Out of Equilibrium, Out of Time Order

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Broad motivation : Non-Equilibrium QFT

- Why non-equilibrium quantum field theory ?
- Standard answer : that generic states in QFT are not in equilibrium.
- Why non-equilibrium QFT in Strings ?
- Some of the fundamental questions of our field are about non-equilibrium physics.
  - Gravity in de-Sitter
  - Black-hole evaporation and information paradox.
  - More generally, time dependent processes in String theory
- First Two are among the central unsolved problems in our field which are inherently non-equilibrium.
- Given the amount of thought and time devoted to them, it is worthwhile to ask what is the correct formalism to ask these Qs ?

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Out of Time Order (OTO) Correlators

Traditional QFT lays great emphasis on single time path-integrals/time-ordered correlators.
Why consider Out of Time Order Correlators(OTOCs)/path integrals with timefolds ?

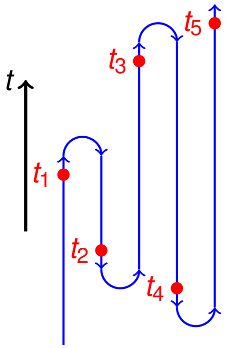

Figure : The timefolded contour necessary to compute the correlator with temporal ordering $t_1 > t_2$, $t_2 < t_3$, $t_3 > t_4$ and $t_4 < t_5$.

Motivations

- Not the first time in this conference : See previous talks about how OTO correlators can be used to
  - diagnose chaos,
  - to study scrambling,
  - to give one measure of how close a theory is to being holographic to classical gravity.
- OTO correlators as being **novel 'observables' in non-equilibrium QFT**.
- Increasing realisation that OTOCs capture certain physical features of a system that are very difficult to extract via usual correlators.
- Picture not yet completely clear : but the relevant features seem to be usually non-local, information theoretic (like entanglement, scrambling, chaos etc.)
- Could very well be that there are many more and this list just the tip of an iceberg. **Need for a systematic study and exploration**

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Generalised Schwinger/Keldysh(SK) Path integrals :

$$\mathcal{Z}_{k-oto}[\mathcal{J}_{\alpha\mathrm{R}},\mathcal{J}_{\alpha\mathrm{L}}]$$
$$= \mathrm{Tr}\left(\cdots U[\mathcal{J}_{3\mathrm{R}}](U[\mathcal{J}_{2\mathrm{L}}])^{\dagger}U[\mathcal{J}_{1\mathrm{R}}]\ \hat{\rho}_{\mathrm{initial}}\ (U[\mathcal{J}_{1\mathrm{L}}])^{\dagger}U[\mathcal{J}_{2\mathrm{R}}](U[\mathcal{J}_{3\mathrm{L}}])^{\dagger}\cdots\right) .$$

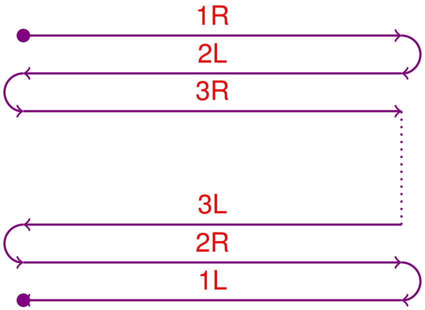

Figure : The k-OTO contour computing the out-of-time-ordered correlation functions. Timefolds of more and more depths required to compute highly out-of-time-order correlators.

How OTO is an OTOC ? Proper OTO number

- Ordering is such that the 1R contour is past-most, the 1L is future-most, and the inner contours with $\alpha > 1$ will nest in between in the order they appear, viz.,
  1R $<$ 2L $<$ 3R $< \cdots <$ 3L $<$ 2R $<$ 1L.
- Minimum timefold depth required to compute a OTO correlator $=$ proper OTO number.
- A rough measure of how inaccessible the information is via usual correlators.
- Not quite - in fact, a current question : Which OTOCs give genuinely new and useful information about the state ? and When ?

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- Let me number the operators in decreasing time-ordering.
- Will use a notation n denote $\widehat{\mathbb{O}}_n(t_n)$ where $t_1 > t_2 > \ldots$.
- For example, $\langle 1234 \rangle$ is the usual time-ordered correlator.
- $\langle 4321 \rangle$ is anti-time-ordered. Both can be computed with 1 time-fold and hence have proper OTO number 1 or proper 1-OTO.
- What about $\langle 1324 \rangle$? Way to do this is to count **future turning point (FTP) operators**.
- FTP operators are those with both their neighbours in the past, e.g., $\langle \hat{1}3\hat{2}4 \rangle$
- So $\langle \hat{1}3\hat{2}4 \rangle$ is actually proper 2-OTO.

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How OTO is an OTOC ?

- Any proper k-OTO correlator has k operators closest to future turning points and $k-1$ operators close to past turning points
- Any proper k-OTOC thus has at least $(2k-1)$ operators.
- A n-pt correlator can have at most proper OTO number $\text{Int}\left(\frac{n+1}{2}\right)$.
- 2-OTO starts from 3-pt fns.
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- Many ways to think about OTOCs - all somewhat complementary
  1. $(2k)^n$ countour(C) correlators : good for path integral, Diagrammatics but 'Yuge' redundancies. One strategy may be to use Column Vector method .
  2. As $n!$ Wightman(W) correlators (simplest)

     $$G_\sigma(t_1, t_2, \cdots, t_n) = \left\langle \widehat{\mathbb{O}}_{\sigma(1)} \widehat{\mathbb{O}}_{\sigma(2)} \cdots \widehat{\mathbb{O}}_{\sigma(n)} \right\rangle , \qquad \sigma \in S_n ,$$

     Complicated diagrammatics. Long answers mixing up all sorts of physics.
  3. As $2^{n-2}\ n!$ nested(N) correlators involving (anti-)commutators (may bring out causal features, discontinuities etc (cf, Caron-Huot's talk))

     $$[\{[\widehat{\mathbb{O}}_1, \widehat{\mathbb{O}}_2], \widehat{\mathbb{O}}_3\}, \cdots] ,$$

     Will abbreviate this to $[123_+ \ldots]$ in the following. Many redundancies (generalised Jacobis) - can be alleviated by a clever basis choice.

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Simple questions about OTOCs

- How many Wightman n-pt fns are proper q-OTO ? Say $g_{n,q}$. What is $g^{Eql}_{n,q}$?
- How to resolve redundancies in the nested correlators ? Classifying $(2^{n-2}-1)n!$ sJacobi relations
- How to classify redundancies in contoue correlators ? Answer decides the structure of EFTs.
- How is equilibrium/thermalisation characterised by OTOCs ? OTO fluctuation dissipation theorem ?
- Is there a OTO kinetic theory ? OTO fluid dynamics ?
- How do we set up experiments to measure OTOCs ? How does OTO structure of the environment affect a quantum system ?

Will try to describe some answers we have. But most questions are open !

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- How to resolve redundancies in the nested correlators ? Classifying $(2^{n-2}-1)n!$ sJacobi relations
- How to classify redundancies in contoue correlators ? Answer decides the structure of EFTs.
- How is equilibrium/thermalisation characterised by OTOCs ? OTO fluctuation dissipation theorem ?
- Is there a OTO kinetic theory ? OTO fluid dynamics ?
- How do we set up experiments to measure OTOCs ? How does OTO structure of the environment affect a quantum system ?

Will try to describe some answers we have. But most questions are open !

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Counting proper q OTOC

- Number of Wightman n-pt fns that are proper q-OTO

$$g_{n,q} = \text{Coefficient of } \mu^q \text{ in } \mathcal{G}_n(\mu)$$
$$\mathcal{G}_n(\mu) \equiv \left(2\sqrt{1-\mu}\right)^{n+1} \mathrm{Li}_{-n}\left(\frac{2}{1+\sqrt{1-\mu}} - 1\right)$$

where

$$\mathrm{Li}_{-n}(z) = \left(z\frac{\partial}{\partial z}\right)^n \frac{z}{1-z}$$

- To count it, realise that proper OTO number is decided by counting turning point operators.
- This reduces the problem to that of counting n-permutations with q maxima .
- An interesting combinatorics problem with the above generating fn.

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Counting proper q OTOC II

- Two ways to add a future n^{th} operator without increasing proper OTO number
  1. Start with $(n-1)$-point correlator whose proper OTO number is less than q and then increase it.
     Done by an insertion in n intervals that exist between previous $(n-1)$ insertions.e.g., $\langle 432\hat{1}\rangle \mapsto \langle \hat{4}532\hat{1}\rangle$
  2. The second way is to start with a proper q-OTO $(n-1)$-point correlator and add an operator without increasing the proper OTO number.
     Done by inserting n^{th} operator just before or just after one of the q future turning point operators. e.g., $\langle 432\hat{1}\rangle \mapsto \langle 5432\hat{1}\rangle$
- This gives

$$\sum_{j=1}^{q} g_{n,j} = n\sum_{j=1}^{q-1} g_{n-1,j} + (2q)g_{n-1,q} \tag{1}$$

which then gives the generating function as advertised.

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Counting proper q OTOC III

$g_{n,q}$	$q = 1$	2	3
$n = 1$	1	0	0
2	2	0	0
3	4	2	0
4	8	16	0
5	16	88	16

Table : The decomposition of the $n!$ Wightman basis correlators into the proper q-OTO correlators for low-lying values of n.

- These counts are useful, since they tell you how many correlators one has to compute before one is done say, diagrammatically.
- Many diagrammatic computations are actually redundant !

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Counting proper q OTOC in thermal state

$g^{Eq}_{n,q}$	$q=1$	2	3
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Table : Same as before but in Equilibrium.

- Things simplify in equilibrium : $\langle\hat{1}4\hat{2}3\rangle$ and $\langle\hat{1}3\hat{2}4\rangle$ are the only independent 2-OTOs till 4-pts.
- The 3-pt 2-OTOs contain no novel information in equilibrium. (Measure of thermalisation ?)
- Note that the rows here add upto $(n-1)!$.

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Reducing OTOs in thermal equilibrium

- The reduction in the thermal state is due to periodicity in imaginary time. Let n_β represent the operator shifted in time by $-i\beta$.
- We can then write

$$\langle 12 \rangle = \langle 2_\beta 1 \rangle = \langle 1_\beta 2_\beta \rangle$$
$$\langle 12_\beta 3 \rangle = \langle 3_\beta 12_\beta \rangle\,,\ \langle 21_\beta 3 \rangle = \langle 3_\beta 21_\beta \rangle$$

  and so on.This reduces $n!$ correlators to $(n-1)!$ independent correlators.
- Great simplification occurs in finite temperature (say in deriving fluctuation disiipation theorem) by moving to general OTO correlators (as opposed to usual discussions tied to Schwinger-Keldysh contour for 1-OTOs).
- The euclidean periodicity (KMS) relates *some* proper q-OTO Wightman functions with proper (q + 1)-OTO correlators.

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General FDTs

- FDTs arise by writing thermal Wightman correlators in terms of commutators. For example,

$$\langle 12\rangle = -\,\mathfrak{f}_2[12]\,.$$

$$\langle 123\rangle = \mathfrak{f}_2\mathfrak{f}_3[123] + (1+\mathfrak{f}_1)(1+\mathfrak{f}_2)[321]$$

$$\begin{aligned}\langle 1234\rangle = &(1+\mathfrak{f}_1)\,\mathfrak{f}_{3,4}\Big\{\mathfrak{f}_4[1234] + (1+\mathfrak{f}_3)\,[1243]\Big\}\\ &+(1+\mathfrak{f}_1)\,\Big\{\,(1+\mathfrak{f}_{2,4})\,\mathfrak{f}_4[1324] + \mathfrak{f}_{2,4}\,(1+\mathfrak{f}_2)\,[1342]\Big\}\\ &+(1+\mathfrak{f}_1)\,(1+\mathfrak{f}_{2,3})\,\Big\{\mathfrak{f}_3[1423] + (1+\mathfrak{f}_2)\,[1432]\Big\}\end{aligned}$$

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Summary

- What we have done is to begin a systematic study of OTO correlators.
- How much/what new information they can give ? is the central question.
- Many equilibrium relations like FDTs for higher point functions seem to enormously simplify once one expands the formalism to include OTOs
- may be an indication that it is the right way to think of thermalisation ?
- Many generalisations to be made, formalisms to be developed, computations to be done.
- E.g., can one use the relations we uncovered to set up a practical diagrammatics at finite T ?
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Thank you !