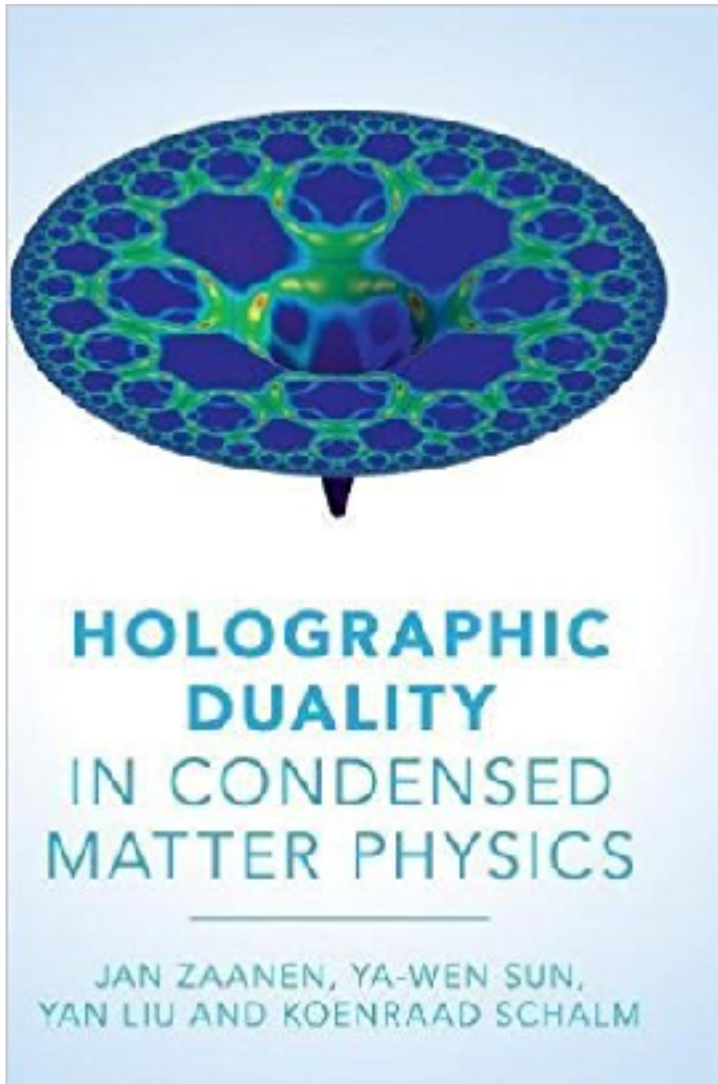
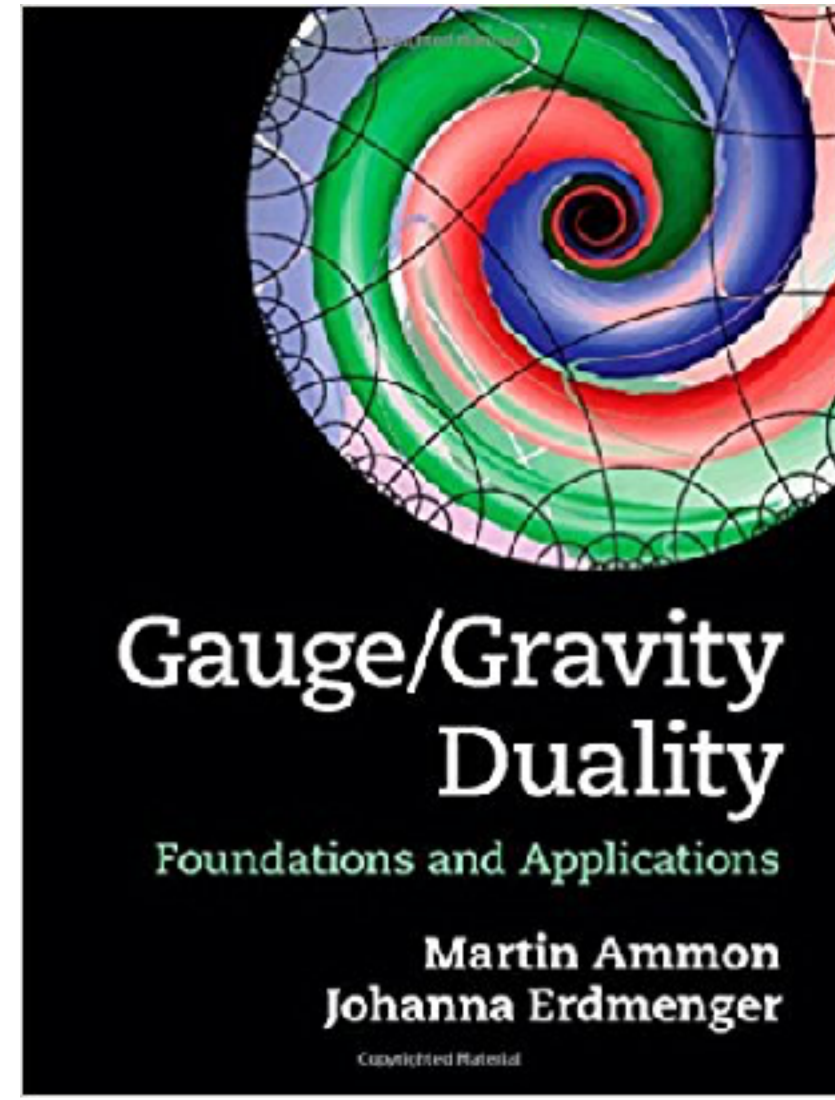
Incoherent Transport and Black Holes

Talk at “Strings 2017”, Tel-Aviv

Aristomenis Donos
Durham University





Holographic quantum matter

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We present a review of theories of states of quantum matter without quasiparticle excitations. Solvable examples of such states are provided through a holographic duality with gravitational theories in an emergent spatial dimension. We review the duality between gravitational backgrounds and the various states of quantum matter which live on the boundary. We then describe quantum matter at a fixed commensurate density (often described by conformal field theories), and also compressible quantum matter with variable density, providing an extensive discussion of transport in both cases. We present a unified discussion of the holographic theory of transport with memory matrix and hydrodynamic methods, allowing a direct connection to experimentally realized quantum matter. We also explore other important challenges in non-quasiparticle physics, including symmetry broken phases such as superconductors and non-equilibrium dynamics.

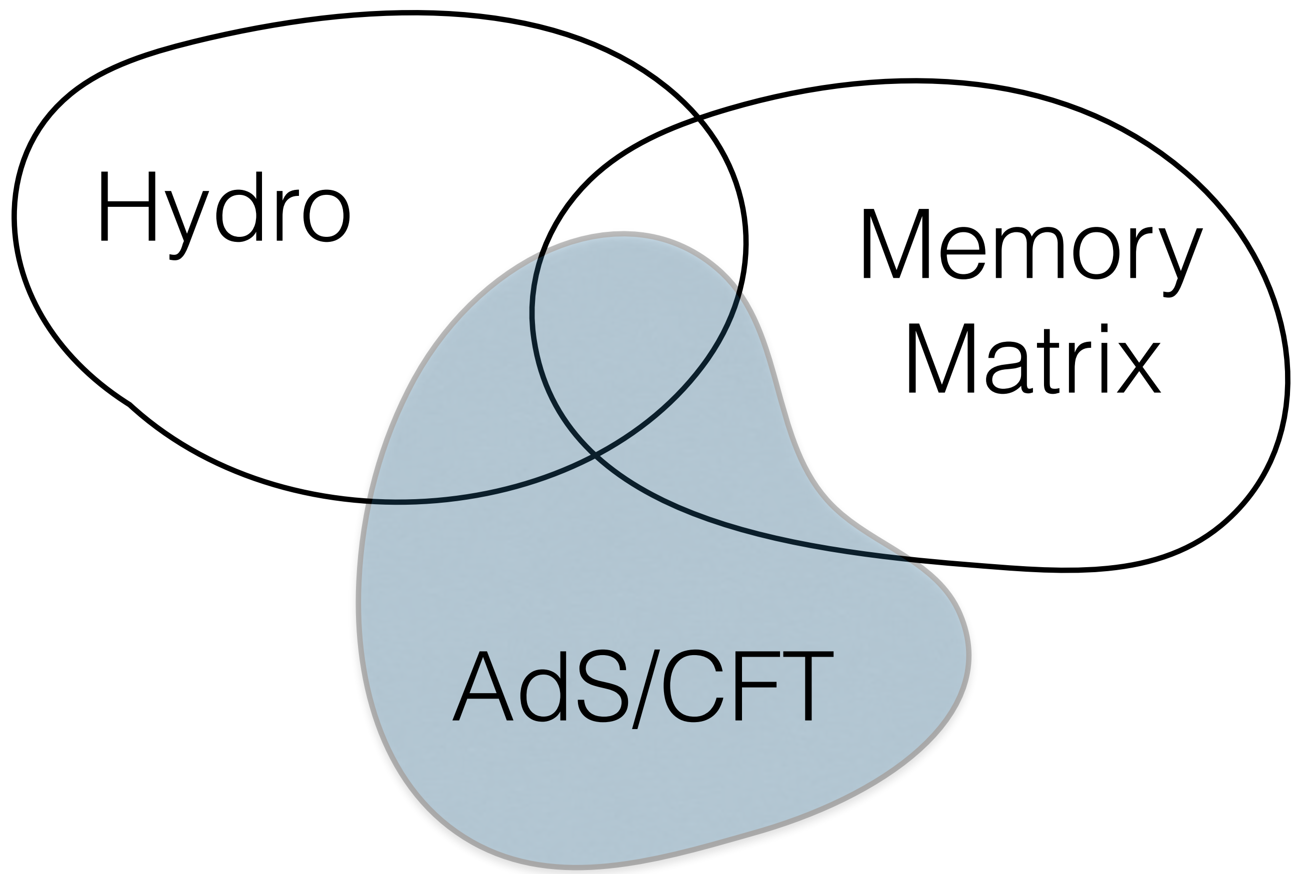
Recent Foci of AdS/CMT

- Transport
  - Linear response
  - Non-Linear, Steady State Flows
- Phase diagrams
  - Black Hole Instabilities
  - Back-reacted geometries
- Quenches

- Strange Metals realise strongly interacting states of matter without “quasi-particles”

- Long-lived modes are long wavelength perturbations of (almost) conserved charges and (pseudo) Goldstone modes

- Holography provides a theoretical framework to gain new insights



Underlying microscopic theory with familiar CMT limits (e.g. hydro, memory matrix formalism)

➡ Realise novel ground states of strongly coupled matter

Transport

- Experiments on transport of conserved charges probe collective degrees of freedom
- Introduce external electric field and temperature gradient
- Measure response currents as a function of frequency, temperature, external parameters

Transport

<u>Sources</u>

$$\vec{E} = \vec{E}_0 \, e^{-i\omega t}$$

$$\vec{\nabla} T = -T \, \vec{\zeta}_0 \, e^{-i\omega t}$$

<u>Response Currents</u>

$$\vec{J} = \vec{J}_0(\omega) \, e^{-i\omega t}$$

$$\vec{J}_Q = \vec{J}_{Q0}(\omega) \, e^{-i\omega t}$$

$$\begin{pmatrix} J_0^i \\ J_{Q0}^i \end{pmatrix} = \begin{pmatrix} \sigma^{ij} & T \, \alpha^{ij} \\ T \, \bar{\alpha}^{ij} & T \, \bar{\kappa}^{ij} \end{pmatrix} \begin{pmatrix} E_{0j} \\ \zeta_{0j} \end{pmatrix}$$

- The transport coefficients are directly related to retarded Green's functions

- The "open circuit" thermal conductivity is $\kappa = \bar{\kappa} - T \, \bar{\alpha} \, \sigma^{-1} \, \alpha$

- "DC" refers to the limit $\omega \to 0$

Quasi-Particle Hallmarks (Fermi Liquid)

- Quasi-Particles that carry heat / charge and are coupled to a lattice

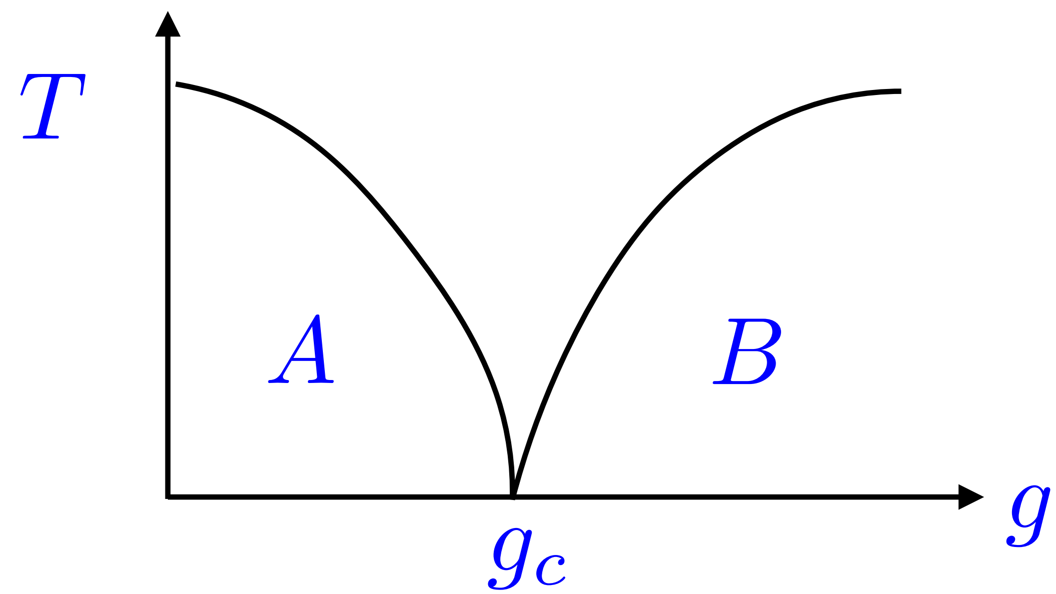
- The heat current follows the electric current (Wiedemann-Franz law) $L \equiv \kappa\,(\sigma\,T)^{-1} = \frac{\pi^2}{3}\frac{k_B^2}{q^2}$

- Quadratic resistivity $\sigma_{DC} \propto T^{-2}$

- Lower bound on σ_{DC} (Mott-Ioffe-Regel)

Strange / Bad metals violate all or some of these

Neutral Quantum Critical Points



Sachdev

- Strange Metals appear in the vicinity of QCPs

- Finite electric DC conductivity σ_Q from Quantum Critical, "incoherent" current at zero charge

Damle, Sachdev

- Recent experiments with Graphene at zero chemical potential

Crossno, Shi, Wang, Liu, Harzheim, Lucas, Sachdev, Kim, Taniguchi, Watanabe, Ohki, Fong

CFT Metals at Finite Density

$$\bar{\kappa}(\omega) = \frac{s^2 T}{\epsilon + P}\left(\delta(\omega) + \frac{i}{\omega}\right) + \frac{\mu^2}{T}\sigma_Q, \quad \sigma(\omega) = \frac{\rho^2}{\epsilon + P}\left(\delta(\omega) + \frac{i}{\omega}\right) + \sigma_Q$$

$$\alpha(\omega) = \frac{\rho s}{\epsilon + P}\left(\delta(\omega) + \frac{i}{\omega}\right) - \frac{\mu}{T}\sigma_Q$$

- Delta function due to momentum conservation

  Hartnoll, Kovtun, Muller, Sachdev **Hartnoll, Herzog** **Kovtun**

- The delta function drops out of certain combinations which probe the "incoherent" current

  - The open circuit thermal conductivity $\kappa_{DC} = \sigma_Q\,(sT + \mu\rho)^2/(T\rho^2)$
  - The diffusive current $J_{inc} \equiv \dfrac{sT\,J - \rho\,J_Q}{\epsilon + P}, \quad \sigma_{DC}^{inc} = \sigma_Q$

    Hartnoll, Davison, Gouteraux
  - The "heat free" electric conductivity $\sigma_{J_Q=0} = \sigma - \alpha\bar{\alpha}T/\bar{\kappa}$

    AD, Gauntlett

Momentum Relaxation Mechanism

$$S \to S + \int \phi(\mathbf{x})\,\mathcal{O}(\mathbf{x}) - \frac{1}{2}\int \delta g_{\mu\nu}(\mathbf{x})\,T^{\mu\nu}(\mathbf{x}) + \int A_t(\mathbf{x})\,J^t(\mathbf{x})$$

- Add time-independent sources which break translations to form a momentum relaxing “lattice”

- Compute retarded Green’s functions for the currents

Universal “lattice” sources:

- Strain (stress tensor)

Herzog, Balasubramanian **AD, Gauntlett, Pantelidou**

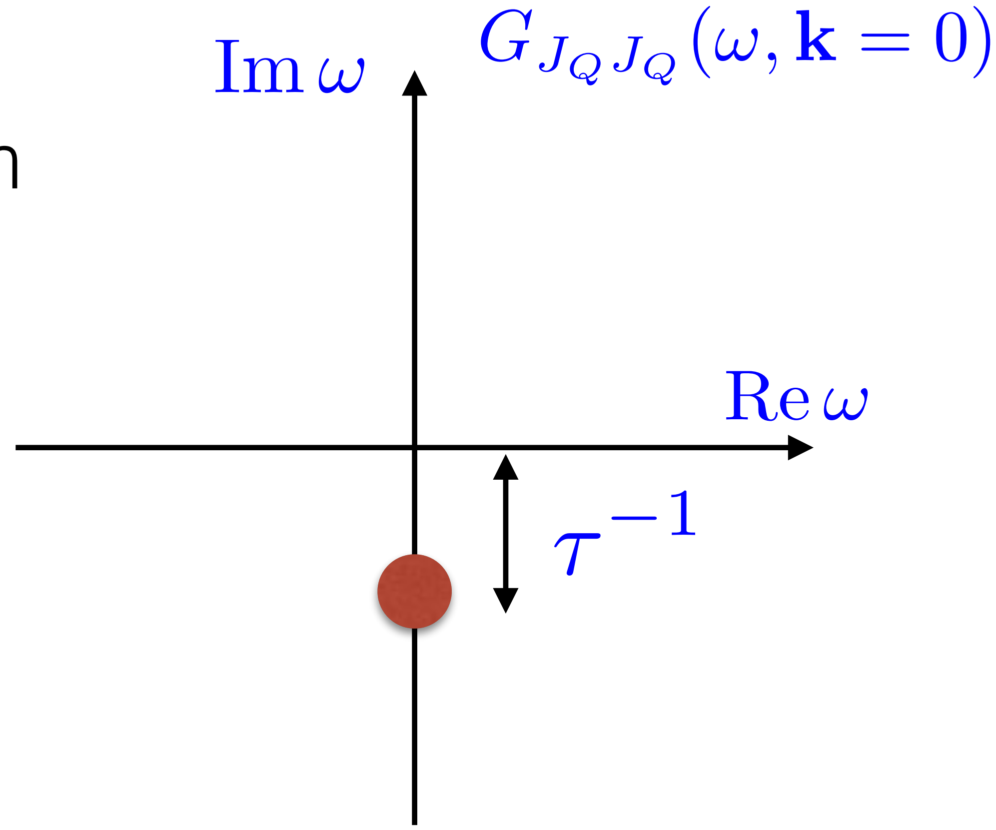
AD, Gauntlett, Griffin, Melgar **Scopelliti, Schalm, Lucas**

- Chemical potential (charge density)

Hartnoll, Hofman **Horowitz, Santos, Tong** **Chesler, Lucas, Sachdev** **Lucas**

Momentum Relaxation

- Breaking translations moves the momentum conservation pole down the imaginary axis and resolves the delta function



$$\delta(\omega) + \frac{i}{\omega} \to \frac{1}{-i\omega + \tau^{-1}}$$

- The momentum relaxation rate τ^{-1} is function of the lattice couplings, temperature and chemical potential

- DC transport is dominated by τ when large compared to microscopic timescales

- Relation between τ and the zero frequency spectral weight of "Lattice" operator

Hartnoll, Hofman

Corners of Universality?

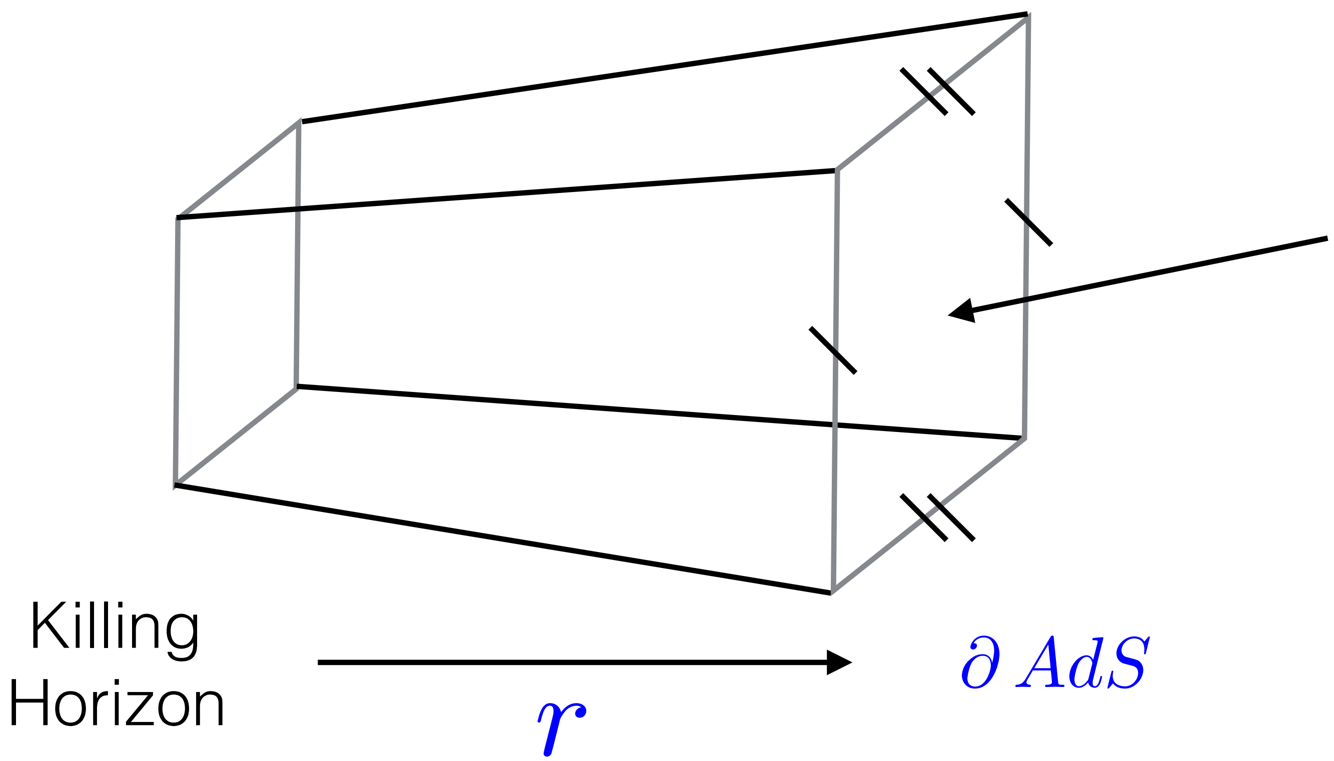
- Break translations weakly and examine incoherent current (e.g. κ_{DC} at finite density)
  - In the DC limit the momentum relaxation time drops out
  - Remnant "Wiedemann-Franz" law $\bar{L} \equiv \bar{\kappa}\,(\sigma\,T)^{-1} = \dfrac{s^2}{\rho^2}$

    Mahajan, Barkeshill, Hartnoll
  - Hall angle dominated by $\theta_H \sim B\,\rho^{-1}\,\sigma_{DC}$

    Hartnoll, Kovtun, Muller, Sachdev **Lucas, Sachdev**
- Break translations strongly and examine DC currents
  - All timescales become microscopic
  - Transport becomes incoherent
  - Existence of "Planckian" bounds on diffusion?

    Hartnoll

Holographic Lattices



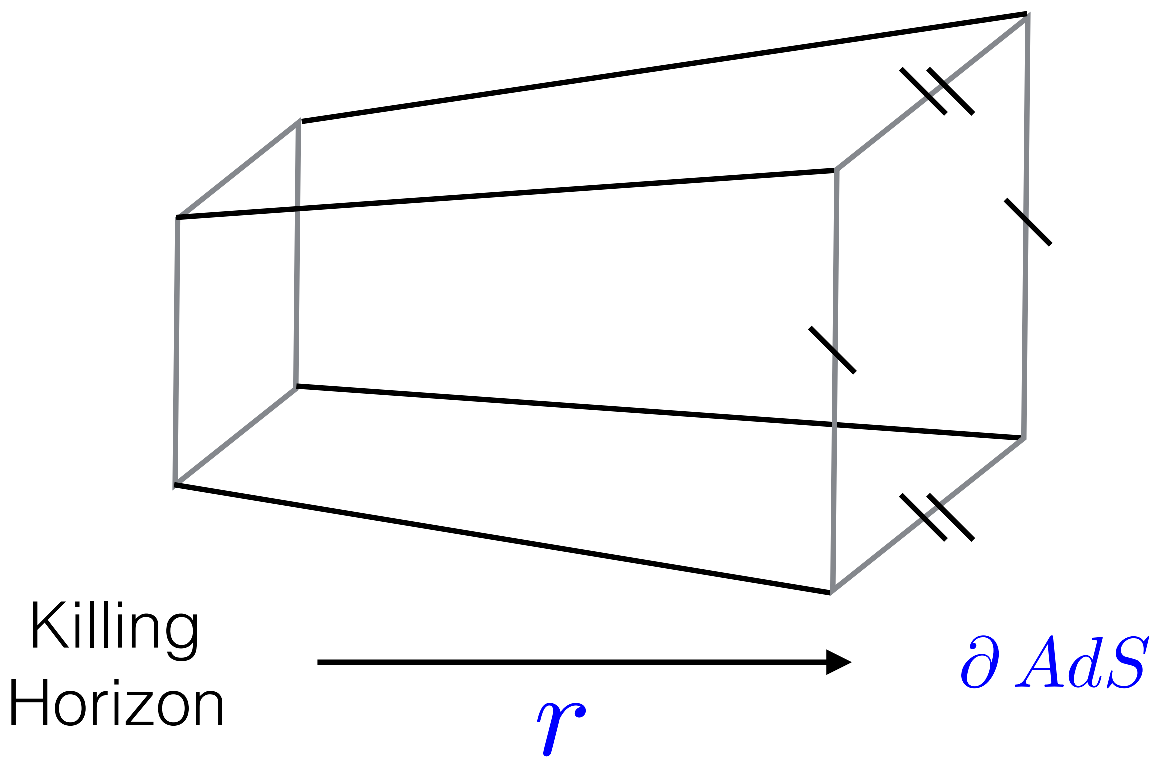
$$A_t(t,\mathbf{x},r) \to \mu(\mathbf{x}) + \cdots$$

$$g_{ij}(t,\mathbf{x},r) \to r^2\, g_{ij}(\mathbf{x}) + \cdots$$

$$\varphi(t,\mathbf{x},r) \to r^{\Delta_\varphi - d}\varphi(\mathbf{x}) + \cdots$$

- Holography realises the thermal state as a black brane solution
- Impose periodic boundary conditions which break translations

Holographic Lattices

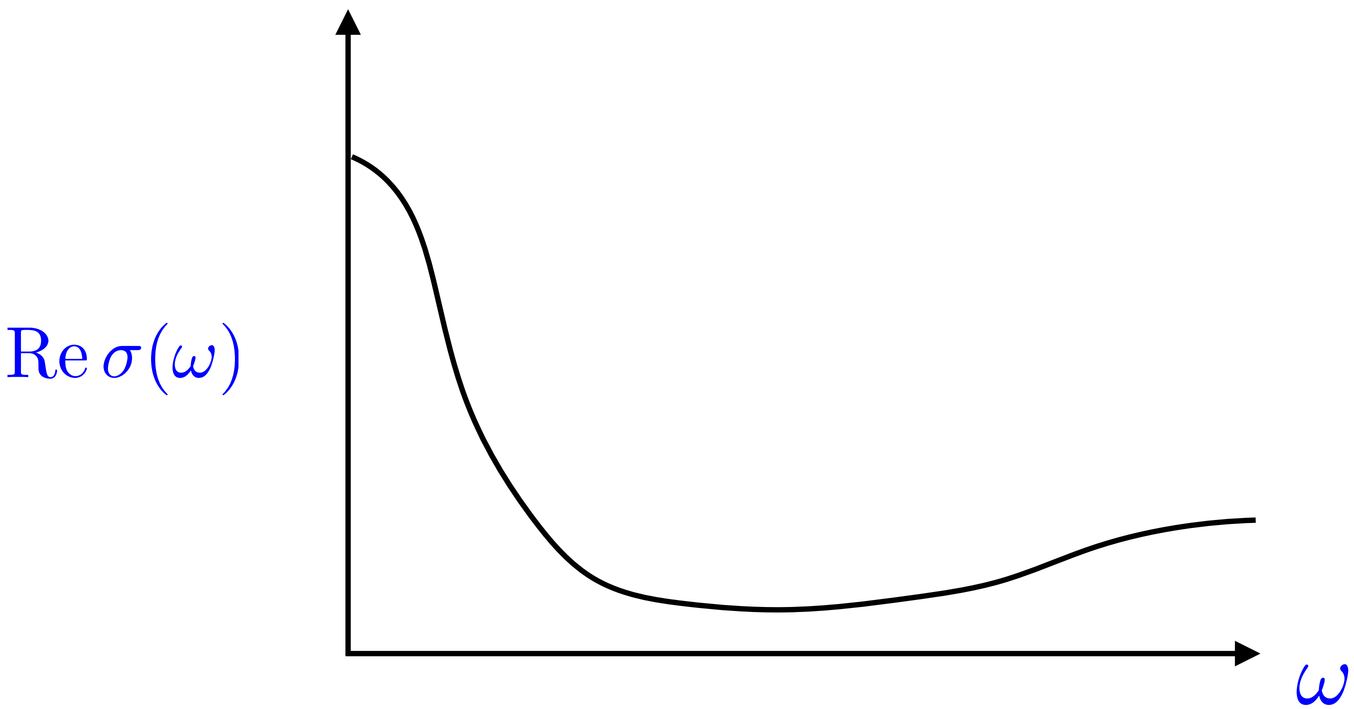


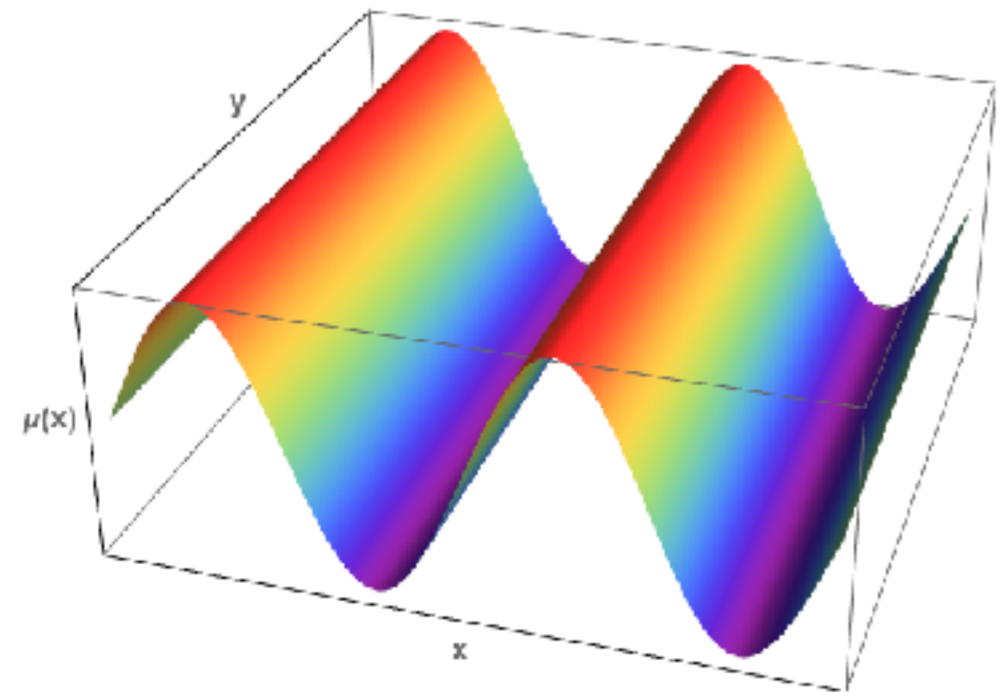
$$\vec{E}(t) = e^{-i\omega t}\,\vec{E}$$

$$\vec{\nabla}T(t) = -e^{-i\omega t}\,T\,\vec{\zeta}$$

- Perturb by boundary electric field and temperature gradient
- Construct perturbation in the bulk and read off electric and heat current from the fall-offs

Weak Momentum Relaxation



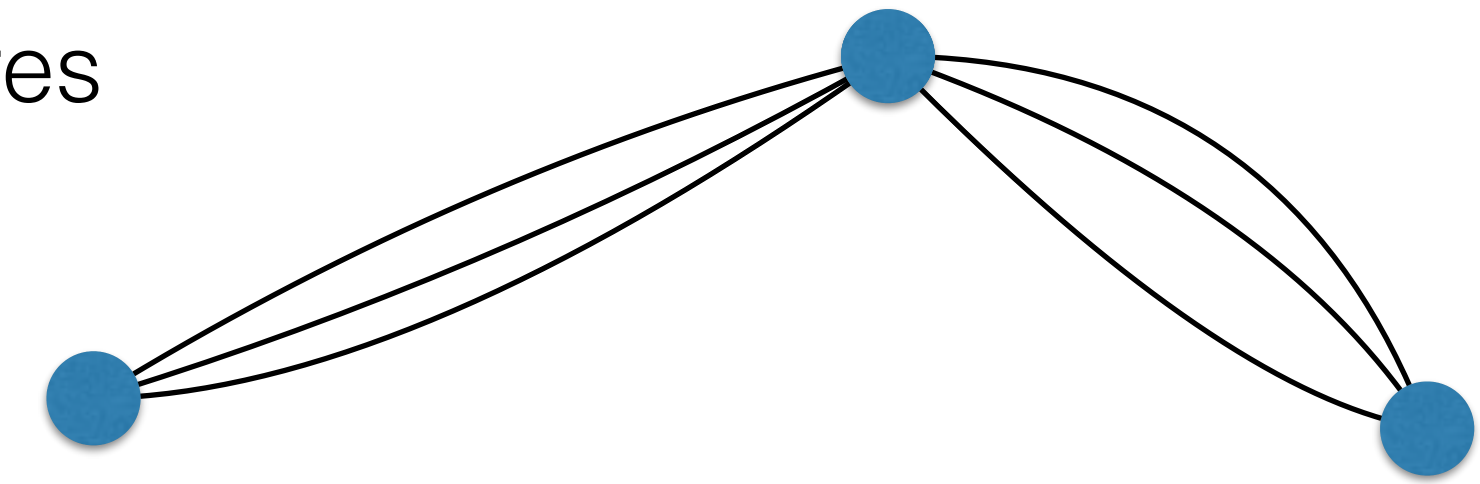


- Hard numerical problem **Horowitz, Santos, Tong** **Chesler, Lucas, Sachdev** **AD, Gauntlett**
- First examples governed by weak momentum relaxation at low temperatures

Strong Momentum Relaxation

UV

Horizon restores translations



Incoherent

- $AdS_2 \times \mathbb{R}^d$
- Lifshitz
- Hyperscale Violating

Kachru, Liu, Mulligan

Charmousis, Gouteraux, Kim, Kiritsis, Meyer

Taylor

Huijse, Sachdev, Swingle

AD, Hartnoll

AD, Gouteraux, Kiritsis

AD, Gauntlett

Gouteraux

Andrade, Withers

- Strong lattices can drastically change the RG flow

  AD, Hartnoll **AD, Gauntlett**

- Novel horizons which don't restore translations lead to incoherent transport

DC Transport

- Introduce constant electric field E_j and temp gradient ζ_j
- Time independent, divergence free, periodic current densities $J^i(x,y)$ and $J_Q^i(x,y)$ will form in e.g. 2+1 dimensions
- The relevant quantity to transport is the total fluxes of currents $\bar{J}^i$ and $\bar{J}_Q^i$
- Fluxes fix the transport coefficients

e.g. $\bar{J}^x = \int dy\, J^x(x,y)$

$$\bar{J}^i = \sigma_{DC}^{ij}\, E_j + T\alpha_{DC}^{ij}\, \zeta_j$$

$$\bar{J}_Q^i = T\,\bar{\alpha}_{DC}^{ij}\, E_j + T\bar{\kappa}_{DC}^{ij}\, \zeta_j$$

AD, Gauntlett

Lucas

Holographic DC Transport

AD, Gauntlett

$$ds^2 = g_{tt}(r,\mathbf{x})\,dt^2 + ds^2_M(r,\mathbf{x})$$
$$A = a_t(r,\mathbf{x})\,dt$$
$$\varphi = \varphi(r,\mathbf{x})$$

$$+$$

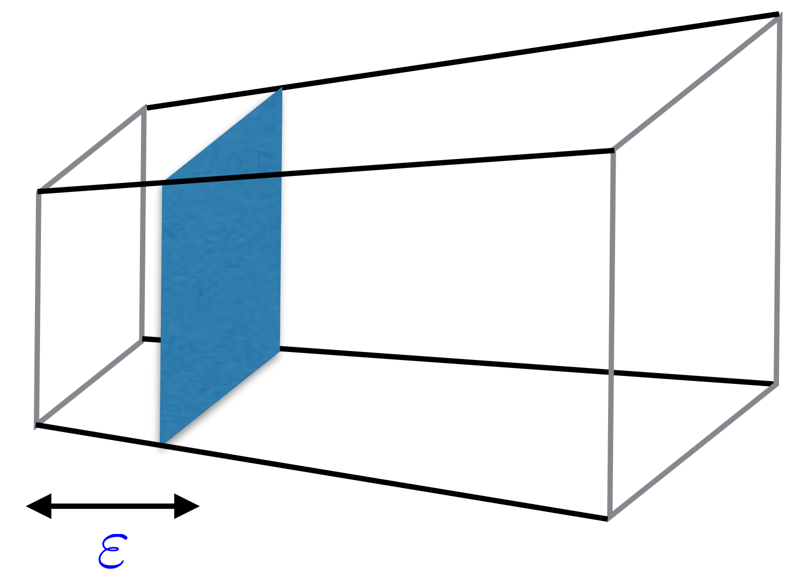
$$\delta g_{\mu\nu}(r,\mathbf{x})dx^\mu dx^\nu$$
$$\delta a_\mu(r,\mathbf{x})\,dx^\mu$$
$$\delta\varphi(r,\mathbf{x})$$

- For simplicity assume static background
- In the bulk this translates to looking for perturbative stationary solutions that preserve ∂_t
- There are current densities J^i_h and J^i_{Qh} on the horizon with fluxes equal to the field theory ones
- Similar to Komar / Smarr formulae $\bar{J}^i = \bar{J}^i_h, \quad \bar{J}^i_Q = \bar{J}^i_{Qh}$
- Extends earlier hints Kovtun, Son, Starinets Iqbal, Liu

Holographic DC Transport

AD, Gauntlett

$$\delta g_{ti}(\varepsilon,\mathbf{x}) = -v_i(\mathbf{x}) + \mathcal{O}(\varepsilon)$$
$$\delta a_t(\varepsilon,\mathbf{x}) = w(\mathbf{x}) + \mathcal{O}(\varepsilon)$$
$$\delta g_{tr}(\varepsilon,\mathbf{x}) = (4\pi T)^{-1} p(\mathbf{x}) + \mathcal{O}(\varepsilon)$$



$$J_h^i = \frac{s}{4\pi}\left(\partial^i w + E^i\right) + \rho\, v^i$$

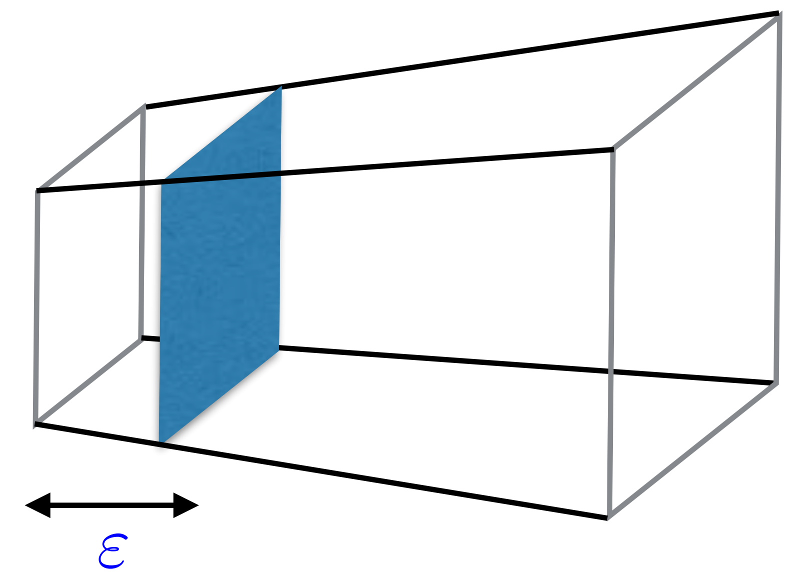
$$J_{Qh}^i = T s\, v^i, \quad s = 4\pi\sqrt{g_{(0)}}, \quad \rho = \sqrt{g_{(0)}}\, a_t^{(0)}$$

- Impose in-falling conditions close to the horizon
- Constitutive relations expressing horizon currents in terms of near horizon expansion coefficients
- Einstein-Maxwell as illustrative example

Holographic DC Transport

AD, Gauntlett

$$H = \int N_\mu \mathcal{H}^\mu + D\,\mathcal{G} + \text{Surface Terms}$$



$$\mathcal{H}^t \Rightarrow \quad \nabla_i v^i = 0$$

$$\mathcal{G} \Rightarrow \quad \nabla^2 w + v^i \nabla_i a_t^{(0)} = -\nabla_i E^i$$

$$\mathcal{H}^j \Rightarrow \quad 2\nabla^i \nabla_{(i} v_{j)} + a_t^{(0)} \nabla_j w - \nabla_j p = -4\pi T\,\zeta_j - a_t^{(0)} E_j$$

- Expanding constraints gives a closed system of equations
- Horizon current densities are fixed by a Stokes flow for an auxiliary, steadily driven fluid on the curved horizon
- Not a derivative expansion!
- Exact utilisation of the old membrane paradigm in holography

Damour **Thorne, Price**

Holographic DC Transport

- More general matter fields

  Banks, AD, Gauntlett

  - Additional "friction" terms $\partial_i\varphi\,\partial_j\varphi\,v^i$ in Stokes equations

- Broken time reversal symmetry - magnetic fields and background magnetisation currents

  AD, Gauntlett, Griffin, Melgar   AD, Gauntlett, Griffin, Lohitsiri, Melgar

  - Additional "Lorentz" and "Coriolis" force terms

  - The relation between the boundary and the horizon fluxes is modified by magnetisation terms

$$\bar{J}^i = \bar{J}^i_h - M_E^{ij}\,\zeta_j\,, \qquad \bar{J}^i_Q = \bar{J}^i_{Qh} - M_E^{ij}\,E_j - 2\,M_T^{ij}\,\zeta_j$$

  - The horizon currents are the correct transport currents!

    Cooper, Halperin, Ruzin

- Higher derivative gravity (e.g. Gauss-Bonnet)

  AD, Gauntlett, Griffin, Melgar

Holographic DC Transport

AD, Gauntlett

$$\nabla_i v^i = 0$$

$$\nabla^2 w + v^i \nabla_i a_t^{(0)} = -\nabla_i E^i$$

$$2\,\nabla^i \nabla_{(i} v_{j)} + a_t^{(0)} \nabla_j w - \nabla_j\, p = -4\pi T\, \zeta_j - a_t^{(0)} E_j$$

- Remnant “Wiedemman-Franz” law for perturbative breaking of translations

$$\bar{\kappa} \approx L^{-1}\, 4\pi s T \qquad \alpha \approx L^{-1}\, 4\pi \rho \qquad \sigma \approx L^{-1}\, 4\pi \rho^2 / s$$

$$\bar{L} = \bar{\kappa}/(T\,\sigma) \approx \frac{s^2}{\rho^2}$$

Holographic Bounds on Conductivity

- Bounds of conductivities extracted from hydro flows

  Lucas

- Applicable to holographic DC conductivities due to similarity to steady fluid flows

- Einstein-Maxwell in D=4 **Grozdanov, Lucas, Sachdev, Schalm**

$$\sigma_{DC} \geq e^{-2}$$

- Einstein-Maxwell-Scalar in D=4 with potential $V(\varphi)$ and global minimum V_{min} **Grozdanov, Lucas, Schalm**

$$T^{-1}\,\kappa_{DC} \geq 8\,\pi^2\,(6 - V_{min})^{-1}$$

Holographic DC Transport

To think about:

- Hydrodynamic/Diffusive modes

  AD, Gauntlett, Ziogas

- Treatment of Supercurrents / Goldstone modes

- Finite but Small Frequency

- What else does the horizon fix?

Holography ➜ Hydrodynamics

- Within fluid / gravity the exact current fluxes from the horizon Stokes flow come from summing an infinite hydro expansion

- Possible to examine the hydrodynamic limit of lattice by:

  - By directly examining the conductivities

    Davison, Gouteraux

  - By systematically performing the hydrodynamic expansion

    Blake

- Clarified the relation between the coefficient σ_Q and holographic formulae for DC conductivity

- Comparison with intuition from treatment of weak momentum relaxation in relativistic hydrodynamics

  Hartnoll, Kovtun, Muller, Sachdev

Q-lattices

$$\mathcal{L} = R - \frac{1}{2}\left(\partial\varphi\right)^2 - \frac{1}{2}\Phi\left(\varphi\right)\left[\left(\partial\chi_1\right)^2 + \left(\partial\chi_2\right)^2\right] + V\left(\varphi\right) - \frac{Z\left(\varphi\right)}{4}F^2$$

$$ds_4^2 = -U(r)\,dt^2 + U(r)^{-1}\,dr^2 + e^{2\,V_1(r)}\left(dx^2 + dy^2\right)$$

$$A = a(r)\,dt, \quad \varphi = \varphi(r)$$

$$\chi_1 = k\,x, \quad \chi_2 = k\,y$$

- Simple models which retain homogeneous metric and relax momentum
- Require global U(1)'s or shift symmetries in the bulk

AD, Gauntlett **Andrade, Withers**

- Translations are restored when either

$\Phi(\varphi) \to 0$ $\quad k \to 0$ (Lattice hydrodynamic limit)

- Higher derivative Q-lattices - Linear Axions

Baggioli, Pujolas **Gouteraux, Kiritsis, Li**

DC Transport for Q-lattices

- The Stokes flow equations simplify to algebraic equations

- Possible to manipulate the bulk ODEs and express DC conductivities in terms of horizon data (Historically happened first!)

$$\sigma_{DC} = Z(\varphi_h) + \frac{4\pi\rho^2}{k^2\Phi(\varphi_h)s}$$

Blake, Tong **Andrade, Withers** **Gouteraux**

$$\alpha_{DC} = \frac{4\pi\rho}{k^2\Phi(\varphi_h)} \qquad \bar{\kappa}_{DC} = \frac{4\pi sT}{k^2\Phi(\varphi_h)}$$

AD, Gauntlett

DC Transport for Q-lattices

- In Cuprates θ_H and σ_{DC} scale differently

- Similar in spirit expressions in the presence of magnetic fields

  AD, Blake **AD, Blake, Lohitsiri**

- For perturbative background magnetic fields

$$\sigma_{DC} = Z(\varphi_h) + \frac{4\pi\rho^2}{k^2\Phi(\varphi_h)s}$$

$$\equiv \sigma_{ccs} + \sigma_{diss}$$

$$\theta_H \propto \frac{B}{\rho}\sigma_{diss}$$

- Can have $\sigma_{ccs} \gg \sigma_{diss}$ at low temperatures

- The Hall angle and the conductivity can scale differently with temperature

  AD, Blake

Q-Lattice Incoherent Ground States

$$\Phi(\varphi) \sim e^{\delta_i \varphi}, \quad Z \sim e^{\gamma\varphi}, \quad V \sim e^{\alpha\varphi}$$

$$ds^2 \approx \rho^{-(2-\theta)} \left(-\rho^{-2(z-1)} d\bar{t}^2 + d\rho^2 + d\bar{x}^2 + d\bar{y}^2\right)$$
$$\varphi \approx s \ln \rho, \quad \chi_1 = \bar{k}\,\bar{x}, \quad \chi_2 = \bar{k}\,\bar{y}$$

- Simple examples of ground states with broken translations

  AD, Gauntlett **Gouteraux**

- Electric charge is carried by irrelevant modes
- Metric similar to translationally invariant hyperscale violating
- Can be found "top down"

Azeyanagi, Li, Takayanagi **Mateos, Trancanelli**

AD, Gauntlett, Rodriguez Sosa **AD, Gauntlett, Rodriguez Sosa, Rosen**

Q-Lattice Incoherent Ground States

Can study cold black hole horizons and find behaviour of transport coefficients:

- Heat and electric currents scale independently

  AD, Gauntlett

- Violation of “Motte-Ioffe-Regel” bounds. Can become power law insulators at low temps.

  AD, Gauntlett **Gouteraux**

- The Hall angle and electric conductivity scale differently

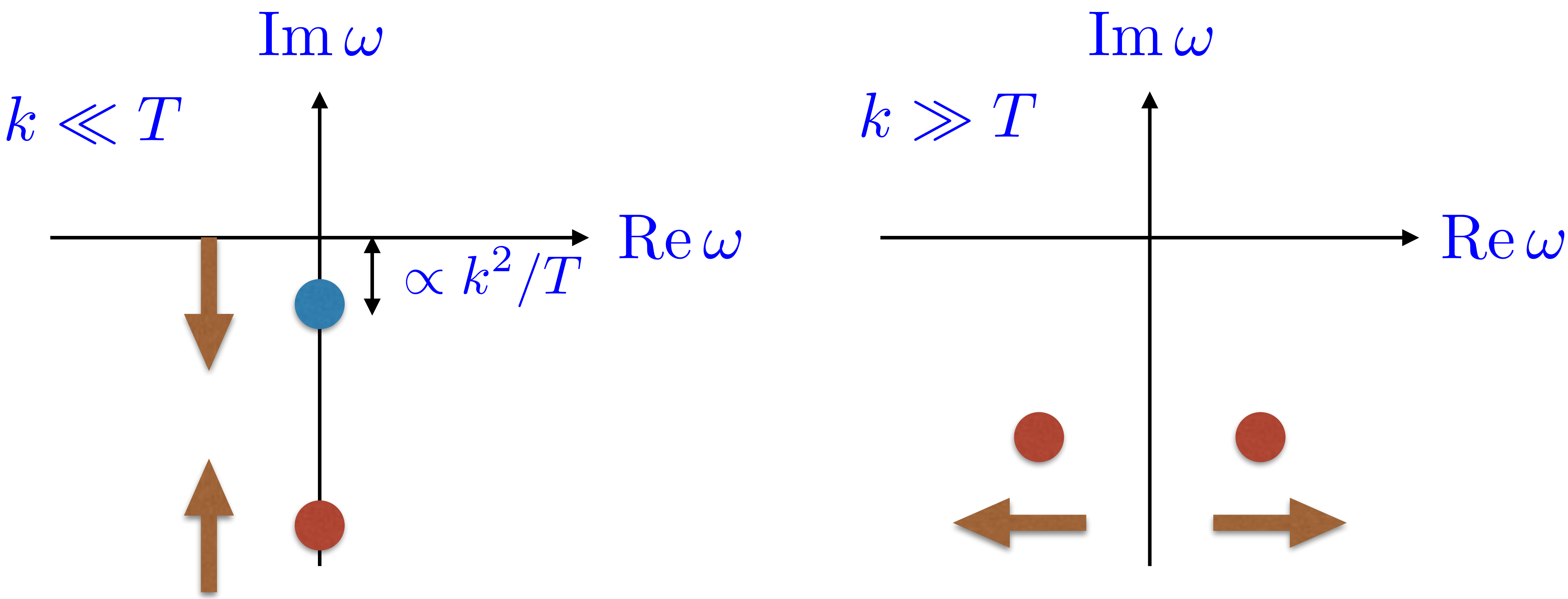
  AD, Blake

- Initial effort to match the temperature scaling of the Cuprates $\sigma \propto T^{-1}, \quad \theta_H \propto T^{-2}$

  Amoretti, Baggioli, Magnoli, Musso

Transition to Incoherent Transport

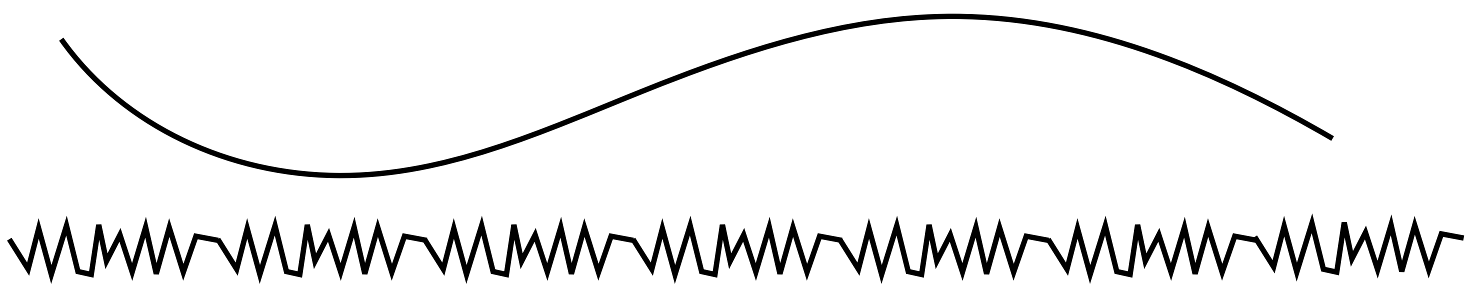
Davison, Gouteraux



- Consider linear axion model at zero charge with dimensionless parameter k/T
- Examine the transition through the pole structure of $\kappa(\omega)$
- Transition when $k \approx T$

Incoherent Hydrodynamics

Hartnoll



$$E,\ \rho \propto e^{-i\omega_\pm(k)t+ikx}$$

$$\omega_\pm(k) = -i\,D_\pm\,k^2$$

- With broken translations, long wavelength excitations of charge and energy become coupled and diffusive

- Diffusion with quasiparticles is fixed by a Fermi velocity and timescale related to the mean free path $D_{FL} \sim v_F^2\,\tau_{FL}$

- For incoherent metals the Mott-Ioffe-Regge bound should be replaced by $D_\pm \gtrsim v^2\,\tau_{eff} \sim v^2\,T^{-1}$ Hartnoll

- Similar to $\eta/s \gtrsim C$ Kovtun, Son, Starinets

Incoherent Hydrodynamics

Hartnoll

- A “standard” hydrodynamics arguments yields the modes

$$\delta\mu(t,\mathbf{x}) = \delta\mu_0\, e^{-i\omega t + i\,\mathbf{k}\mathbf{x}} \qquad \delta T(t,\mathbf{x}) = \delta T_0\, e^{-i\omega t + i\,\mathbf{k}\mathbf{x}}$$

$$D_+ \, D_- = \frac{\sigma}{\chi}\frac{\kappa}{c_\rho} \qquad \omega_\pm(\mathbf{k}) = -i\, D_\pm\, \mathbf{k}^2$$

$$D_+ + D_- = \frac{\sigma}{\chi} + \frac{\kappa}{c_\rho} + \frac{T}{c_\rho \chi^2 \sigma}\left(\zeta\,\sigma - \chi\,\alpha\right)^2$$

- The diffusion constants are fixed in terms of conductivities and thermodynamic susceptibilities

- Easy to extract in holographic theories

Diffusion and Chaos

Blake

- Which scales set $D_{\pm} \sim v^2 \, \tau$ for incoherent transport?

- Appealing proposal to identify v with the butterfly velocity v_B and a Planckian timescale $\tau \sim 1/T$

  Blake

- Intuition comes from Q-lattice - Linear axion models

$$ds^2 = -U(r)\, dt^2 + \frac{dr^2}{U(r)} + V(r)\, d\mathbf{x}_d^2$$

- The butterfly velocity is given terms of horizon data

$$v_B = \frac{4\pi T}{d\, V'(r_h)}$$

Shenker, Stanford

Roberts, Stanford, Susskind

Blake

Roberts, Swingle

Diffusion and Chaos

Blake

- The Q-Lattices can flow to families of ground states which are incoherent or restore translations
- The diffusion constants simplify at low temperatures
- By inspection, for all these cases:

$$D_T \equiv D_- = \kappa/c_\rho = E\,\frac{v_B^2}{2\pi T}$$

With $E > 1/2$ a temperature independent number

Blake AD, Blake Davison, Fu, Georges, Gu, Jensen, Sachdev Blake, Davison, Sachdev

Diffusion and Chaos

- Similar results from non-Einstein Gravity dual models (?)
  - Sachdev-Ye-Kitaev chain models

    Gu, Stanford, Qi **Davison, Fu, Georges, Gu, Jensen, Sachdev**
  - Systems of Electrons/Phonons

    Werman, Kivelson, Berg
  - Bose-Hubbard Models

    Bohrdt, Endrel, Mendes, Knap
  - O(N) Models

    Chowdhury, Swingle
  - Critical Fermi Surface $D_T \sim v_B^2 \, \lambda_L^{-1}$

    Patel, Sachdev
  - Diffusive Metals

    Patel, Chowdhury, Sachdev, Swingle
- Food for thought
  - Sachdev-Ye-Kitaev chain with varying inter-site couplings

    Gu, Lucas, Qi

Conclusions / Outlook

- Discussed Transport and Momentum Relaxation in Holography
- DC conductivities from black hole horizons
- Holographic Ground States of Incoherent Transport
- Hints of Universality for Incoherent Transport
- Vast variety of Ground States with broken translations

Nakamura, Ooguri, Park　　**Ooguri, Park**　　**AD, Gauntlett**　　**Bergman, Jokela, Lifschytz, Lippert**

AD, Gauntlett, Pantelidou　　**Withers**　　**Iizuka, Kachru, Kundu, Narayan, Sircar, Trivedi**

Rozali, Smyth, Sorkin, Stang　　**Cremonini, Li Li, Jie Ren**　　**Cai, Li Li, Wang, Zaanen**　　**Andrade, Krikun**