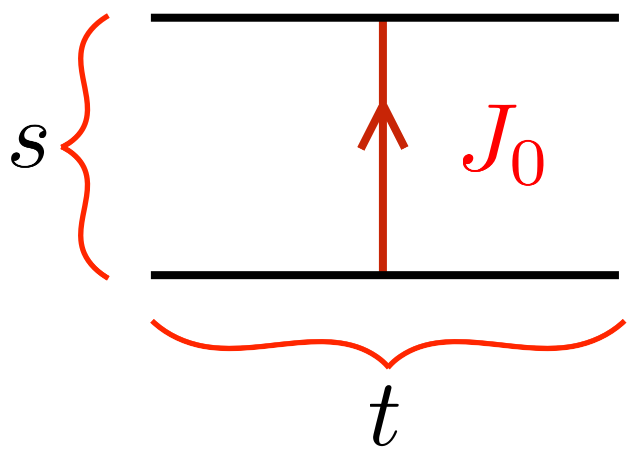
- crossing

$$A(s,t) = A(t,s)$$

What is WIHS?

- soft high energy limit (causality for HS) [talk Caron-Huot]



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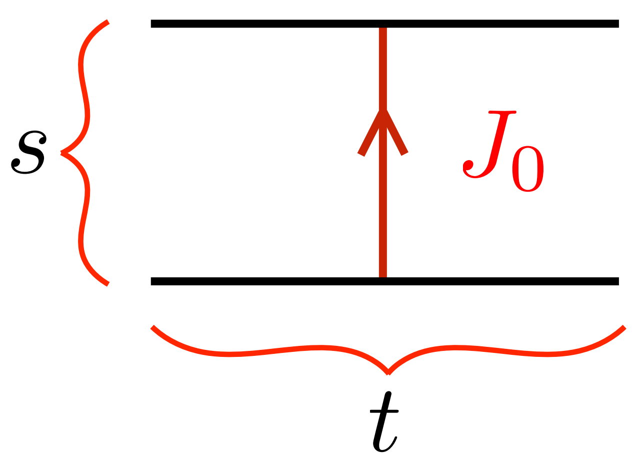

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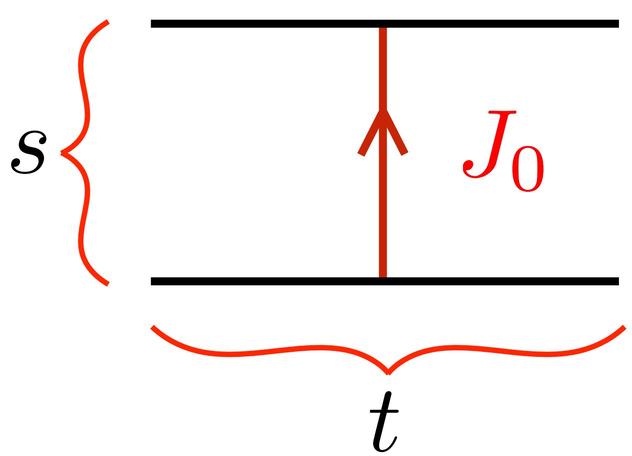

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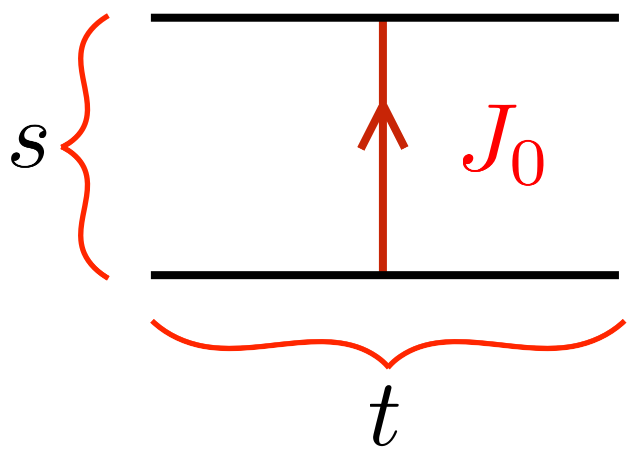

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Solutions:
fundamental strings,
large N confining gauge theories, …

$$\frac{\Gamma(-s)\Gamma(-t)}{\Gamma(-t-s)}$$

[Veneziano]
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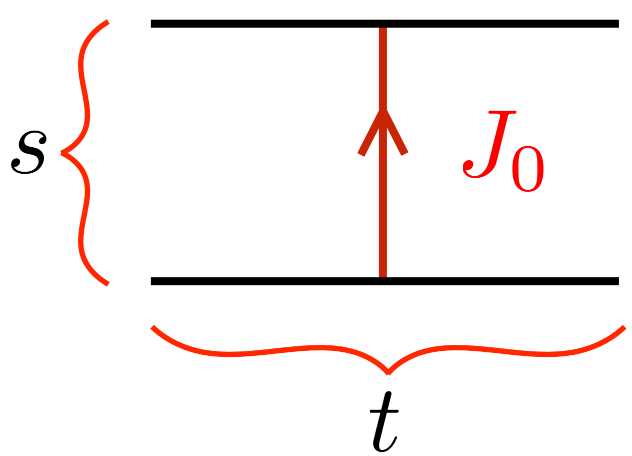

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very non-generic

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The Regge Trajectory $j(t)$

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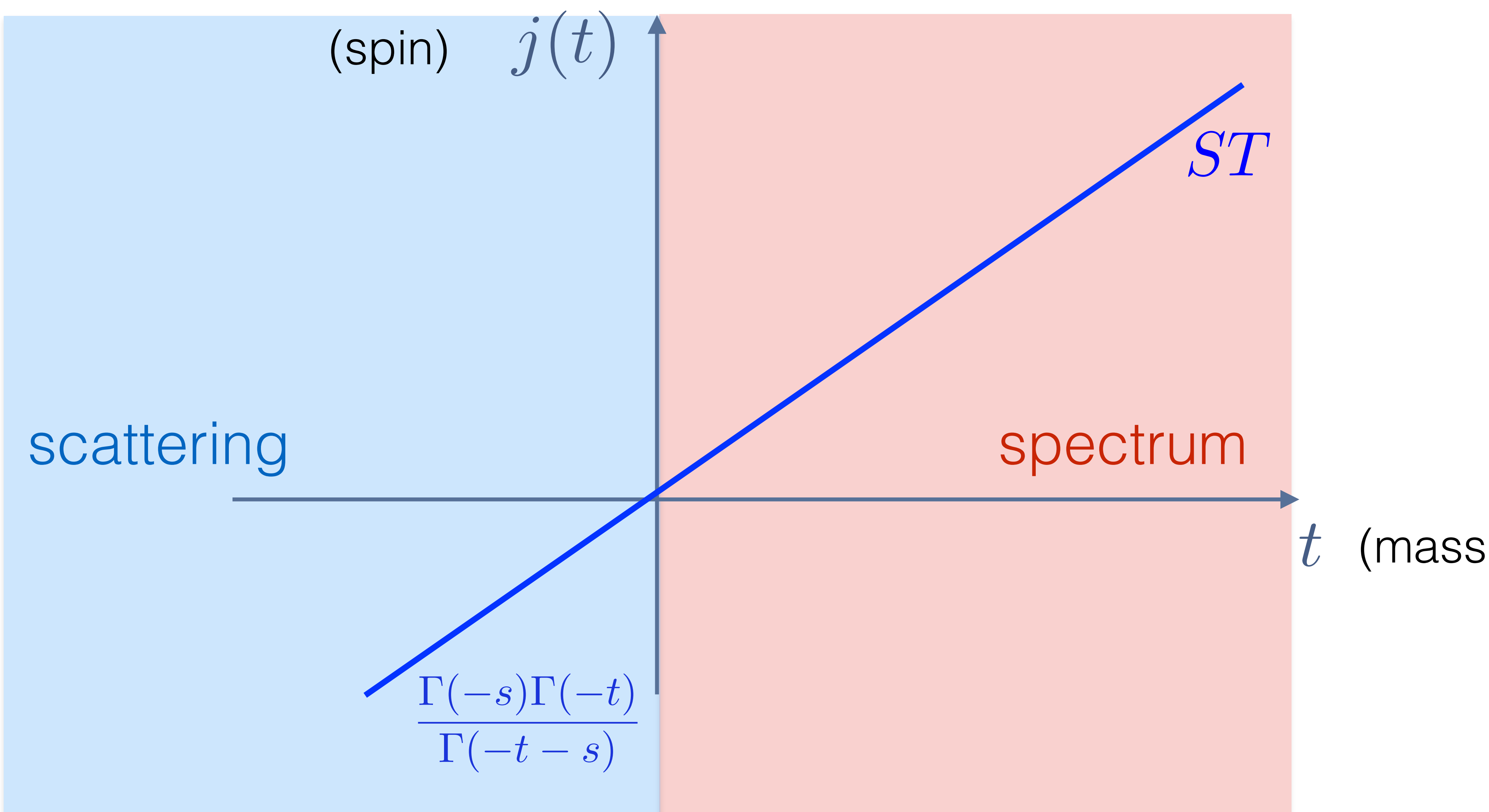

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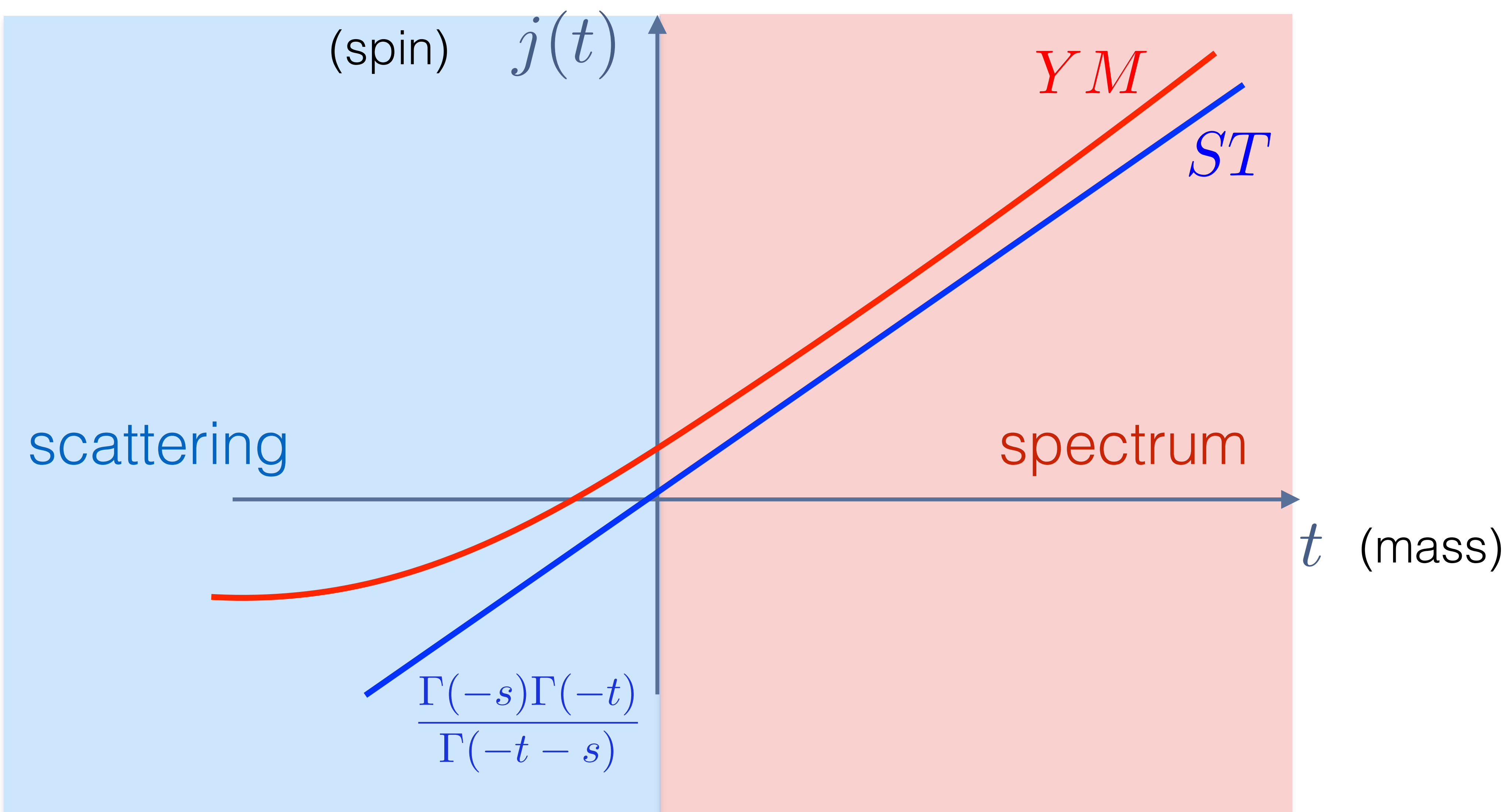

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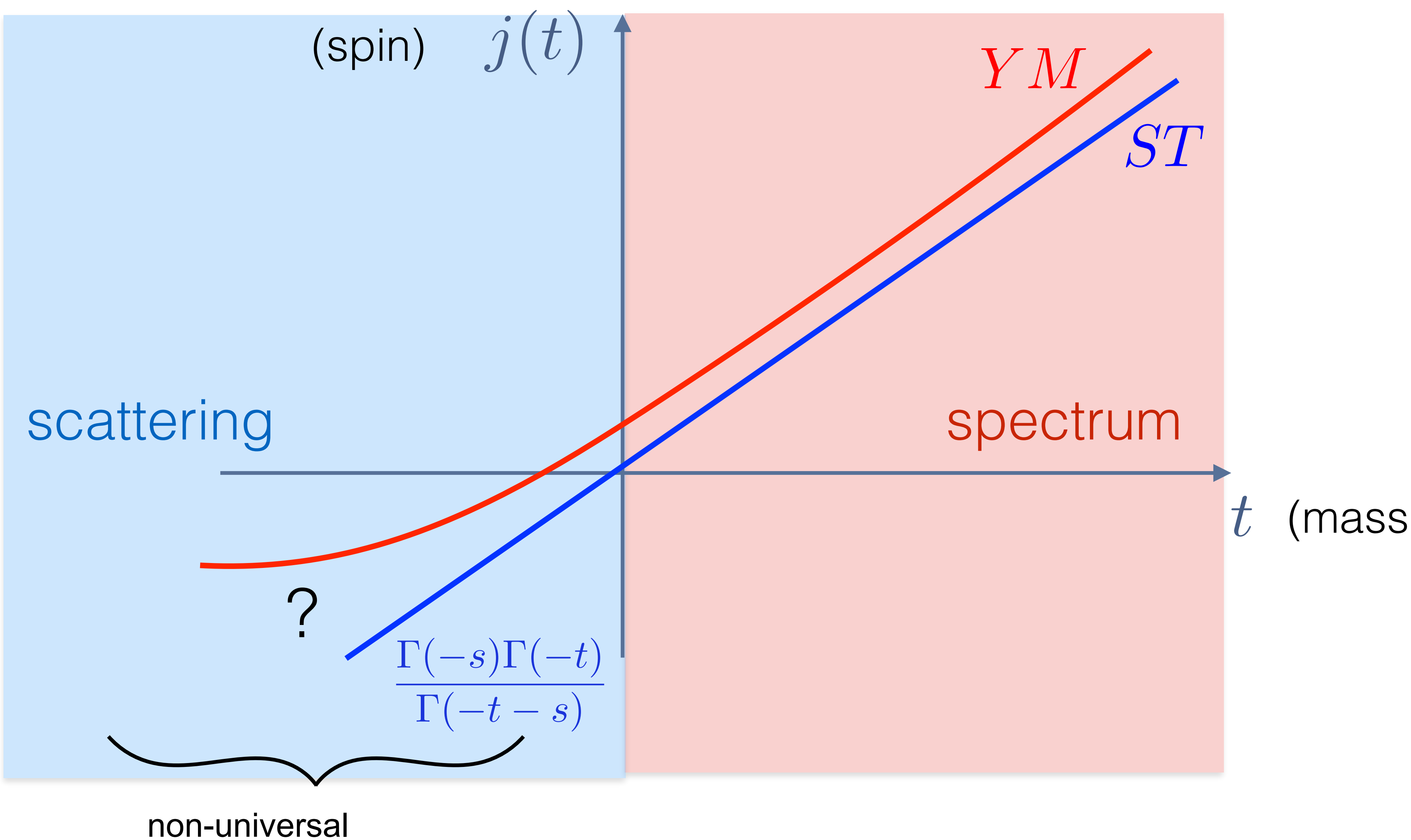

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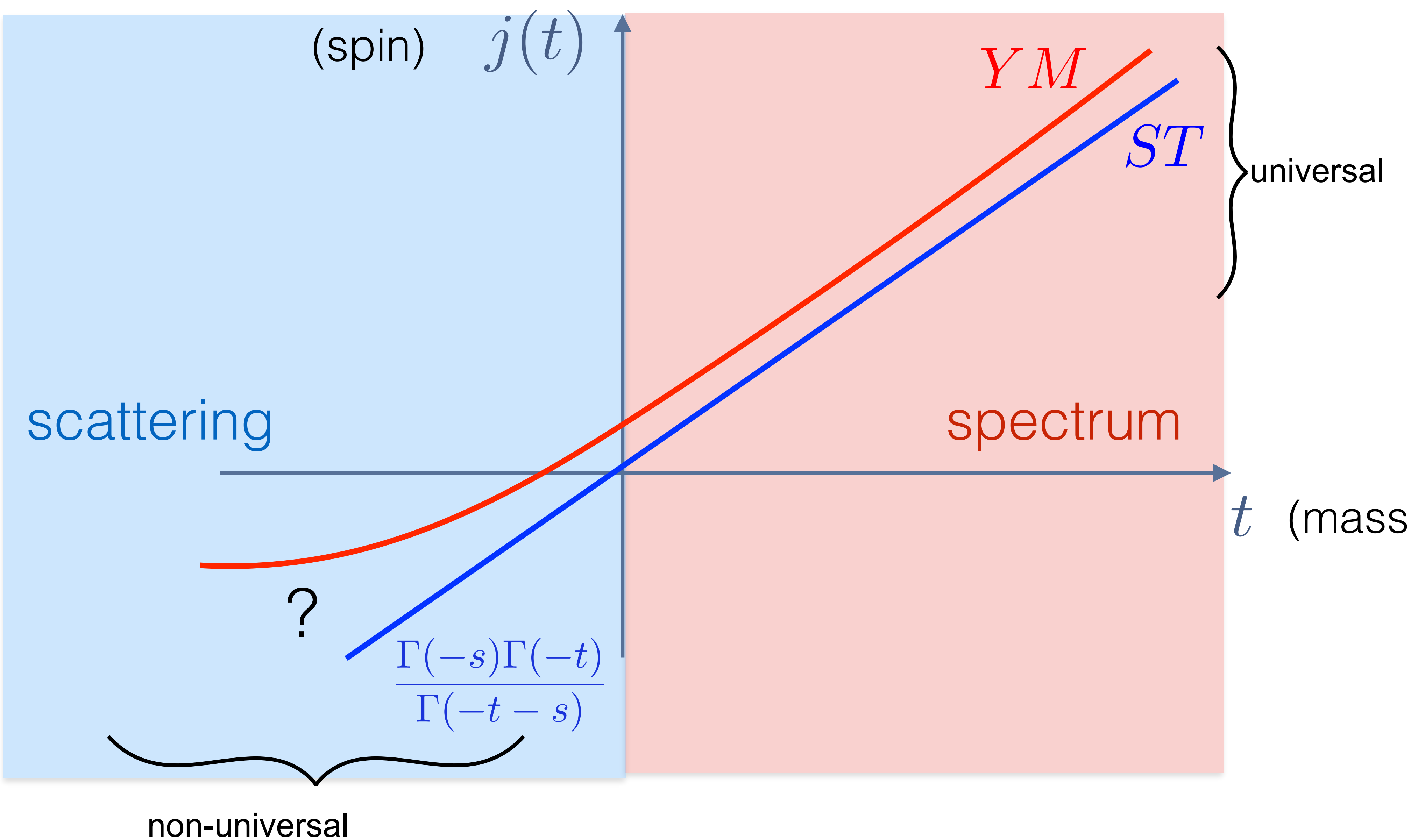
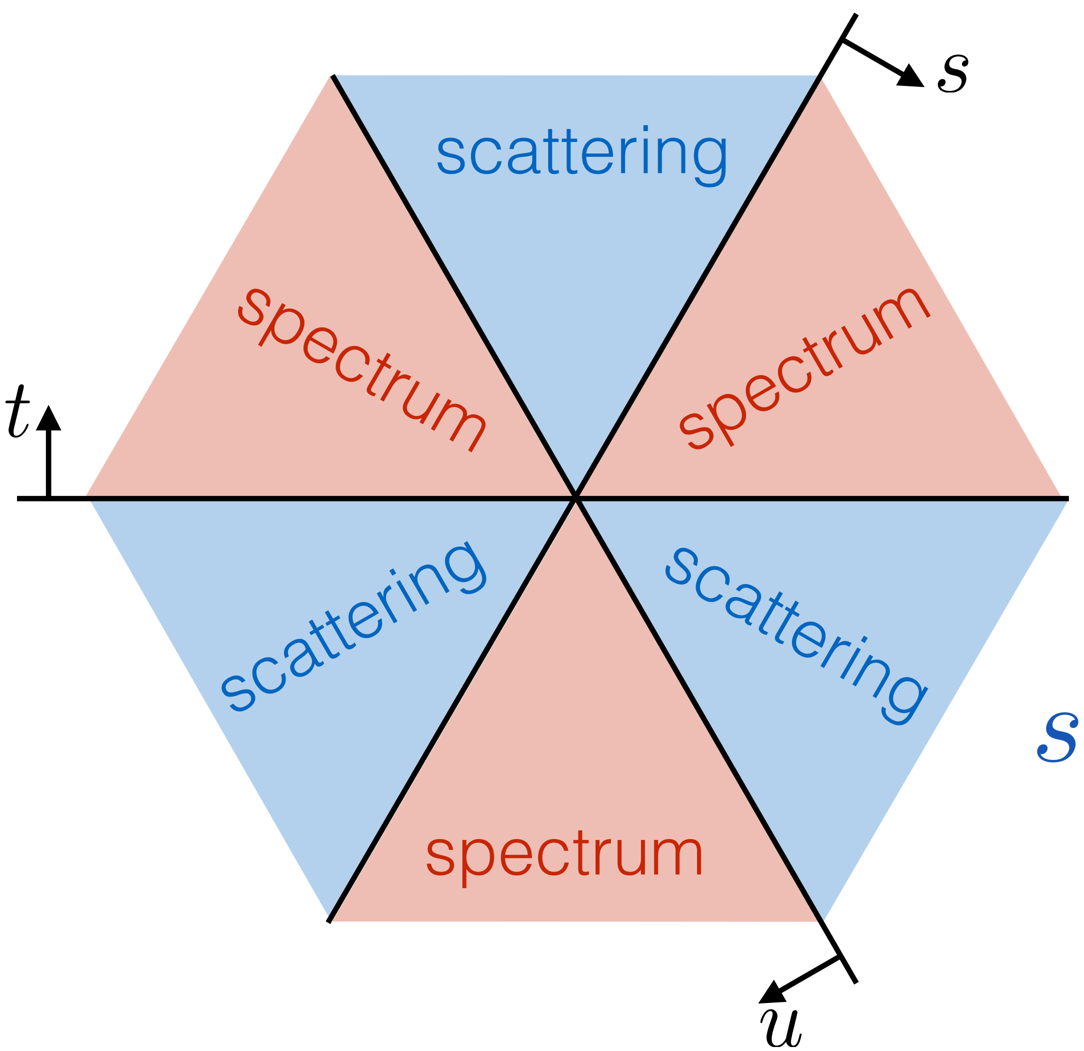


Mandelstam Plane

WIHS amplitudes are universal at imaginary scattering angles



$$s, t > 0$$

$$s > 0, \quad -s < t < 0$$

Non-universal/Real Angles (numerical methods)

Universal/Imaginary Angles (analytic methods)

Results

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limit of the Veneziano amplitude
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correction due to the slowdown of the string
(massive endpoints)/spectrum non-degeneracy

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corrections are O(log E)

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$$\begin{aligned} j(t) &= \alpha' t + \text{corrections} \\ &\quad +\text{parallel trajectories} \end{aligned} \quad \Rightarrow \quad \text{Im } A(s,b) \sim e^{-\frac{b^2}{\alpha' \log s}}$$

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- insensitive to the microscopic spectrum degeneracy

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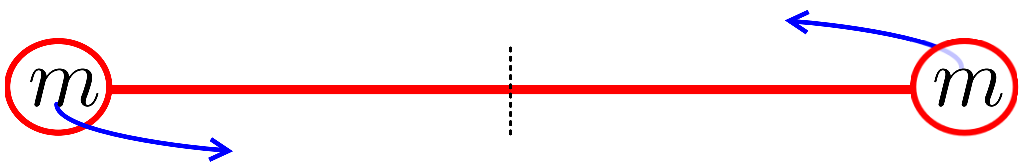
- worldsheet: slowdown of the string endpoints



$$j(t) = \alpha'\left(t - \frac{8\sqrt{\pi}}{3}m^{3/2}t^{1/4}+\ldots\right)$$

[Chodos, Thorn, 74']
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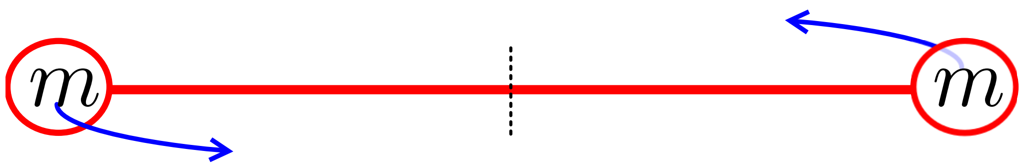

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- bootstrap: removal of the spectrum degeneracy

$$j_{\mathrm{sub-leading}}(t) \neq j_{\mathrm{leading}}(t) + \mathrm{integer}$$

Computing the Correction

- Scattering of Strings With Massive Endpoints
- Universality (Holography & EFT of Long Strings)
- Bootstrap

Worldsheet Computation (review)

$$\lim_{\substack{|s|,|t| \to \infty \\ s/t \text{ fixed}}} A(s,t) = e^{-S_E(s,t)}$$

[Gross, Mende]
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- imaginary scattering angles (amplitude is large) $-S_E \gg 1$

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Flat space

$$S_E = \frac{1}{2\pi\alpha'} \int d^2z \, \partial x \cdot \bar{\partial} x - i \sum_j k_j \cdot x(\sigma_j)$$

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$s, t > 0$

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[Chodos, Thorn]

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Modified boundary condition:

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The expansion reorganizes itself in terms of $\sqrt{m}$:

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The on-shell action evaluates to

$$S_E = S_{GM} + \frac{2}{3} m L_b + \ldots$$

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reparameterization invariant

For four external particles

$$\delta \log A(s,t) = -\frac{16\sqrt{\pi}}{3} \alpha' m^{3/2} \left(\frac{s\,t}{s+t}\right)^{\frac{1}{4}} \left[\mathrm{K}\left(\frac{s}{s+t}\right) + \mathrm{K}\left(\frac{t}{s+t}\right)\right] + \mathcal{O}(m^{5/2})$$

Adding The Mass

The on-shell action evaluates to

$$S_E = S_{GM} + \frac{2}{3} m L_b + \ldots$$

Gross-Mende solution

$$L_b = \sqrt{2\pi\alpha'\, m} \int d\sigma \, (\partial_\sigma^2 x_0 \cdot \partial_\sigma^2 x_0)^{1/4}$$

reparameterization
invariant

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$$\delta \log A(s,t) = -\frac{16\sqrt{\pi}}{3} \alpha' m^{3/2} \left(\frac{s\, t}{s+t} \right)^{\frac{1}{4}} \left[\mathrm{K}\left(\frac{s}{s+t} \right) + \mathrm{K}\left(\frac{t}{s+t} \right) \right] + \mathcal{O}(m^{5/2})$$

non-universal $O(t^{-1/4})$

Emergent s-u Crossing Symmetry

$$\lim_{\substack{s,t\to\infty \\ s/t \text{ fixed}}} \log A(s,t) = \alpha' \left[(s+t)\log(s+t) - s\log(s) - t\log(t)\right]$$
$$- \frac{16\sqrt{\pi}}{3}\alpha' m^{3/2}\left(\frac{s\,t}{s+t}\right)^{\frac{1}{4}}\left[\mathrm{K}\left(\frac{s}{s+t}\right) + \mathrm{K}\left(\frac{t}{s+t}\right)\right] + \dots$$

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The **s-t** crossing is manifest: $\log A(s,t) = \log A(t,s)$

What about the **s-u** crossing? $\log A(s,t) = \mathrm{Re}[\log A(u,t)]$

$$u = -s - t$$

Emergent s-u Crossing Symmetry

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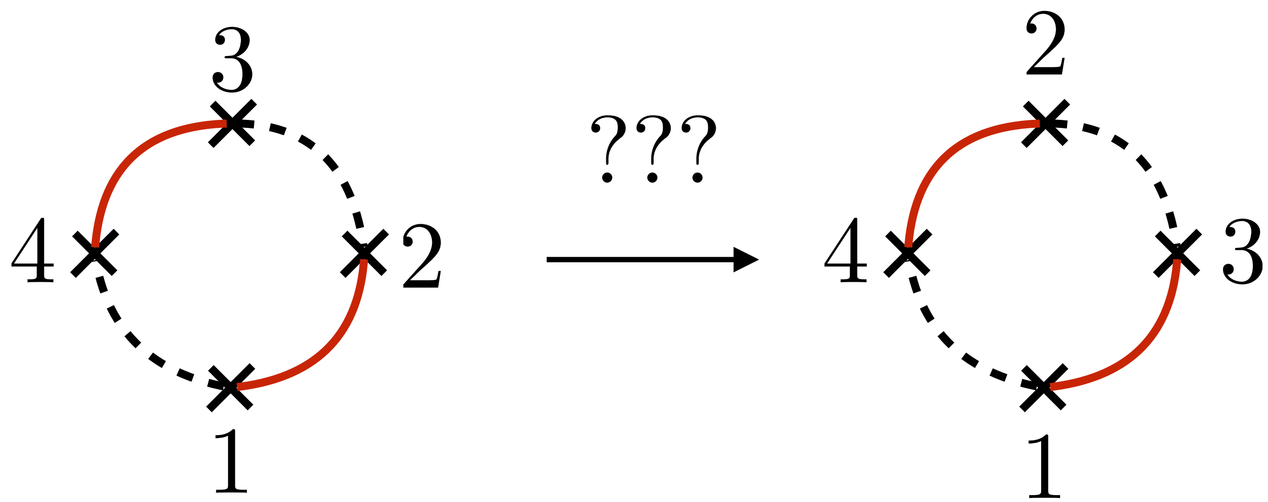
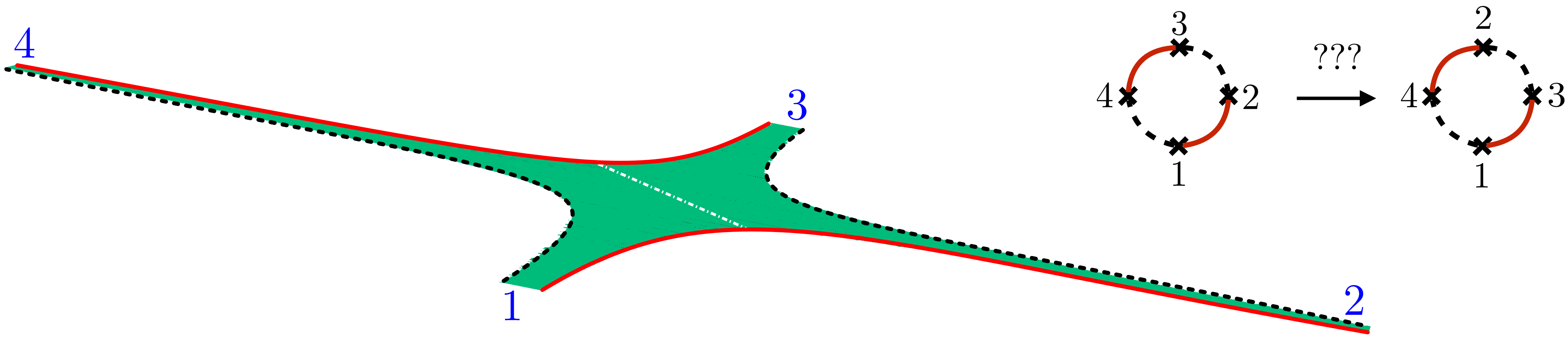


Emergent s-u Crossing Symmetry

[Komatsu]



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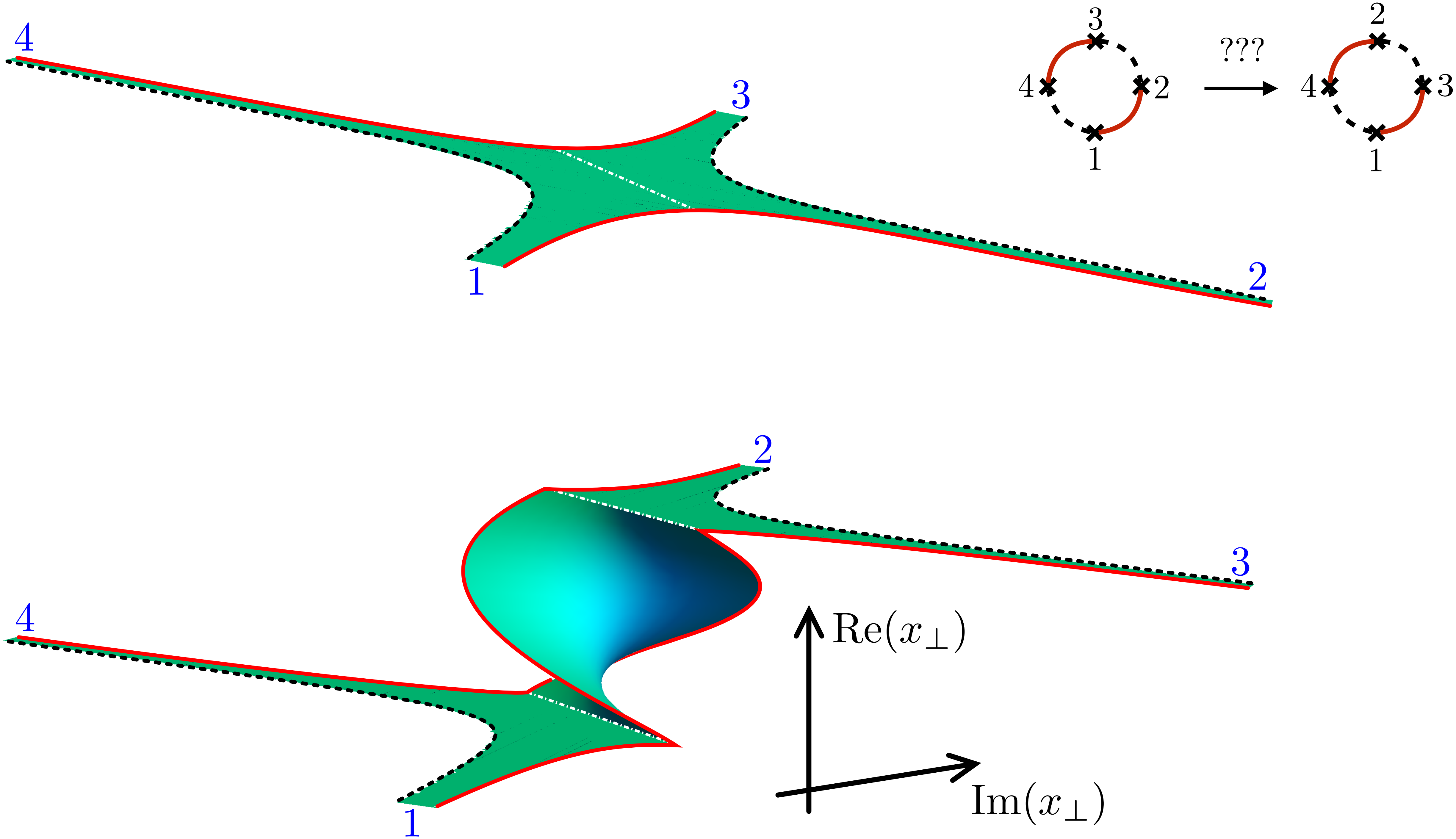


Asymptotic s-u Crossing

Equivalently, the asymptotic **s-u** crossing is:

$$\mathrm{dDisc}_s \log A(s,t) \equiv$$

$$\log A(-s-t+i\epsilon,t) + \log A(-s-t-i\epsilon,t) - 2\log A(s,t) = 0$$

Double discontinuity is zero!

Universality

Why is the correction universal?

Why is the correction universal?

Why is the massive ends model physical?

Holographic Argument

[Erdmenger et al.]
[Sonnenschein]

Holographic dual of a confining gauge theory:

$$ds^2 = dr^2 + f(r)\, dx^2_{1,d-1}$$

r

Holographic
radial direction

- AdS in the UV $\qquad \lim_{r\to\infty} f(r) = e^{2r}$

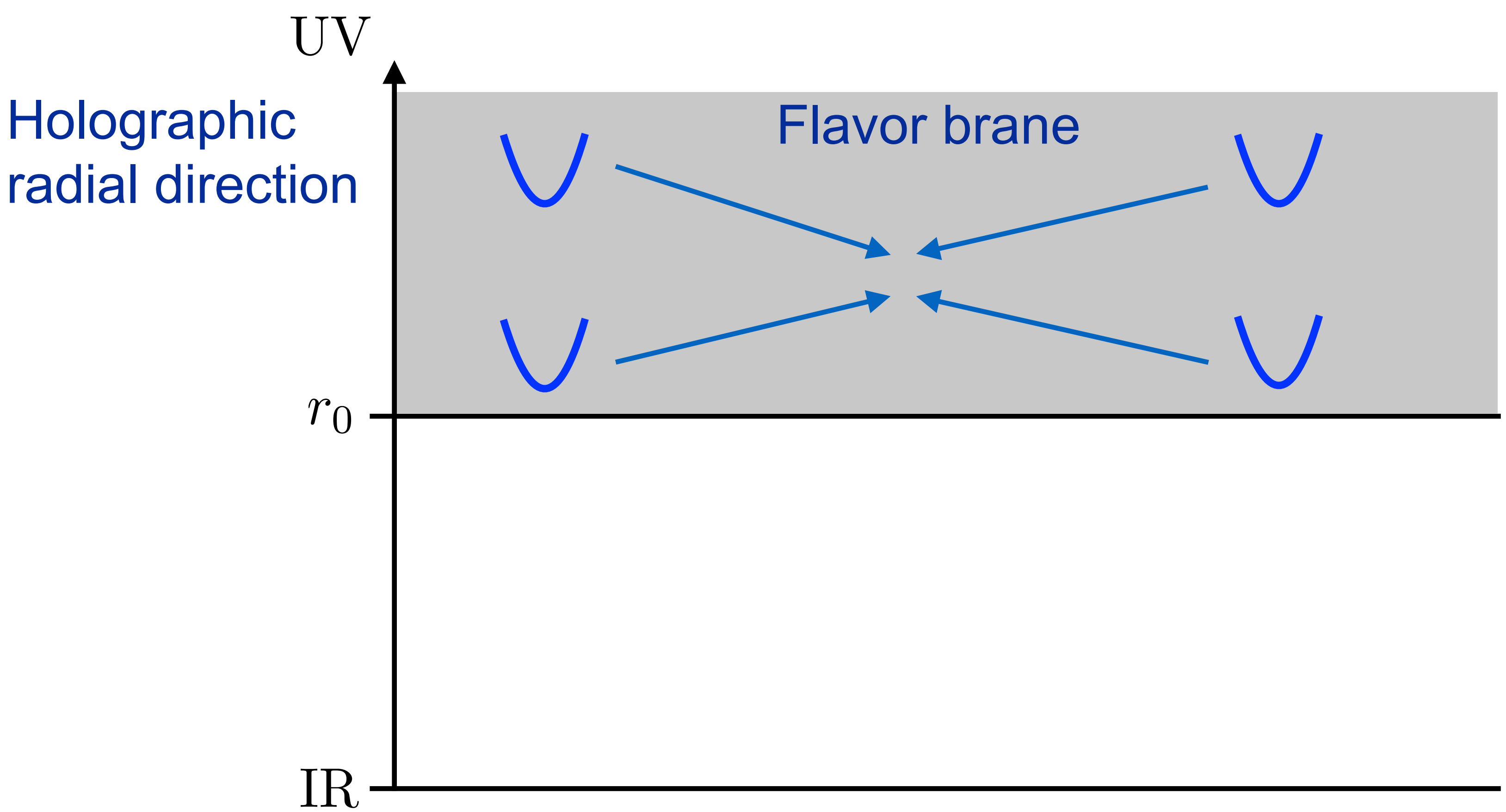
- Cutoff in the IR $\qquad f(0) = 1$

x

Polchinski-Strassler Mechanism

[Polchinski, Strassler]

For mesons we add a space-filling flavor brane



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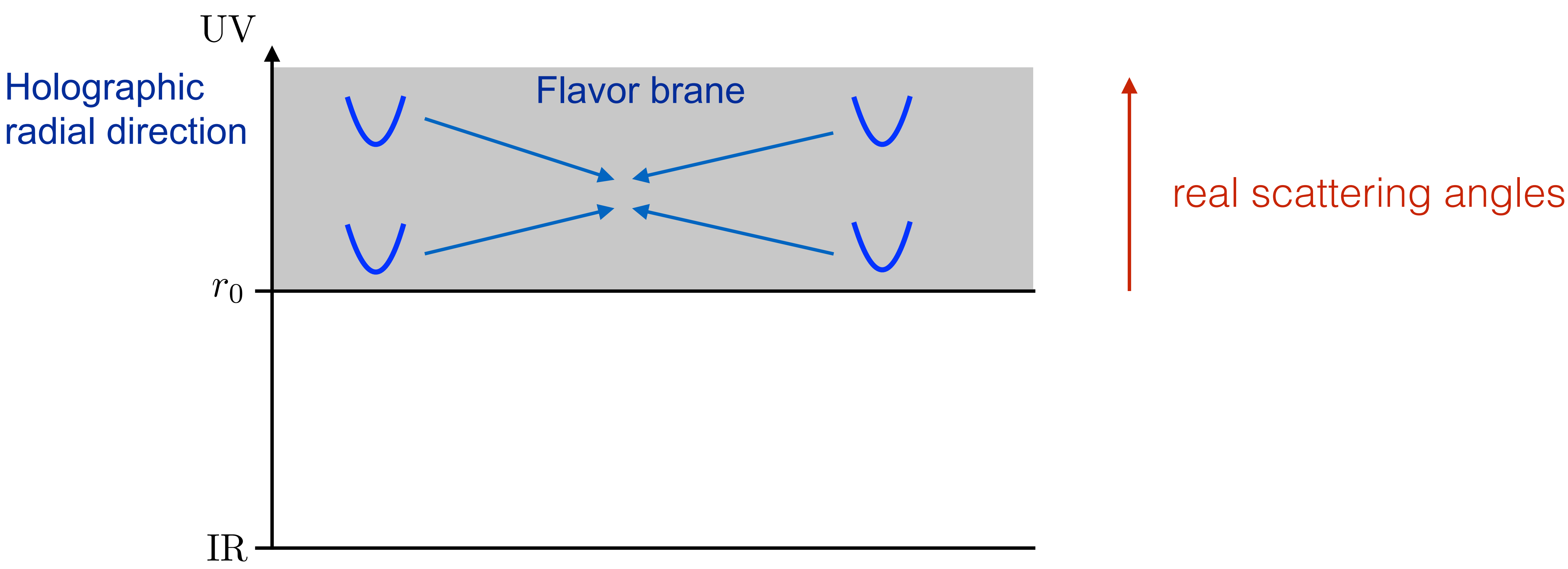

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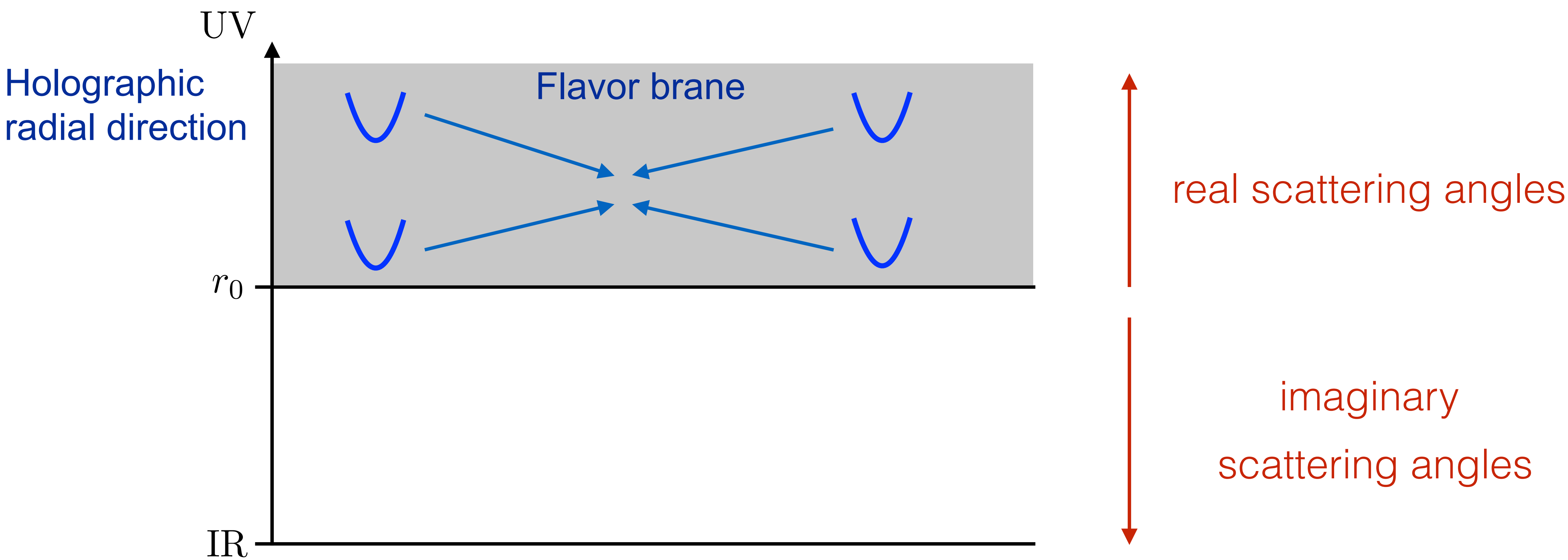

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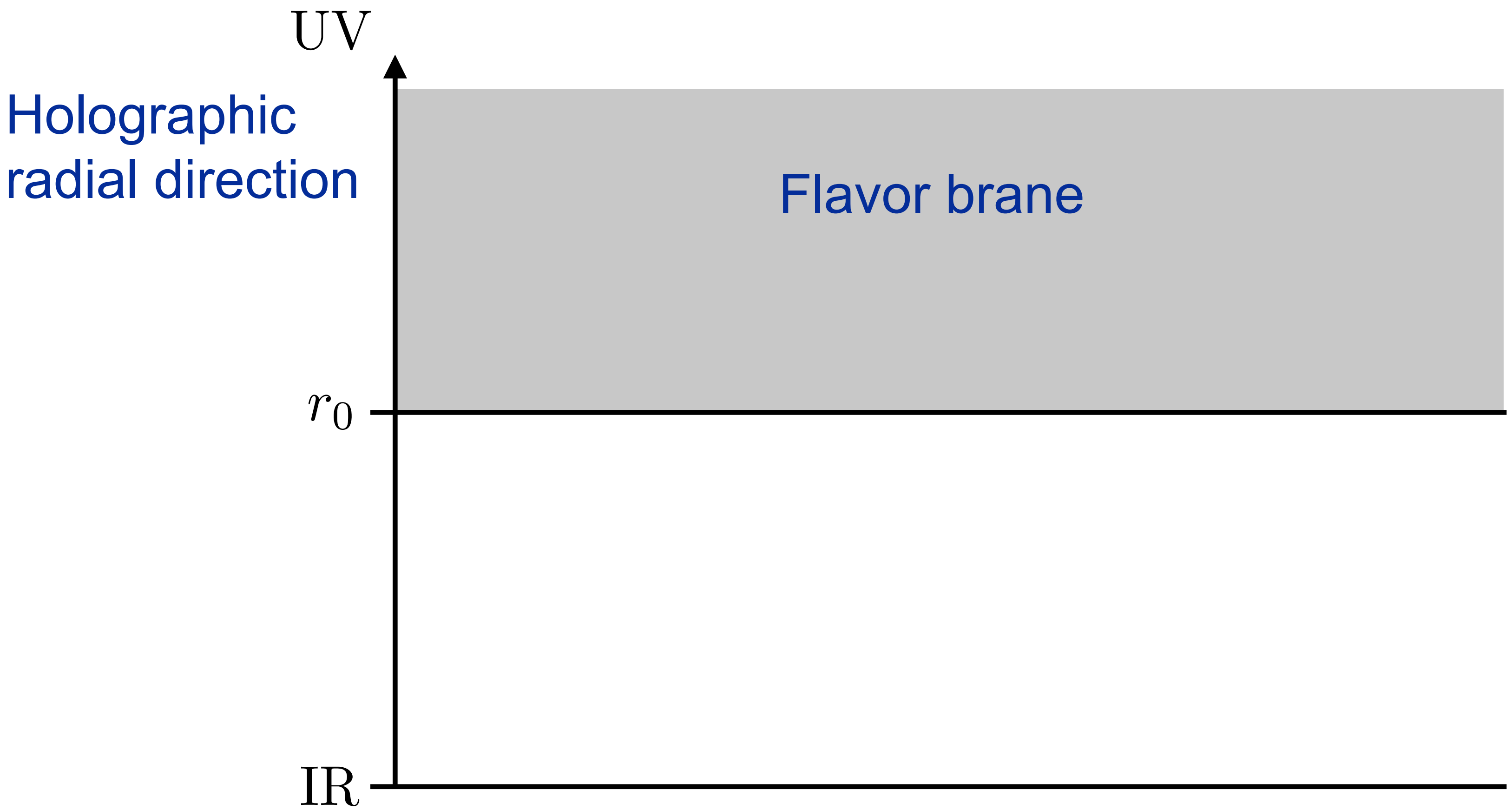
At high energies the string acquires the characteristic shape



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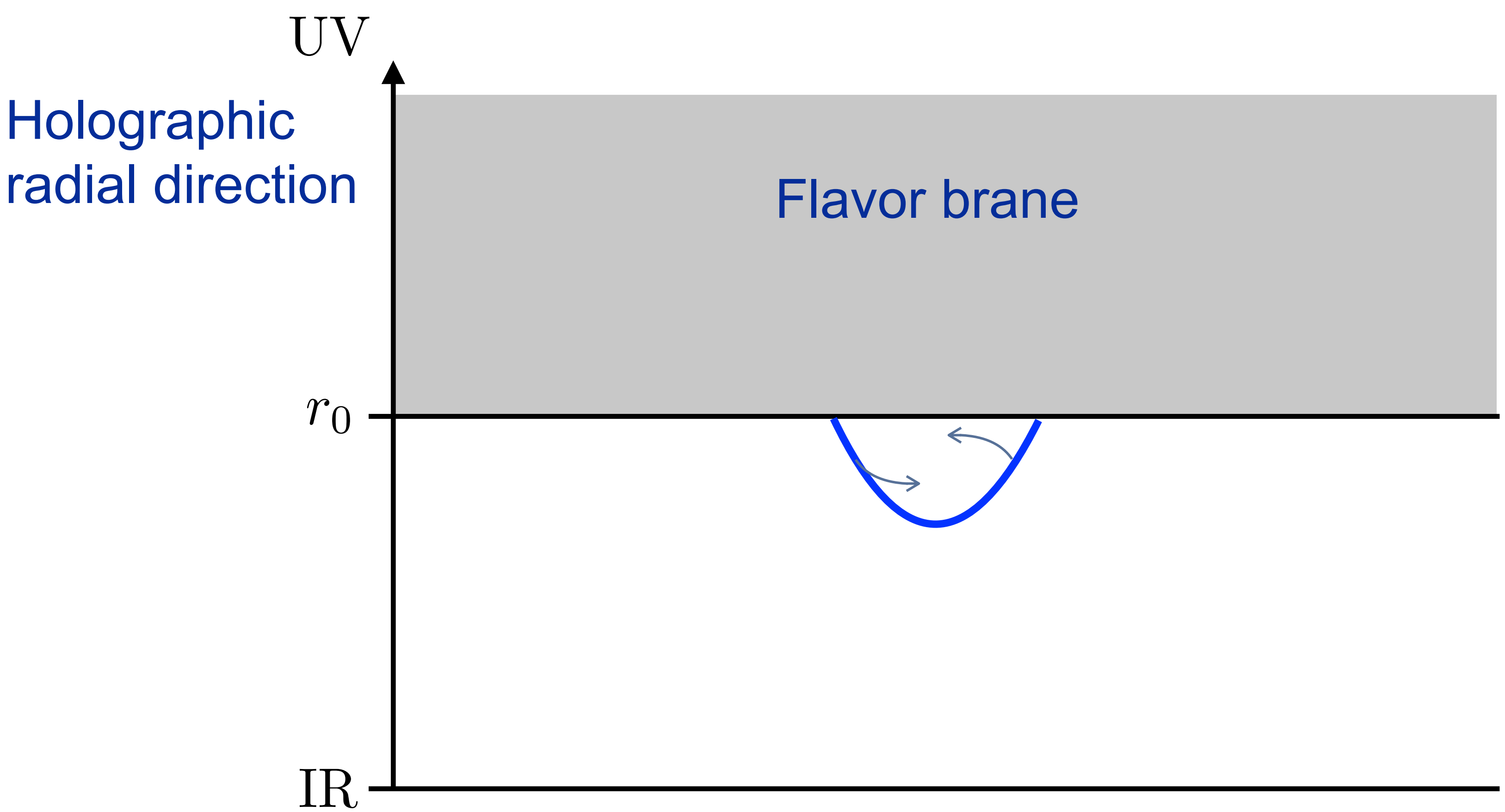

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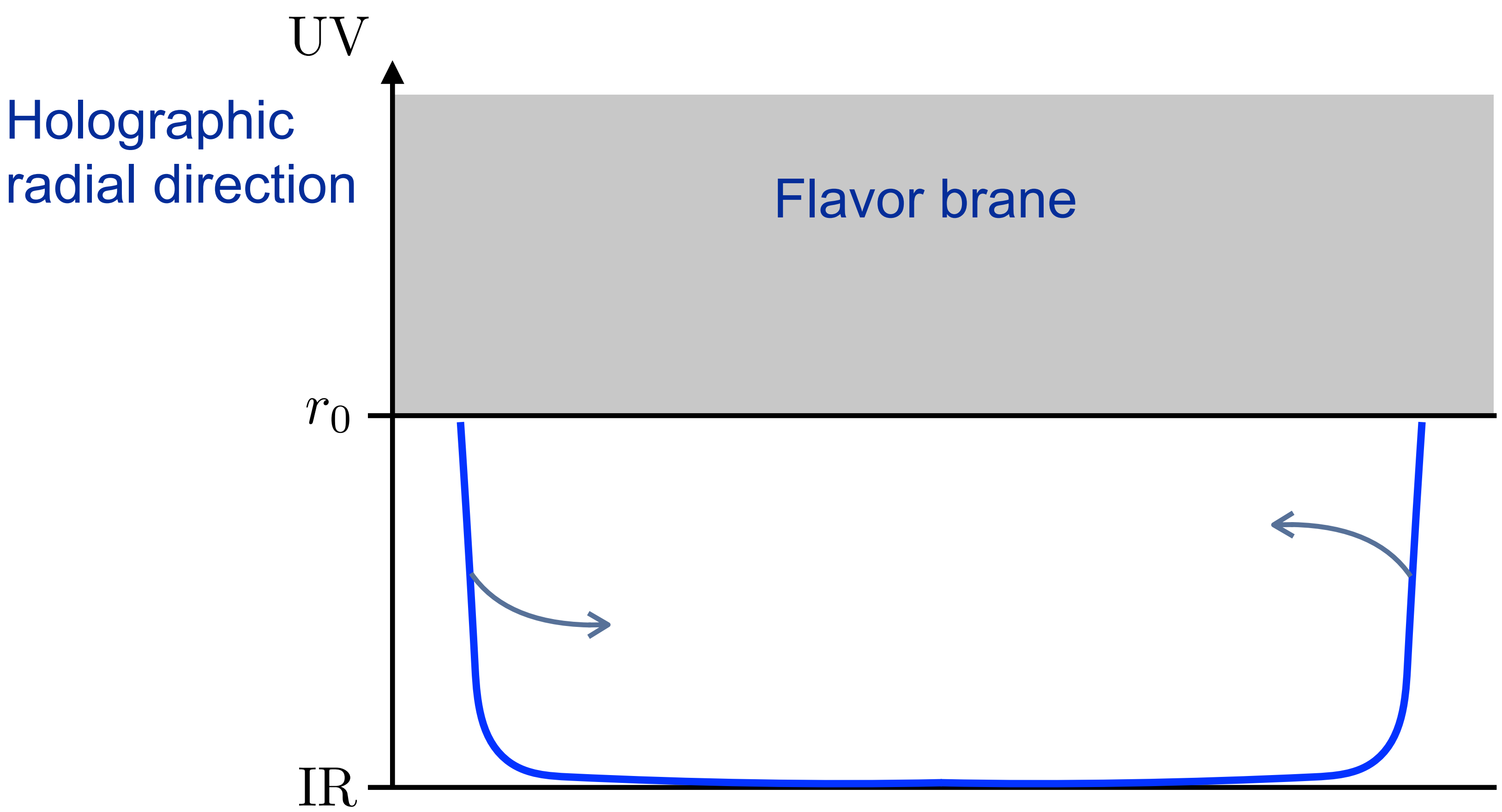

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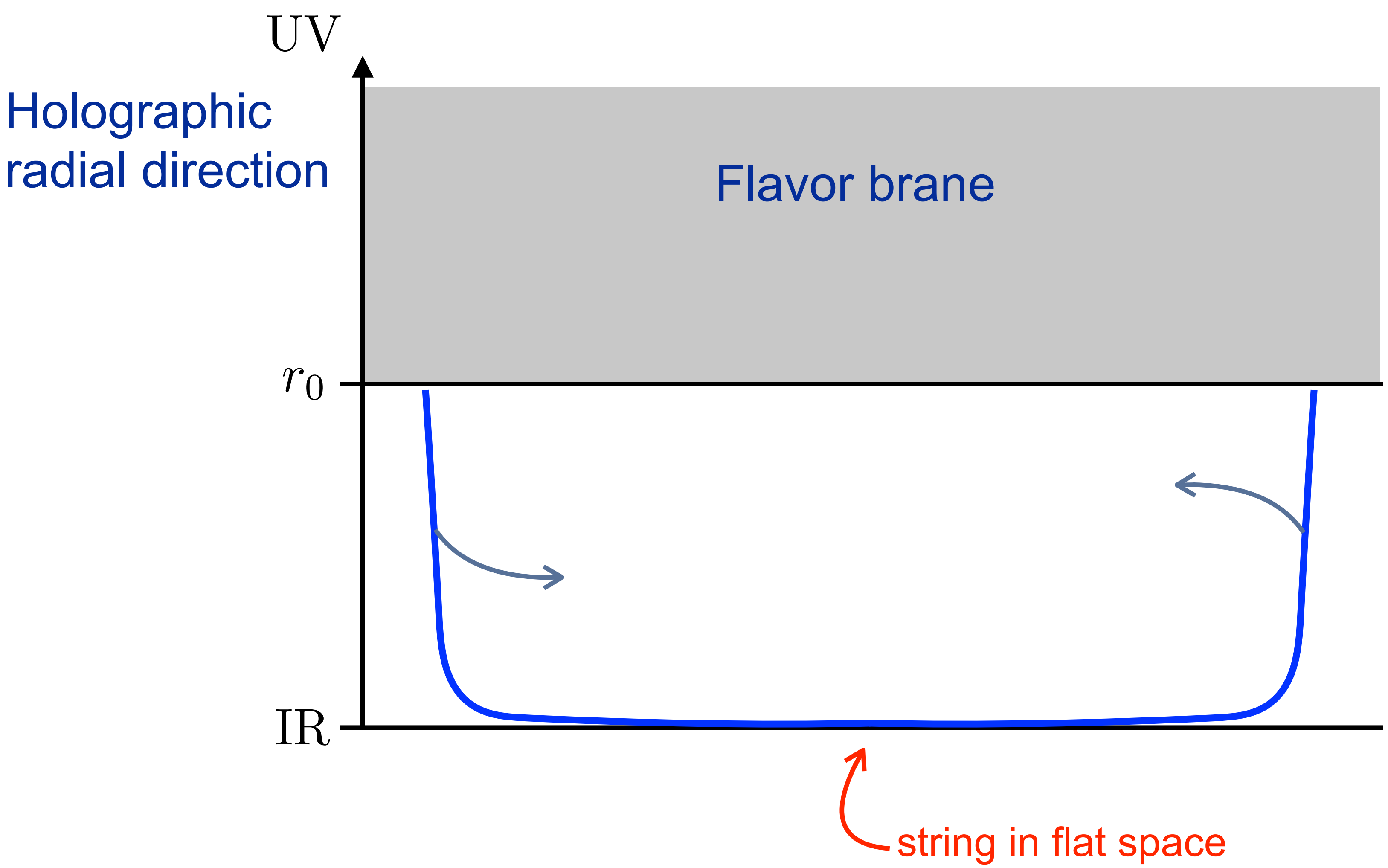

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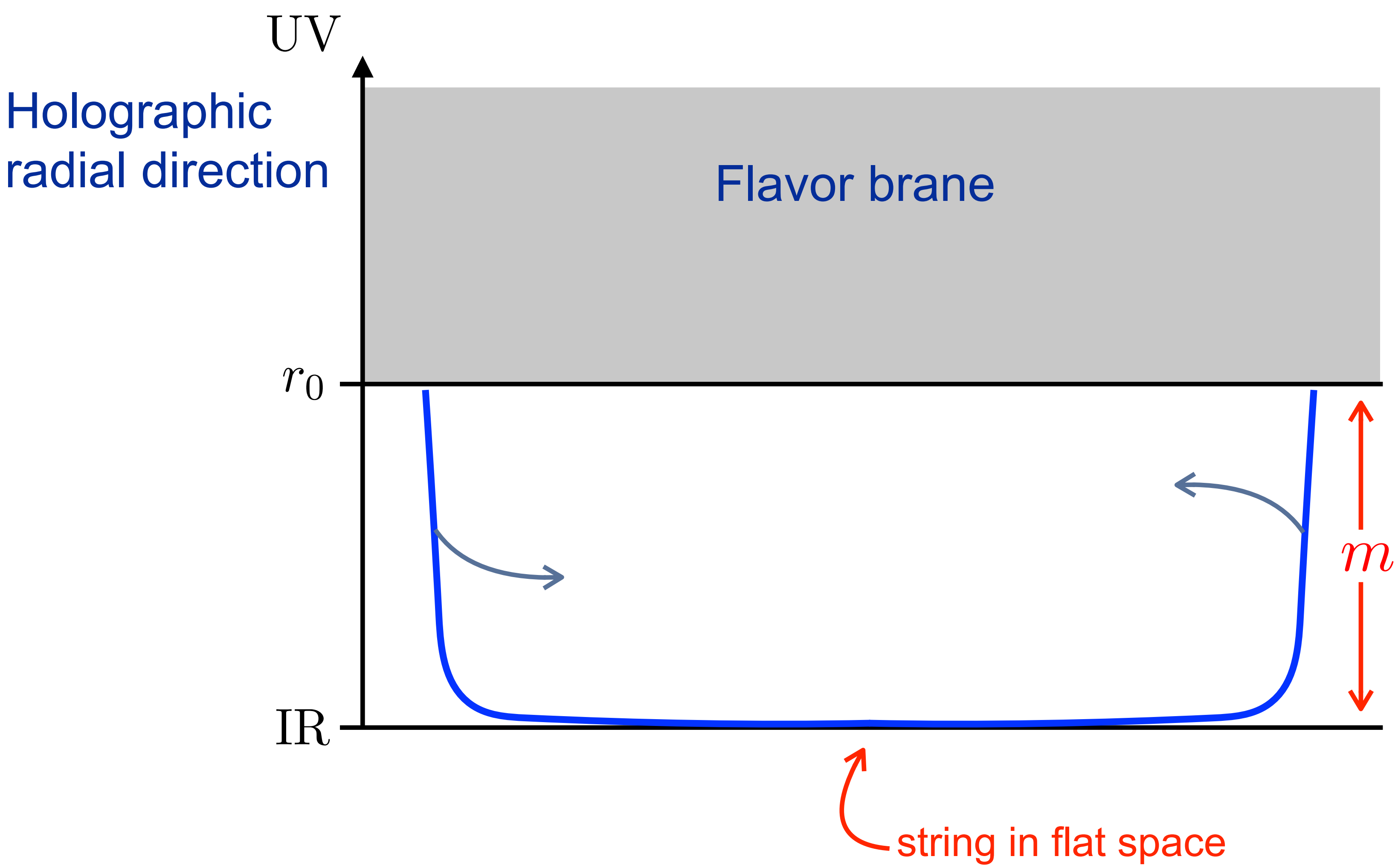

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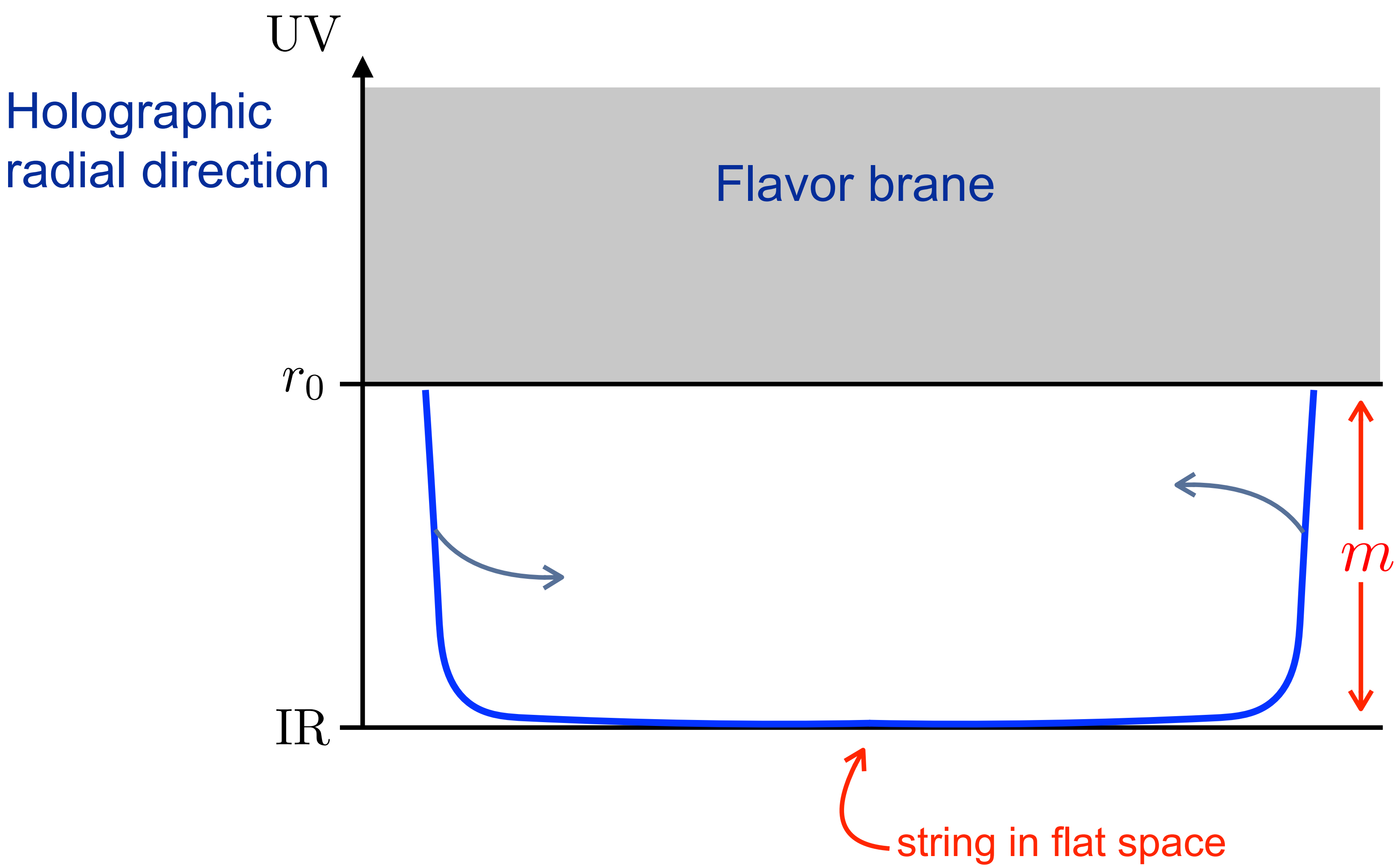
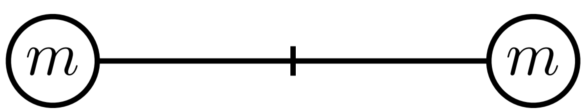


$$m^2\alpha' \gg 1$$

$\Rightarrow$



effective description

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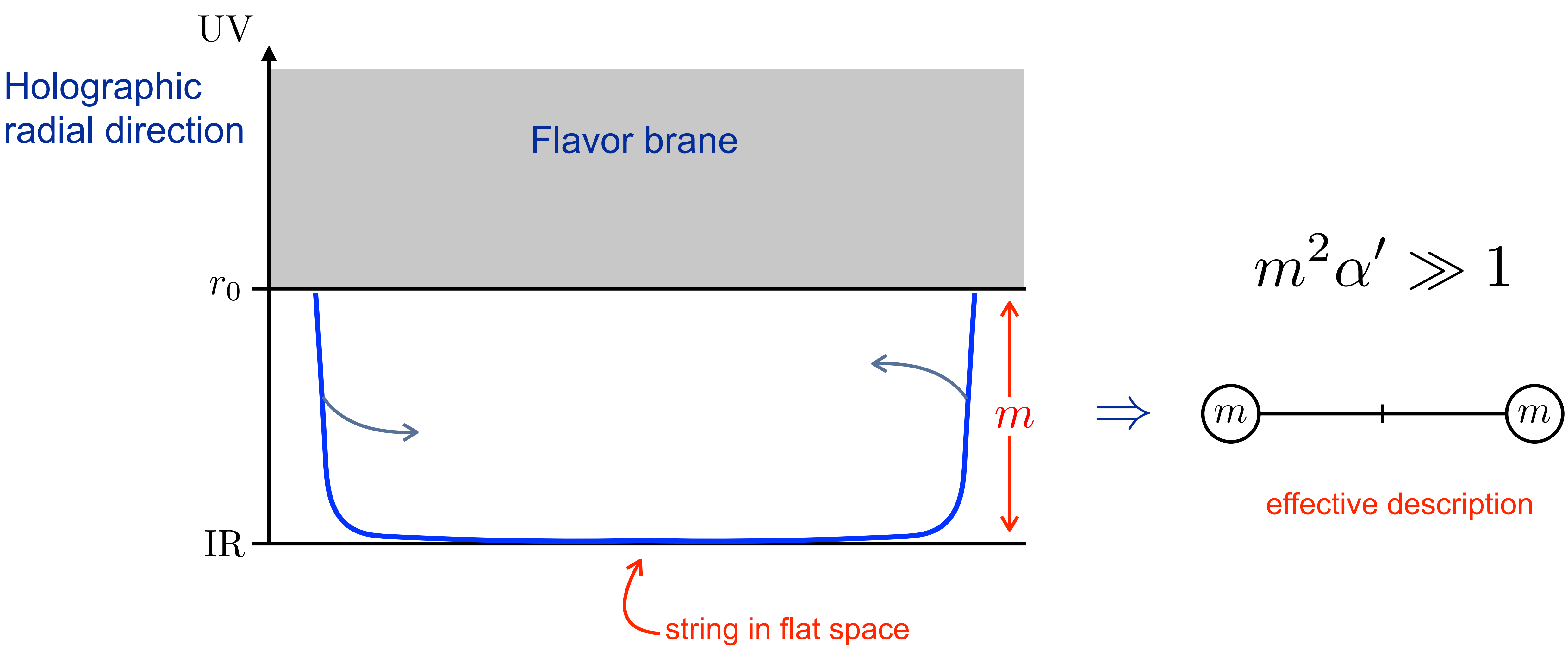

- At $s, t \gg 1$ the holographic model reduces to the string with massive ends

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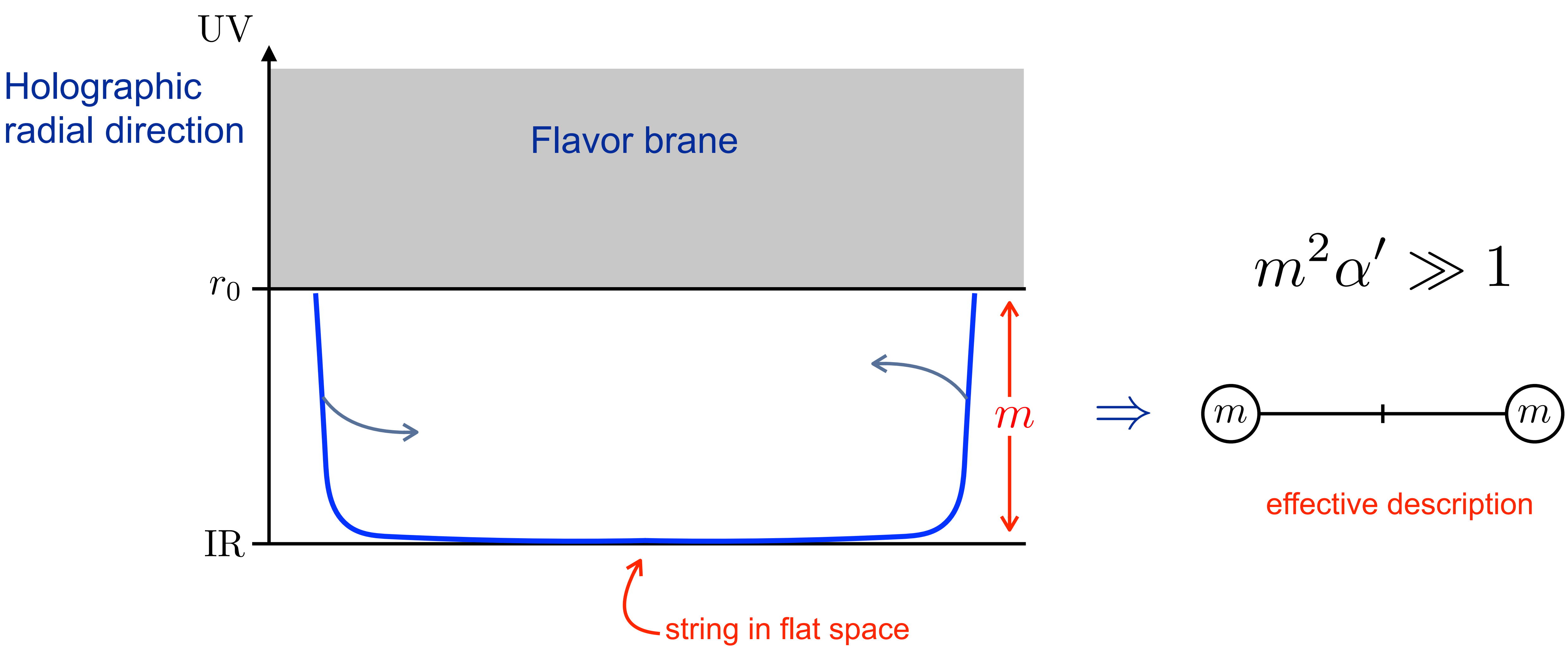

- At $s, t \gg 1$ the holographic model reduces to the string with massive ends
- Insensitive to the details of the background

EFT of Long Strings

[Polchinski, Strominger]
[Aharony et al.]
[Dubovsky et al.]
[Hellerman et al.]

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- boundary corrections (open strings)

Unique in the effective theory of open strings

$$j(t) = \alpha' \left(t - \frac{8\sqrt{\pi}}{3} m^{3/2} t^{1/4} \right)$$

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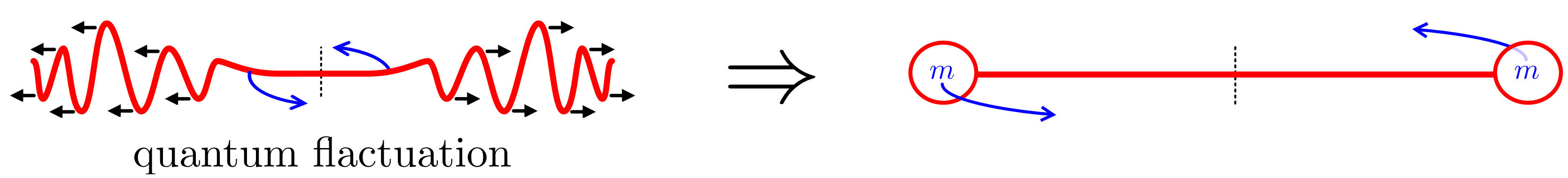
$$\frac{(\partial^2 x \cdot \bar{\partial} x)(\partial x \cdot \bar{\partial}^2 x)}{(\partial x \cdot \bar{\partial} x)^2}$$

Polchinski-Strominger term



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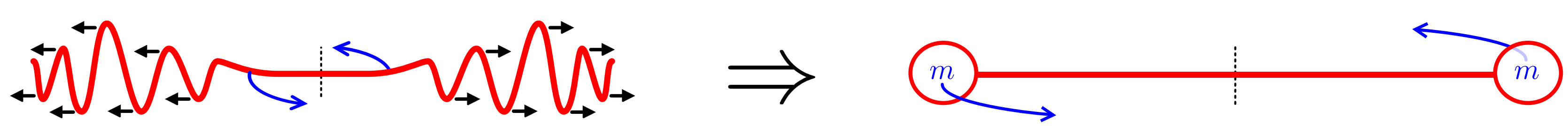
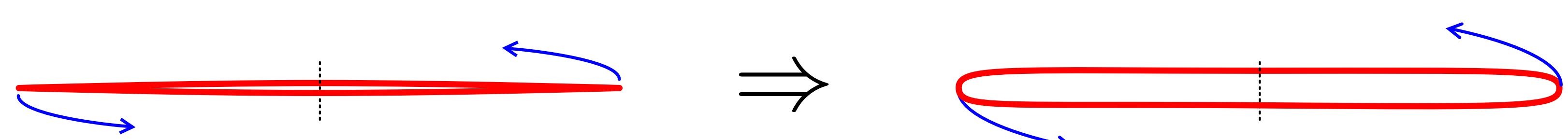


quantum flactuation

- higher derivative corrections (closed strings)



folded closed string

[hopefully somebody in progress]

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[Aharony et al.]
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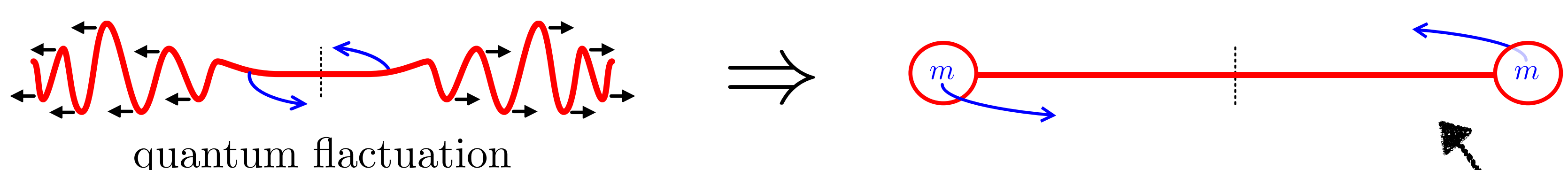

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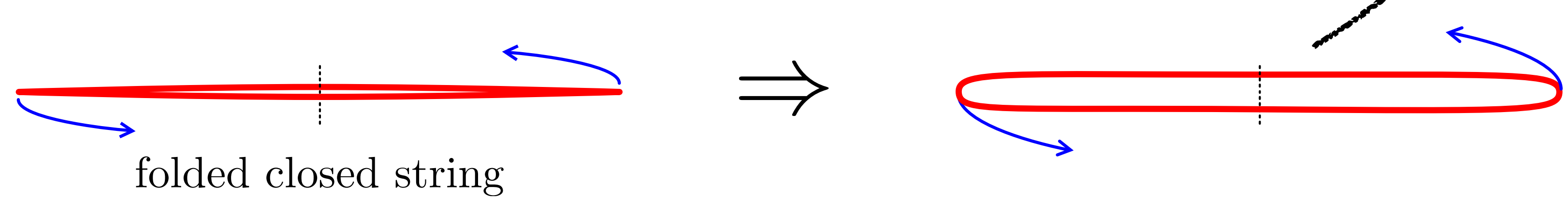


[hopefully somebody in progress]

Bootstrap

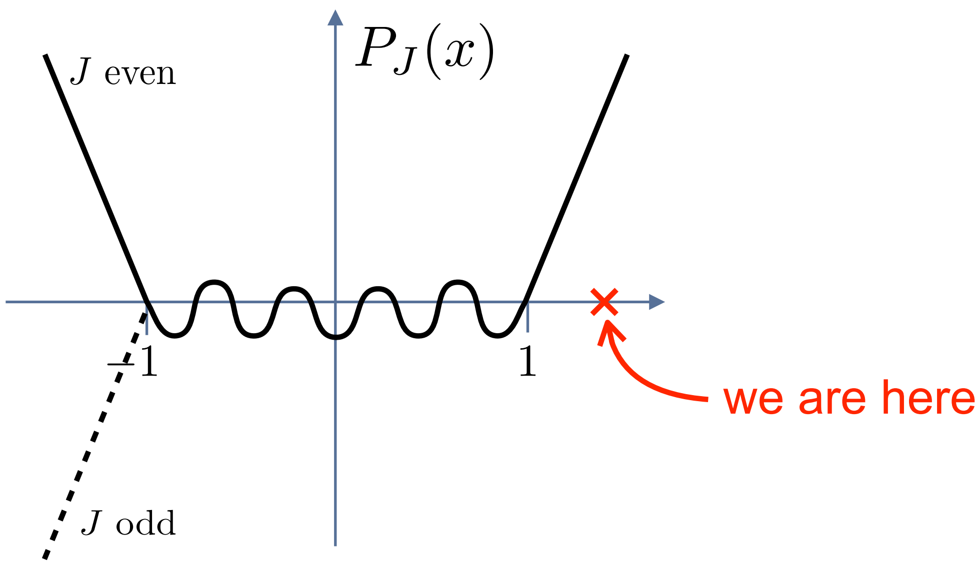
Bootstrap (Leading Order)

For s,t large and positive a thermodynamic picture emerges

$$\lim_{\substack{s,t\to\infty \\ s/t \text{ fixed}}} \log A\left((1+i\epsilon)s, (1+i\epsilon)t\right)$$

[Caron-Huot, Komargodski, Sever, AZ]

Partial wave



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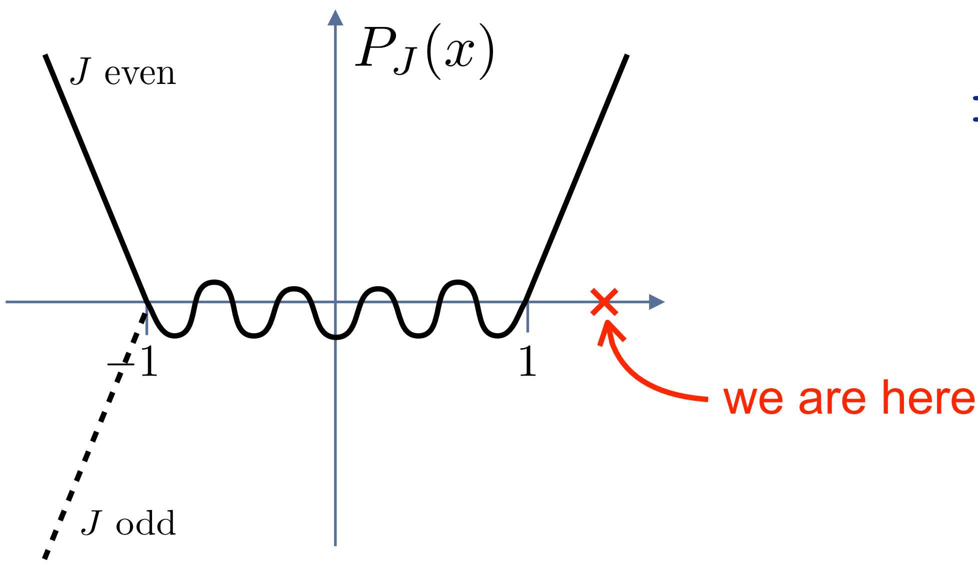

$\Longrightarrow$ • All residues are positive

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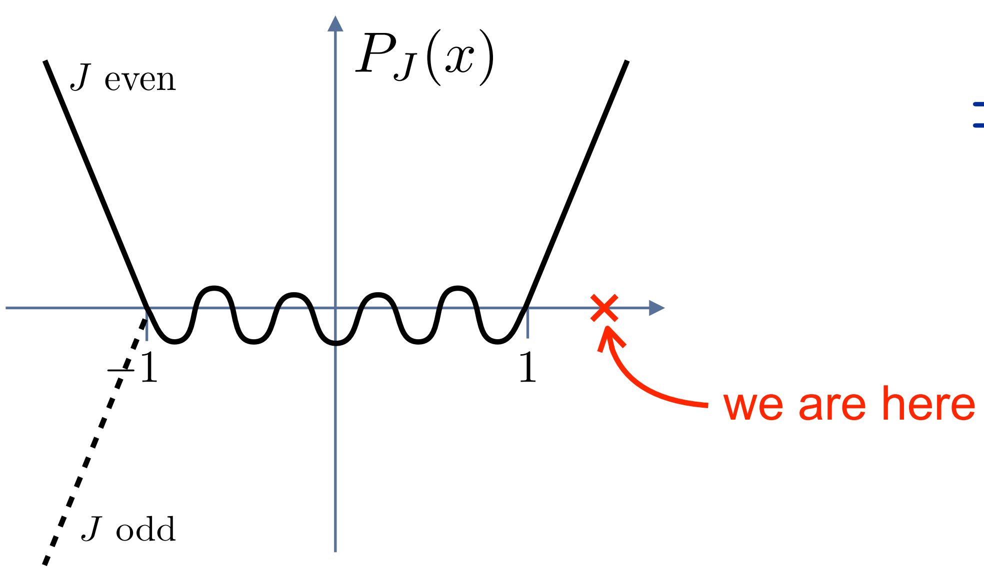

$\Rightarrow$ • All residues are positive

$\Rightarrow$ • At least one zero between every two poles

• There could be more zeros

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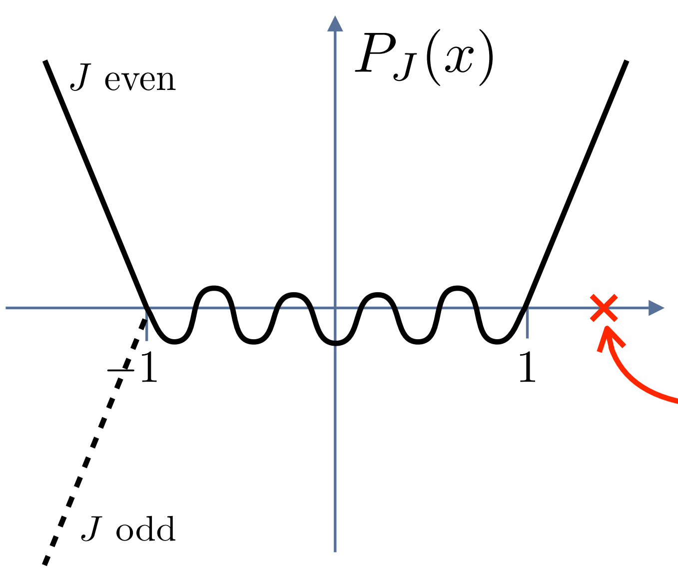

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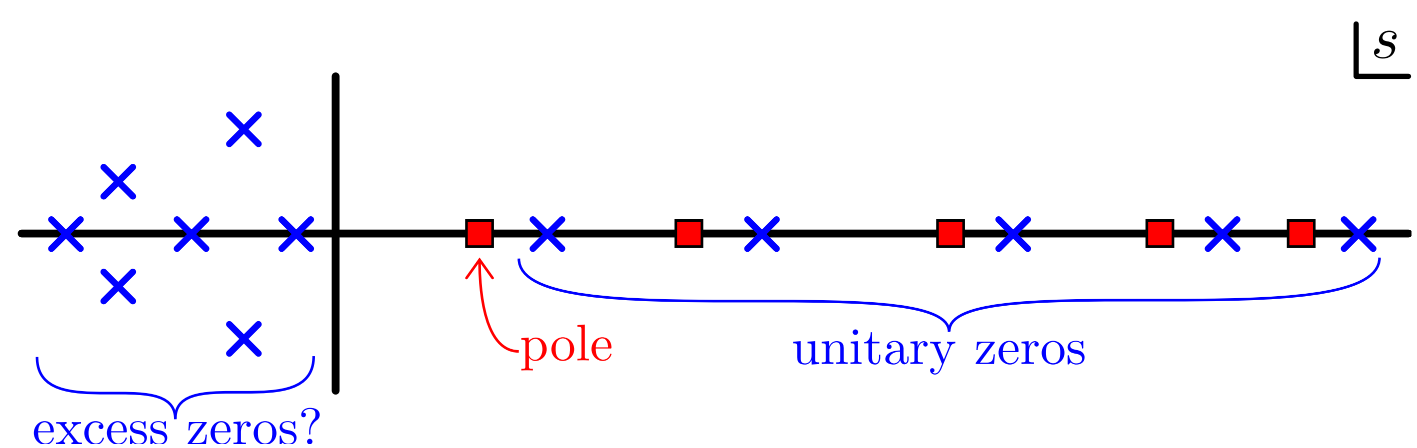
Complex s plane
at fixed real t



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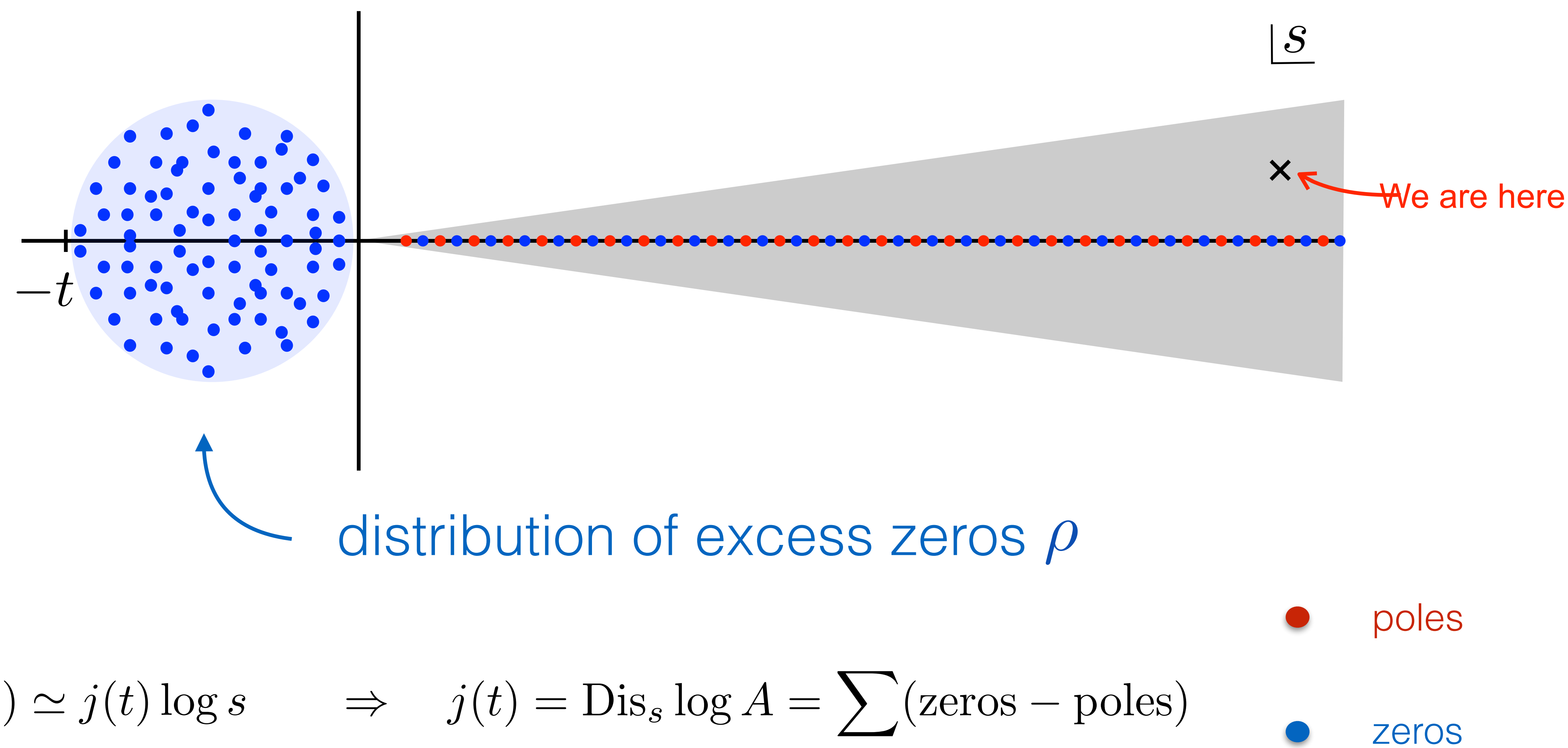


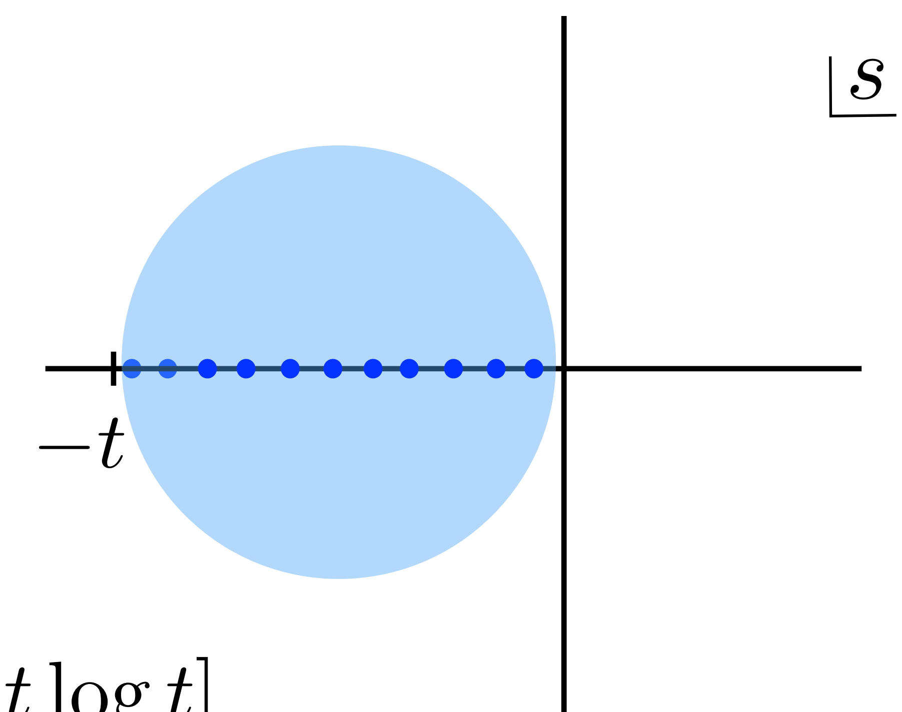
$$\log A(s,t) \simeq j(t) \log s \quad \Rightarrow \quad j(t) = \mathrm{Dis}_s \log A = \sum (\text{zeros} - \text{poles})$$

Bootstrap (Leading Order)

To leading order we have

$$\log A(s,t) \simeq \log \int_0^{j(t)} dj\ c_j(t)\ P_j\left(1+\frac{2s}{t}\right)\ , \qquad c_j(t) \geq 0$$

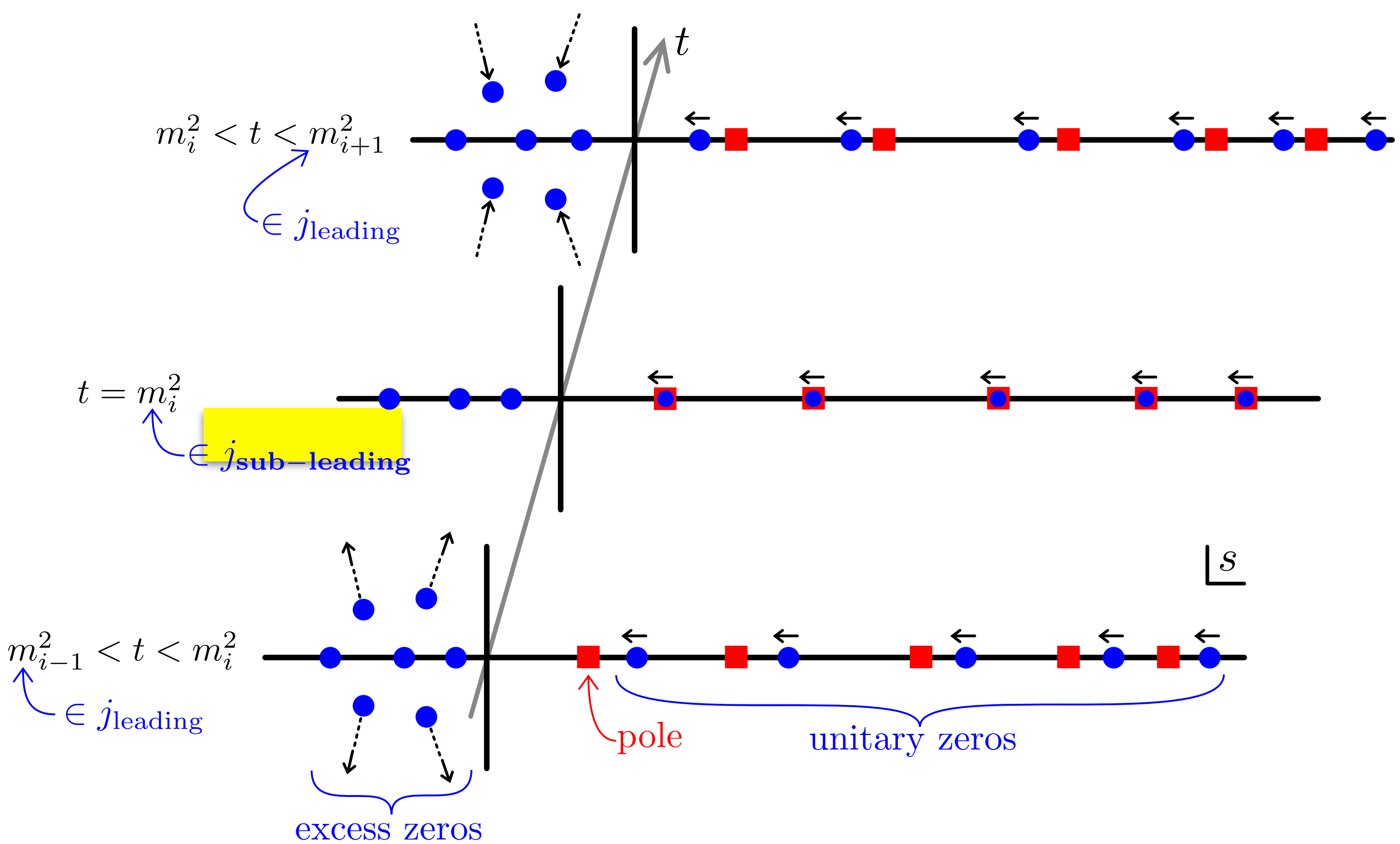
The unique solution is

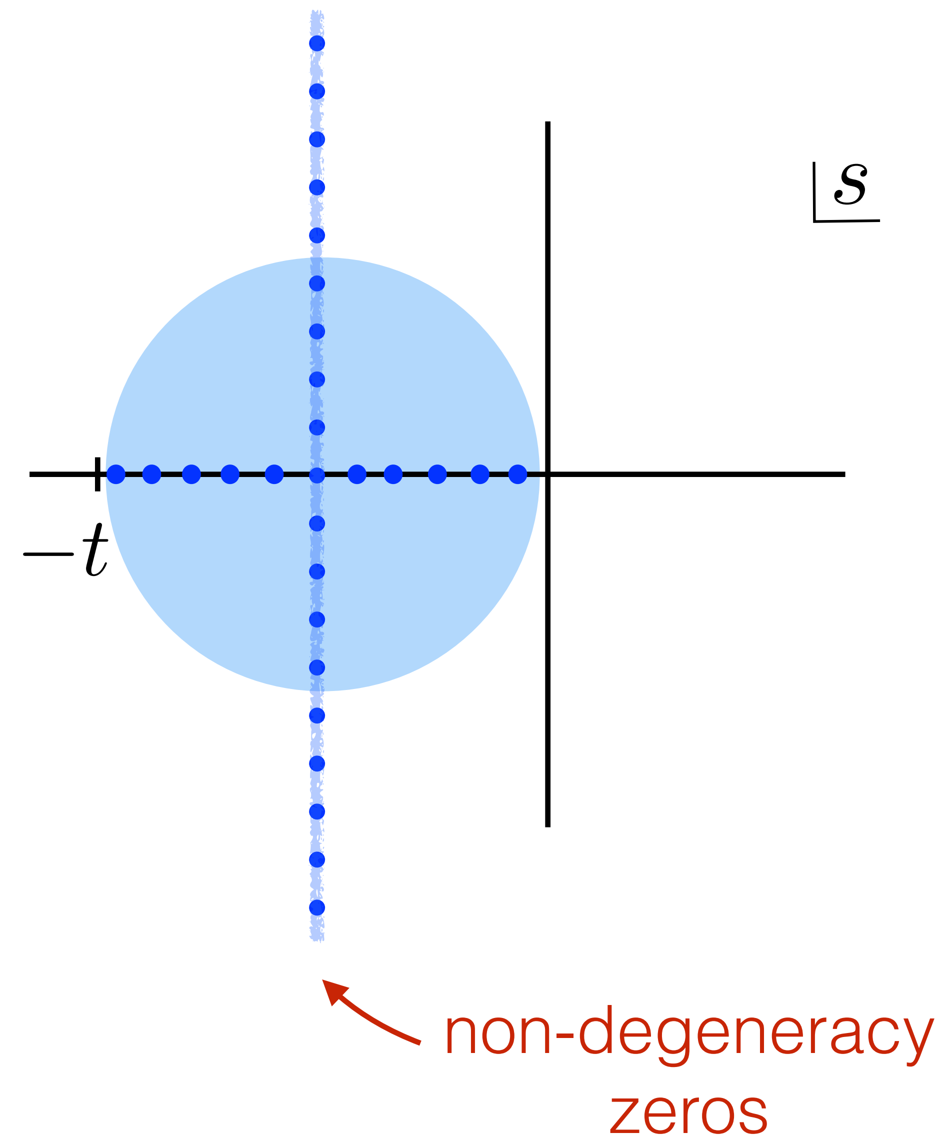


$$\log A(s,t) = \alpha' t \int_0^1 dx \rho(x) \log\left(1+\frac{s}{tx}\right)$$
$$= \alpha' \left[(s+t)\log(s+t) - s\log s - t\log t\right]$$

Bootstrap (Correction)

- Spectrum Non-degeneracy/Support of excess zeros





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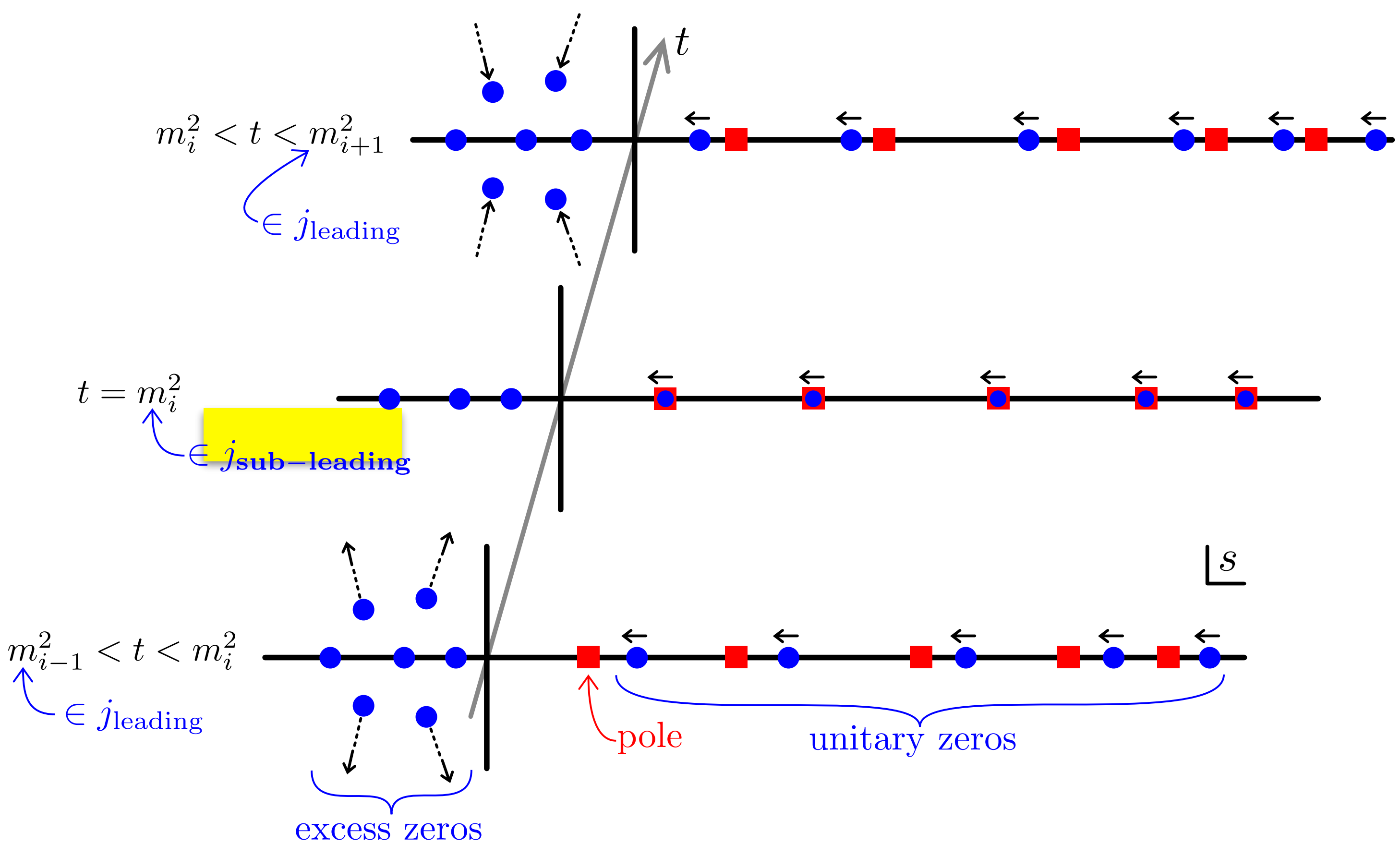

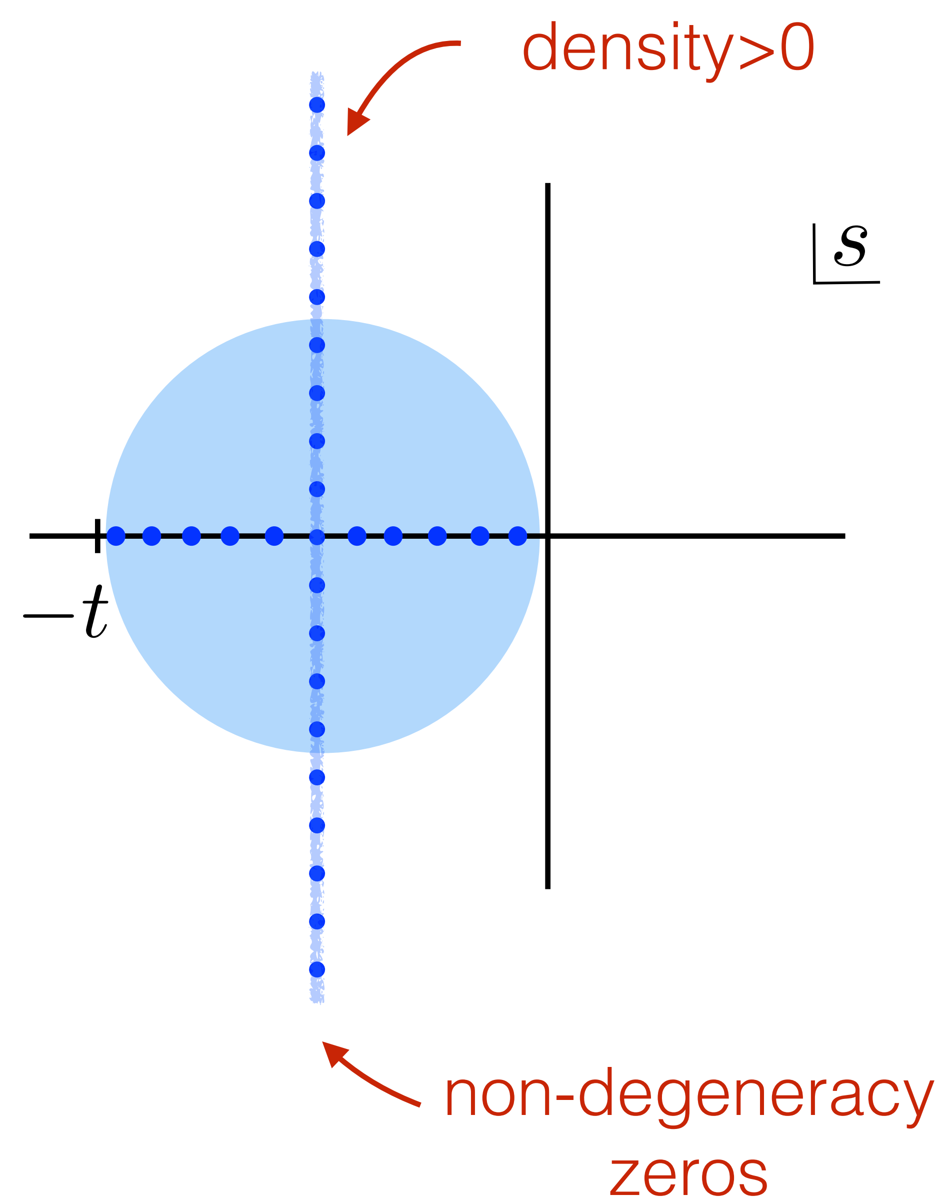

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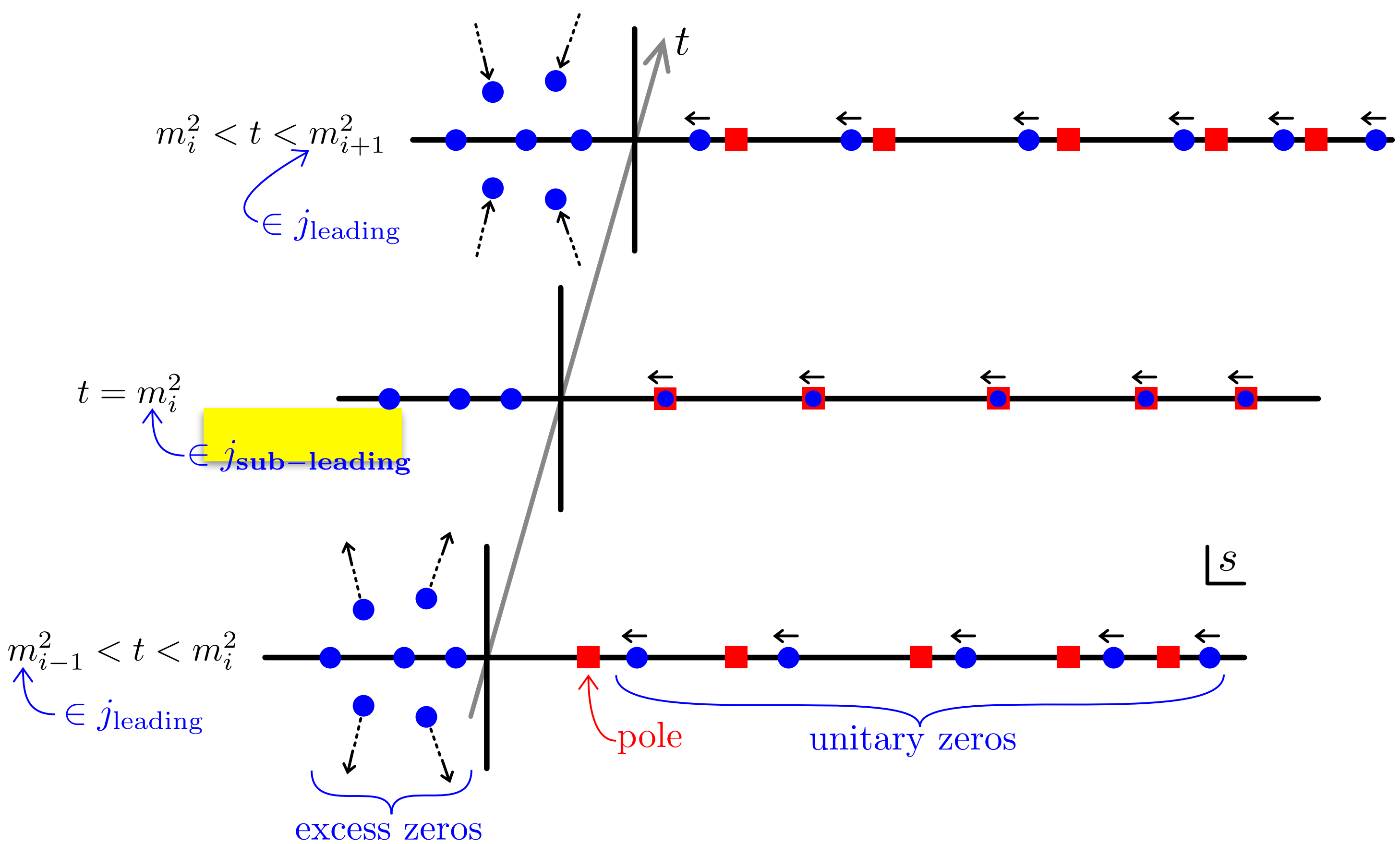

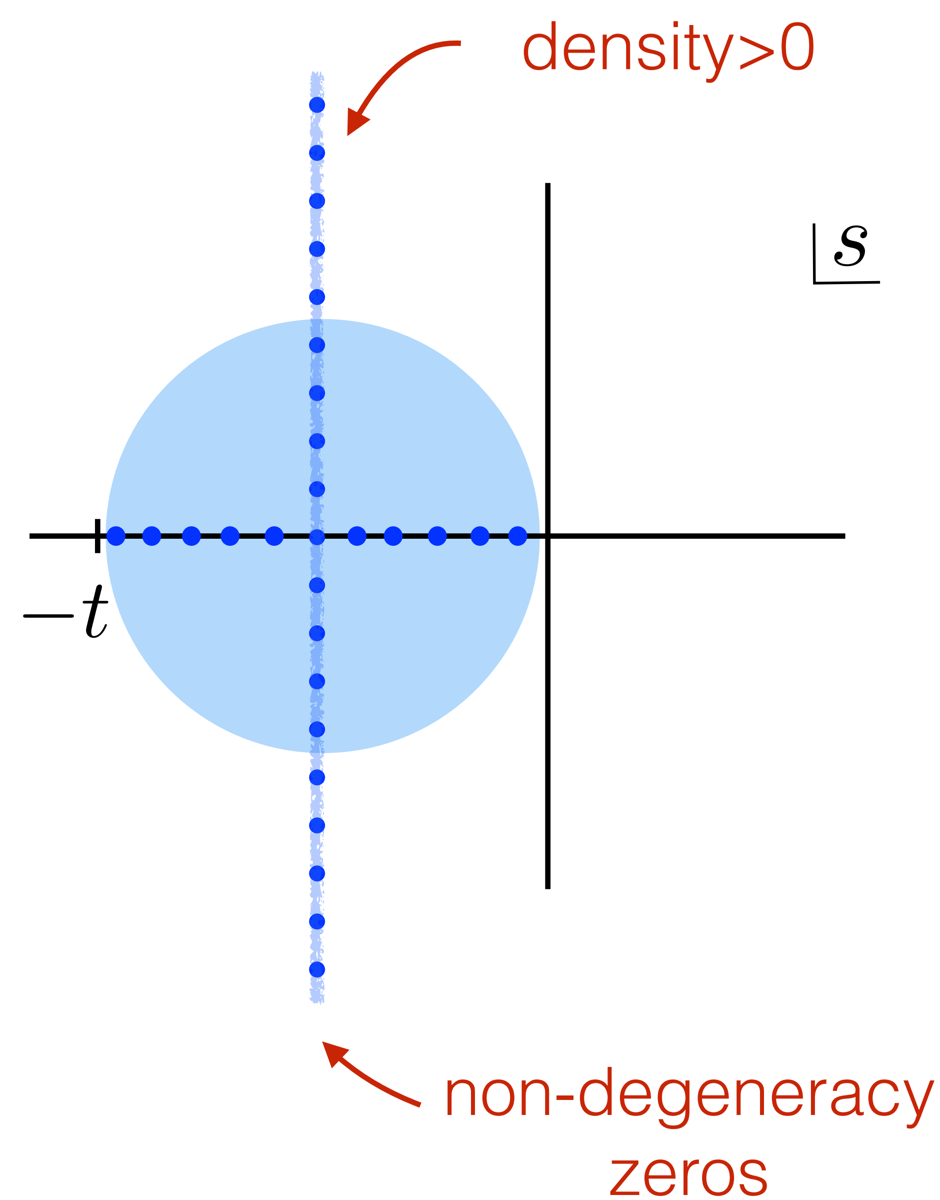

Indeed, the massive ends correction is of this form!

Bootstrap (Correction)

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- Unitarity and the **s-t** crossing are not enough

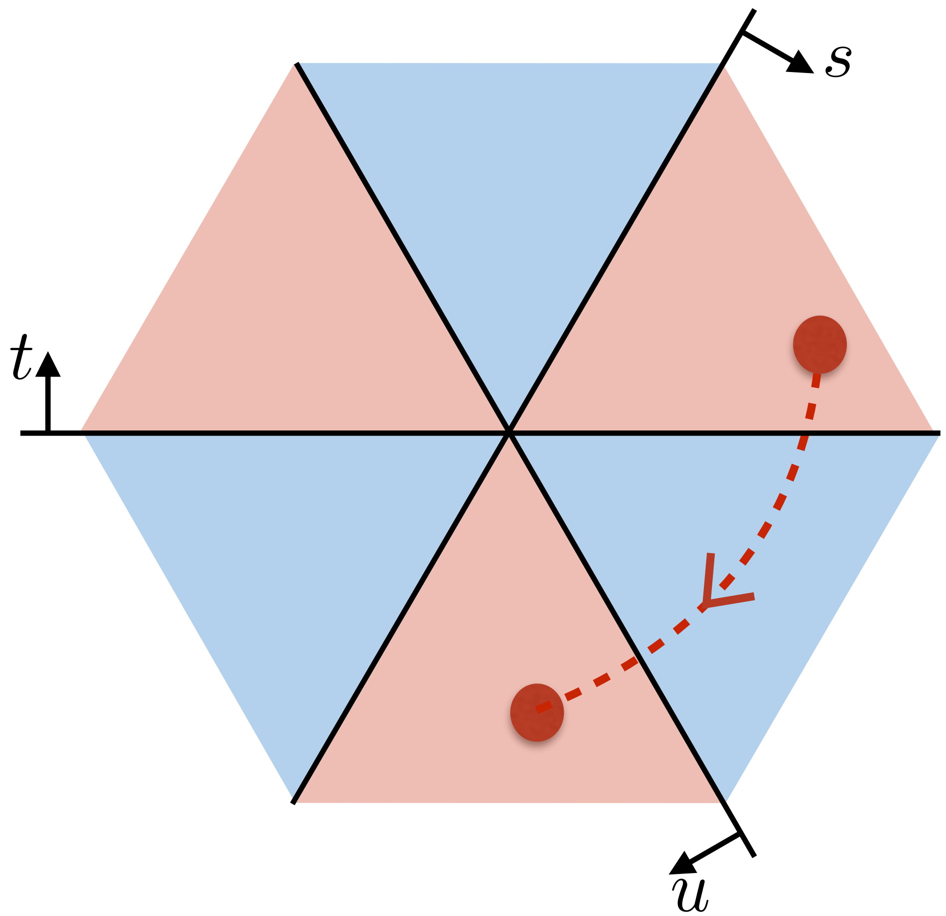
$$\log A(s,t) = \log A(t,s)$$

Bootstrap (Correction)

- Unitarity and the **s-t** crossing are not enough

$$\log A(s,t) = \log A(t,s)$$

- Impose the **s-u** crossing



$$\mathrm{dDisc}_s \log A(s,t) = 0$$

Bootstrap (Integral Equation)

The extra condition leads to an integral equation

$$\delta j(t) = t^k$$

correction to the trajectory

$$\delta\rho_k(y) = \int_0^1 dx\,\left[K(y,x) + K(1-y,1-x)\right]\delta\rho_k(x)$$

correction to the distribution

$$K(y,x) = \frac{\cot\pi k}{\pi}\left(y\,\mathrm{P}\frac{1}{x-y} - k\log\frac{x}{|x-y|}\right)$$

$$+\frac{(1-y)^{k-1}}{\pi\sin\pi k}\left(\frac{y}{x+y-x\,y} + k\log\frac{x\,(1-y)}{x+y-x\,y}\right)$$

Bootstrap (Integral Equation)

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- The correction we found obeys the equation

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- Easy to show that only **k=1/4, k=3/4** are possible

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The solution is unique? (in progress)

Conclusions and Open questions

- WIHS is an exciting, unexplored and stringy territory

[Hagedorn?]

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- Bootstrap in AdS (Mellin space)

  [Theories with accumulation?]

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- Bootstrap in AdS (Mellin space)

  [Theories with accumulation?]

- Quantum theories [talk Penedones]

  [Universal?]

Conclusions and Open questions

- WIHS is an exciting, unexplored and stringy territory

  [Hagedorn?]

- Subject to the analytic bootstrap [talk Alday]

  [Systematic expansion, EFT+holography?]

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  [Theories with accumulation?]

- Quantum theories [talk Penedones]

  [Universal?]

thank you!

Bootstrap Method

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Learn things that ``never happen''
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Clearly, the final result is independent of the starting point.

[Simmons-Duffin]

Bootstrap in Mellin space

Mack polynomials at large spin take the form

$$M(s,t) \simeq \frac{c^2_{\Delta\Delta\tau}\,\mathcal{Q}^{\Delta,\tau,d}_{J,m}(s)}{t-(\tau+2m)} + \ldots$$

Crossing equation takes the form

$$M(s,t) \simeq \sum^{J(t)} c_J J^s = \sum^{J(s)} c_J J^t \simeq M(t,s)$$

The solution is

$$\log M(s,t) = \frac{1}{c}\ s\ t$$

$$\tau(J) = c \log J$$

Worldsheet Computation

Worldsheet Computation

$$S_E = \frac{1}{2\pi\alpha'} \int d^2z \, \partial_z x^\mu \partial_{\bar{z}} x_\mu + \frac{1}{2} \int d\sigma \left(e \, \partial_\sigma x^\mu \partial_\sigma x_\mu + \frac{m^2}{e} \right) + i \sum_j k_j^\mu x_\mu(\sigma_j)$$

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Gross-Mende solution

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- impose the Virasoro constraint

$$\frac{1}{(2\pi\alpha')^2} \frac{m^2}{e^2} = e^2 \partial_\sigma^2 x_0^\mu \partial_\sigma^2 x_0^\mu - m^2 \left[\partial_\sigma \log e\right]^2 + e^2 \left[2\partial_\sigma^2 x_0^\mu \partial_\sigma^2 y^\mu + \partial_\sigma^2 y^\mu \partial_\sigma^2 y^\mu\right]$$

Worldsheet Computation

Consider the small m expansion

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The leading solution is

$$\boxed{e_*(\sigma)^2 = \frac{m}{2\pi\alpha'\sqrt{\partial_\sigma^2 x_0 \cdot \partial_\sigma^2 x_0}}} \qquad \boxed{e_* \sim \sqrt{m}}$$