Six-Dimensional $(2,0)$ Scattering Amplitudes: The M5 Brane and IIB Supergravity on $K3$

Matthew Heydeman

Caltech

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Based on [1812.06111] with John H. Schwarz, Congkao Wen, Shun-Qing Zhang

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- Gives an S-matrix definition of these well-known but subtle theories. Understand interactions of tensors, symmetry breaking, the $K3$ moduli space, and webs of dualities in lower dimensions.

6D Spinor Helicity and On-Shell SUSY

- Momentum vectors can be described as bispinors of $Spin(5,1) \sim SU^*(4)$, $p^\mu \sim p^{AB}$ with $A, B = 1, \ldots, 4$.
- Little group = $SU(2) \times SU(2)$; a,$\hat{a} = 1, 2$. Introduce spinors λ^A_{ia}:

On-shell momentum for particle i:

$$p_i^{AB} = \langle \lambda_i^A \lambda_i^B \rangle = \epsilon_{ab} \lambda_i^{A,a} \lambda_i^{B,b} = \lambda_i^{A+} \lambda_i^{B-} - \lambda_i^{A-} \lambda_i^{B+}.$$

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- In 6D, may also have independent anti-chiral spinors $\tilde{\lambda}_{A\hat{a}}$.
- Also introduce Grassmann numbers η_i^{Ia}, on-shell superfields $\Phi(\eta_i)$ so that:

$(2,0)$ SUSY + R-charges:

$$q_i^{AI} = \langle \lambda_i^A \eta_i^I \rangle, \quad \bar{q}_{iI}^A = \langle \lambda_i^A \frac{\partial}{\partial \eta_i^I} \rangle, \quad USp(4) \text{ R-charges} \sim \eta^2, \eta\partial\eta, \partial\eta^2$$

$$\Phi(\eta_i) = \phi + \eta_a^I \psi_I^a + \eta_a^I \eta^{aJ} \phi_{IJ} + \eta_a^I \eta_{bI} B^{ab} + \eta_b^I \eta_J^b \eta_a^J \bar{\psi}_I^a + \eta^4 \bar{\phi}.$$

Simplest Amplitudes

- Lorentz invariants (similar expressions for anti-chiral w/ $[\dots]$):

$$\langle \lambda_i^a \lambda_j^b \lambda_k^c \lambda_l^d \rangle = \epsilon_{ABCD} \lambda_i^{A,a} \lambda_j^{B,b} \lambda_k^{C,c} \lambda_l^{D,d},$$

Components of superamplitudes:

$$A_4^{\mathrm{M5-brane}} = \delta^8 \left(\sum_i \lambda_{ia}^A \eta_i^{Ia} \right) \equiv \delta^8(Q) \longleftrightarrow \mathcal{L}_{\mathrm{DBI}} = \mathcal{L}_{\mathrm{free}} + H^4 + \dots$$

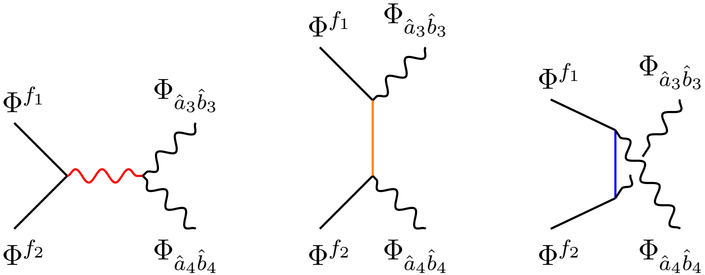
- Amplitudes techniques allow for explicit computation of higher points without a Lagrangian (for example, 6 tensors):

$$A_6(B_{++}) = \frac{1}{s_{123}} \left(\epsilon^{ab} \langle 1_+ \, 2_+ \, 3_+ \, pa \rangle \langle p_b \, 4_+ \, 5_+ \, 6_+ \rangle \right)^2 + \text{perms}$$

Allowed for many checks of leading $U(1)$ M5-brane; symmetries, soft theorems, reduction to 4D $\mathcal{N} = 4$, abelian E/M duality . . .

$(2,0)$ Supergravity - Interacting tensor multiplets and supergravitons

- Comes from Type IIB Sugra on $K3$ at 2-derivative order, away from strongly coupled cusps. There are 21 $(2,0)$ tensor multiplets required by $K3$/anomaly cancellation, so we must introduce an $SO(21)$, $f = 1 \ldots 21$ flavor symmetry.
- Ex: 2 supergravitons, 2 tensors:



$$A^{(2,0)}_{2,2} = \delta^8(Q)\left(\delta^{f_1 f_2}\frac{[1_{\hat{a}_1}2_{\hat{a}_2}3_{\hat{a}_3}4_{\hat{a}_4}][1^{\hat{a}_1}2^{\hat{a}_2}3_{\hat{b}_3}4_{\hat{b}_4}]}{s_{12}\, s_{23}\, s_{31}}\right) + \text{sym}\,.$$

n-point Amplitude as a Worldsheet Integral

- Introduce n-punctured $\sigma_i \in \mathbb{CP}^1$, target space is the 6D super-null cone parametrized by polynomial holomorphic maps $\mathcal{Z}(\sigma) = (\lambda_a^A(\sigma), \eta^{Ia}(\sigma))$. Amplitude may be written as a localized integral over these moduli, weighted by 'integrands'.

Schematic 6D Amplitude:

$$\mathcal{A}_{6D} = \int \frac{\prod d\sigma_i \, d\mathcal{Z}_k}{\text{Vol}(G)} \prod_{i=1}^{n} \delta'\left(\mathcal{Z}_i - \frac{\mathcal{Z}(\sigma_i)}{\prod_{j \neq i} \sigma_i - \sigma_j} \right) \times \mathcal{I}_L \mathcal{I}_R$$

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- For $(2,0)$ Sugra w/ n_1 supergravitons, n_2 tensors, we find $\mathcal{I}_L \mathcal{I}_R \sim \mathrm{Pf}\, \mathcal{X}_{n_2} \times M^{n_1}_{\hat{a}\hat{b}}$. Here $[\mathcal{X}_n]_{ij} = \frac{\delta_{f_i f_j}}{\sigma_{ij}}$ is a sort of flavored fermion determinant and $M^{n_1}_{\hat{a}\hat{b}}$ is a determinant of graviton polarization spinors.
- The general formulas may be compared with the explicit expressions and pass all consistency checks.

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- Double soft limits lead to flavor and R-symmetry rotated lower point amplitudes.

Expected $(2,0)$ Sugra $SO(21)$ Double Soft Theorem:

$$A_n(\phi_1^{f_1}, \bar{\phi}_2^{f_2}, \ldots) \to \frac{1}{2} \sum_{i=3}^{n} \frac{p_i \cdot (p_1 - p_2)}{p_i \cdot (p_1 + p_2)} R_i^{f_1 \, f_2} A_{n-2} \, .$$

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- Also find new mixed soft theorems when scalars form no singlets

New $(2,0)$ Mixed Flavor and R-symmetry Double Soft:

$$A_n(\bar{\phi}_1^{f_1}, \phi_2^{f_2, IJ}, \ldots) \to \sum_{i=3}^{n} \frac{p_1 \cdot p_2}{p_i \cdot (p_1 + p_2)} R_i^{f_1 f_2} R_i^{IJ} A_{n-2} \, .$$

- This allows us to recover the moduli space $\mathcal{M}_{\text{Sugra}} = \frac{SO(5,21)}{SO(5)\times SO(21)}$!

Discussion:

- Main point: New perspective on perturbative 6D $(2,0)$ amplitudes, including both explicit and general formulas. These pass all consistency checks and circumvent the usual difficulties of these theories. Also connects with many other topics at this conference:
- New formula for 4D $\mathcal{N}=4$ supergravity amplitudes. Reduction of tensor and graviton multiplets implies this Sugra is consistent with Type II/Heterotic duality in 4D.
- Applications to $\mathrm{AdS}_3 \times \mathrm{S}^3 \times K3$. Flat space limit of Mellin amplitudes $\rightarrow$ Sugra amplitudes, leading to 2D CFT constraints.
- Possibility of extensions to more interesting $(1,0)$ supergravities with less SUSY and richer moduli spaces.
- Loops, higher dimension operators, and Coulomb branch amplitudes in 4/6D.