S-matrix Bootstrap revisited

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S-matrix Bootstrap I: QFT in AdS [arXiv:1607.06109] S-matrix Bootstrap II: two-dimensional amplitudes [arXiv:1607.06110] S-matrix Bootstrap III: higher dimensional amplitudes [to appear]

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Motivation

Bootstrap Philosophy: bound the space of theories by imposing consistency conditions on physical observables. Bootstraphy

Goal: extend recent success in CFT to massive QFT.

Outline

- S-matrix Bootstrap in $D=2$
- S-matrix Bootstrap in D>2
- S-matrix from Conformal Bootstrap
- Open questions

S-matrix Bootstrap in 2D QFT

2 to 2 Scattering Amplitude

$$
k_i^2 = m^2
$$

\n
$$
s \equiv (k_1 + k_2)^2
$$

\n
$$
t \equiv (k_2 - k_3)^2 = 4m^2 - s
$$

\n
$$
u \equiv (k_3 - k_1)^2 = 0
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2 to 2 Scattering Amplitude quantum field theory. We will further focus on the elastic scattering process involving iden-

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Crossing symmetry: $S(s) = S(4m^2 - s)$ energy-momentum conservation in the independent Mandelstam variable such management Mandelstam variable such a
Analyticity follows from mass spectrum Analyticity follows from mass Analyticity follows from mass spectrum.

at special points. We hope these results will constitute the first steps in a general program In a companion paper [5] we analyzed this problem from the conformal bootstrap point Γ_{Ω} σ m_b *m* 2*m* 0 $\int_{-\infty}^{2\pi i}$ em_b and m_b will not be considered here). Sometimes, symmetry alone forbids such cuts or poles. In those case, the ³Interchanging particles 3 and 4 leads to *^t* = 0, *^u* = 4*m*² *^s* and the same amplitude *^S*(*s*).

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Constraints + question

Crossing: $S(s) = S(4m^2 - s)$

Analyticity: S

$$
S(s^*) = \left[S(s)\right]^*
$$

Constraints + question

Constraints + question oomon amno question

Unitarity: $|S(s)|^2 \leq 1$, $s > 4m^2$.

Constraints + question oomon amno question

$$
S_{opt}(s) = \frac{\sqrt{s(4m^2 - s)} + \sqrt{m_b^2(4m^2 - m_b^2)}}{\sqrt{s(4m^2 - s)} - \sqrt{m_b^2(4m^2 - m_b^2)}} \equiv [m_b](s)
$$
 [Symanzik '61]
CDD factor
Pole at $s = m_b^2 > 2$ [Castillejo, Dalitz, Dyson]

 $|S_{opt}(s)|$ No particle production $|S_{opt}(s)|^2 = 1$, $s > 4m^2$.

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Proof: $h(s) \equiv$ *S*(*s*) $[m_b](s)$

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Maximum cubic coupling

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Numerical approach dimensions.³ Then *S*(*s, t*) is a function with a cut for *s >* 4*m*², another cut for *t >* 4*m*² as well as poles for single-particle processes in the *s*- and *t*- channels. Next we use a very convenient change of variable which maps the full complex plane with those cuts removed

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Numerical approach if they were independent variables; they are not since *s* + *t* + *u* = 4*m*² and *u* = 0 in two dimensions.³ Then *S*(*s, t*) is a function with a cut for *s >* 4*m*², another cut for *t >* 4*m*² as well as poles for single-particle processes in the *s*- and *t*- channels. Next we use a very convenient change of variable which maps the full complex plane with those cuts removed

Crossing symmetry and analyticity are automatic. Unitarity gives quadratic constraints: s : to stability \overline{a}

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|S_{ext}(s, 4m^2 - s)|^2 \le 1, \qquad s > 4m^2
$$

Numerical approach

Ansatz:

$$
S_{ext}(s,t) = \frac{g_b^2}{s - m_b^2} + \frac{g_b^2}{t - m_b^2} + \sum_{a,b=0} c_{(ab)} \rho_s^a \rho_t^b
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[Simmons-Duffin '15]

Use semidefinite programming (SDPB) to maximize g_b^2 subject to these constraints. This reproduces the analytic solution as $N_{\rm max}\to\infty$

S-matrix Bootstrap in d+1 QFT

2 to 2 Scattering Amplitude 2 to 2 Scattering Amplitude ϵ conventions the Secretary elements is

 $\langle \mathbf{p}_3, \mathbf{p}_4 | S | \mathbf{p}_1, \mathbf{p}_2 \rangle = 1 + i(2\pi)^{d+1} \delta^{(d+1)}(p_1 + p_2 - p_3 - p_4) T(s, t, u)$

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T(s,t,u) = \frac{g_b^2}{s - m_b^2} + \frac{g_b^2}{t - m_b^2} + \frac{g_b^2}{u - m_b^2} + \sum_{a,b,c=0} \alpha_{(abc)} \rho_s^a \rho_t^b \rho_u^c
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$$

Partial waves: Gegenbauer polynomial

$$
S_{\ell}(s) = 1 + i \frac{(s - 4m^2)^{\frac{d-2}{2}}}{\sqrt{s}} \int_{-1}^{1} dx (1 - x^2)^{\frac{d-3}{2}} P_{\ell}^{(d)}(x) T(s, t, u)|_{t \to -\frac{1-x}{2}(s - 4m^2)} \prod_{u \to -\frac{1+x}{2}(s - 4m^2)}^{1-x}
$$

 $\frac{1}{2}$ ⇒ $\frac{1}{2}$ $> 4n$ $\frac{2}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ **Unitarity:** $|S_{\ell}(s)|^2 \le 1$, $s > 4m^2$, $\ell = 0, 2, 4, ...$ $, \quad \ell=0,2,4,\ldots$

2 to 2 Scattering Amplitude 2 to 2 Scattering Amplitude ϵ conventions the Secretary elements is that ⇢*^s* = ⇢*^t* = ⇢*^u* = 0 corresponds to the crossing symmetric point *s* = *t* = *u* = $rino$ Amplitude minus the starting s unit disk, we see that in the cuts lie outside the poly

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Unitarity:
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|S_{\ell}(s)|^2 \le 1
$$
, $s > 4m^2$, $\ell = 0, 2, 4, ... \ell_{\text{max}}$

Physical values of the Mandelstam invariants are therefore 4 *s* and 4 *s t* 0. We can \Rightarrow Quadratic constraints on the variables $\{g_{b}^{\tau}, \alpha_{(abc)}\}$ \Rightarrow Quadratic constraints on the variables $\{g_b^2, \alpha_{(abc)}\}$

 $a + b + c \leq N_{\text{max}}$

 \int exception \mathbb{R}^n

Maximal cubic coupling in 3+1 QFT

Maximal quartic coupling

Ansatz with **no poles**. Maximize $\lambda =$ $(e.g. \pi^0 \pi^0 \to \pi^0 \pi^0)$ 1 32π $T(s=t=u=$ 4 3 $m^2)$

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Maximal quartic coupling the analyticity constraints on the surface *s* + *t* + *u* = 4 can be extended to a function on $\frac{3}{4}$ and the poles of $\frac{3}{4}$ and $\frac{3}{4}$. This follows can be written in the following can be written in the follow

Improved ansatz with threshold bound state: vanishing of higher cohomologies of cohomologies of coherent analytic sheaves on Stein manifolds $\vert \nu_S\vert$ – in the manifolds $\vert \nu_S\vert$ Satz with the shold bound state. The surface \mathbb{R}^n

$$
T(s,t,u) = \beta \left(\frac{1}{\rho_s - 1} + \frac{1}{\rho_t - 1} + \frac{1}{\rho_u - 1} \right) + \sum_{a,b,c=0} \alpha_{(abc)} \rho_s^a \rho_t^b \rho_u^c
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No particle production?

S-matrix from the Conformal Bootstrap

 \mathbf{F} is expansion of \mathbf{F} and global coordinates (right). Surfaces (right). Surfaces of \mathbf{F} Correlation functions of boundary operators

$$
\langle \mathcal{O}(x) \dots \rangle = \lim_{z \to 0} z^{-\Delta} \dots \langle \phi(z, x) \dots \rangle
$$

bulk operator

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Isometry group of AdS = SO(d+1,1) = Conformal group example, for a scalar particle of mass *m* at rest in the center of AdS we have the familiar

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bulk operators to conformal boundary. More precisely, we can write a local bulk $\mathcal{A}(\mathcal{O})$ Jse conformal bootstrap to study $\langle C \rangle$ \Rightarrow Use conformal bootstrap to study $\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$

Flat space limit of AdS

AdS radius $R \to \infty$

Flat space limit of AdS particle perceives its surrounding as flat space. So we are interested in taking *miR* ! 1 so that space limit of AdS \mathbf{t} was well-dimensions the following simple relation between the operators of t

AdS radius $R \to \infty$

Mass spectrum: $\Delta_i \sim m_i R$

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AdS radius $R \to \infty$ \mathcal{L}

definition is recalled below. The claim is that the *n*-particle flat scattering space amplitude Scattering amplitudes: Mellin amplitude $(m_1)^a T(k_i) = \lim_{\Delta_i \to \infty}$ $(\Delta_1)^a$ $\frac{1}{\mathcal{N}}M$ $\overline{1}$ $\gamma_{ij} =$ $\Delta_i \Delta_j$ $\Delta_1 + \cdots + \Delta_n$ $\overline{1}$ 1 + $k_i \cdot k_j$ $m_i m_j$ $(\Delta_1)^a$ \blacktriangleright $(\Delta_1)^a$ \blacktriangleright $(\Delta_i\Delta_i$ $(k_i\Delta_i)$ $\alpha = n(d-1)/2d-1$ $\Lambda = \frac{1}{2}\pi^{\frac{d}{2}}\Gamma\left(\sum \Delta_i - d\right)\prod_{i=1}^{n} \sqrt{\mathcal{C}_{\Delta_i}}$ $\alpha = \Gamma(\Delta)$ $a = n(d-1)/2 - d-1$ $\mathcal{N} = \frac{1}{2} \pi^{\frac{3}{2}} \Gamma\left(\frac{\Delta - i}{2}\right) \prod_{i=1}^d \frac{\sqrt{2} \Delta_i}{\Gamma(\Delta_i)},$ $\mathcal{C}_{\Delta} \equiv \frac{1}{2 \pi^{\frac{d}{2}} \Gamma(\Delta_i)}$ definition is recalled below. The claim is that the *n*-particle flat scattering space amplitude (*m*1) *^a ^T*(*ki*) = lim *i*!1 $\lambda_i \rightarrow \infty$ \mathcal{N} $\left(\begin{array}{c} \lambda_1 + \cdots + \Delta_n \\ \lambda_n \end{array}\right)$ $m_i m_j$ *^N ^M ij* = ¹ + *···* + *ⁿ* 1 + *mim^j* factor is given by a combination of \mathcal{L} $(\Lambda)^\alpha$ $(\Lambda)^\alpha$ IVICIIIII aliipiituuc ¹ + *···* + *ⁿ* 1 + *mim^j* $\Delta_i P^{(n)}$ and $\Delta_i P^{(n)}$ and $\Delta_1 + \cdots + \Delta_n$ (and $m_i m_j$) factor is given by a combination of gamma function of gamma functions, which is given by a combination of gamm
The combinations, which is given by a combination of gamma functions, which is given by a combination of gamma $\mathcal{N} =$ 1 2 $\pi^{\frac{d}{2}}\Gamma$ $\int \sum \Delta_i - d$ 2 $\bigwedge^n \frac{n}{\prod_{i=1}^n}$ *i*=1 $\sqrt{\mathcal{C}_{\Delta_i}}$ $\Gamma(\Delta_i)$ $, \qquad \mathcal{C}_{\Delta} \equiv$ $\Gamma(\Delta)$ $\left(\frac{\partial}{\partial z} \right) \prod_{i=1}^{\infty} \frac{\nabla^2 \Delta_i}{\Gamma(\Delta_i)}, \qquad \mathcal{C}_{\Delta} \equiv \frac{\Gamma(\square)}{2\pi^{\frac{d}{2}} \Gamma(\Delta - \frac{d}{2} + 1)}.$ \mathbf{C} , and the CT's under consideration – where all operators acquire a parametrically acquire a parametrically a parameters actually acquire a parameter \mathbf{C} $(\Lambda_1)^a$ \blacklozenge $(\Lambda_2)^a$ \blacklozenge $(\Lambda_i)^a$ $(\Lambda_i)^a$ $(\Lambda_i)^a$ $r(m_1)^{a}T(k_i) = \lim_{\Delta_i \to \infty} \frac{1}{\Delta_i}M\left(\gamma_{ij} = \frac{1}{\Delta_i + \cdots + \Delta_i}\left(1 + \frac{1}{m_i m_i}\right)\right)$ dictionary, each with its own advantages and limitations. The contract of the contract of the contract of the c
The contract of the contract o **g**₁₂ α ¹ $\Delta_i\Delta$ $123 k \cdot k$ $\left(1\right)$ $\left\{\frac{k_i \cdot k_j}{k_i}\right\}$ We can increase the relation, independent leads the relation, independent leads to any M $\sum_{i=1}^{\infty}$ i, $\sum_{i=1}^{n}$ \sqrt{a} $\sum_{i=1}^{n}$ $\sum_{i=1}^{n}$ $\left(\frac{\sum\Delta_i-a}{\sum\Delta_i-a}\right)\prod\frac{\sqrt{c_{\Delta_i}}}{\sqrt{c_{\Delta_i}}}$ is $\left(\frac{\sum\Delta_i-a}{\sum\Delta_i-a}\right)$ $\begin{split} \mathcal{L} & \quad \mathcal{L} = \int \prod_{i=1}^d \Gamma(\Delta_i) \end{split}$ $\begin{split} \begin{split} \mathcal{L} & \Delta = \int \partial_i \pi^{\frac{d}{2}} \Gamma\left(\Delta - \frac{d}{2} + 1\right) \end{split}$ \mathcal{C} in fact relation, independently of any Mellin transform, independently of any Mellin transform, by con*g*ˆ¹²³ 12³ in AdS*d*+1. Notice that the coupling ˆ*g*¹²³ is dimensionful; we measure it in units $(m_i)^a T(k_i) = \lim_{n \to \infty} \frac{(\Delta_1)^a}{M} \left(\gamma_{ii} = \frac{\Delta_i \Delta_j}{\gamma_{ii}^2} \left(1 + \frac{k_i \cdot k_j}{2} \right) \right)$ ¹ . The $\Delta_i \rightarrow \infty$ \mathcal{N} $\left(\begin{array}{c} \lambda_1 \end{array} \right)$ $\Delta_1 + \cdots + \Delta_n$ $\left(\begin{array}{c} \lambda_2 \end{array} \right)$ = *i*(*ⁱ d*). The tree level boundary three-point function is given by the $\frac{1}{4}$ diagram shown in $\frac{1}{4}$ diagram $\left(\sum_{i=1}^{n} \Delta_i\right)$ ι ¹ $2n - 1$ $\left($

$\mathcal{O}_1 \times \mathcal{O}_1 = 1 + \lambda_{112}\mathcal{O}_2 + \ldots$ (operators with $\Delta > 2\Delta_1)\ldots$

¹¹². If this interaction would be very strong then we would expect a

mediates an attractive force between the particles of mass *m*¹ with a strength that is

$\mathcal{O}_1 \times \mathcal{O}_1 = 1 + \lambda_{112}\mathcal{O}_2 + \ldots$ (operators with $\Delta > 2\Delta_1)\ldots$ made use of the well-established numerical bootstrap algorithms \mathcal{S} . The basic idea is always in all \mathcal{S} $\mathcal{O}_1 \times \mathcal{O}_1 = 1 + \lambda_{112} \mathcal{O}_2 + \ldots$ (operators with $\Delta > 2 \Delta_1 \right) \ldots$

$$
\langle \mathcal{O}_1(0)\mathcal{O}_1(z)\mathcal{O}_1(1)\mathcal{O}_1(\infty)\rangle = \frac{1}{z^{2\Delta_1}}\sum_k \lambda_{11k}^2 G_{\Delta_k}(z)
$$

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 $\mathcal{O}_1 \times \mathcal{O}_1 = 1 + \lambda_{112}\mathcal{O}_2 + \ldots$ (operators with $\Delta > 2\Delta_1)\ldots$ $\langle O_1(0)O_1(z)O_1(1)O_1(\infty)\rangle = \frac{1}{2\Delta} \sum_{n=1}^{\infty} \lambda_{11k}^2 G_{\Delta_k}(z)$ $G_{\Delta_k}(z) := z^{\Delta_k} {}_2F_1(\Delta_k, \Delta_k, 2z)$ $\zeta^{2\Delta_1}$ $\frac{1}{k}$ $\frac{1}{2\Delta_k}$ $\frac{1}{2\Delta_k}$ made use of the well-established numerical bootstrap algorithms \mathcal{S} . The basic idea is always in all \mathcal{S} $\mathcal{O}_1 \times \mathcal{O}_1 = 1 + \lambda_{112}\mathcal{O}_2 + \ldots$ (operators with $\Delta > 2\Delta_1$)... $\langle \mathcal{O}_1(0)\mathcal{O}_1(z)\mathcal{O}_1(1)\mathcal{O}_1(\infty)\rangle =$ 1 $z^{2\Delta_1}$ \sum *k* $\lambda_{11k}^2 G_{\Delta_k}(z) \qquad G_{\Delta_k}(z) := z^{\Delta_k} {}_2F_1(x)$ with $G_{\Delta_{\bm{k}}}(z) \vcentcolon= z^{\Delta_{\bm{k}} }{}_2F_1(\Delta_{\bm{k}}, \Delta_{\bm{k}}, 2\Delta_{\bm{k}}, z)$ and with *z* = (*x*12*x*34)*/*(*x*13*x*24) the only independent cross-ratio. Since all four operators are conformal block

mediates an attractive force between the particles of mass *m*¹ with a strength that is

G^k (*z*) (*z* ! 1 *z*)

Extrapolation2

Extrapolation 2

Open questions

Future work

- Bootstrap multiple amplitudes
- Anomalous thresholds (Landau diagrams)
- Particles with spin (internal and external)
- Particles with flavour (global symmetries)
- Use analyticity beyond the physical sheet
- Connect with conformal bootstrap for D>2
- Other interesting questions? Maximize particle production?
- Can we input UV data about the QFT? Hard scattering?

Thank you!

2D Conformal bootstrap - preliminary

RG flows from QFT in AdS

