

# Holographic duals for 5d SCFTs

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UCLA

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[arXiv:1606.01254](https://arxiv.org/abs/1606.01254), [arXiv:1611.09411](https://arxiv.org/abs/1611.09411), [arXiv:1703.08186](https://arxiv.org/abs/1703.08186),  
[arXiv:1705.01561](https://arxiv.org/abs/1705.01561), [arXiv:1706.00433](https://arxiv.org/abs/1706.00433)

with Eric D'Hoker, Michael Gutperle, Andreas Karch,  
Chrysostomos Marasinou, Andrea Trivella

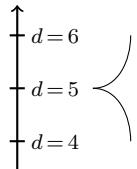
# Introduction

## 5d SCFTs

Classification of superconformal algebras allows for  $d > 4$  [Nahm '78]  
 $\text{tr } F^2$  relevant, gauge theories non-renormalizable ( $\sim \sqrt{g}R$  in 4d)

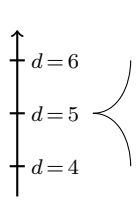
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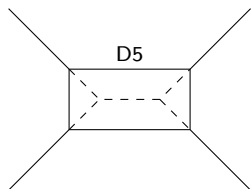
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Asymptotically safe gauge theories, parents to isolated  $4d$  theories, relations to  $6d$ , exceptional global symmetries, dualities, ...

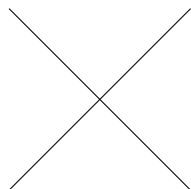
## 5d SCFTs from 5-brane webs

[Aharony, Hanany, Kol '97]

5-brane web: arrangement of  $(p, q)$  5-branes, junctions with fixed angles and conserved charges

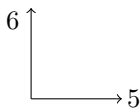


Coulomb branch  
finite gauge coupling



UV fixed point CFT

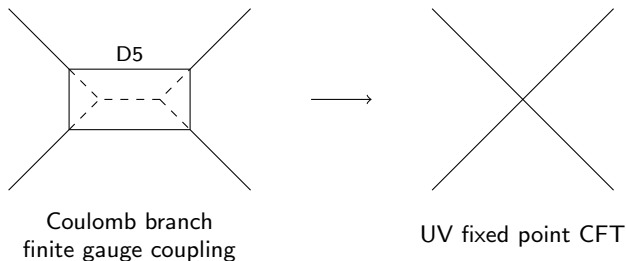
	0	1	2	3	4	5	6	7	8	9
D5	x	x	x	x	x	x				
NS5	x	x	x	x	x		x			



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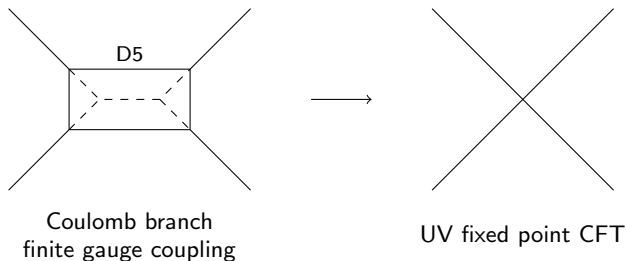


Large classes of 5d SCFTs, with and without gauge theory deformations, beyond initial classification

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Large classes of 5d SCFTs, with and without gauge theory deformations, beyond initial classification

Coulomb branches, relevant deformations and RG flows, . . .



## 5d SCFTs from 5-brane webs: supergravity duals?

AdS/CFT for EE, correlators, . . . needs corresponding  $\text{AdS}_6$  solutions in IIB supergravity: not a standard near-horizon limit

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This talk:  $\text{AdS}_6$  solutions in IIB and connection to 5-brane webs

# AdS<sub>6</sub> solutions in type IIB supergravity

## Outline:

- Ansatz and general local solution
- Global solutions on the disc
- Connection to 5-brane webs
- Entanglement entropy vs. free energy

AdS<sub>6</sub> solutions in type IIB supergravity  
– ansatz and local solution –

## Symmetries and ansatz

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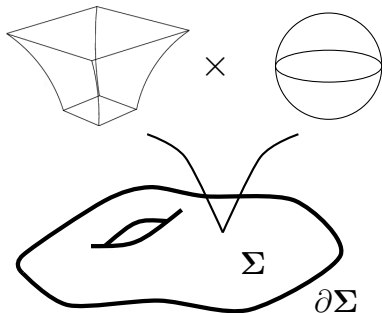
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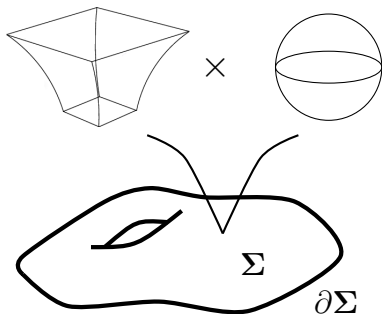


General ansatz:  $\text{AdS}_6$  and  $\text{S}^2$   
warped over Riemann surface  $\Sigma$

$$\mathcal{M} = (\text{AdS}_6 \times \text{S}^2) \times_{\text{w}} \Sigma$$

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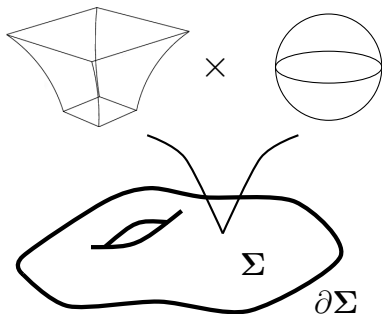
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Remaining bosonic fields:  $C_{(4)} = 0$      $C_{(2)} \propto \text{vol}_{\text{S}^2}$      $\tau = \chi + ie^{-2\phi}$

## General local solution

With complex coordinate  $w$  on  $\Sigma$

$$ds^2 = f_6(w)^2 ds_{\text{AdS}_6}^2 + f_2(w)^2 ds_{\text{S}^2}^2 + 4\rho(w)^2 |dw|^2$$

$$C_{(2)} = \mathcal{C}(w) \text{vol}_{\text{S}^2} \quad B(w) = \frac{1 + i\tau(w)}{1 - i\tau(w)}$$

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Decomposing Killing spinors, reducing BPS eq. on  $\text{AdS}_6$  and  $S^2$   
→ coupled PDEs on  $\Sigma$  for supergravity fields & Killing spinors

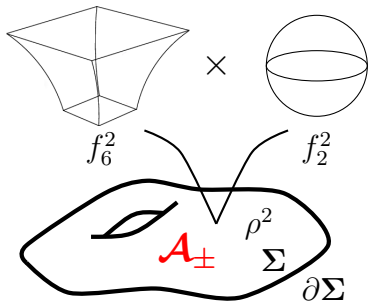


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Arbitrary locally holomorphic

$$\mathcal{A}_\pm : \Sigma \rightarrow \mathbb{C}$$

yield metric functions  $f_6^2$ ,  $f_2^2$ ,  $\rho^2$ ,  
axion-dilaton  $B$ , two-form field  $\mathcal{C}$   
and Killing spinors solving BPS eq.

$SU(1,1) \otimes \mathbb{C}$  transf. of  $\mathcal{A}_\pm$  induce  $SL(2, \mathbb{R})$  on supergravity fields

## General local solution

Explicit solution for supergravity fields:

$$f_6^2 = c_6^2 \sqrt{6\mathcal{G}} \left[ \frac{1+R}{1-R} \right]^{1/2}$$

$$f_2^2 = \frac{c_6^2}{9} \sqrt{6\mathcal{G}} \left[ \frac{1-R}{1+R} \right]^{3/2}$$

$$\rho^2 = \frac{\kappa^2}{\sqrt{6\mathcal{G}}} \left[ \frac{1+R}{1-R} \right]^{1/2}$$

$$B = \frac{\partial_w \mathcal{A}_+ \partial_{\bar{w}} \mathcal{G} - R \partial_{\bar{w}} \bar{\mathcal{A}}_- \partial_w \mathcal{G}}{R \partial_{\bar{w}} \bar{\mathcal{A}}_+ \partial_w \mathcal{G} - \partial_w \mathcal{A}_- \partial_{\bar{w}} \mathcal{G}}$$

$$C = \frac{4ic_6^2}{9} \left[ \frac{\partial_{\bar{w}} \bar{\mathcal{A}}_- \partial_w \mathcal{G}}{\kappa^2} - 2R \frac{\partial_w \mathcal{G} \partial_{\bar{w}} \bar{\mathcal{A}}_- + \partial_{\bar{w}} \mathcal{G} \partial_w \mathcal{A}_+}{(R+1)^2 \kappa^2} - \bar{\mathcal{A}}_- - 2\mathcal{A}_+ \right]$$

with composite quantities

$$\kappa^2 = -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2$$

$$\partial_w \mathcal{B} = \mathcal{A}_+ \partial_w \mathcal{A}_- - \mathcal{A}_- \partial_w \mathcal{A}_+$$

$$\mathcal{G} = |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}}$$

$$R + \frac{1}{R} = 2 + 6 \frac{\kappa^2 \mathcal{G}}{|\partial_w \mathcal{G}|^2}$$



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## General local solution

General type IIB supergravity solution with 16 supersymmetries on  $\text{AdS}_6 \times S^2$  warped over  $\Sigma$ , in terms of locally holomorphic  $\mathcal{A}_\pm$  on  $\Sigma$ .

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→ global solutions

AdS<sub>6</sub> solutions in type IIB supergravity  
– global solutions on the disc –

## Regularity conditions

Demanding real, geodesically complete geometry with consistent spacetime signature and  $\text{Im}(\tau) > 0$  imposes constraints:

$$\kappa^2|_{\text{int}(\Sigma)} > 0 \qquad \mathcal{G}|_{\text{int}(\Sigma)} > 0$$

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For 10d geometry w/o boundary, collapse  $S^2$  on  $\partial\Sigma$  (AdS<sub>6</sub> finite):

$$\kappa^2|_{\partial\Sigma} = 0 \qquad \mathcal{G}|_{\partial\Sigma} = 0$$

Not all independent,  $\mathcal{G}|_{\text{int}(\Sigma)} > 0$  implied by the other conditions.

## Solving the regularity conditions

Fix topology of  $\Sigma$ , 1) construct locally holomorphic  $\mathcal{A}_\pm$  producing regular  $\kappa^2$ , 2) implement additional constraints for regular  $\mathcal{G}$ .



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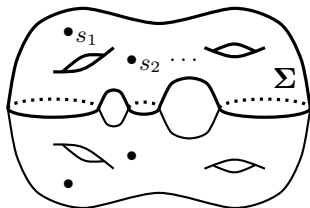
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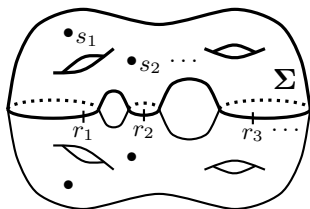
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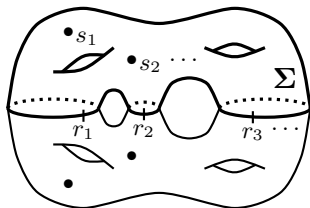
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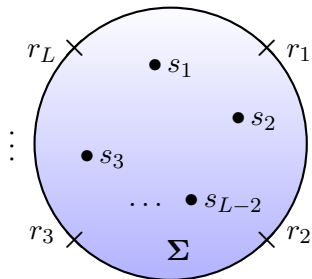
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- 1b) constructing  $\partial_w \mathcal{A}_{\pm}$ ,  $\mathcal{A}_{\pm}$  adds  $L$  poles  $r_{\ell}$  on  $\partial\Sigma$  + constants
- 2) further integration for  $\mathcal{G}$ ,  $\mathcal{G}|_{\partial\Sigma} = 0$  constrains parameters

## Regular solutions on the disc

$\Sigma = \text{disc}/\text{upper half plane}$ ,  $L \geq 3$  poles,  $N = L - 2$  "charges"



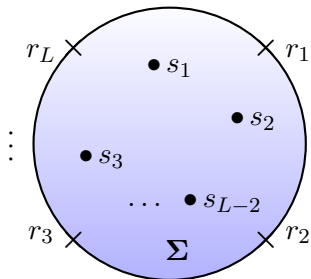
$$\mathcal{A}_+ = \mathcal{A}_+^0 + \sum_{\ell=1}^L Z_+^\ell \ln(w - r_\ell)$$

$$Z_+^\ell = \sigma \prod_{n=1}^{L-2} (r_\ell - s_n) \prod_{k \neq \ell}^L \frac{1}{r_\ell - r_k}$$

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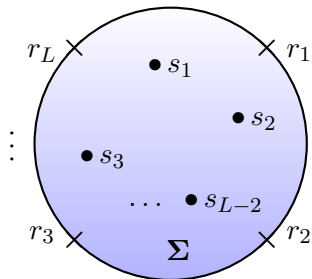
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$\mathcal{G}|_{\partial\Sigma} = 0 \sim$  one local condition per pole  $\rightarrow 2L - 2$  free parameters

$$\mathcal{A}_+^0 Z_-^k - \mathcal{A}_-^0 Z_+^k + \sum_{\ell \neq k} Z^{[\ell, k]} \ln |p_\ell - p_k| = 0$$

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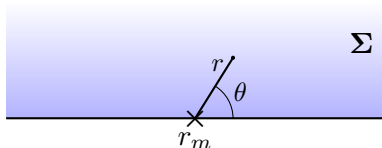
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Solutions regular everywhere, except for possibly the poles...



## Regular solutions on the disc – behavior near poles



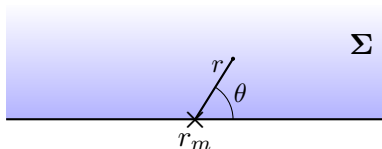
Supergravity fields in terms of  $\kappa^2$ ,  $\mathcal{G}$ ,  $R$

$$\mathcal{G} = \sum_{\ell \neq \ell'}^L Z^{[\ell, \ell']} \left[ \frac{1}{2} \ln \left\{ \frac{w - r_{\ell'}}{(r_{\ell} - r_{\ell'})^2} \right\} \overline{\ln \left\{ \frac{w - r_{\ell}}{(r_{\ell} - r_{\ell'})^2} \right\}} + \int_{\infty}^w dz \frac{\ln(z - r_{\ell})}{z - r_{\ell'}} - \text{c.c.} \right]$$

$$\kappa^2 = -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2 \qquad R + \frac{1}{R} = 2 + 6 \frac{\kappa^2 \mathcal{G}}{|\partial_w \mathcal{G}|^2}$$

simplify drastically in  $r \ll 1$  expansion...

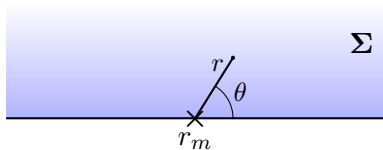
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string frame:

$$\tilde{d}s^2 \approx \frac{2}{3} |Z_+^m - Z_-^m| \left[ 3c_6^2 |\ln r| ds_{\text{AdS}_6}^2 + \overbrace{\frac{dr^2}{r^2} + d\theta^2}^{\Sigma} + c_6^2 \sin^2 \theta ds_{\text{S}^2}^2 \right]$$

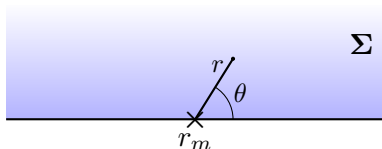
## Regular solutions on the disc – behavior near poles



string frame:

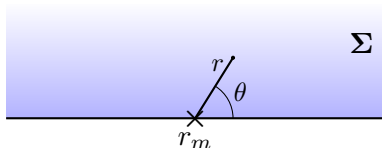
$$\tilde{d}s^2 \approx \frac{2}{3} |Z_+^m - Z_-^m| \left[ \underbrace{3c_6^2 |\ln r| ds_{\text{AdS}_6}^2}_{\rightarrow \mathbb{R}^{1,5}} + \overbrace{\frac{dr^2}{r^2} + d\theta^2 + c_6^2 \sin^2 \theta ds_{\mathbb{S}^2}^2}^{\Sigma} \right]$$

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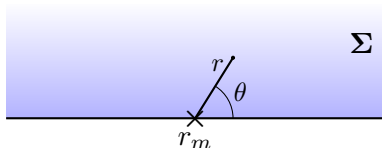
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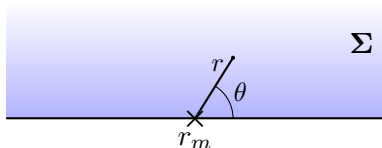
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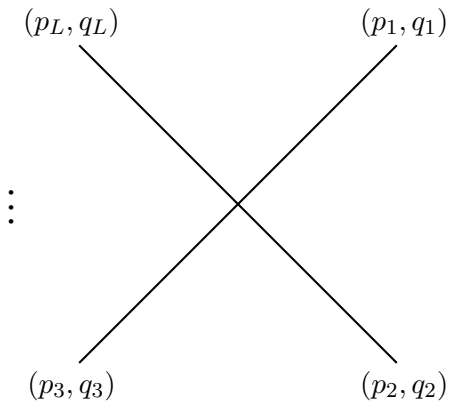
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Solutions regular with isolated poles corresponding to 5-branes ✓

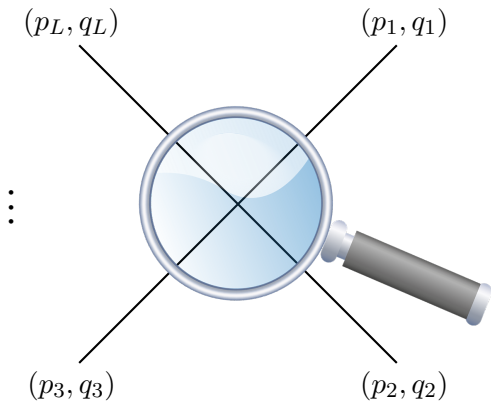
AdS<sub>6</sub> solutions in type IIB supergravity  
– connection to 5-brane webs –



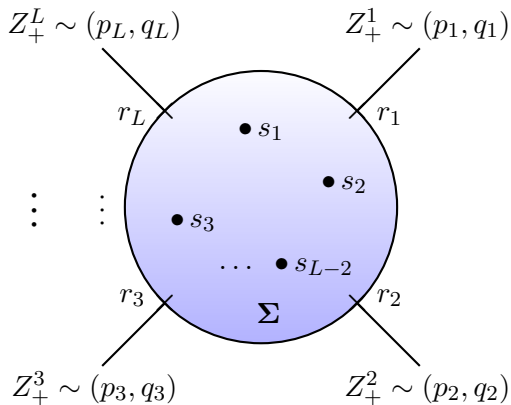
## 5-brane web picture



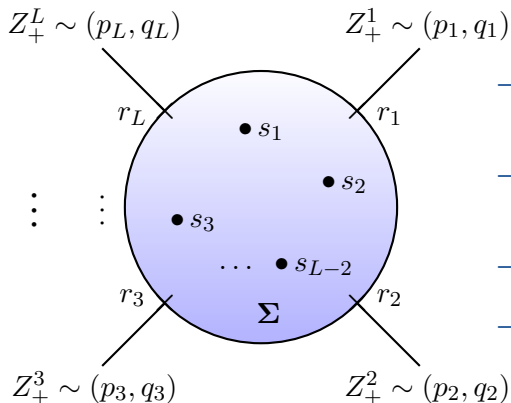
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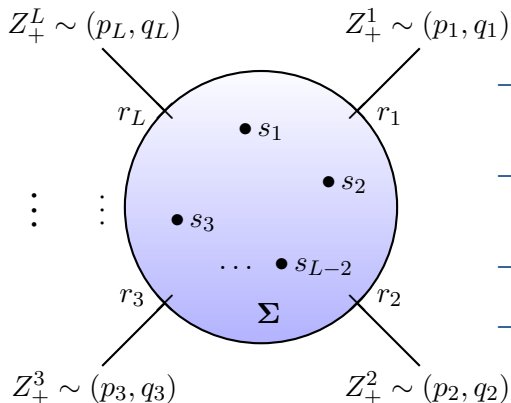


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- external 5-branes explicitly  $(p, q)$  charge conserved
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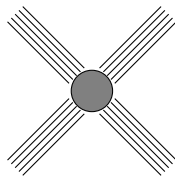
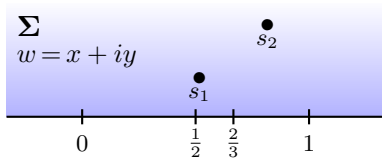
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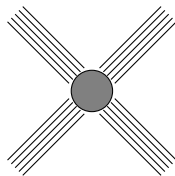
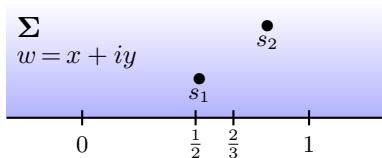
Supergravity solutions for fully localized 5-brane intersections. ✓

## 4-pole example



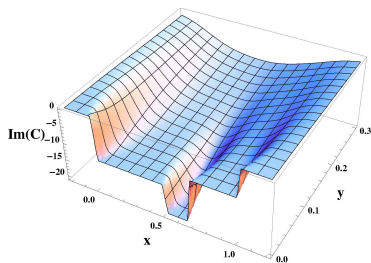
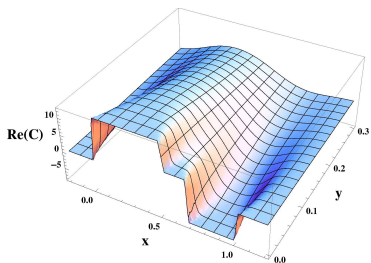
$$\begin{aligned} -Z_+^1 &= Z_+^3 = 2 + 2i \\ -Z_+^4 &= Z_+^2 = 3 - 3i \end{aligned}$$

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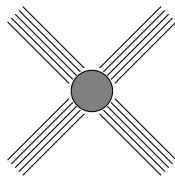
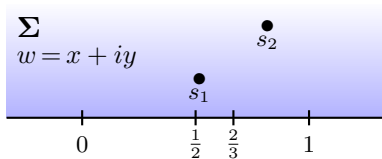
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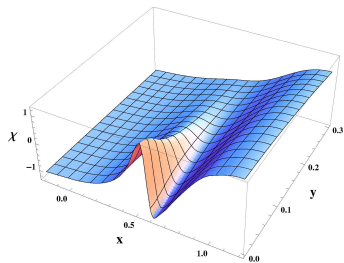
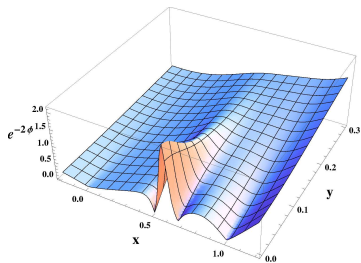
$$\Delta\mathcal{C} \sim p - iq, \text{ charge conservation} \sim \sum \Delta\mathcal{C} = 0$$

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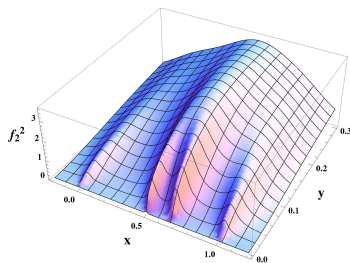
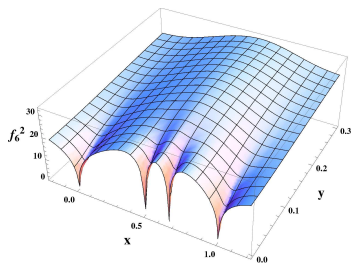
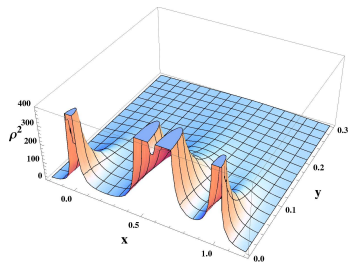
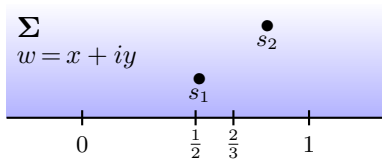
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$\exp(-2\phi) \rightarrow 0$  at poles,  $\chi$  finite



## 4-pole example



Einstein-frame metric as needed for string-frame 5-brane metric ✓

AdS<sub>6</sub> solutions in type IIB supergravity  
– entanglement entropy vs. free energy –

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First steps in AdS/CFT: Holographic interpretation consistent?  
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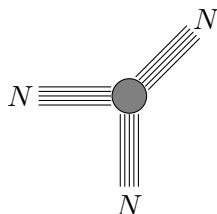
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Computation straightforward for both – no divergences from poles, no finite contributions. Related as expected. ✓

## Entanglement entropy vs. free energy – examples



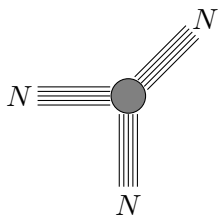
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$$N - SU(N - 1) \times \cdots \times SU(2) - 2$$

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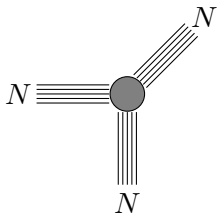
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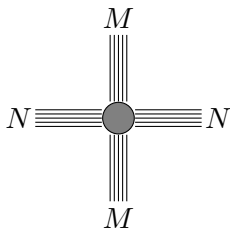
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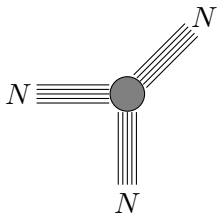
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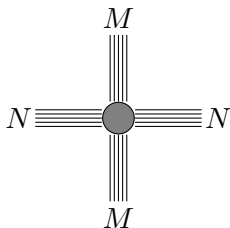
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## Summary & Outlook

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Supergravity solutions for fully localized 5-brane intersections in type IIB. Holographic duals for the corresponding 5d SCFTs.

Solutions regular everywhere except for isolated physically meaningful singularities, tame enough for AdS/CFT.

Free energy and entanglement entropy satisfy expected relation, first quantitative indications on dual SCFTs.

[ Extension to disc with punctures and  $SL(2, \mathbb{R})$  monodromy, for 5-brane intersections with (mutually local) 7-branes. ]

→ [arXiv:1706.00433](https://arxiv.org/abs/1706.00433)

## Outlook

Establish relation to dual SCFTs: compare free energy to field theory computation using supersymmetric localization.

AdS/CFT with warped products: spectra, correlation functions, defects, Wilson lines, truncation to 6d  $F(4)$  gauged sugra. . .

More general Riemann surfaces  $\Sigma$ : Annulus worked out explicitly, no solutions (so far). Single boundary and higher genus?

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