The Apparent Horizon in AdS/CFT

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Black Holes Thermodynamics

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Really a question about quantum gravity... let's use holography

which was initially motivated by black hole thermodynamics anyways.

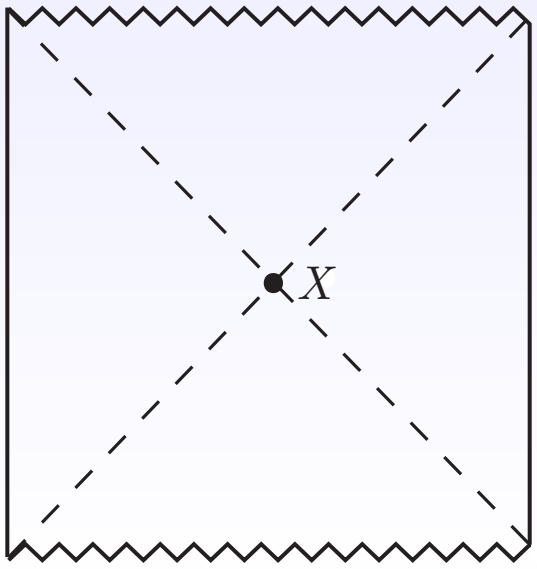
Entropy in AdS/CFT

Ryu, Takayanagi; Hubeny, Rangamani, Takayanagi:

Holographic Entanglement Entropy

$$S[\rho] \equiv -\mathrm{tr}\,(\rho \ln \rho) = \frac{\mathrm{Area}[X]}{4G\hbar},$$

where ρ is the state of a full boundary, X is a codimension-2, surface homologous to the boundary which is an extremum of the area functional; minimal area one if there's more than one.



Entanglement Entropy in Black Hole Collapse

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$$S[\rho] = 0$$

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But entropy of black hole is growing.

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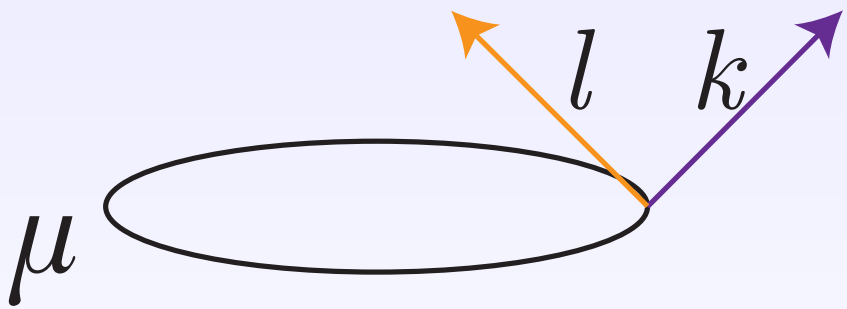
$$S^{(\text{outer})} = \max_{\rho'} \left[-\text{tr}\rho' \ln \rho' \ : \ \text{black hole exterior fixed} \right]$$

How to qualify the interior region?

Event horizon is natural notion (with an area law!), but thermodynamics defined this way have an uncomfortable acausality. Let's use the **apparent horizon**.

Qualifying the Black Hole Interior

On a time slice, it is the outermost surface with light rays that do not bend into its exterior. Hawking



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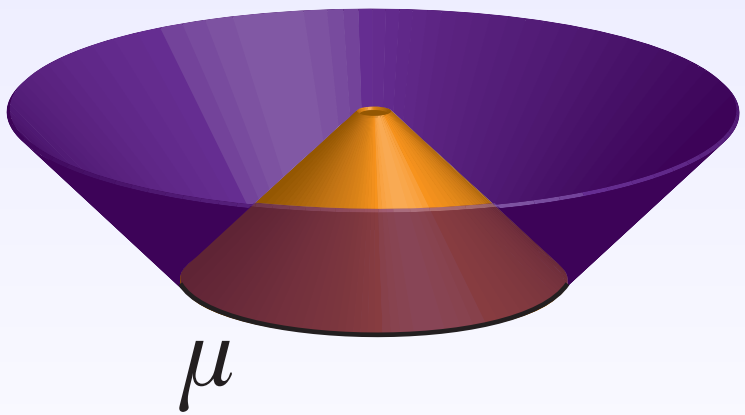
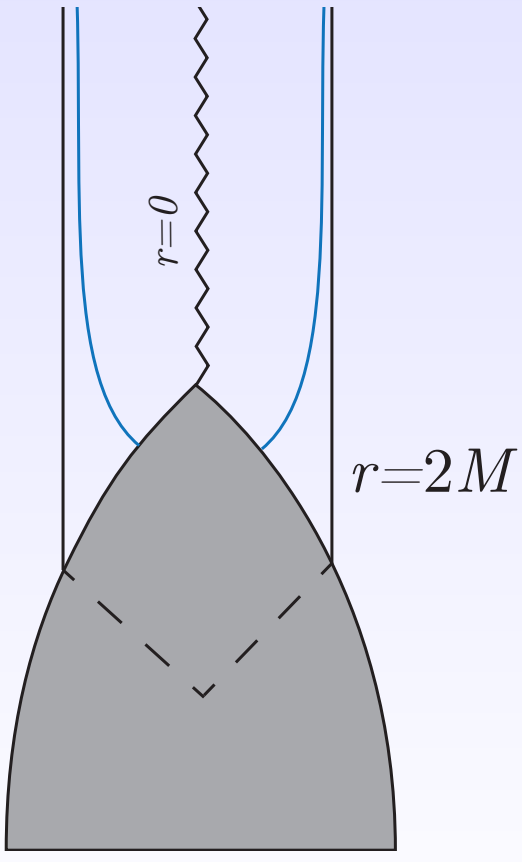
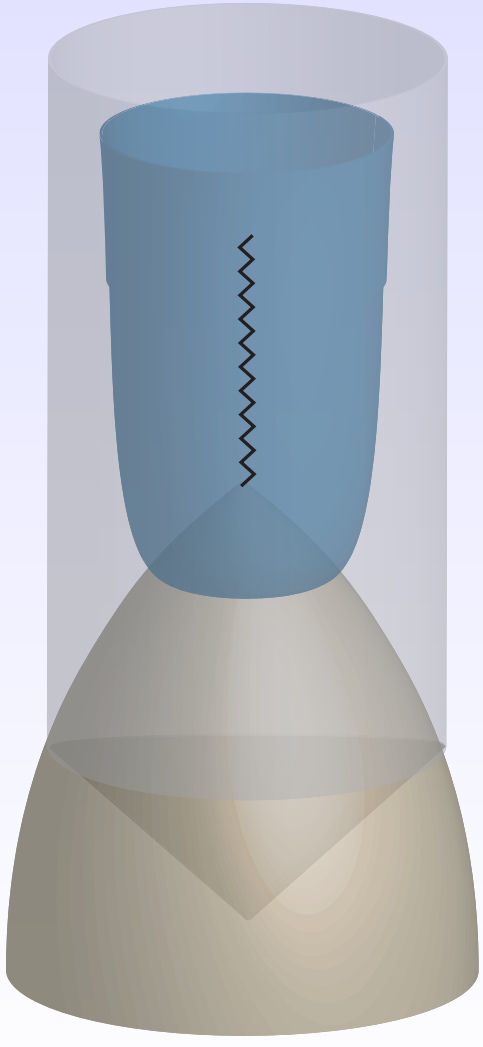


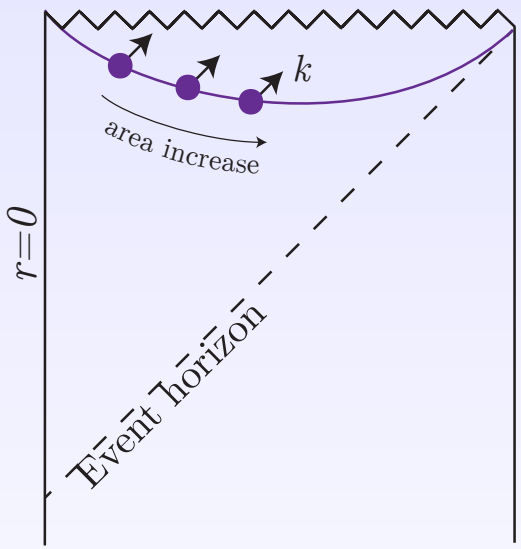
Apparent horizon in Black Hole Collapse





Apparent Horizon

A $D-1$-dim'l surface foliated by apparent horizons (under the right assumptions) *does* obey an area law. Hayward; Ashtekar, Krishnan



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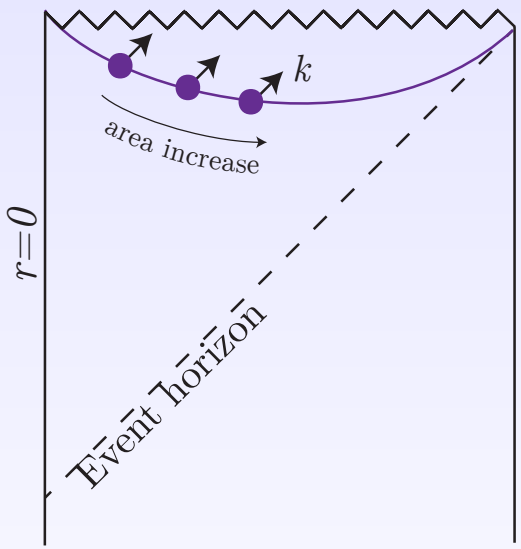
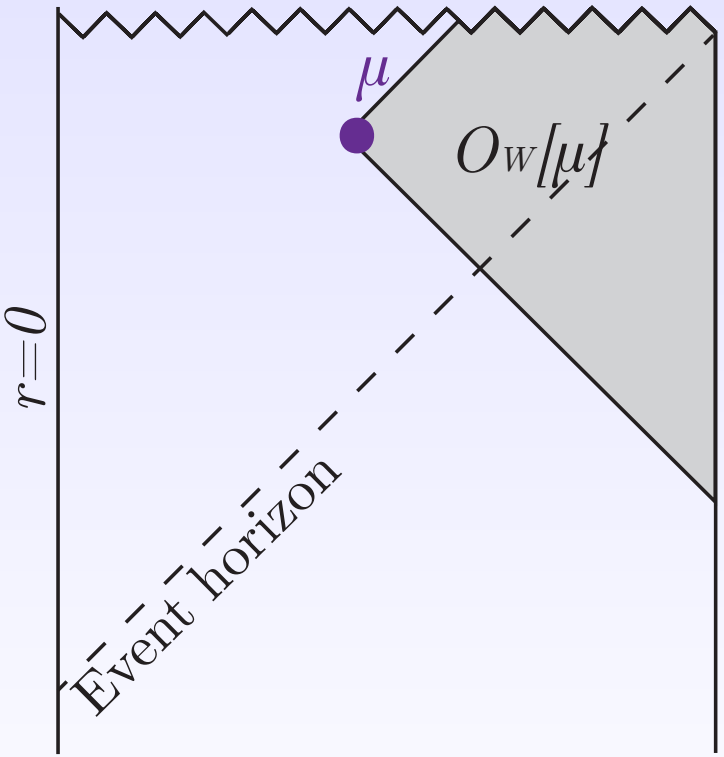


In a static black hole, the event horizon is precisely such a $D-1$-dim'l surface.

Coarse-Grained Entropy

What is the entropy associated to coarse graining over the region behind the apparent horizon μ?



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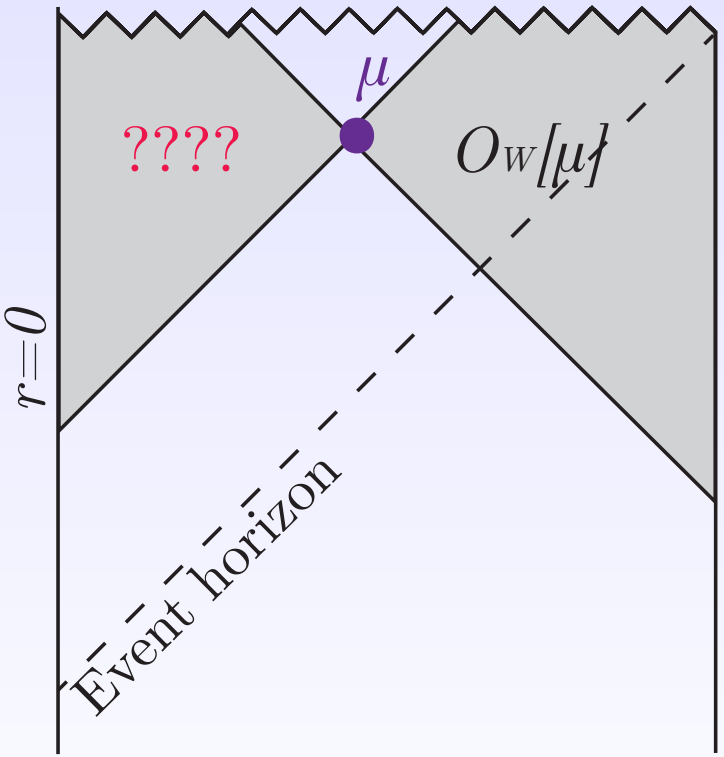

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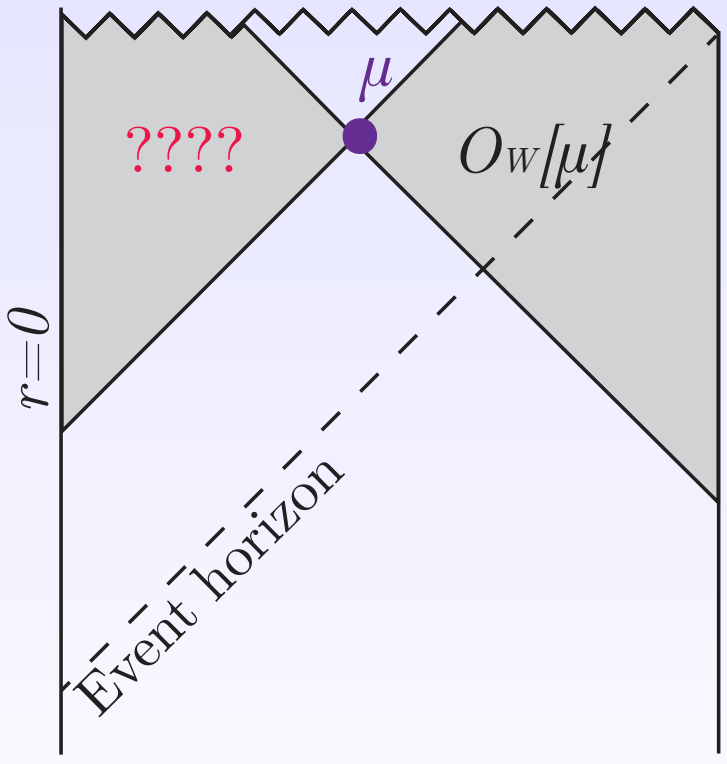


$$S^{(\text{outer})}[\mu] \equiv \max_{\rho'} \left[-\text{tr}\rho' \ln \rho' \ : \ O_W[\mu] \text{ fixed} \right]$$

Area of the Apparent Horizon in AdS/CFT

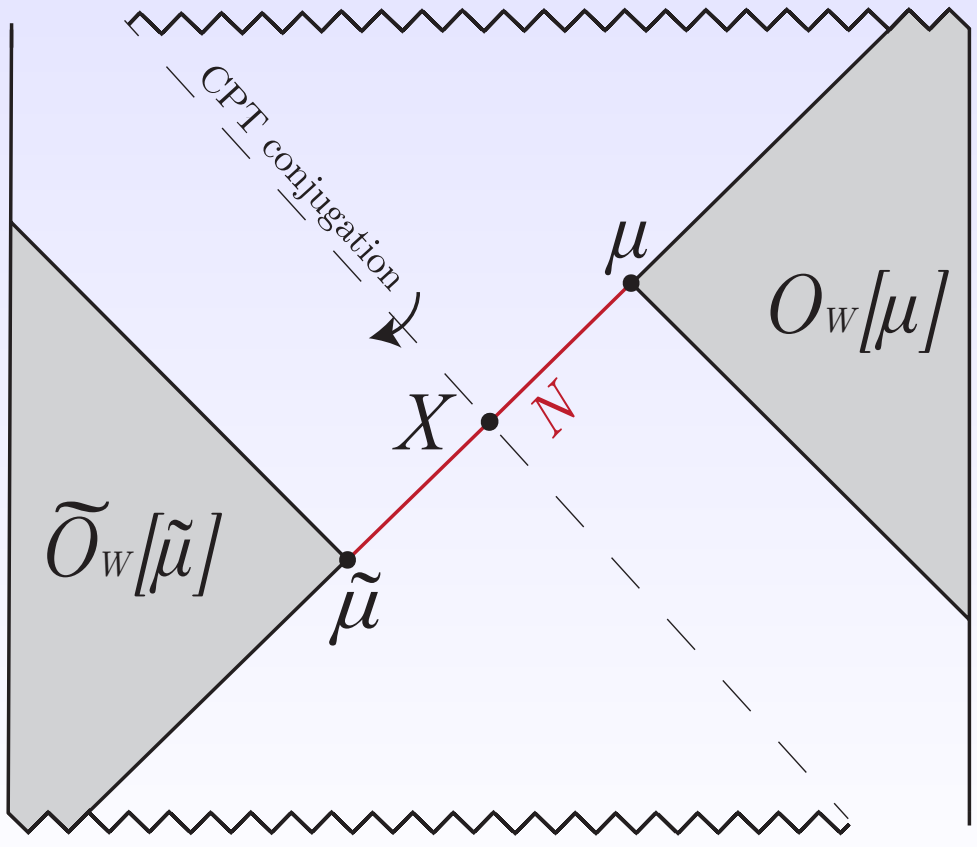
Area = Coarse-Grained Entanglement Entropy

$$
\begin{aligned}
S^{\text{outer}}[\mu] &\equiv \max_{\rho'} \left(-\text{tr}[\rho' \ln \rho'] : O_W[\mu]\text{fixed} \right) \\
&= \max_{\rho'} \left(\frac{\text{Area}[X_{HRT}]}{4G\hbar} : O_W[\mu]\text{fixed} \right) \\
&= \frac{\text{Area}[\mu]}{4G\hbar}
\end{aligned}
$$

where μ is an apparent horizon, ρ' are field theory states dual to a classical bulk with $O_W[\mu]$ fixed, but any (consistent) interior geometry.

Coarse-Grained Spacetime

What is the maximizing state ρ'?



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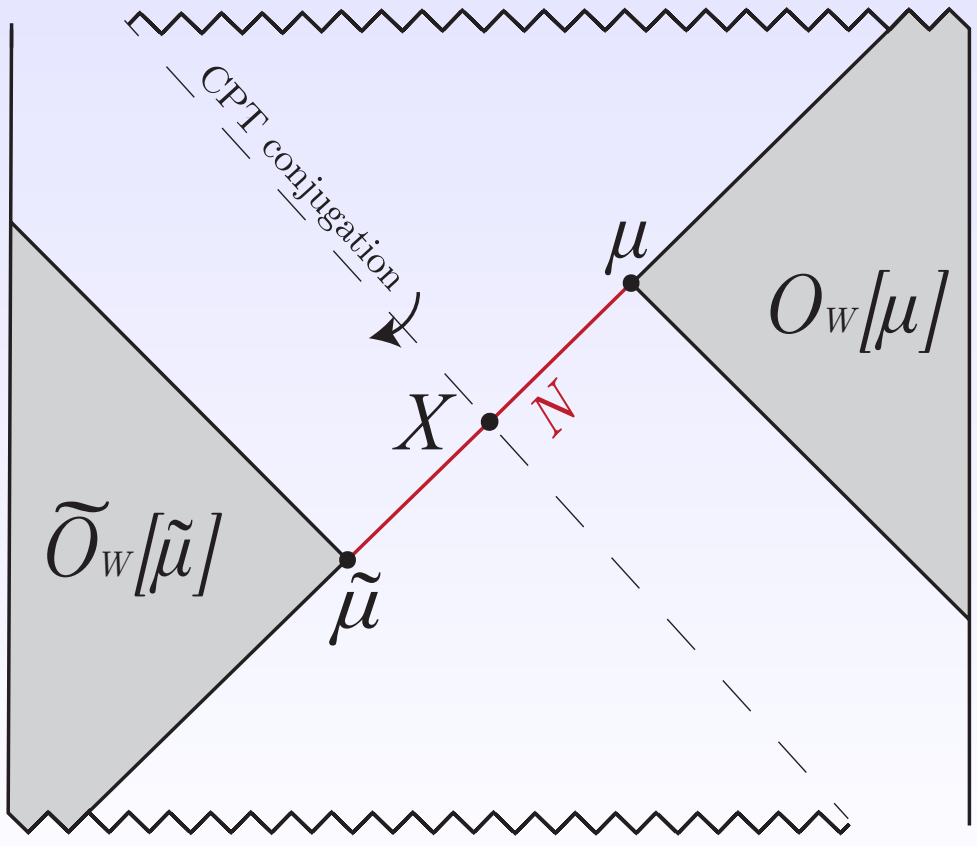

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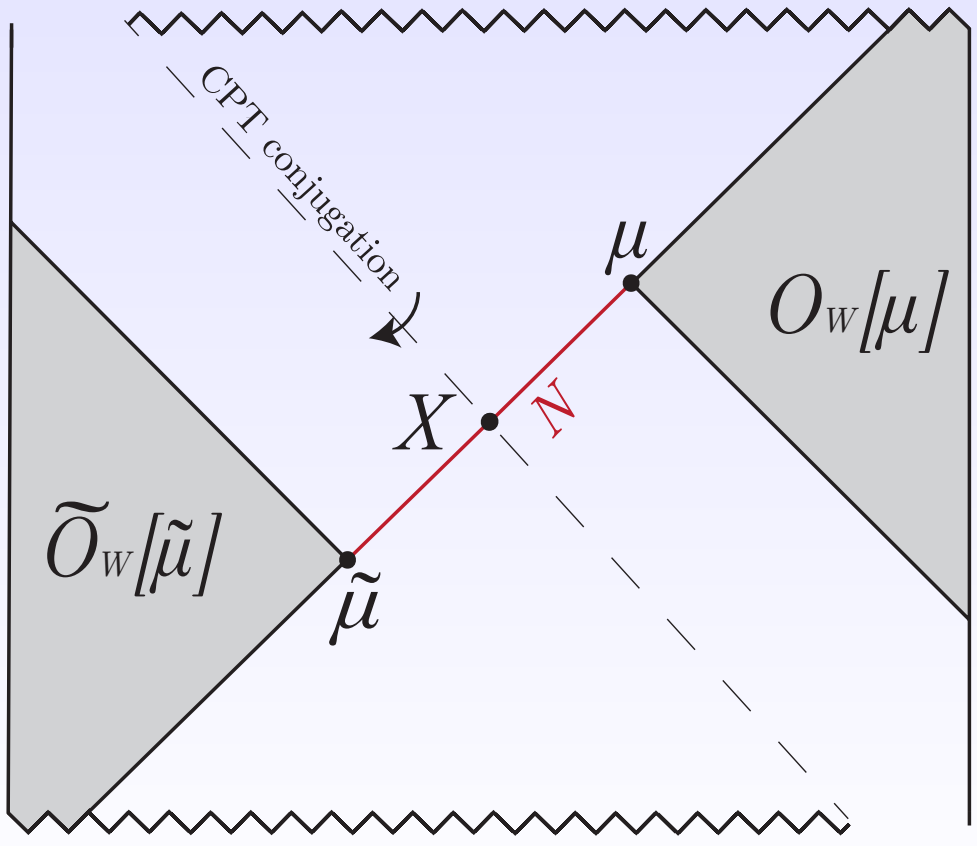

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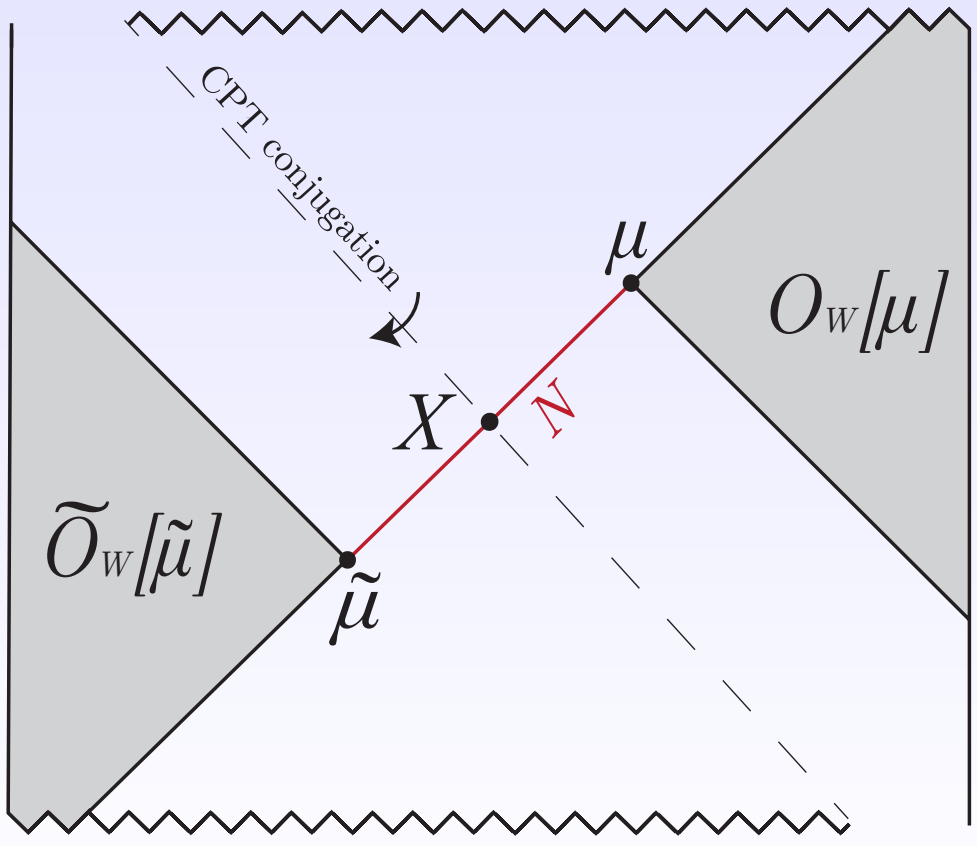

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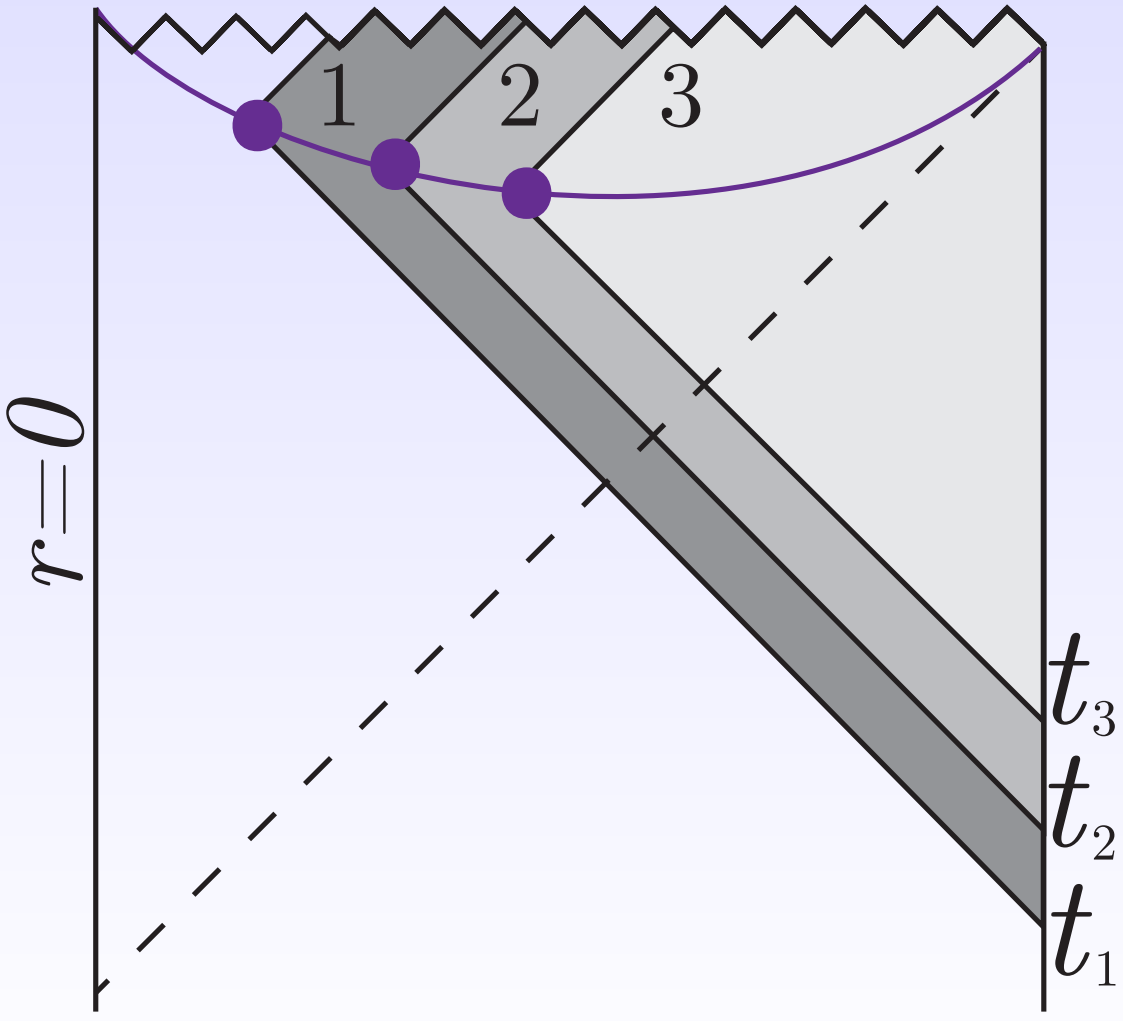
$$\Rightarrow \quad S[\rho'] = \frac{\text{Area}[\mu]}{4G\hbar}$$

Explanation of Area Law



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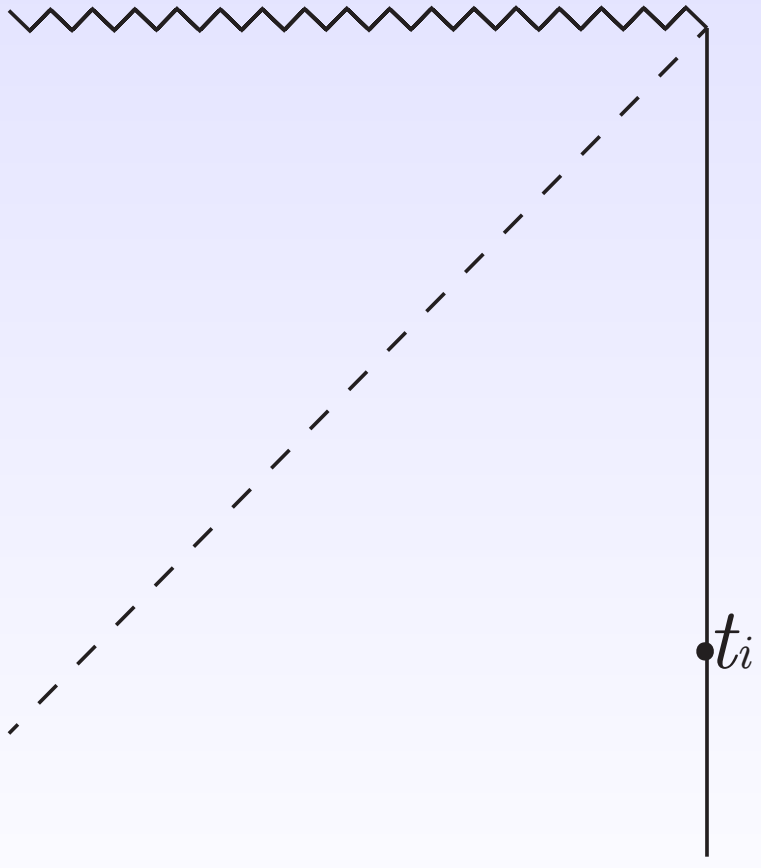

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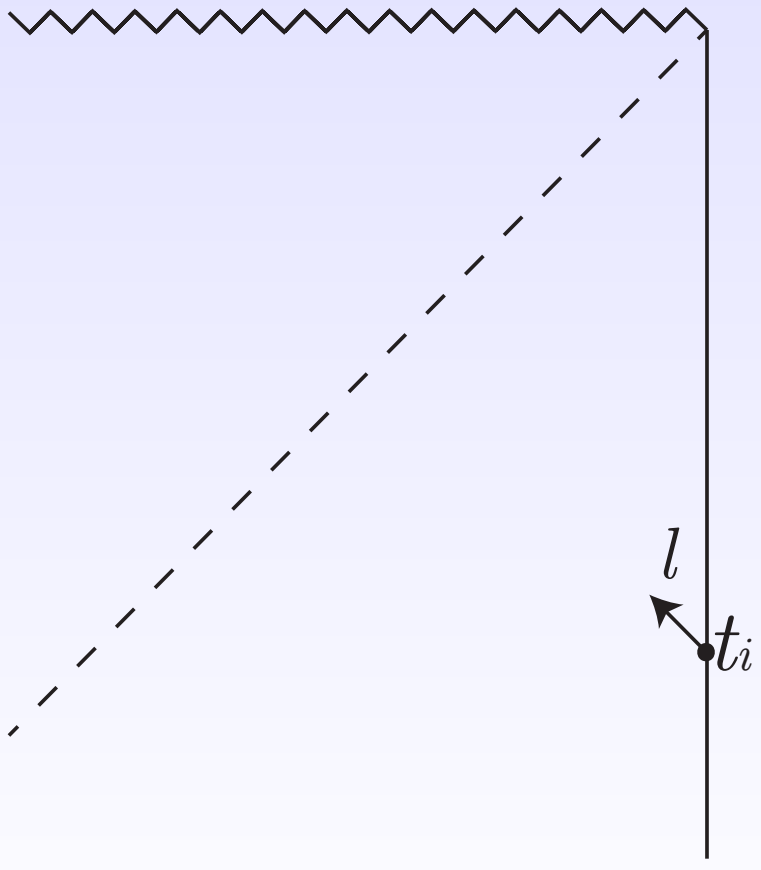

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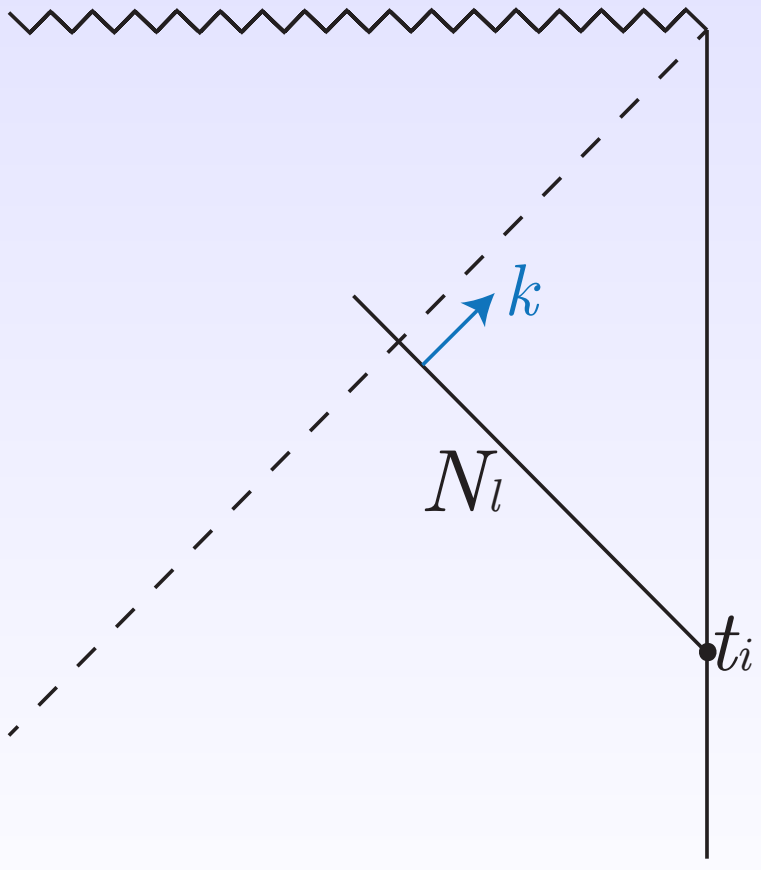

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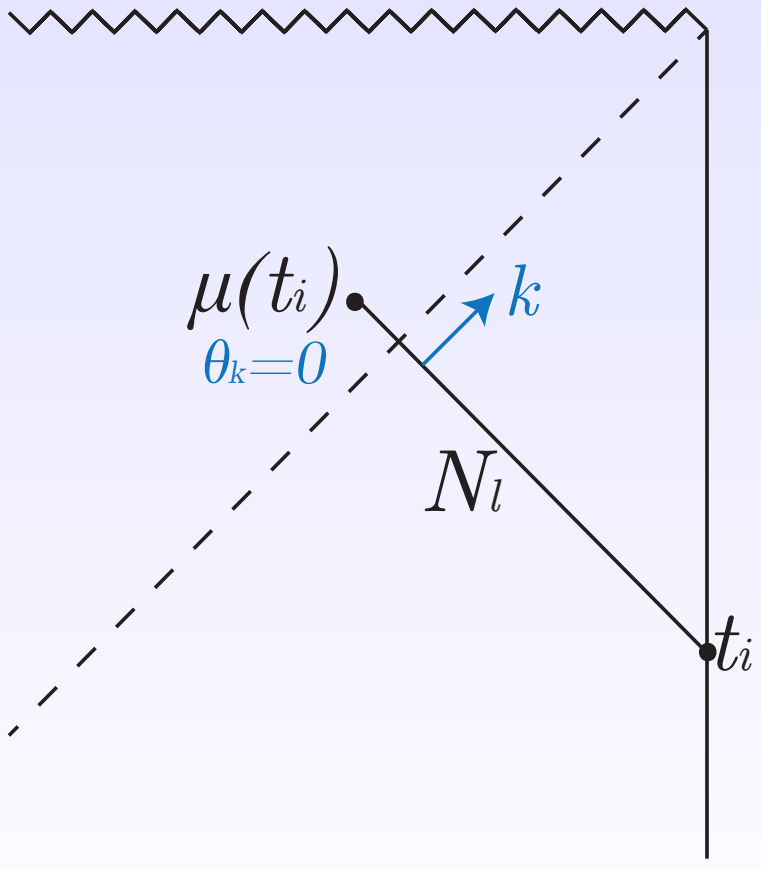

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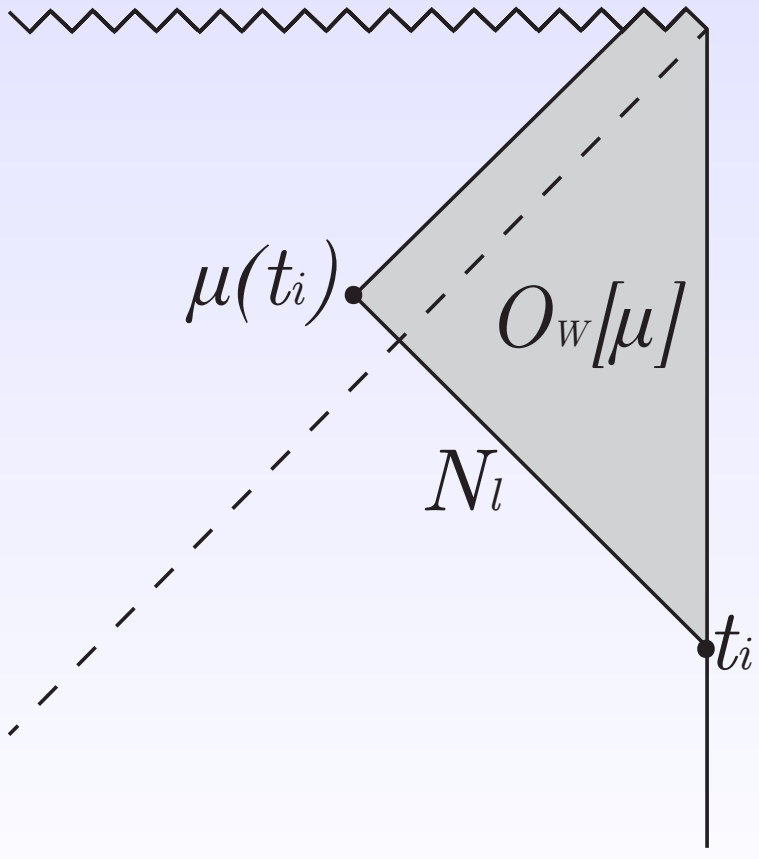

Boundary Dual

When black hole is near equilibrium: (apparent horizon is perturbatively close to the event horizon)

Assume: If O and J are "simple" (correspond to locally propagating bulk perturbations), then (in the strictly classical Einstein gravity limit):

$$S^{(\text{dual})}(t_i) = \max_{\rho'}(S[\rho'] : \text{fixing } \langle O \rangle_J \text{ after } t_i)$$

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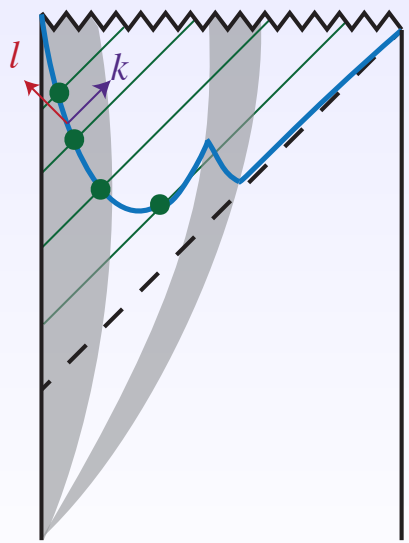
Summary

- Area of apparent horizon is entropy associated to ignorance of region behind it.
- Can construct explicitly the "coarse-grained" spacetime dual to state maximizing $S[\rho]$ subject to fixed exterior of apparent horizon.
- First holographic explanation of area law in the bulk.
- Possible boundary dual via fixing one point functions (with sources), but unclear how general that is.

Future Directions and Applications

- Boundary-anchored surfaces? Subtleties with divergence structure Marolf, White - to appear
- Interpretation of non-minimal area extremal surfaces (not entwinement Balasubramanian, Chowdhury, Czech, de Boer; Lin; Balasubramanian,Bernamonti, Craps, De Jonckheere, Galli...)
- Interpretation of area law for arbitrary signature "apparent horizons" NE, Bousso



- $1/N$ corrections