Unreasonble Simplicity of AdS_5 x S_5 correlators

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N=4 super-Yang-Mills:
toy model for 4D field theory and AdS gravity

remains rich&intricate: laboratory for new tools

planar integrability; localization; perturbation theory,…

this talk: supergravity limit.
study correlators dual to KK modes on AdS_5xS^5.

beyond planar limit

Covariant propagator in $AdS_5 \times S^5$ superspace

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ABSTRACT: We give an explicit superspace propagator for the chiral scalar field strength of 10D IIB supergravity on an $AdS_5 \times S^5$ background. Because this space is conformally flat, the propagator is very simple, almost identical to that of flat space. We also give an explicit expansion over the Kaluza-Klein modes of S^5. The fact that the full propagator is so much simpler suggests that, as in 2D conformal field theory, AdS/CFT calculations would be simpler without a mode expansion.

KEYWORDS: AdS/CFT, coset superspace, covariant propagator.

We'll fulfill this dream for four-point correlators

Recipe:

1. Take correlator of lowest KK-mode (four axidilatons)

$$\langle 0|\phi\phi\bar{\phi}\bar{\phi}|0\rangle = \frac{G(u,v)}{x_{12}^4 x_{34}^4}$$

[D'Hoker, Freedman, Mathur, Matusis& Rastelli '99;

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}$$

2. Uplift all distances to 10D

$$x_{ij}^2 \mapsto (x_i - x_j)^2 + (y_i - y_j)^2$$

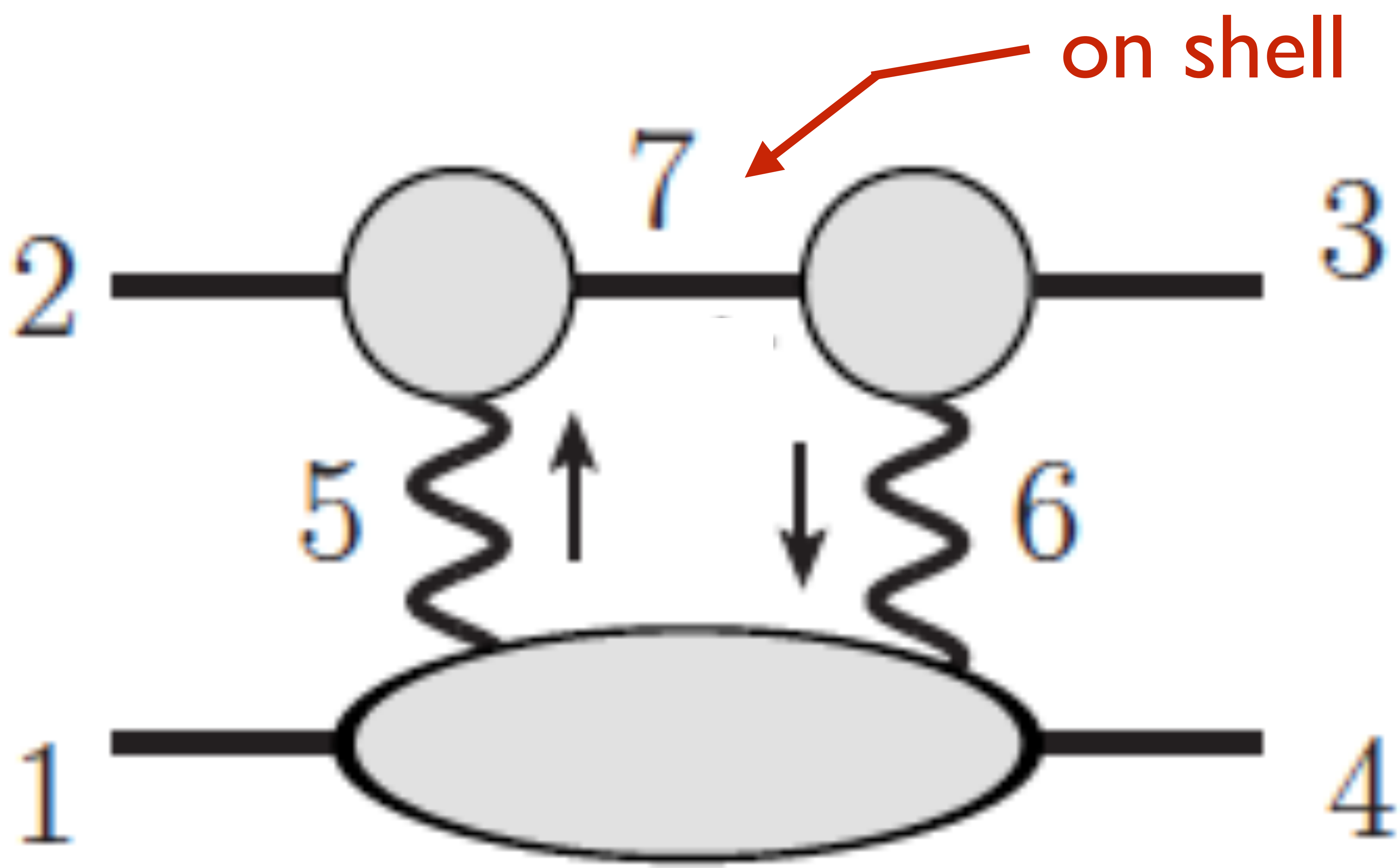
Claim: this gives all KK mode correlators!

Outline

- Calculation technique
- Result: 10D IIB sugra ≃ CFT [for some tree correlators]
- Generalizations

Supergravity Feynman rules are hard.

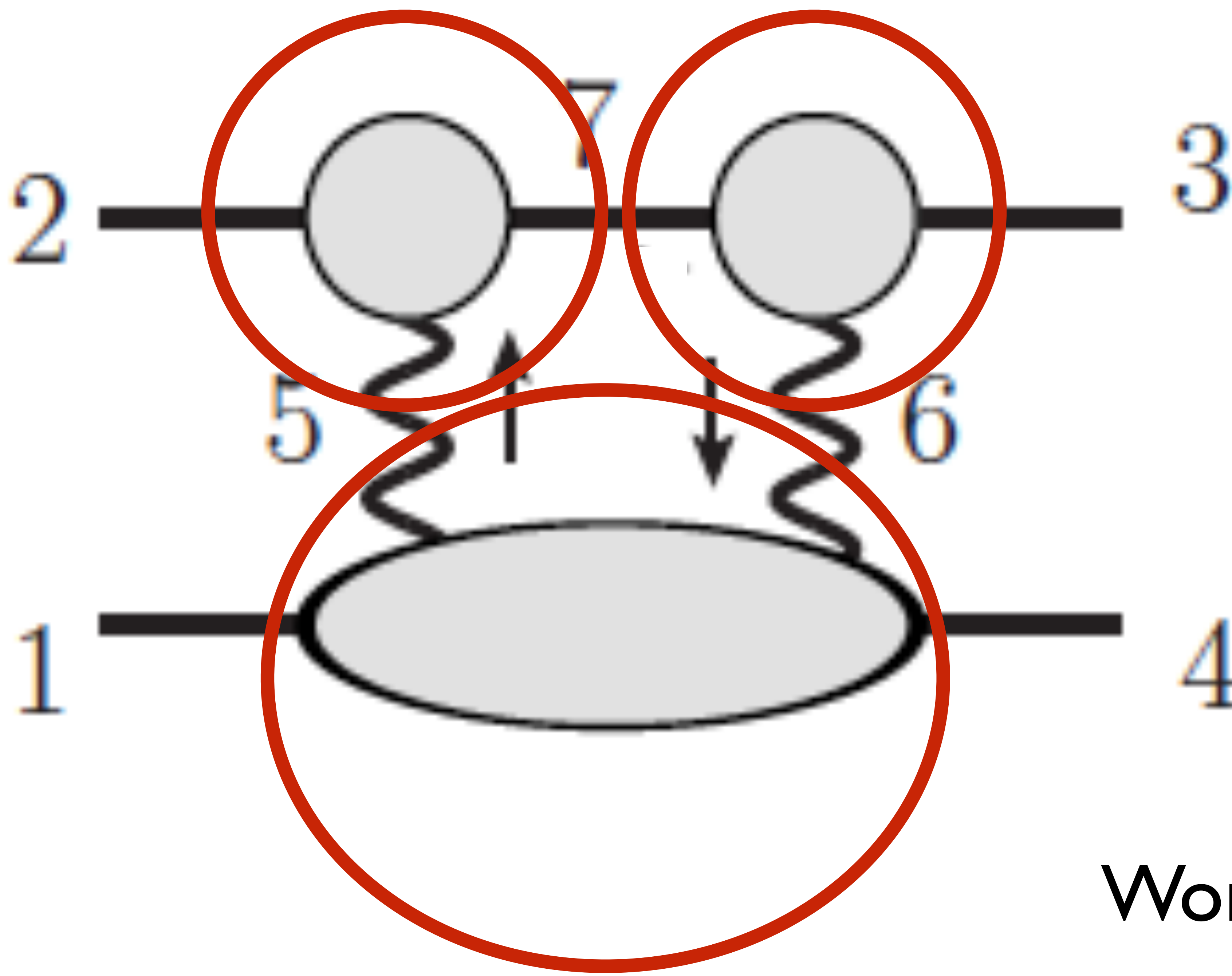
break correlators into physical CFT building blocks



[cf Zvi Bern's talk]

Supergravity Feynman rules are hard.

break correlators into physical CFT building blocks



Work in CFT

CFT: N=4 SYM @ large N_c & strong coupling λ

Input: -symmetries: conformal, $SO(6)_R$ global, susy

-all unprotected operators acquire large dimensions

CFT: N=4 SYM @ large N_c & strong coupling λ

Input: -symmetries: conformal, $SO(6)_R$ global, susy

-all unprotected operators acquire large dimensions

⇒Minimal spectrum: 1/2-BPS single-traces (& their products)

$$\mathcal{O}^p \simeq \mathrm{Tr}[\phi^{i_1} \dots \phi^{i_p}] \qquad \Delta = p \geq 2,$$

p=2: stress tensor

p>2: S_5 graviton spherical harmonics

Goal: compute $<O^pO^qO^rO^s>$ at tree-level ($1/N_c^2$)

Tool: Lorentzian inversion formula

$$c(J,\Delta) = \int_{\Diamond} \left[\text{Inverse block}\right] \times \left[\text{dDisc}\, G\right]$$

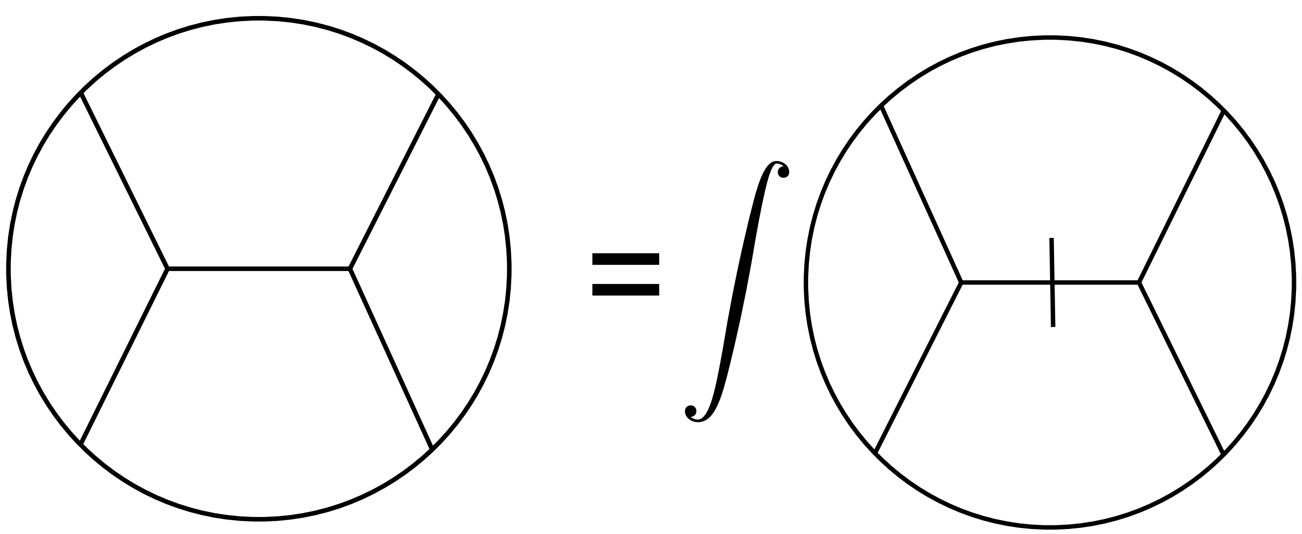
s-channel OPE data

group theory

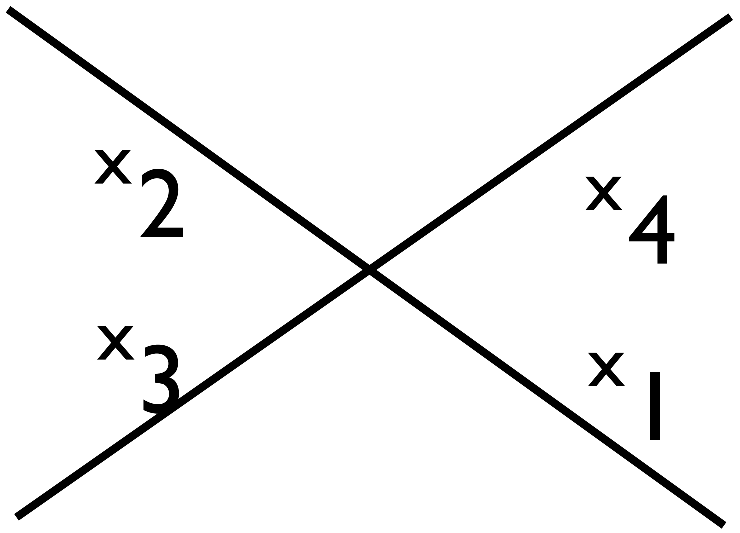
absorptive part = cut diagram

[SCH 17; Simmons-Duffin, Stanford, Witten; Simmons-Duffin, Kravchuk]

gives OPE data from 'absorptive part'



What's 'absorptive part'?



$$\langle 0|T\phi_1\cdots\phi_4|0\rangle \equiv G = G_E + i\mathcal{M}$$

$$\langle 0|\bar{T}\phi_1\cdots\phi_4|0\rangle \equiv G^* = G_E - i\mathcal{M}^*$$

$$\langle 0|\phi_2\phi_3\phi_1\phi_4|0\rangle \equiv G_E$$

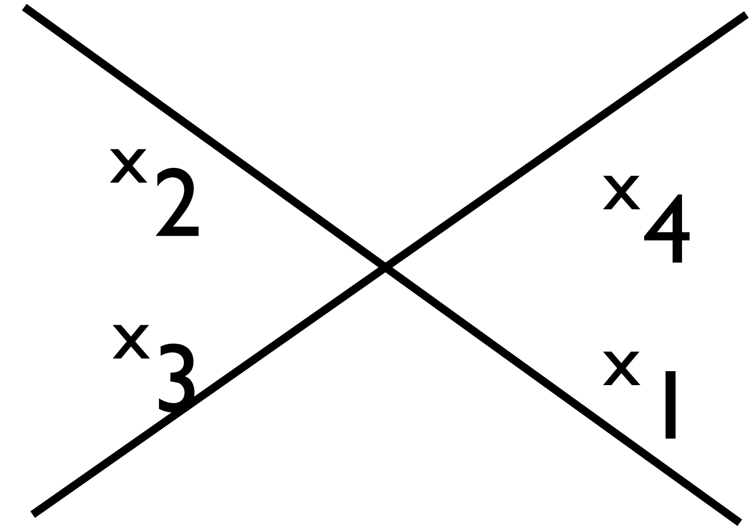
double-commutator:

$$\mathrm{dDisc}\, G \equiv \frac{1}{2}\langle 0|[\phi_2,\phi_3][\phi_1,\phi_4]|0\rangle = \text{“Im } \mathcal{M}\text{”}$$

Positive & bounded

cf: [Maldacena, Shenker&Stanford '15 'bound on chaos']
[Hartman,Kundu&Tajdini '16 'proof of ANEC']

dDisc kills cross-channel double-traces:

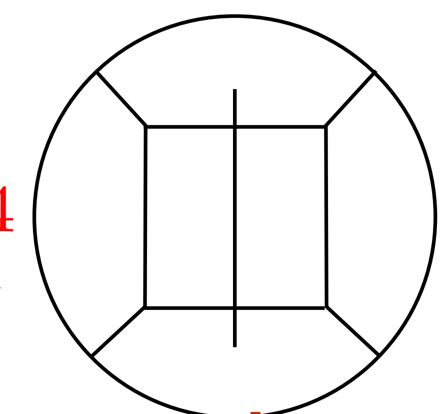


$$\phi_1\phi_4 \sim \sum_{j,\Delta} c_{j,\Delta}((x_1-x_4)^2)^{\frac{\Delta-\Delta_1-\Delta_4}{2}}$$

$$[\phi_1,\phi_4] \sim \sum_{j,\Delta} c_{j,\Delta}|(x_1-x_4)^2|^{\frac{\Delta-\Delta_1-\Delta_4}{2}} \sin\left(\pi\frac{\Delta-\Delta_1-\Delta_4}{2}\right)$$

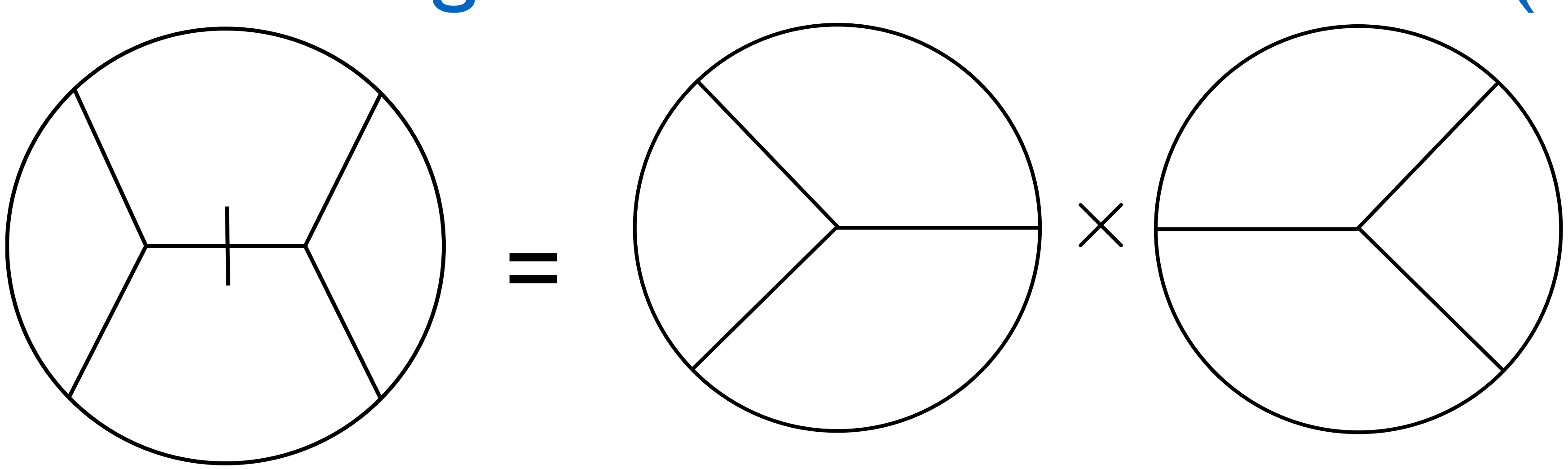
$\propto \gamma/N_c^2$

$$\Rightarrow \mathrm{dDisc}G\Big|_{\mathrm{double-traces}} \sim \langle[\phi_1,\phi_4][\phi_2,\phi_3]\rangle \sim \gamma^2/N_c^4$$



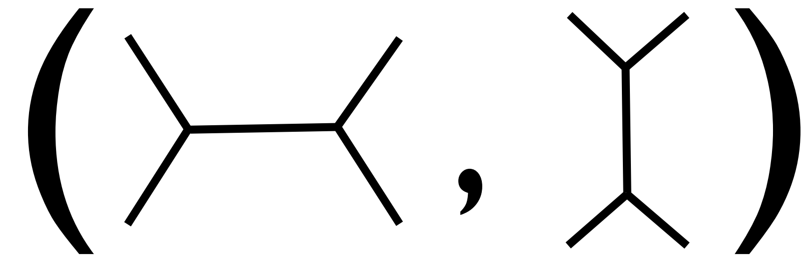
one-loop

cut tree = single-trace conformal block (=known)



How to integrate?

Conceptually: 6j symbol for SO(D,2)



-explicit expressions in D=2, D=4 [Liu,Perlmutter, Rosenhaus & Simmons-Duffin '18]

-Large spin expansion: lightcone limit $(z, \bar{z}) \to (0, 1)$

[Komargodski&Zhiboedov '12; Fitzpatrick,Kaplan,Poland&Simmons Duffin '12, Alday&Bissi&…, Kaviraj,Sen,Sinha&…,Alday,Bissi,Perlmutter&Aharony,…]

-Any D: leading-twist known [Cardona, (Guha, Kanumilli)& Sen '18, Albayrak, Meltzer& Poland '19; Li '19]

-N=4SYM: special case (integer Δ): just Γ-functions

SUSY Ward identities

[Nirschl& Osborn '04]

$$\mathcal{G}_{\{p_i\}}(z,\bar{z},\alpha,\bar{\alpha}) = k\chi(z,\alpha)\chi(\bar{z},\bar{\alpha}) + \frac{(z-\alpha)(z-\bar{\alpha})(\bar{z}-\alpha)(\bar{z}-\bar{\alpha})}{(\alpha-\bar{\alpha})(z-\bar{z})} \times \left(-\frac{\chi(\bar{z},\bar{\alpha})f(z,\alpha)}{\alpha z(\bar{z}-\bar{\alpha})} + \frac{\chi(\bar{z},\alpha)f(z,\bar{\alpha})}{\bar{\alpha} z(\bar{z}-\alpha)} + \frac{\chi(z,\bar{\alpha})f(\bar{z},\alpha)}{\alpha \bar{z}(z-\bar{\alpha})} - \frac{\chi(z,\alpha)f(\bar{z},\bar{\alpha})}{\bar{\alpha}\bar{z}(z-\alpha)}\right) + \frac{(z-\alpha)(z-\bar{\alpha})(\bar{z}-\alpha)(\bar{z}-\bar{\alpha})}{(z\bar{z})^2(\alpha\bar{\alpha})^2} H_{\{p_i\}}(z,\bar{z},\alpha,\bar{\alpha}),$$

Analogous to flat-space S-matrix

$$A_4^{\text{sugra}} = G_N \delta^{16}(Q) \times \frac{1}{stu}$$

apply 1D inversion to f, 4D inversion to H

[non-perturbative convergence for J>-3: ? convergent sum rule for stress tensor.] $\Rightarrow$

[Gillioz, Lu& Luty '18; Zhiboedov's talk]

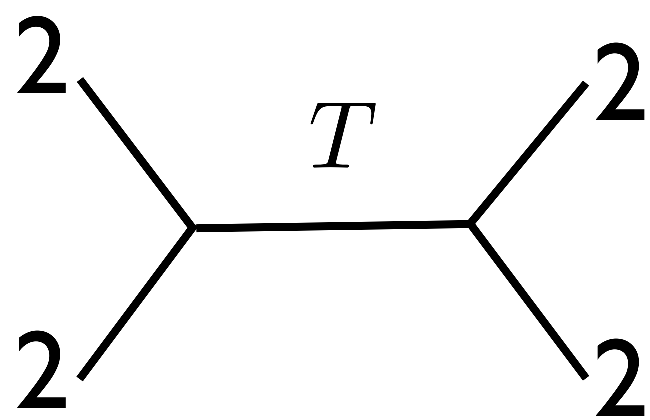
Example: 2222

(four stress tensor multiplets)

The double-discontinuity is simple:

$$\mathrm{dDisc}\left[\frac{\bar{z}-z}{z\bar{z}}\mathcal{H}^{(1)}(z,\bar{z})\right] = \left(\frac{z}{1-z} - \frac{2z^2}{(1-z)^2} - \frac{2z^3\log z}{(1-z)^3}\right)\mathrm{dDisc}\left[\frac{\bar{z}}{1-\bar{z}}\right].$$

comes from pole of stress tensor exchange



$$\sim \frac{1}{1-\bar{z}} \propto \frac{1}{x_{14}^2 x_{23}^2}$$

From here, two routes:

1. Feed dDisc into inversion integral, get OPE data

$$c^{(1)}(h,\bar{h}) = \frac{\pi^2}{\sin(\pi h)^2}\frac{(h-2)(h-1)h(h+1)}{2}.$$

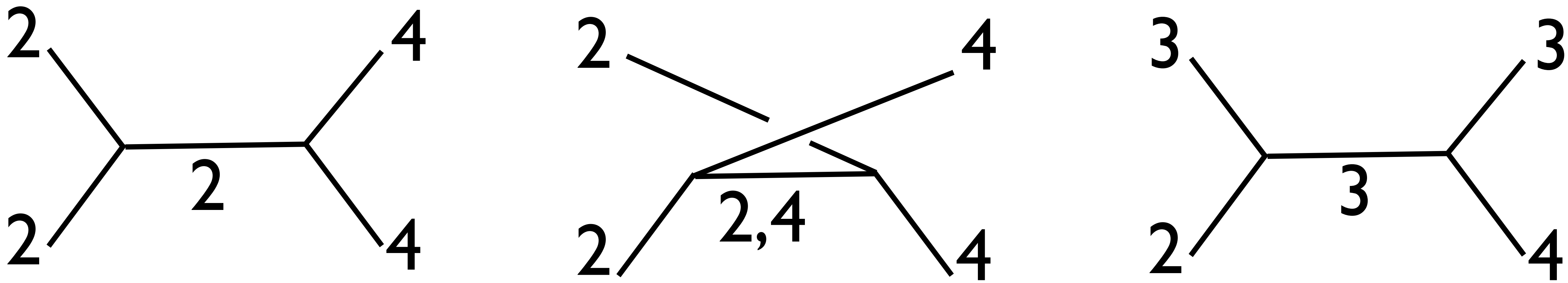
2. Find unique function:
- single-valued
- vanishes in Regge limit (z,zb->0)
- has predicted poles at zb=1 and infinity

Answer: $H^{(1)} = \frac{u^2}{v} - u^4\bar{D}_{2,4,2,2}(z,\bar{z})$

$$\bar{D}_{2422} = uv\partial_u\partial_v(1+u\partial_u+v\partial_v)\frac{2\text{Li}_2(z)-2\text{Li}_2(\bar{z})+\dots}{z-\bar{z}}$$

Matches supergravity calculations!

Straightforward to apply to other correlators



get mixing matrices. e.g.: at twist 6 and R-symmetry[0,2,0]:

$$\gamma^{(1)}_{6,0,2} = \begin{pmatrix} \gamma^{(1)}_{24,24} & \gamma^{(1)}_{24,33} \\ \gamma^{(1)}_{33,24} & \gamma^{(1)}_{33,33} \end{pmatrix}$$ with: $$\gamma^{(1)}_{pq,rs} \equiv \frac{\langle a^{(0)}\gamma^{(1)}\rangle_{pqrs}}{\sqrt{\langle a^{(0)}\rangle_{pqqp}\langle a^{(0)}\rangle_{rssr}}}$$

Amazingly, the *eigenvalues* are nice:

$$\left(\gamma^{(1)}\right)^{+}_{6,0,2} = \left\{\frac{-\Delta^{(8)}_{0,2}(4,\bar{h})}{(\bar{h}-4)_6}, \frac{-\Delta^{(8)}_{0,2}(4,\bar{h})}{(\bar{h}-2)_6}\right\} \qquad (\bar{h} = j+5)$$

we looked at many cases, ie twist=8 [0,2,0]:

$$\begin{pmatrix} (2442) & (2433) & (2435) & (2444) \\ (3342) & (3333) & (3335) & (3344) \\ (3542) & (3533) & (3553) & (3544) \\ (4442) & (4433) & (4453) & (4444) \end{pmatrix}$$

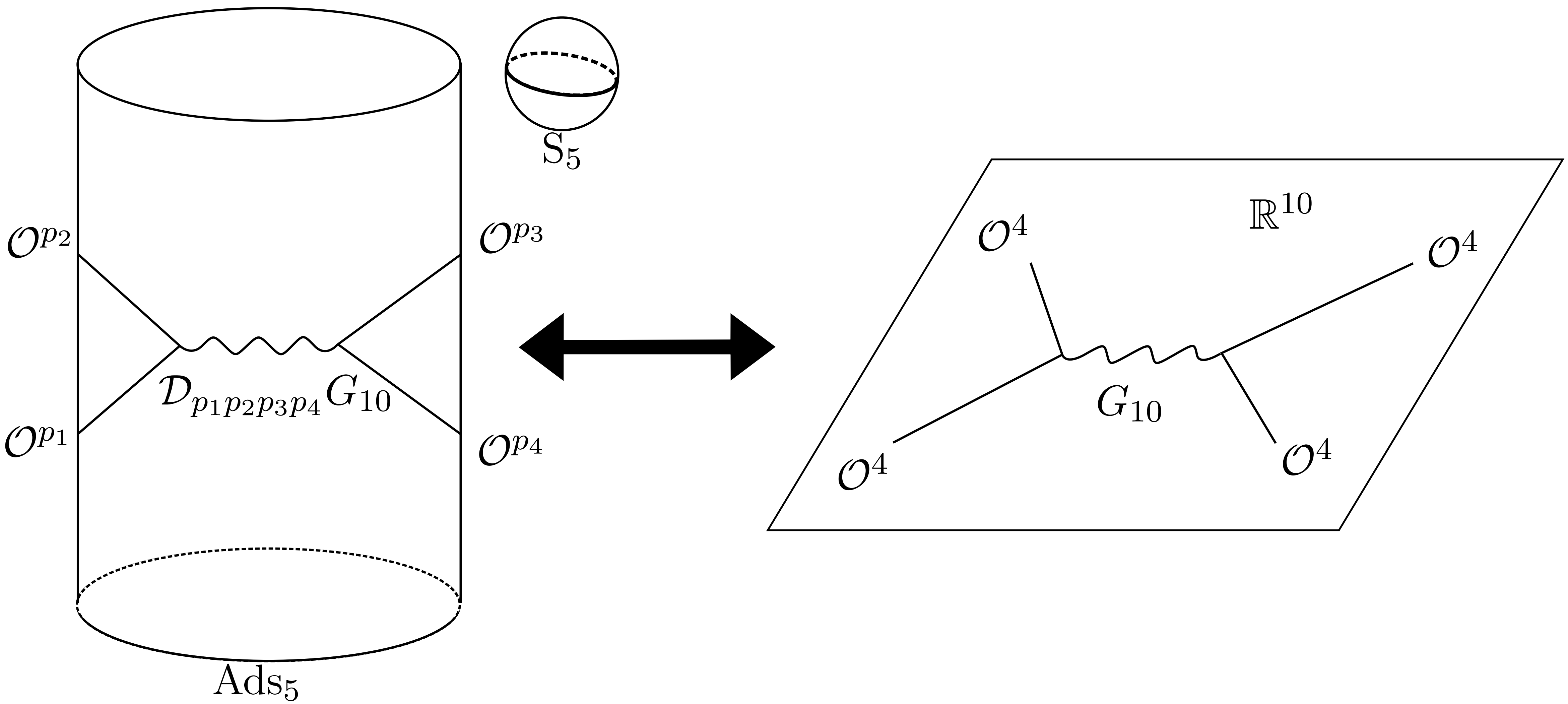
All eigenvalues take the form:

$$\gamma^{(1)} = -\frac{1}{c} \frac{\Delta^{(8)}}{(j+1+\text{integer})_6} \quad !$$

confirm a recent conjecture ✓

[Aprile, Drummond, Heslop&Paul '18]

Part II:
10D IIB supergravity ≃ CFT



Crazy that complicated matrix has rational eigenvalues!

Symmetry explanation:

1. $AdS_5 \times S^5$ is conformally flat

$$ds^2_{AdS_5\times S^5} = z^{-2}(dz^2 + dx_\mu dx^\mu + z^2 d\Omega_5^2)$$

2. IIB supergravity amplitude ~ conformal

$$A_4^{\text{sugra},10D} = 8\pi G_N \delta^{16}(Q) \times \frac{1}{stu}$$

dimensionless 'coupling'

conformal amplitude

$$G_N^{(10D)} \sim 1/m_{\text{pl}}^8$$

Evidence of SO(10,2) symmetry [in 4-pt correlator]

1. Eigenvalue matches flat space phase shift

$$\delta_\ell^{(10)}(s) = -\frac{(L\sqrt{s}/2)^8}{c}\frac{1}{(j+1)_6}$$

used:
$8\pi G_N = \frac{\pi^5 L^8}{c}$

2. Offsets match SO(4,2)⊂SO(10,2) reduction

$$\gamma = -\frac{\Delta^{(8)}}{c}\frac{1}{(j+1+m)_6}$$

3. 10D dilaton correlator generates all KK correlators

$$\langle\phi(w_1)\phi(w_2)\bar{\phi}(w_3)\bar{\phi}(w_4)\rangle_{10} \equiv \frac{G_{10}(u_{10},v_{10})}{((x_{12}^2-y_{12}^2)(x_{34}^2-y_{34}^2))^4}$$

$$= \sum_{p,q,r,s} y_1^p y_2^q y_3^r y_4^s \langle O^p O^q O^r O^s\rangle$$

3* To get a given <$O^pO^qO^rO^s$>: take terms with correct exponent of R-symmetry variables y's

$$\tilde{H}_{p_1p_2p_3p_4}(u,v,\sigma,\tau) = \oint \prod_{i=1}^{4}\left[\frac{da_i\, a_i^{1-p_i}}{2\pi i}\right] \frac{(u/\sigma)^{\frac{p_1+p_2}{2}-2}}{(1-\frac{\sigma}{u}a_1a_2)^4(1-a_3a_4)^4}$$
$$\times\, G_{10}\left(u\frac{(1-\frac{\sigma}{u}a_1a_2)(1-a_3a_4)}{(1-a_1a_3)(1-a_2a_4)}, v\frac{(1-\frac{\tau}{v}a_2a_3)(1-a_1a_4)}{(1-a_1a_3)(1-a_2a_4)}\right)$$

predicts differential relations:

$$\tilde{H}_{pqrs} = \mathcal{D}_{pqrs}\tilde{H}_{2222}$$

$$\mathcal{D}_{2222} = 1,$$
$$\mathcal{D}_{2332} = -\frac{\sqrt{u}}{\sqrt{\sigma}}\tau\partial_v,$$
$$\mathcal{D}_{2233} = 4 - u\partial_u,$$
$$\mathcal{D}_{3333} = 16 - 8u\partial_u + \frac{u+\sigma}{\sigma}(u\partial_u)^2 + 2\frac{u}{\sigma}u\partial_u v\partial_v + \frac{u(v+\tau)}{\sigma v}(v\partial_v)^2\,.$$

satisfied by sugra results!

[D'Hoker, Freedman, Mathur, Matusis& Rastelli '99; Arutyunov, Dolan, Osborn&Sokatchev '02-; Berdichevsky& Naaijkens '03; Dolan, Nirschl&Osborn '06; Uruchurtu '08-; Arutyunov, Klabbers&Savin '18]

The correct guess is tricky in the details.

([L]=4: like 10D free field)

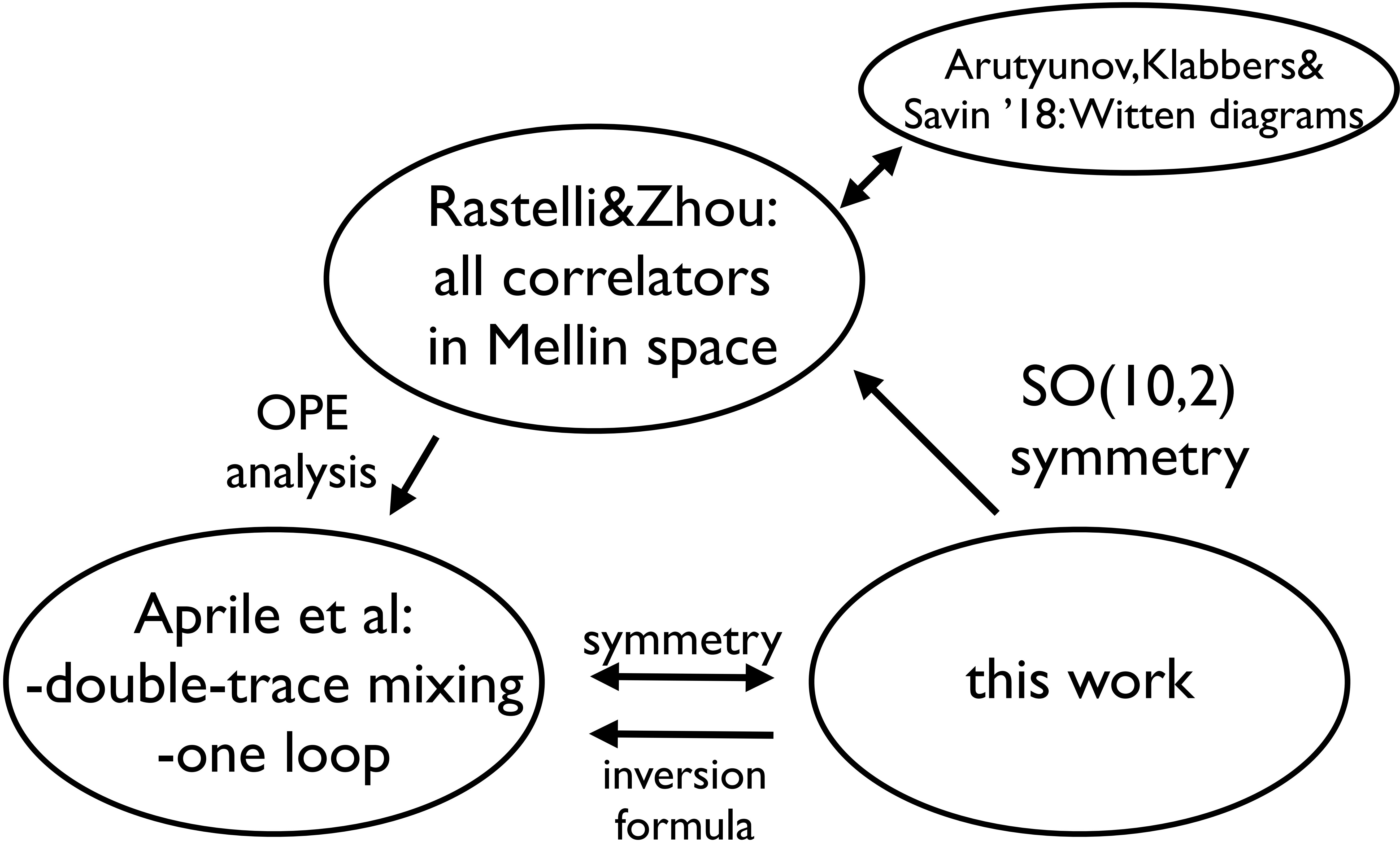
(4D) correlator of Lagrangians has 8-derivative Casimir

$$\langle \mathcal{L}\mathcal{L}\bar{\mathcal{L}}\bar{\mathcal{L}}\rangle^{(4)} = \Delta^{(8)} H$$

(10D) dilaton amplitude is s^4 times conformal

$$A^{(10)} = 8\pi G_N s^4 \times \frac{1}{stu}$$

successful guess needs $\Delta^{(8)} \leftrightarrow (s/4)^4,$ so $H^{\mathrm{tree}} \leftrightarrow 1/(stu)$



complete agreement between all methods!

Further implications of 10D conformal symmetry:

[SCH&Trinh '18]

4. simple Mellin transform of correlators:

$$M_{pqrs}(s,t) \sim \sum \frac{1/(i!j!k!\ell!m!n!)}{(s-i)(t-j)(u-k)}$$

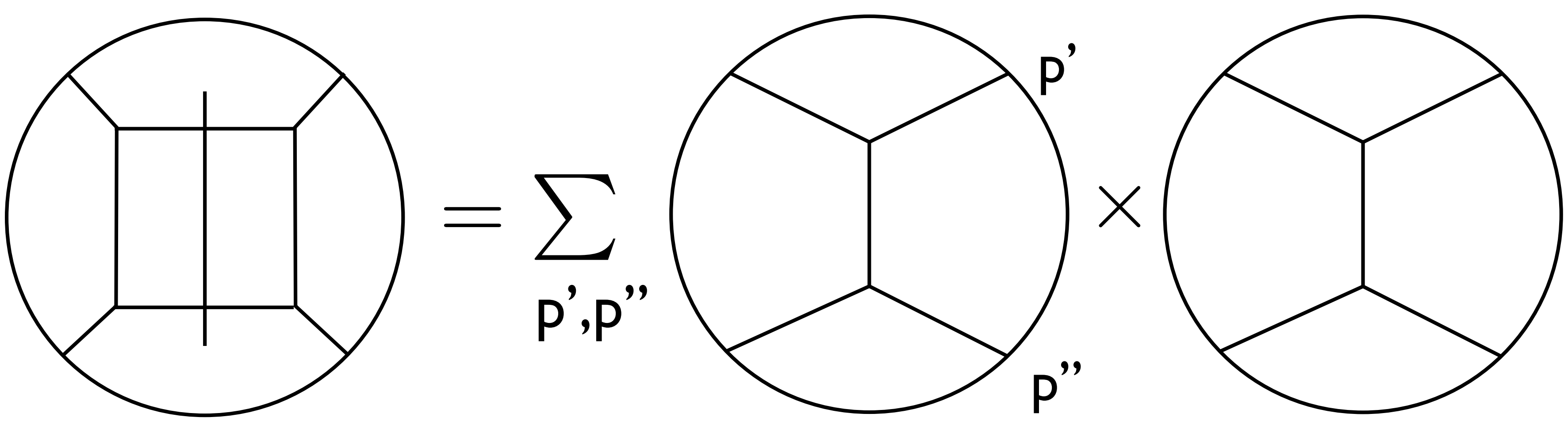
[Rastelli&Zhou '16]

5. Proof reduced to group theory: check poles of dDisc

$$\tilde{H}^{(1)}_{pqrs}\Big|_{v-\text{poles}} \stackrel{?}{=} \mathcal{D}_{pqrs}\left[-\frac{2u^4\log u}{(1-u)^3 v} - \frac{u^3(1+u)}{(1-u)^2 v}\right]$$

checked
to p,q,r,s~10

6. Double-trace mixing diagonalized by 10D blocks



6* Explicit formula for leading-log at each loop order:

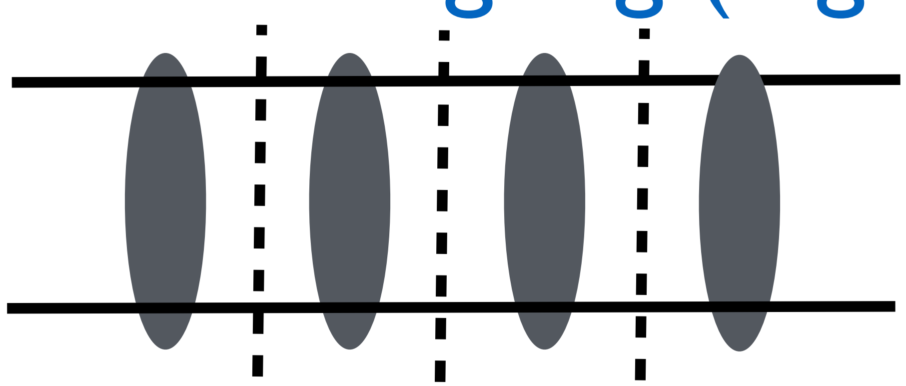
$$\mathcal{H}^{(k)}_{pqrs}(z,\bar{z},\alpha,\bar{\alpha})\Big|_{\log^k u} = \left[\Delta^{(8)}\right]^{k-1} \cdot \mathcal{D}_{pqrs} \cdot \mathcal{D}_{(3)} \cdot h^{(k)}(z).$$

$$h^{(k)}(z) \equiv \frac{1}{k!}\left(\frac{-1}{2}\right)^k \sum_{\ell=0,\ \text{even}}^{\infty} \frac{960\Gamma(j+1)\Gamma(j+4)}{\Gamma(2j+7)} \frac{1}{[(\ell+1)_6]^{k-1}} z^{j+1}{}_2F_1(j+1, j+4, 2j+8, z).$$

Example:

$$h^{(2)}(z) = \frac{\mathrm{Li}_2(z) - (1-z)^5\mathrm{Li}_2(z/(1-z))}{4z^5} - \frac{(1-z)(2z^2-7z+7)\log(1-z)}{8z^4} + \frac{(z-2)(1-z)}{z^3} + \frac{235}{576}\frac{z-2}{z}.$$

gives leading-log ($\log^{L+1}$) terms for *all* correlators



matches 2222 from: [Alday & Bissi '17, Aprile,Drummond,Heslop&Paul '17]

Generalizations 1.

Correlator of tensor multiplets in $AdS_3 x S^3 x K3$

$$A^{(6)} \sim G_N^{(6)} \delta^8(Q) \times \left[\frac{1}{s} + \frac{1}{t} + \frac{1}{u} \right]$$

[Giuso, Russo & Wen '19,
Rastelli, Roumpedakis& Zhou '19]

dimensionless
'coupling'

conformal
amplitude

Also, All S^3 harmonics packaged together!

(what about stress-tensor?)

Generalization 2: stringy corrections?

22pp in Mellin space:

[from flat space+SUSY localization]

$$\mathcal{M}_p(s,t) = \frac{4p}{\Gamma(p-1)}\frac{1}{c}\Bigg[\frac{1}{(s-2)(t-p)(u-p)} + \frac{(p+1)_3}{4}\zeta(3)\frac{1}{\lambda^{3/2}} + \frac{(p+1)_5}{32}\zeta(5)\left[s^2+t^2+u^2+\frac{2p(p-2)}{p+5}s+\left(-2p^2+\frac{50+20p(p+2)}{(p+4)(p+5)}\right)\right]\frac{1}{\lambda^{5/2}} + \cdots\Bigg] + O(c^{-2})\,.$$

[Alday,Bissi&Perlmutter; Binder,Chester,Pufu&Wang '19]

[cf Bissi's talk]

only j_{10}=0 (or 2) eigenvalues change, not eigenvectors!!

$$\gamma = -\frac{\Delta^{(8)}}{c}\frac{1}{(j+1+m)_6} + \frac{\lambda^{-3/2}}{c}\delta_{j,0}\delta_{m,0}B_4(\tau) + \frac{\lambda^{-5/2}}{c}\left(\delta_{j+m,2}B_5(\tau) + (\delta_{j+m,0}B_5'(\tau)\right) + \ldots$$

[Drummond,Nandan,Paul&Rigatos]

(Suggestive of 4- and 6-order Casimir acting on 10D object?)

Summary

-Studied double-trace mixing in strongly coupled N=4 SYM using Lorentzian inversion formula

-SO(10,2) symmetry: formula for all spherical harmonics!

-Leading logs to all orders in $1/N_c$

Further questions

- Simplify other AdS_5xS_5 computations?

- Higher loops/higher points? [Goncalves, Pereira & Zhou '19]

cf: [Loebbert, Mojaza& Plefka '18: hidden conformal symmetry]

[cf Maldacena' 11: Einstein vs conformal gravity]

-Less conformal-looking theories: 6D (2,0), ABJM?