Anomalous Transport: from Anti de Sitter Space to Weyl semi-metals

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arXiv:1610.04413 [hep-th]

arXiv:1703.10682 [cond-mat.mtrl-sci]

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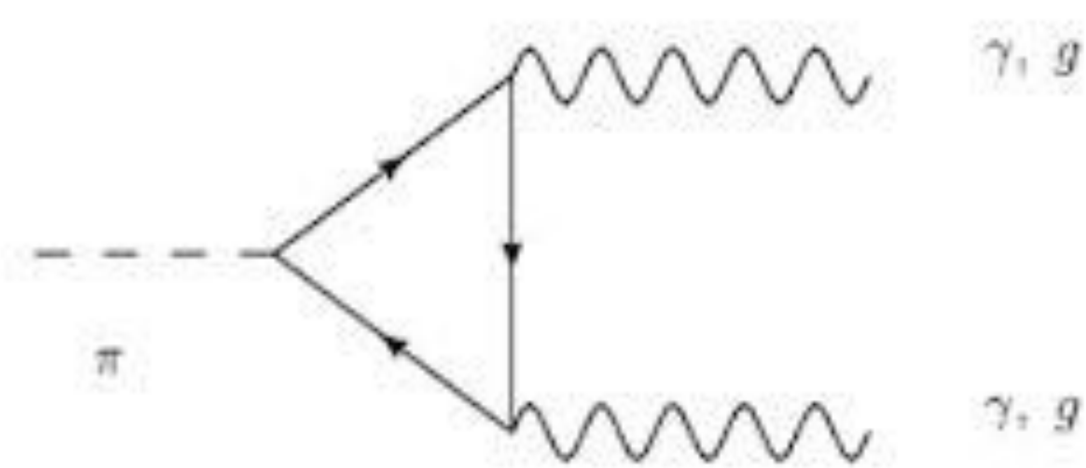
Outline

- Anomalies and Quantum Currents
- WEYL semi-metals
- Anomalous transport in WSMs
- Summary

Anomalies

$$D_\mu J^\mu_a = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{abc}}{96\pi^2}F^b_{\mu\nu}F^c_{\rho\lambda} + \frac{b_a}{768\pi^2}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right)$$



$$d_{abc} = \sum_r q^r_a q^r_b q^r_c - \sum_l q^l_a q^l_b q^l_c$$

$$b_a = \sum_r q^r_a - \sum_l q^l_a$$

- Good old fashioned garden variety triangle anomalies
- Anomaly coefficients detect presence of anomalies
- In 3+1 no pure gravitational anomaly
- consistent Anomalies can always be shifted into currents

Anomalous Transport

CME	CVE
$\vec{J}_a = \frac{d_{abc}}{4\pi^2}\mu_b \vec{B}_c$	$\vec{J}_a = \left(\frac{d_{abc}}{4\pi^2}\mu_b\mu_c + \frac{b_a}{12}T^2\right)\vec{\Omega}$
$\vec{J}_\epsilon = \left(\frac{d_{abc}}{8\pi^2}\mu_b\mu_c + \frac{b_a}{24}T^2\right)\vec{B}_a$	$\vec{J}_\epsilon = \left(\frac{d_{abc}}{6\pi^2}\mu_a\mu_b\mu_c + \frac{b_a}{6}\mu_a T^2\right)\vec{\Omega}$

- Dissipationless currents via magnetic field and rotation
- Proportional to anomaly coefficients
- Anomaly effects at finite density and temperature
- Formulas are correct but need proper interpretation
- Clearest in Holography!!

Anomalous Transport

CME	CVE
$\vec{J}_a = \frac{d_{abc}}{4\pi^2}\mu_b \vec{B}_c$	$\vec{J}_a = \left(\frac{d_{abc}}{4\pi^2}\mu_b\mu_c + \frac{b_a}{12}T^2\right)\vec{\Omega}$
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[Vilenkin] ’79, ‘80
[Alekseev, Chaianov, Fröhlich] ‘98
[Giovannini, Shaposhnikov] ’97
[Newman, Son]
[Kharzeev, Zhitnisky]
[Kharzeev, Fukushima, Warringa]
[Son, Surowka] [Golkar, Son]
[Neiman, Oz] [Hou,Liu,Ren]
[Jensen, Loganayagam, Yarom]
[Megias, K.L.,Pena-Benitez]
[Chowdhury, David]

[Newman]
[Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganayagam, Surowka]
[Ermenger, Haack, Kaminski, Yarom]
[Yee] [Rebhan, Schmitt, Stricker]
[Gynther, K.L.,Pena-Benitez,Rebhan]
[Megias, Melgar, K.L.,Pena-Benitez]

Anomalous Transport

Conservation laws

$$\dot{\epsilon} + \vec{\nabla} \cdot \vec{J}_\epsilon = \vec{E} \cdot \vec{J}$$

$$\dot{\rho} + \vec{\nabla} \cdot \vec{J} = c\vec{E} \cdot \vec{B}$$

Constitutive relations

$$\begin{pmatrix} \vec{J}_\epsilon \\ \vec{J} \end{pmatrix} = L \cdot \begin{pmatrix} \vec{\nabla}\left(\frac{1}{T}\right) \\ \frac{\vec{E}}{T} - \vec{\nabla}\left(\frac{\mu}{T}\right) \end{pmatrix} + \begin{pmatrix} \hat{\sigma}_B \\ \sigma_B \end{pmatrix} \vec{B}$$

Entropy current

$$\vec{J}_s = \frac{1}{T}\vec{J}_\epsilon - \frac{\mu}{T}\vec{J} + \eta_B \vec{B}$$

$$\dot{s} + \vec{\nabla} J_s \geq 0$$

$$\sigma_B = c\mu \qquad \hat{\sigma}_B = \left(c\frac{\mu^2}{2} + c_g T^2\right) \qquad \eta_B = \left(c\frac{\mu^2}{2T} + c_g T\right)$$

Integration constant

[Son, Surowka] [Neiman, Oz]

Anomalous Transport

Interpretation? Why?

Example: proper CME for Dirac fermion

$$d_{5VV} = d_{V5V} = d_{VV5} = 2$$

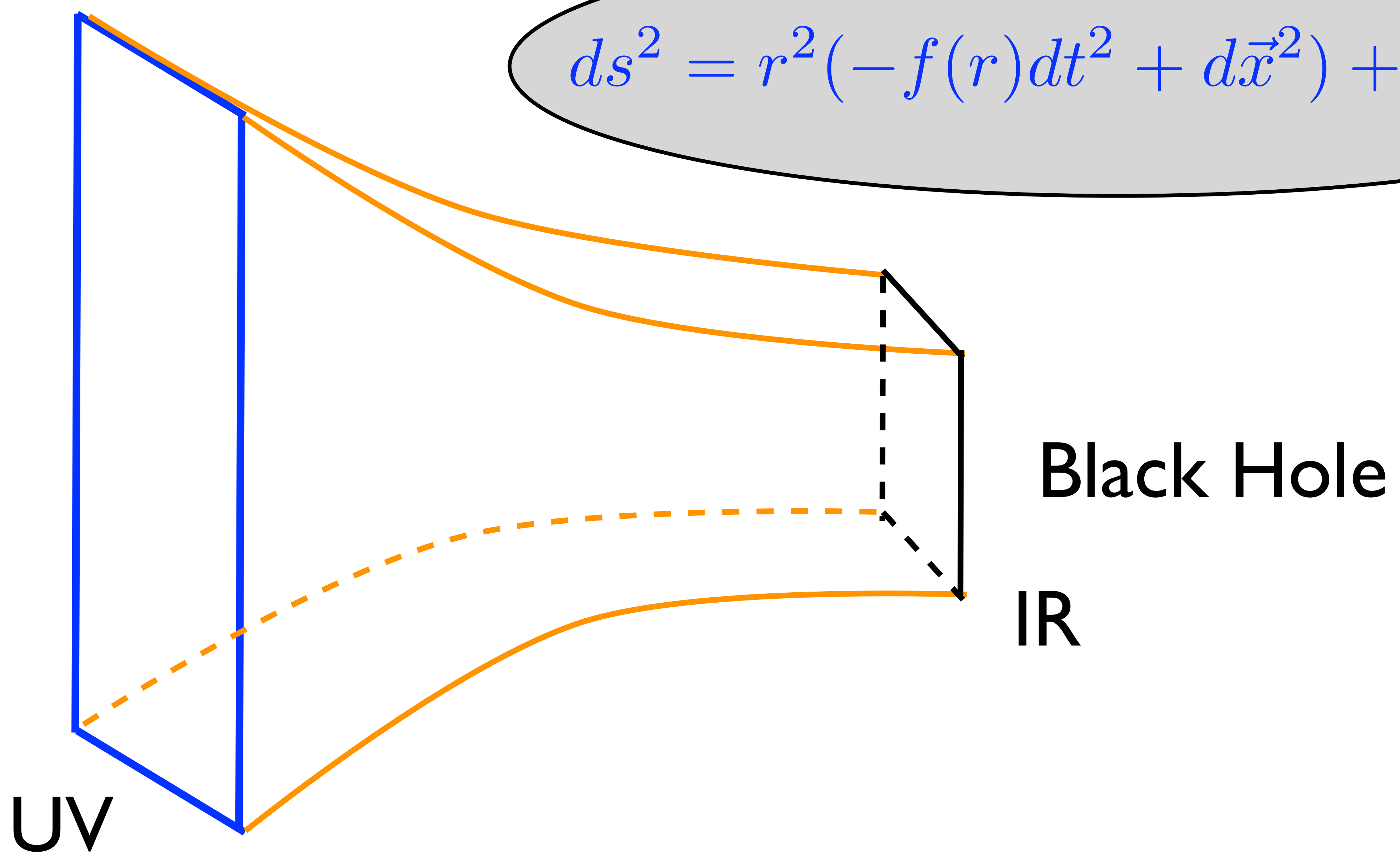
CME	$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$	⟵	Too good to be true
CSE	$\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B}$	⟵	OK, but useful?
AME	$\vec{J} = \frac{\mu}{2\pi^2}\vec{B}_5$	⟵	Edge currents (Fermi arcs)

Theorem due to Bloch $\vec{J} = 0$ in ground state [N.Yamamoto]

Quantum Currents from 5D

Strongly coupled QFT = gravity in 5D

$$ds^2 = r^2(-f(r)dt^2 + d\vec{x}^2) + \frac{dr^2}{f(r)r^2}$$



Quantum Currents from 5D

Anomaly = Chern Simons term

Left-right basis:

$$\int d^5x\, A_L \wedge (F_L \wedge F_L) + A_R \wedge (F_R \wedge F_R)$$

Axial-Vector basis:

$$\int d^5x\, A_5 \wedge (3F \wedge F + F_5 \wedge F_5)$$

Difference is Bardeen counter term

$$\int_\partial d^4x\, c\, A_5 \wedge A \wedge F$$

Quantum Currents from 5D

Holographic dictionary:

$$J^\mu = \sqrt{-g}F^{\mu r} + \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}A^5_\nu F_{\rho\lambda}$$

$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B} - \frac{A^5_0}{2\pi^2}\vec{B}$$

[Gynther, K.L.,Pena-Benitez,Rebhan]

Strict thermodynamic equilibrium: $\mu_5 = A^5_0$

(Proper) CME vanishes in strict equilibrium for physical current

Covariant anomaly vs. Consistent anomaly

[Bardeen, Zumino]

Quantum Currents from 5D

- Anomaly = dynamics of Chern-Simons terms

$$\int_{\mathrm{AdS}} d^5x A \wedge F \wedge F \to \int_{\partial(\mathrm{AdS})} d^4x \theta(F \wedge F)$$

- Grav. Anomaly = dynamics of grav. Chern-Simons terms

$$\int_{AdS} d^5x\, A \wedge R^{(5)} \wedge R^{(5)} \to \int_{\partial(AdS)} d^4x\, \theta \left(R^{(4)} \wedge R^{(4)} + D(K \wedge DK) \right) .$$

- "Extrinsic curvature" K appears
- QFT anomaly knows only 4D curvature
- K-terms does not contribute at r=∞ (UV)

Quantum Currents from 5D

- BUT: at the black hole horizon

$$ds^2 = r^2 f(r)(-dt^2 + 2\vec{A}d\vec{x}dt) + r^2 d\vec{x}^2 + \frac{dr^2}{r^2 f(r)}$$

$$\nabla K \wedge \nabla K \propto r_H^4 f'(r_H)^2 \frac{\partial \vec{A}}{\partial t}(\nabla \times \vec{A})$$

- Suggestive form

$$\nabla K \wedge \nabla K = 64\pi^2 T^2 \vec{E}_g.\vec{B}_g$$

- Holography suggests in deep IR $\quad \partial_\mu J^\mu = \frac{T^2}{12}\vec{E}_g.\vec{B}_g$

- A new anomaly ??? $\quad \partial_\mu J^\mu = \frac{e^2}{4\pi^2}\vec{E}.\vec{B}$

Quantum Currents from 5D

- NO! not anomaly but quantum current (Luttinger)

$$\frac{T^2}{12}\vec{E}_g.\vec{B}_g = \vec{\nabla}\vec{J}_q$$

$$\vec{E}_g \leftarrow -\frac{\vec{\nabla}T}{T} \qquad \vec{B}_g \leftarrow \vec{\nabla}\times\vec{v}$$

$$\vec{J}_q = \frac{b}{12}T^2\vec{\omega}$$

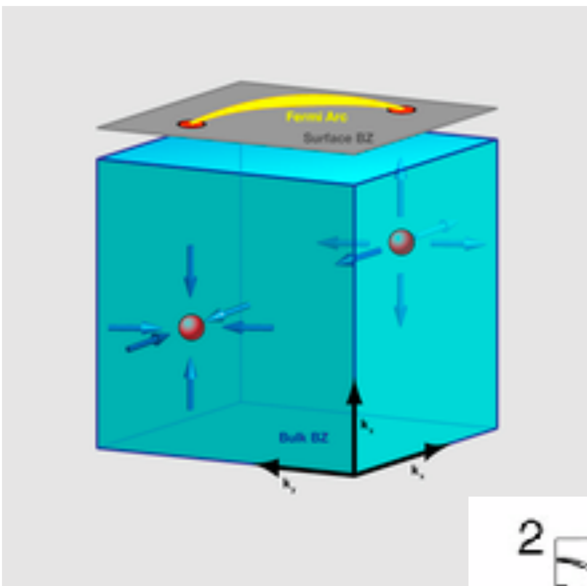
- The Chiral Vortical Effect (CVE) !!

- Current $$J^\mu = \sqrt{-g}F^{\mu r} - \frac{\lambda}{2\pi G}\epsilon^{\mu\nu\rho\lambda}K^\sigma_\nu D_\rho K_{\lambda\sigma}$$

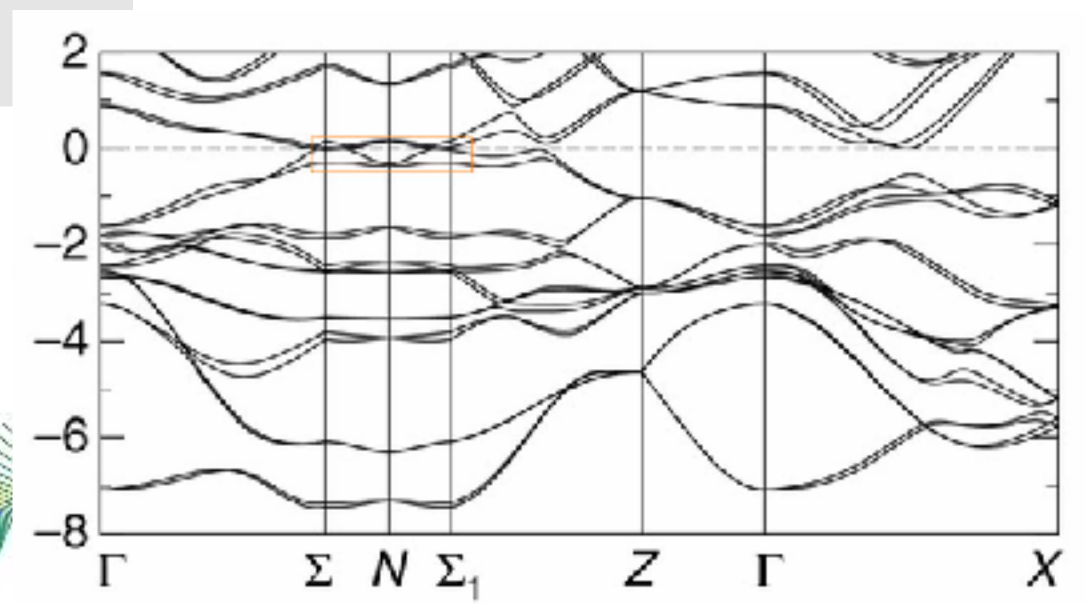
[Copetti, Fernandez-Pendas, K.L.]

[Megias, Melgar, K.L.,Pena-Benitez][Azayanagi, Loganayagam, Ng, Rodriguez],[Chapman, Neiman, Oz],[Grozdanov, Poovuttikul]
hydro+geometry: [Jensen,Loganayagam,Yarom], global anomaly: [Golkar, Sethi]

Applications: WeylSemiMetal

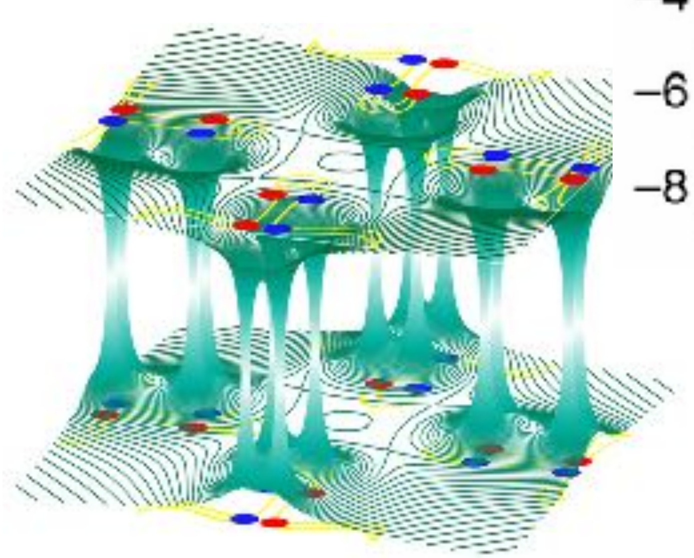


Wikipedia

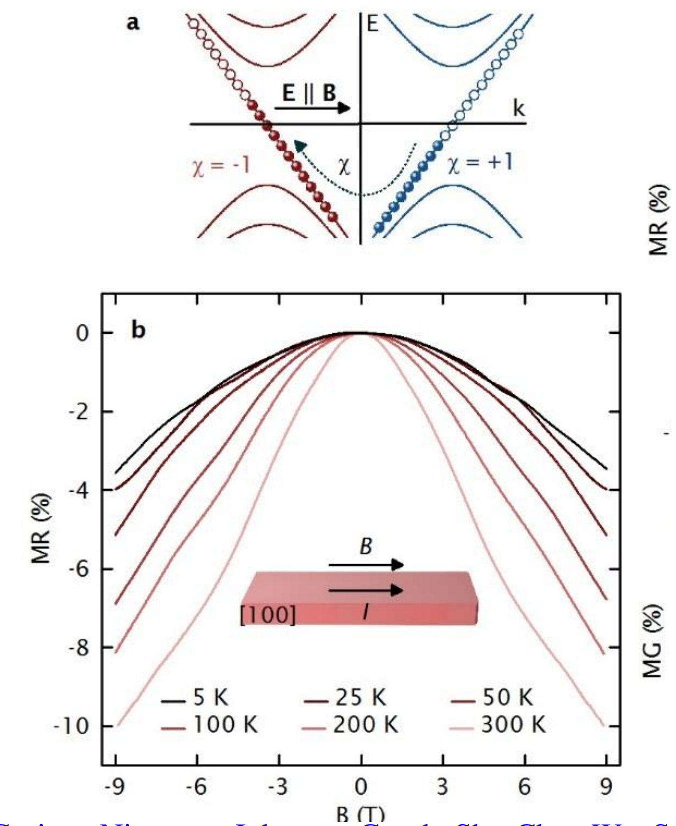


TaAs

[Huang, Xu, Belopolski, Hasan] Nature Comm.

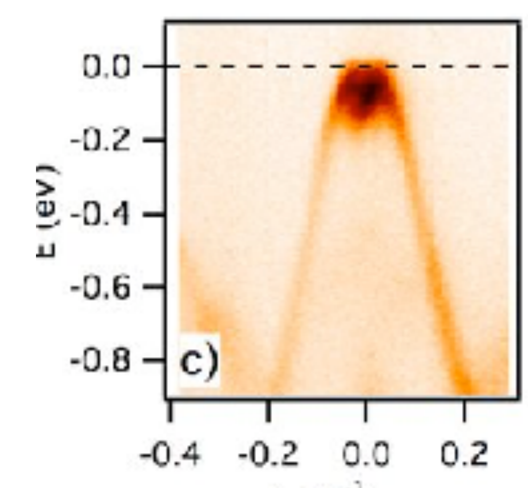


Hiroyuki Inoue, András Gyenis, Zhijun Wang, Jian Li, Seong Woo Oh, Shan Jiang,
Ni Ni, B. Andrei Bernevig,and Ali Yazdani, was published in the March 11, 2016 issue of the journal *Science*



NbP

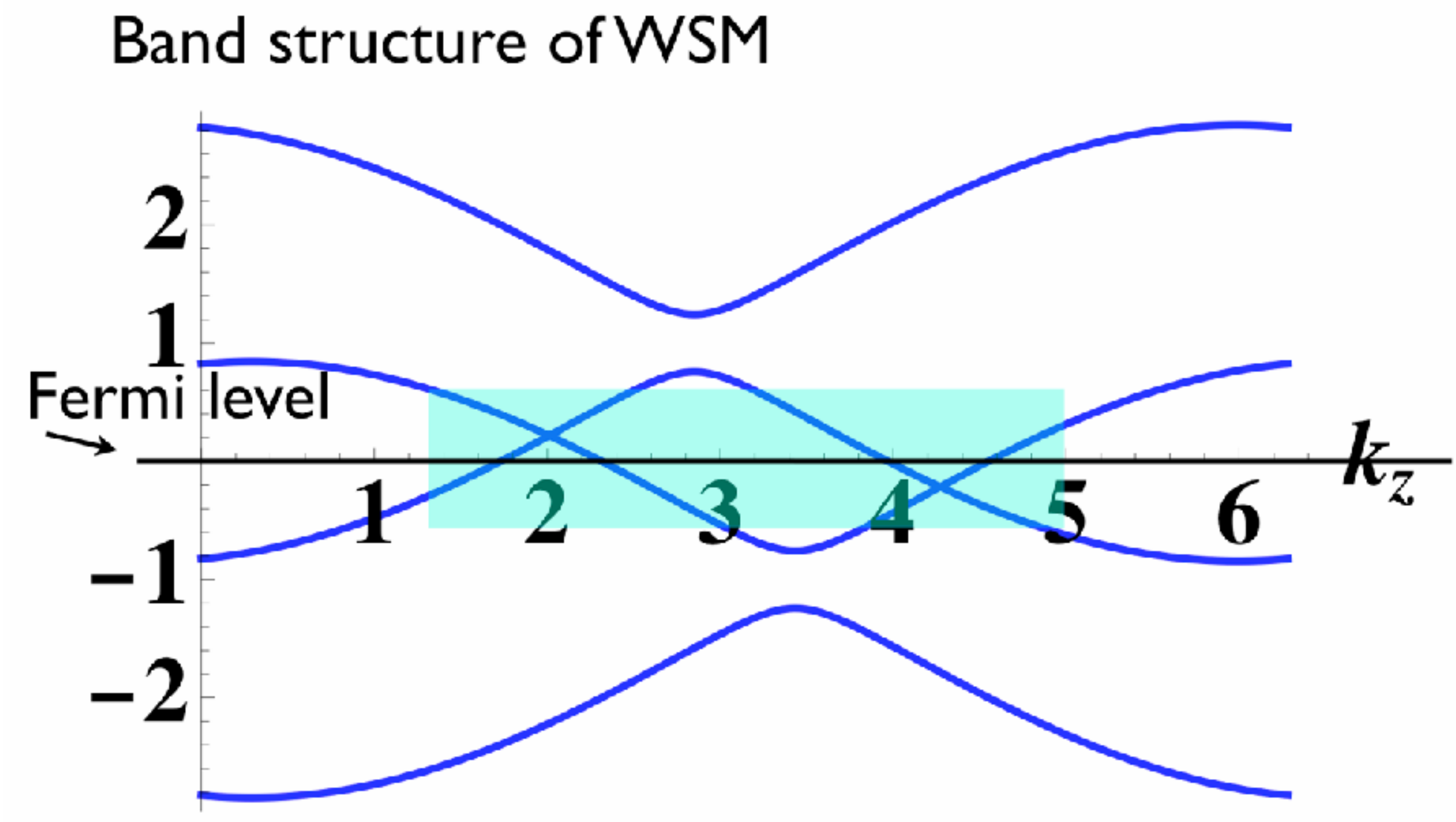
Anna Corinna Niemann, Johannes Gooth, Shu-Chun Wu, Svenja Bäßler, Philip Sergelius, Ruben Hühne, Bernd Rellinghaus, Chandra Shekhar, Vicky Süß, Marcus Schmidt, Claudia Felser, Binghai Yan, Kornelius Nielsch



Zr_5Te

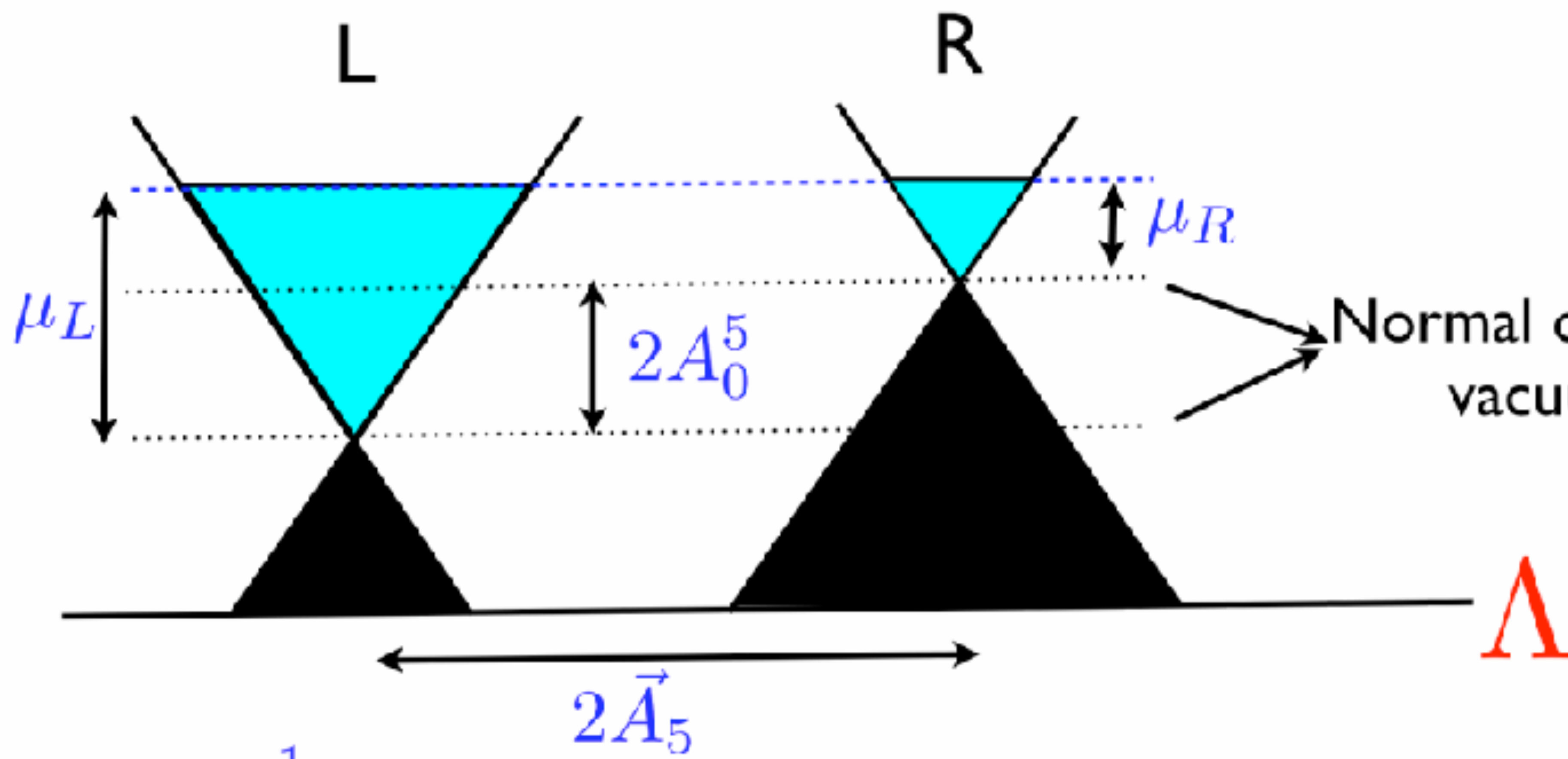
Qiang Li (Brookhaven Natl. Lab.), Dmitri E. Kharzeev (Brookhaven Natl. Lab. & SUNY, Stony Brook), Cheng Zhang, Yuan Huang (Brookhaven Natl. Lab.), I. Pletikosic (Brookhaven Natl. Lab. & Princeton U.), A.V. Fedorov (LBNL, ALS), R.D. Zhong, J.A. Schneeloch, G.D. Gu, T. Valla

Applications: WSM



$$\gamma^{\mu}\left(iD_{\mu}+\gamma_5 A^5_{\mu}\right)\Psi=0$$

CME in WSM



$$\mu_5 = \frac{1}{2}(\mu_L - \mu_R)$$

$$\mu = \frac{1}{2}(\mu_R + \mu_L)$$

CME: $$\vec{J} = \frac{1}{2\pi^2}\left(\mu_5 - A_0^5\right)\vec{B} = 0$$

CME vanishes in equilibrium in WSM "Bloch theorem"

NMR and NTMR in WSM

NMR = Negative Magnetoresistivity

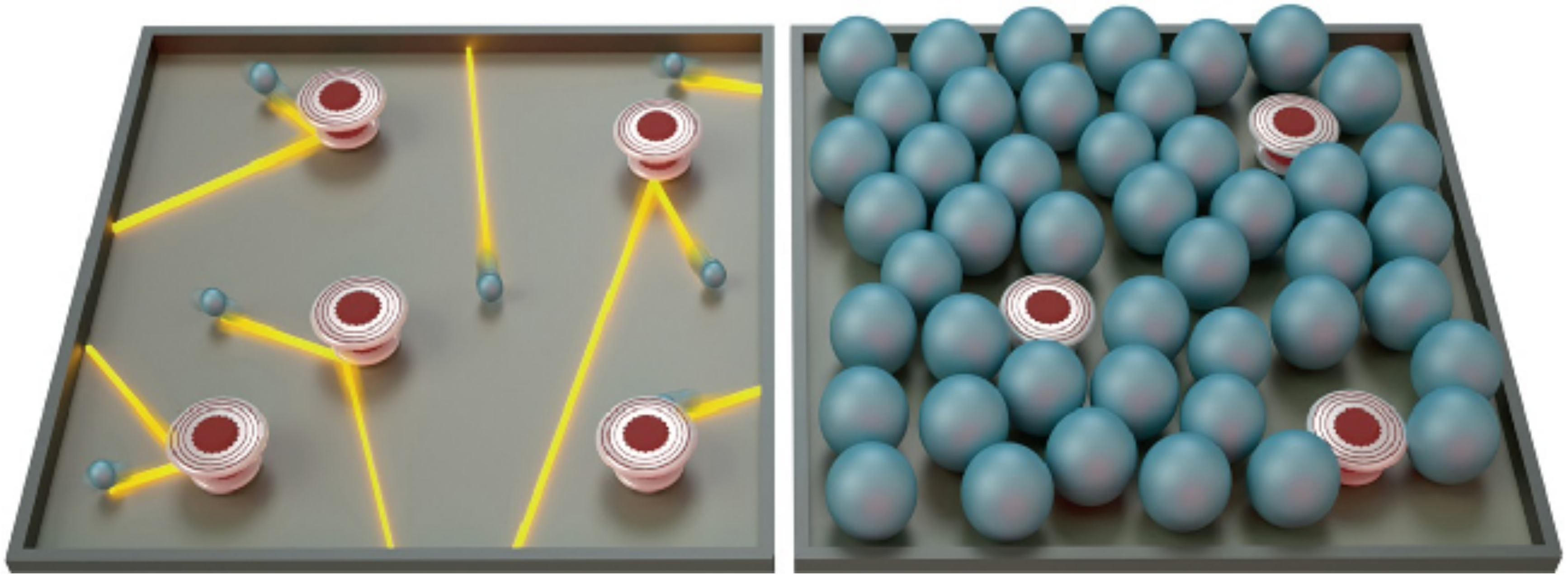
In equilibrium CME vanishes,
Induce non-equilibrium steady state

$$\dot{\rho}_5 = \frac{1}{2\pi^2}\vec{E}.\vec{B} - \frac{1}{\tau_5}\rho_5$$

$$\rho_5 = \chi_5 \mu_5 \qquad \vec{J} = \sigma\vec{E} + \frac{\mu_5}{2\pi^2}\vec{B}$$

$$\vec{J} = \left(\sigma + \frac{\tau_5 B^2}{(2\pi^2)\chi_5}\right)\vec{E}$$

NMR and NTMR in WSM

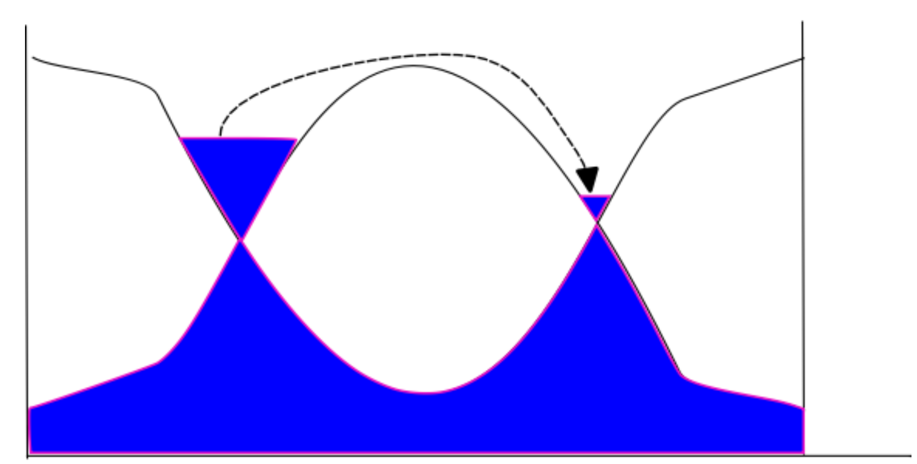


[J. Zaanen, "Electrons go with the flow in exotic materials", *Science* Vol. 351, 6277]

If WSM is not strongly coupled,
hierarchy of scattering times

$$\tau_{\text{inner}} < \tau_{\text{inter}} < \tau_{ee}$$

Kills $\vec{P}$ — Kills ρ_5, ϵ_5 — Is irrelevant



NTMR via CME

Coupled charge and energy transport of chiral currents

$$\vec{J}_\epsilon = \left(\frac{a_\chi}{2}\mu^2 + a_g T^2\right)\vec{B}$$

$$\vec{J} = a_\chi \mu \vec{B}$$

$$\Rightarrow \quad \vec{J} = G_E \vec{E} + G_T \vec{\nabla} T$$

$$G_E = \tau_5 \frac{a_\chi^2}{\det(\Xi)}\left(\frac{\partial \epsilon}{\partial T} - \mu \frac{\partial \rho}{\partial T}\right) B^2$$

$$G_T = \tau_5 \frac{2 a_g a_\chi}{\det(\Xi)} \frac{\partial \rho}{\partial T} B^2$$

arXiv:1703.10682

Large B (ultraquantum limit): $\rho = \frac{|B|}{4\pi^2}\mu$

- G_E linear in B
- G_T vanishes

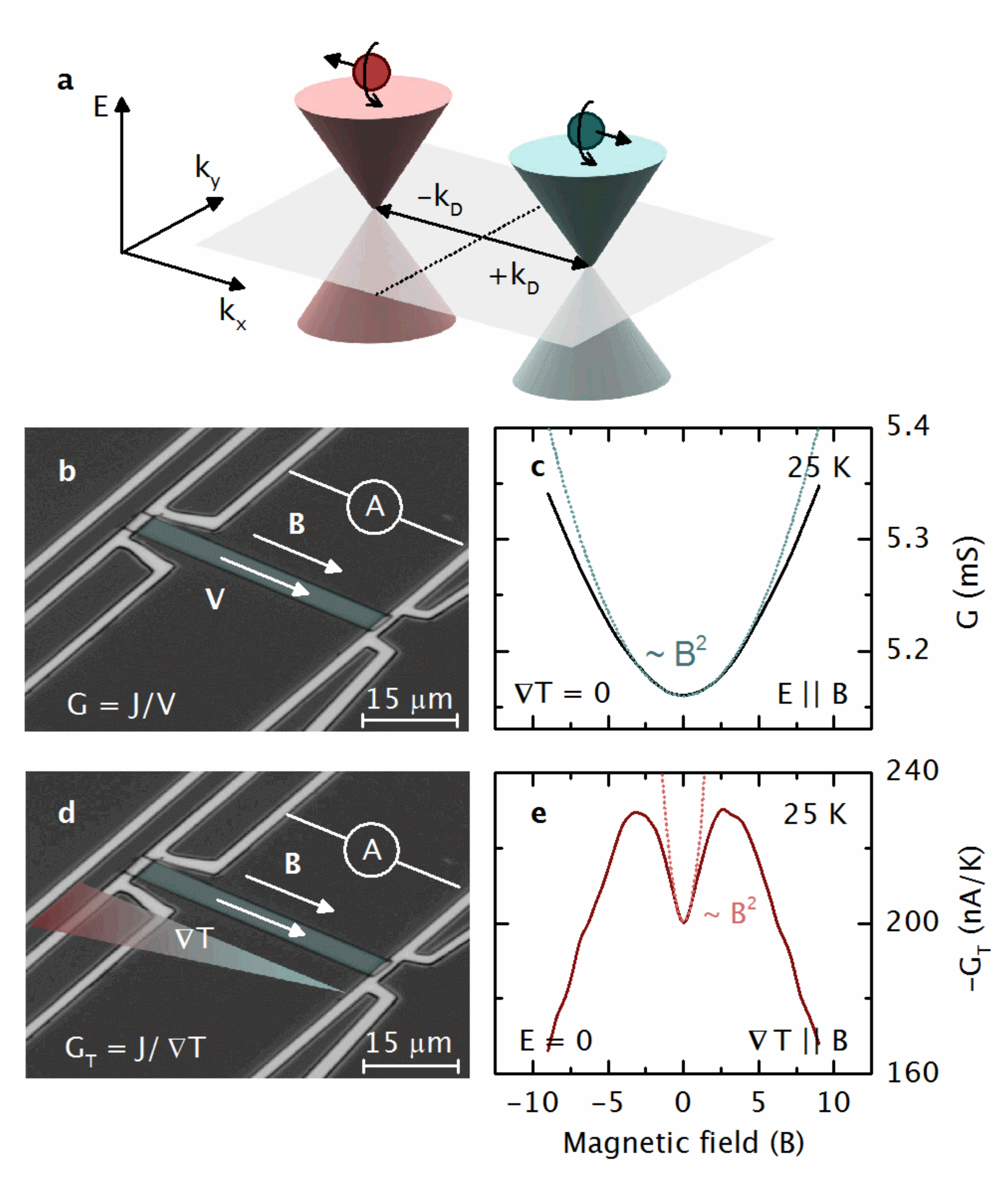
[Spivak, Andreev], [Lundgren, Laurell, Fiete] *kinetic theory*
[Lucas, Davison, Sachdev] *chiral fluids*

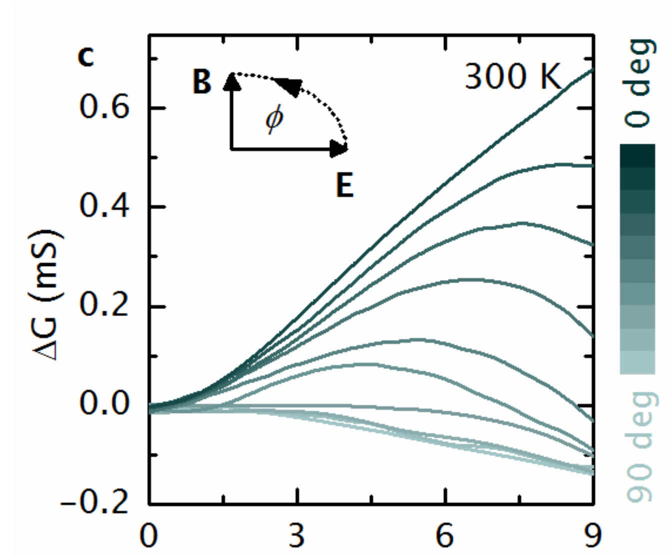
NMR and NTMR in NbP

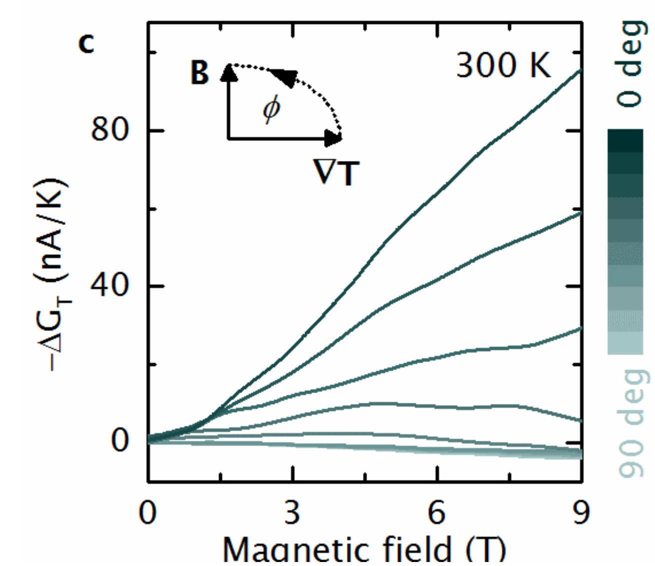
Experimental signatures of the mixed axial-gravitational anomaly in the Weyl semimetal NbP

Johannes Gooth, Anna Corinna Niemann, Tobias Meng, Adolfo G. Grushin, Karl Landsteiner, Bernd Gotsmann, Fabian Menges, Marcus Schmidt, Chandra Shekhar, Vicky Süß, Ruben Huehne, Bernd Rellinghaus, Claudia Felser, Binghai Yan, Kornelius Nielsch

arXiv:1703.10682 [cond-mat.mtrl-sci]





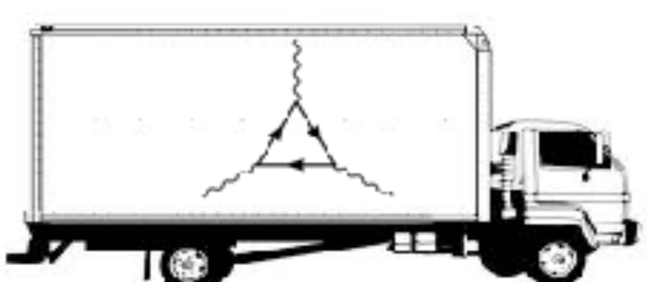


- Angle dependence
- NMR and NTMR show B^2 at small B
- NMR ~ linear for large B field
- NTMR vanishes for large B field

Summary

- Anomalies have moved from Hep-Th to Cond-Mat
- Rich anomaly induced transport phenomenology (CME, CVE, CSE, AME, AHE, NMR, and now NTMR)
- WSMs allow experimental observation of effect of gravitational anomaly
- Holography is ideal tool to investigate anomalous transport
- Holographic model of WSM (Poster of J. Fernandez-Pendas) with Band-bending predicts Hall viscosity

[Y. Liu, K.L.], [Y. Liu, K.L., Y.W. Sun]^2, [Copetti,Fernandez-Pendas,K.L.]
[Grignani, Marini, Pena-Benitez, Speziali],[Ammon, Heidrich, Jimenez-Alba, Moeckel]



Thank You!