Gravitational collapse in SYK model and Choptuik-like phenomenon

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with

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The SYK model is a system of N Majorana fermions and its Hamiltonian is

$$H_0 = - \sum_{1 \le a < b < c < d \le N} j_{abcd} \; \hat{\psi}_a \hat{\psi}_b \hat{\psi}_c \hat{\psi}_d. \tag{1}$$

- The eigenstates of the "Chirality operator" $\Gamma = (2i)^{N/2} \left(\prod_{j=1}^{N} \hat{\psi}_j \right)$ forms a complete basis for the SYK Hilbert space.
- We will denote these normalized states as

$$|B_{\mathbf{s}}\rangle = |\pm, \pm, \cdots, \pm\rangle, \quad \text{which is a state of } N/2 \text{ spins with the spin vector} \quad \mathbf{s} = (\pm, \pm, \cdots, \pm), \tag{2}$$

- We denote states obtained from a $|B_{\mathbf{s}}\rangle$ by evolving it along imaginary time for a distance l with H_0 as

$$|B_{\mathbf{s}}(l)\rangle = e^{-lH_0} \, |B_{\mathbf{s}}\rangle \quad \forall \text{ spin vectors } \mathbf{s}. \tag{3}$$

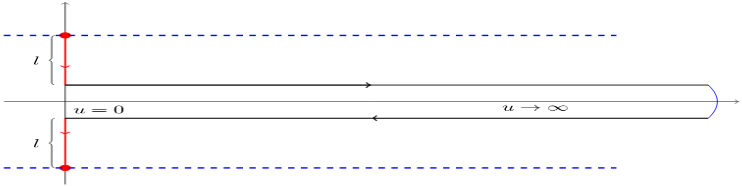
- The SYK model was modified in a paper by Maldacena and Kourkoulou by adding a spin dependent operator (1707.02325)

$$H = H_0 + \hat{\epsilon}(u) \; H_M^{(\mathbf{s}')}, \quad H_M^{(\mathbf{s}')} = -iJ \sum_{k=1}^{N/2} s'_k \hat{\psi}_{2k-1} \hat{\psi}_{2k}, \quad u \text{ is time.} \tag{4}$$

- *We are interested in the classical dynamics of the Schwarzian degree of freedom in this modified theory when we work in a $|B_{\mathbf{s}}(l)\rangle$ state.*

Modified dynamics of Schwarzian mode.

- The path integral contour in the complex time plane for the expectation values of operators in a $|B_{\mathbf{s}}(l)\rangle$ is



- At large N, after disorder averaging, the system is symmetric under Flip group (subgroup of $O(N)$). This symmetry allows us to convert the $|B_{\mathbf{s}}\rangle$ boundary conditions into trace boundary conditions when we consider expectation values of Flip invariant operators.

In the near conformal limit

- The dynamics along the euclidean part (red segments) of the contour is governed by Schwarzian action.
- The action along the real time part of the contour is modified due to the added interaction. In a convenient variable $\phi(u)$ which is related to the Schwarzian mode, $t(u)$ by $\exp(\phi) = t'(u)$, this modified action is given by

$$S = \frac{N\alpha}{J} \int du \left[\frac{\phi'^2}{2} + \frac{J\lambda(u)}{2} \left(e^{\phi} - t' \right) + \frac{J^2 \epsilon(u)}{2} \, e^{\phi/2} \right]. \tag{5}$$

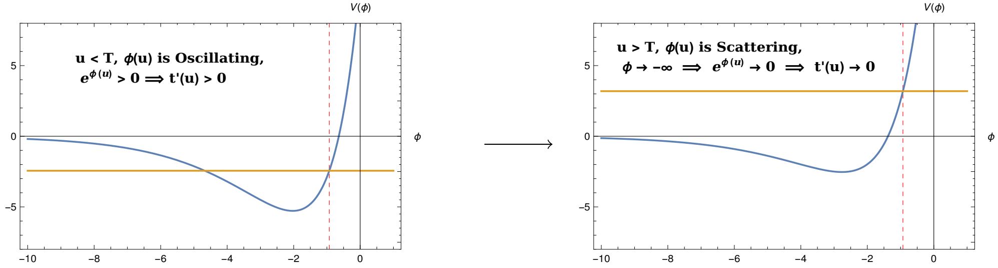
Classical dynamics of the Schwarzian mode in $|B_{\mathbf{s}}(l)\rangle$ state.

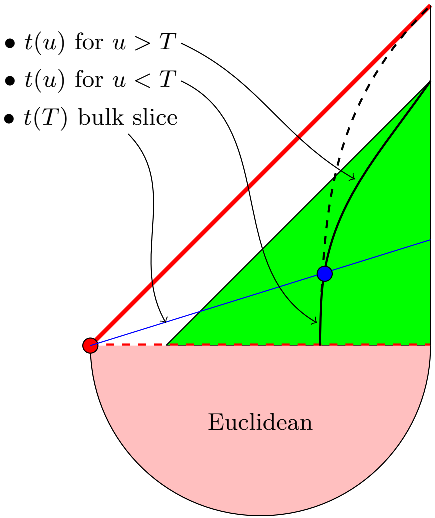
- This modified action is that of a particle in a potential well, with

$$V(\phi) = 2J^2 e^{\phi} - \left(\frac{J^2 \epsilon(u)}{2}\right) e^{\phi/2}, \text{ here } e^{\phi(u)} = t'(u), \text{ with } t(u) \text{ is the Schwarzian mode.} \tag{6}$$

- Classically, given initial conditions $\phi(u_0)$, $\phi'(u_0)$. It was found that ϵ has a critical value ϵ_{cr} such that when $\epsilon > \epsilon_{cr}$ we get oscillating solution and when $\epsilon < \epsilon_{cr}$ we have scattering solution.

- We will take a step function profile for $\epsilon(u)$ (as is considered in quantum quench scenarios), where $\epsilon(u)$ suddenly changes from ϵ_1 to ϵ_2 at $u = T$. The ϵ_1 and ϵ_2 will be chosen such that $\epsilon_1 > \epsilon_{cr}$ giving us oscillatory behaviour when $u < T$ and $\epsilon_2 < \epsilon_{cr}$ giving us scattering behaviour when $u > T$.





The Schwarzian dynamics is dual to the boundary dynamics in the bulk.

- The boundary curve is defined by $(t, z) = (t(u), \delta\ t'(u))$, where $t(u)$ is the Schwarzian mode and bulk coordinates are (t, z).
- If for some u, $t'(u) = 0$, then the boundary curve touches $z = 0$ (solid boundary curve), which implies an existence of a horizon. The light rays from the white region cannot reach the solid boundary curve.
- If $t'(u) \neq 0$ for any u, then the geometry does not have a horizon (dashed boundary curve).
- The change of $\epsilon(u)$ at $u = T$ changes behaviour of $\phi(u)$ from oscillating to scattering when $\phi(u)$ has a scattering behaviour, $t'(u) \to 0$, creating a horizon for the boundary observer.

Choptuik-like transition.

- The Schwarzian mode, $t(u)$ for the step function quench profile is

$$t(u) = A \, \frac{\sinh\left(|c_2|\,(u-T)\right)}{\left(b_2 \cosh\left(|c_2|\,(u-T)\right) - J\epsilon_2\right)} + B \, \tan^{-1}\left(C \; \tanh\left(\frac{1}{2}\left(|c_2|\,(u-T)\right)\right)\right) \xrightarrow{u\to\infty} \sim \tanh\left(\frac{\pi}{\beta_{\text{eff}}}\, u\right)$$

  which is also the value of the Schwarzian mode for a black hole with inverse temperature β_{eff}, from here we extract the ϵ_2 dependence of the black hole temperature.

$$\boxed{T_{bh} = \beta_{\text{eff}}^{-1} \sim \left(\epsilon_{cr} - \epsilon_2\right)^{1/2}} \tag{7}$$

- This critical power law is akin to Choptuik Phenomenon. For $\epsilon_2 > \epsilon_{cr}$, black hole does not form.
- This temperature can also be read off from the rate of thermalization of the two point function.
- *In this model, an exponential fine tuning is required between the state and the perturbation, it is only then that the perturbation can contribute to the large N physics hence this is a perturbation by a state-dependent operator. This was required in order to get the horizoneless geometry. We found critical collapse in the same model with a different sign of the state-dependent operator however a generic gravitational collapse does not require any such fine tuning.*

Thank you!