Exact Bulk Reconstruction, Bulk Locality, and Black Hole Horizons

Hongbin Chen（陈宏彬）

Johns Hopkins University

Based on arXiv:
1708.04246,
1712.02351,
1810.02436,
1905.00015

Collaboration with
Nikhil Anand, Liam Fitzpatrick,
Jared Kaplan, Daliang Li,
Utkarsh Sharma

Strings 2019

Motivation and Approach

- **Goal**: Use 2d CFTs to understand 3d Quantum Gravity (especially non-perturbative effects)
  - How does bulk locality break down?
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◉ **Today's talk**: focus on the bulk proto-field ϕ, i.e. a scalar field interacting with gravity

Bulk Proto-fields in AdS_3/CFT_2 at Finite c

A bulk scalar proto-field in AdS_3 is fixed uniquely by Virasoro symmetry:

1708.04246 Anand, HC, Fitzpatrick, Kaplan, Li

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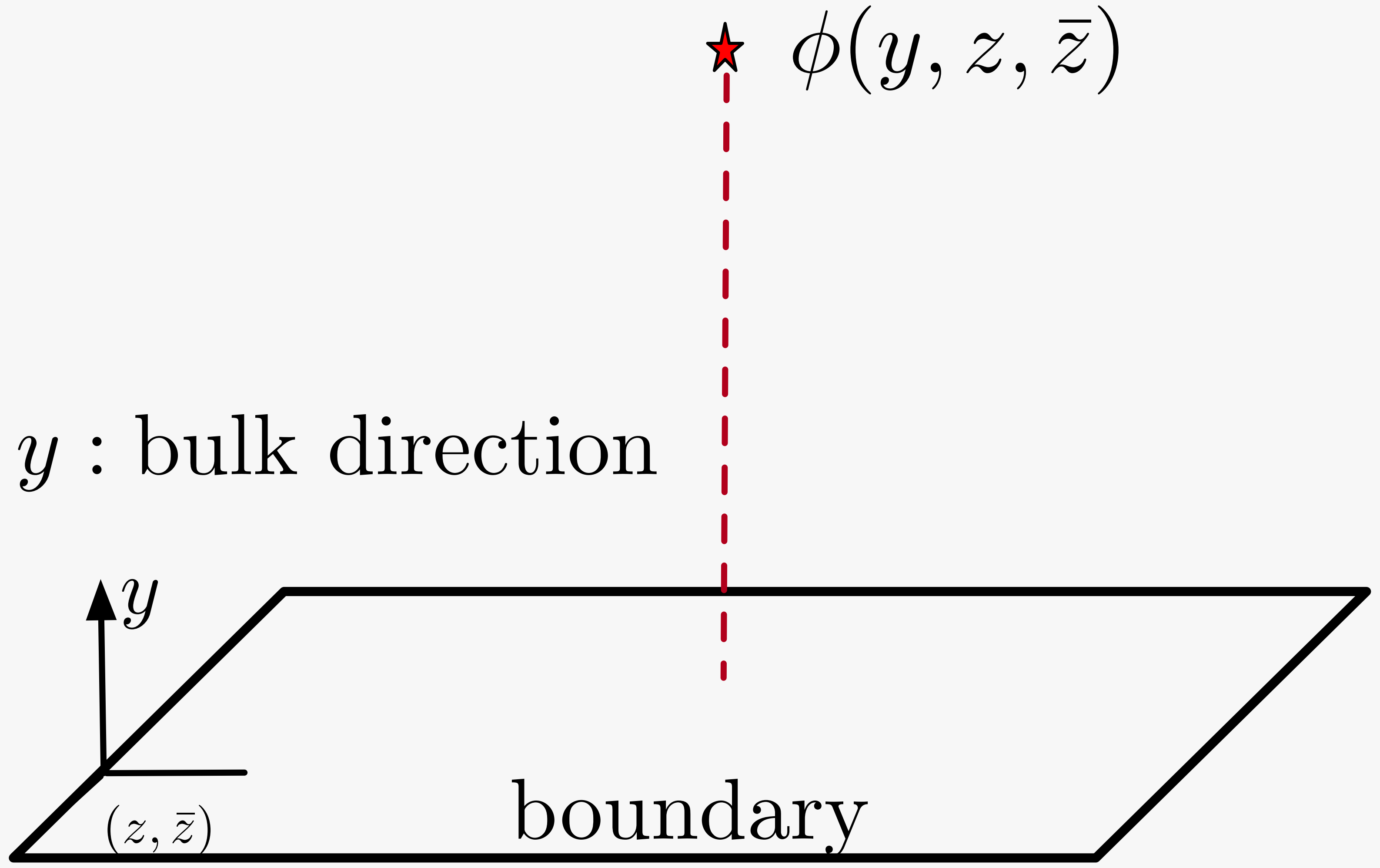

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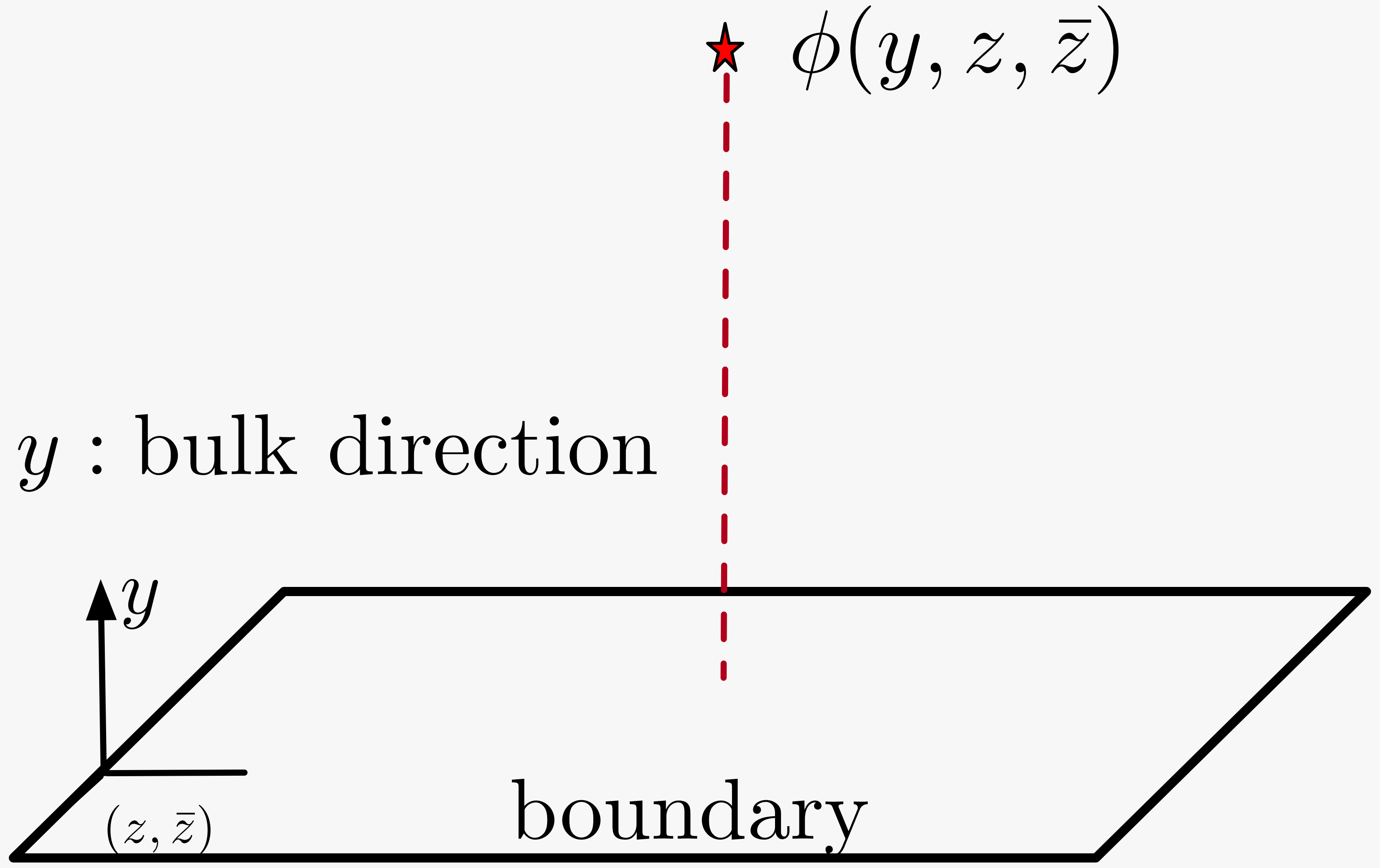

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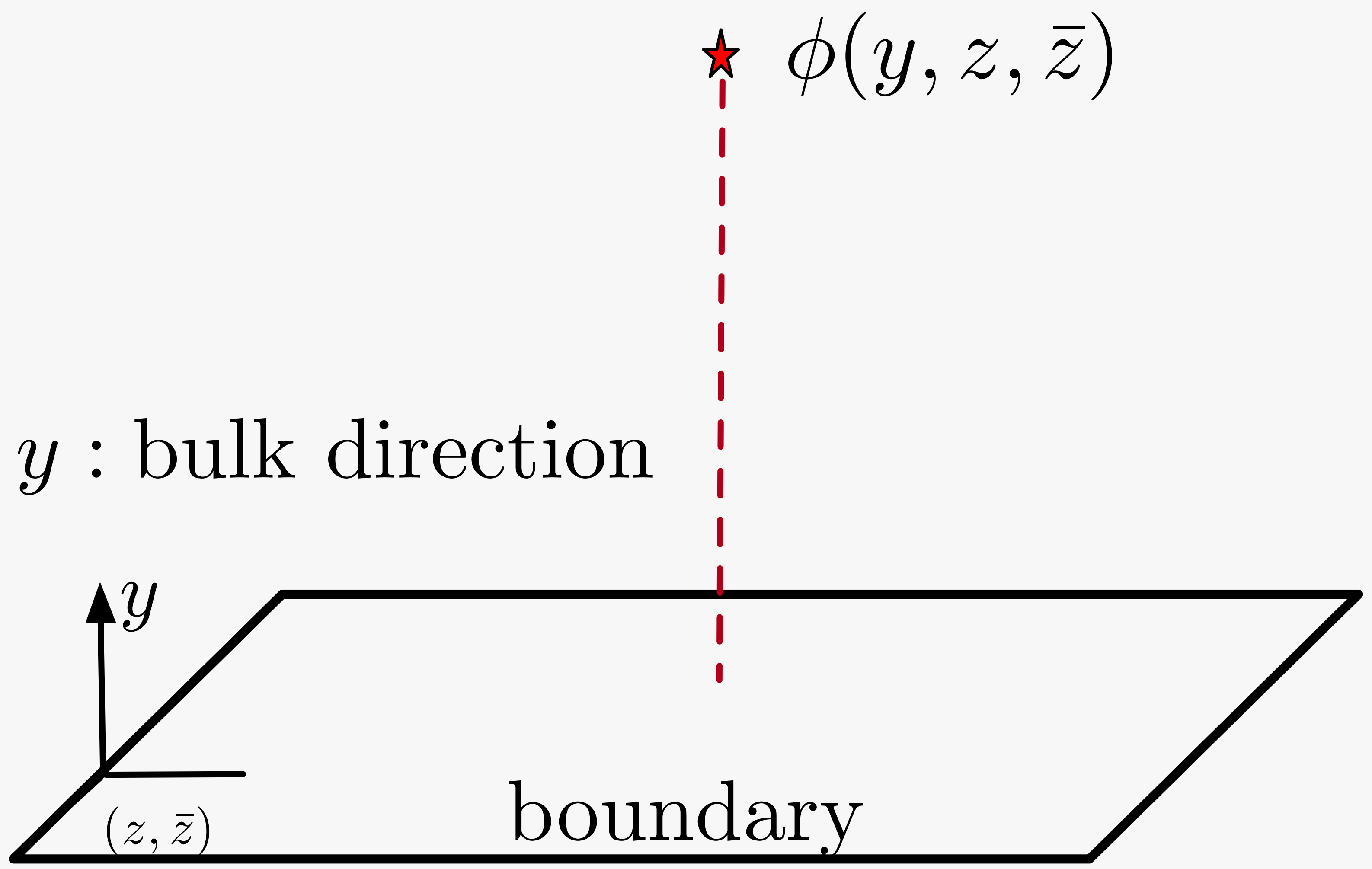

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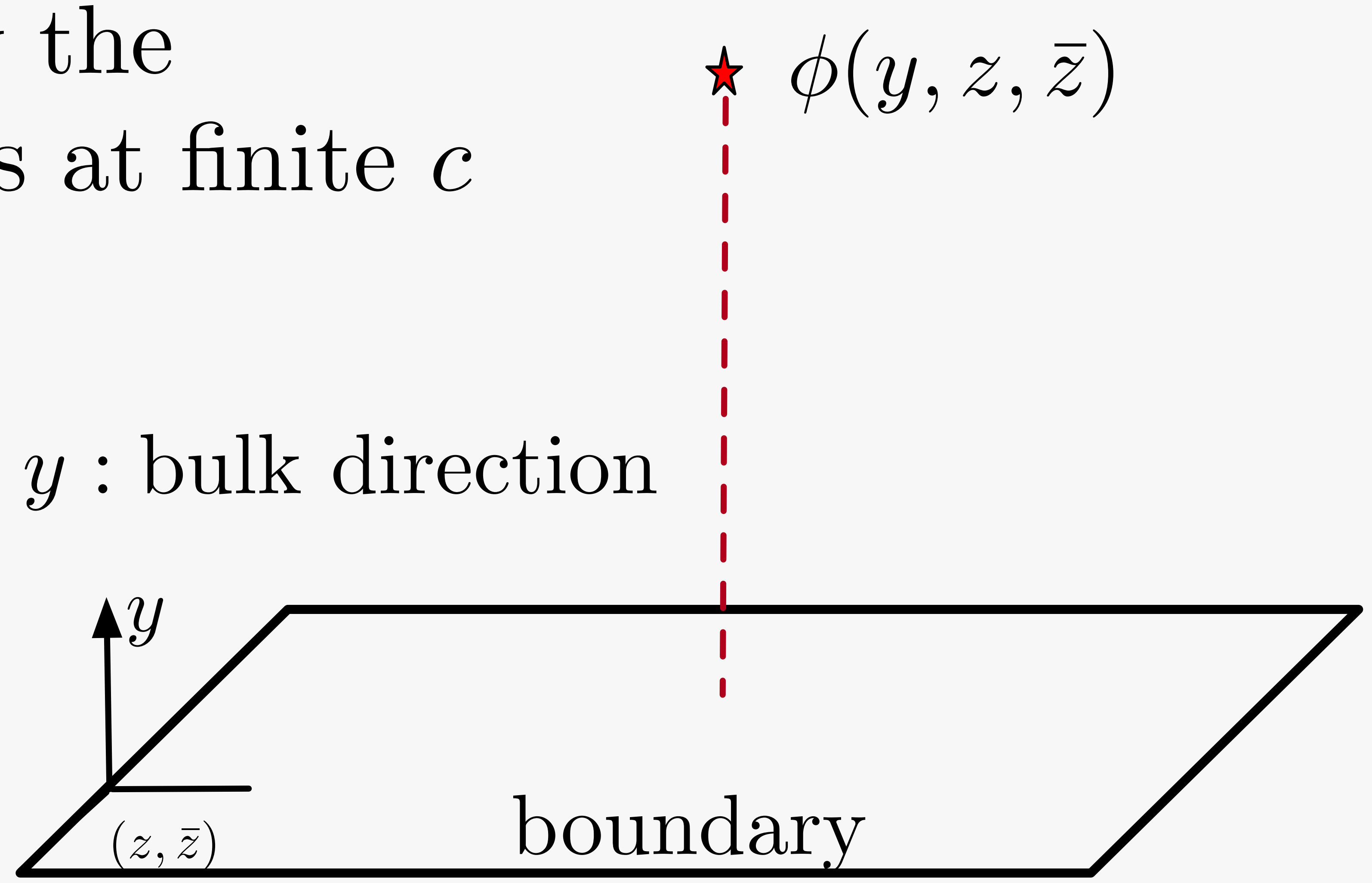

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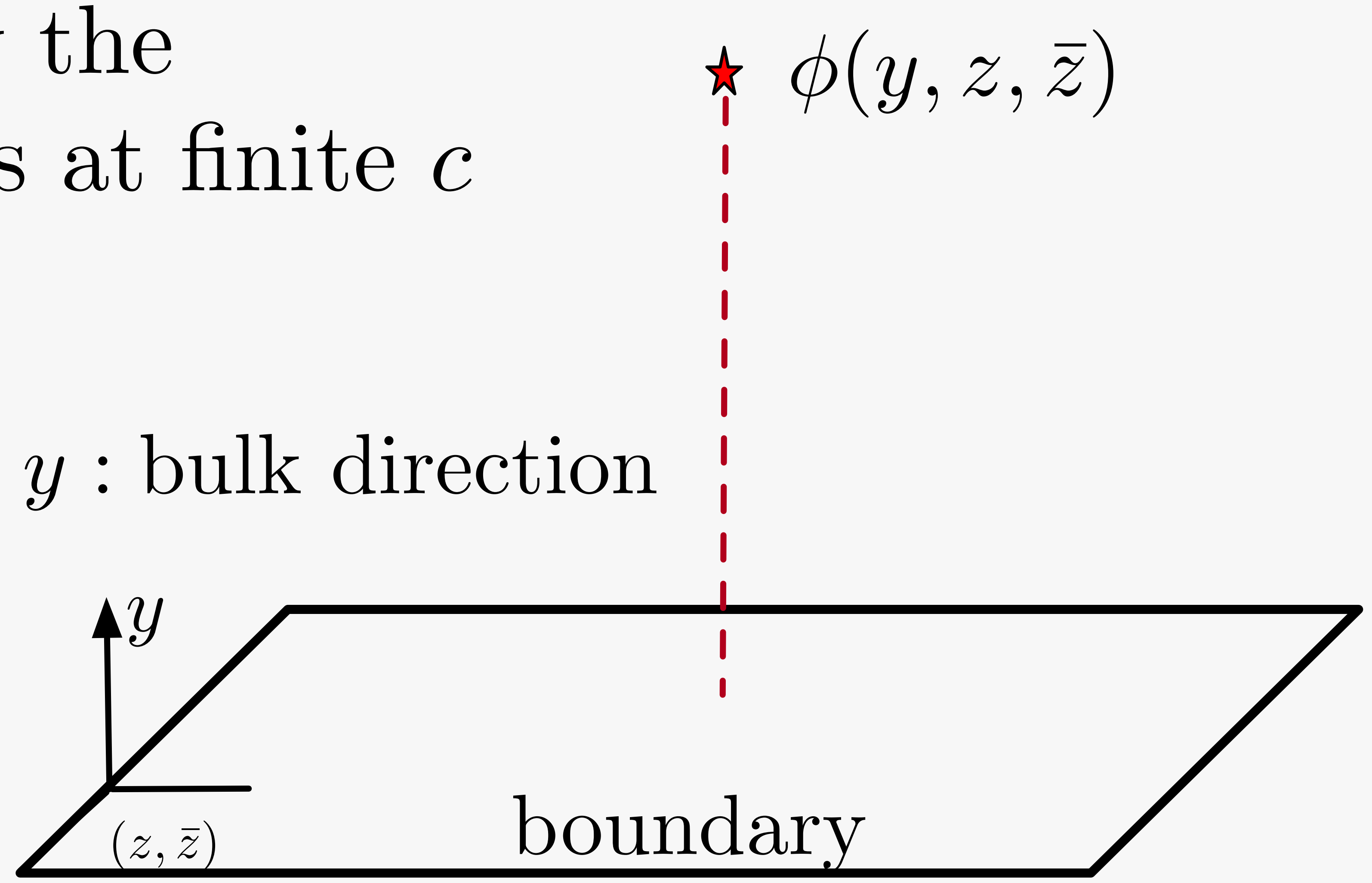

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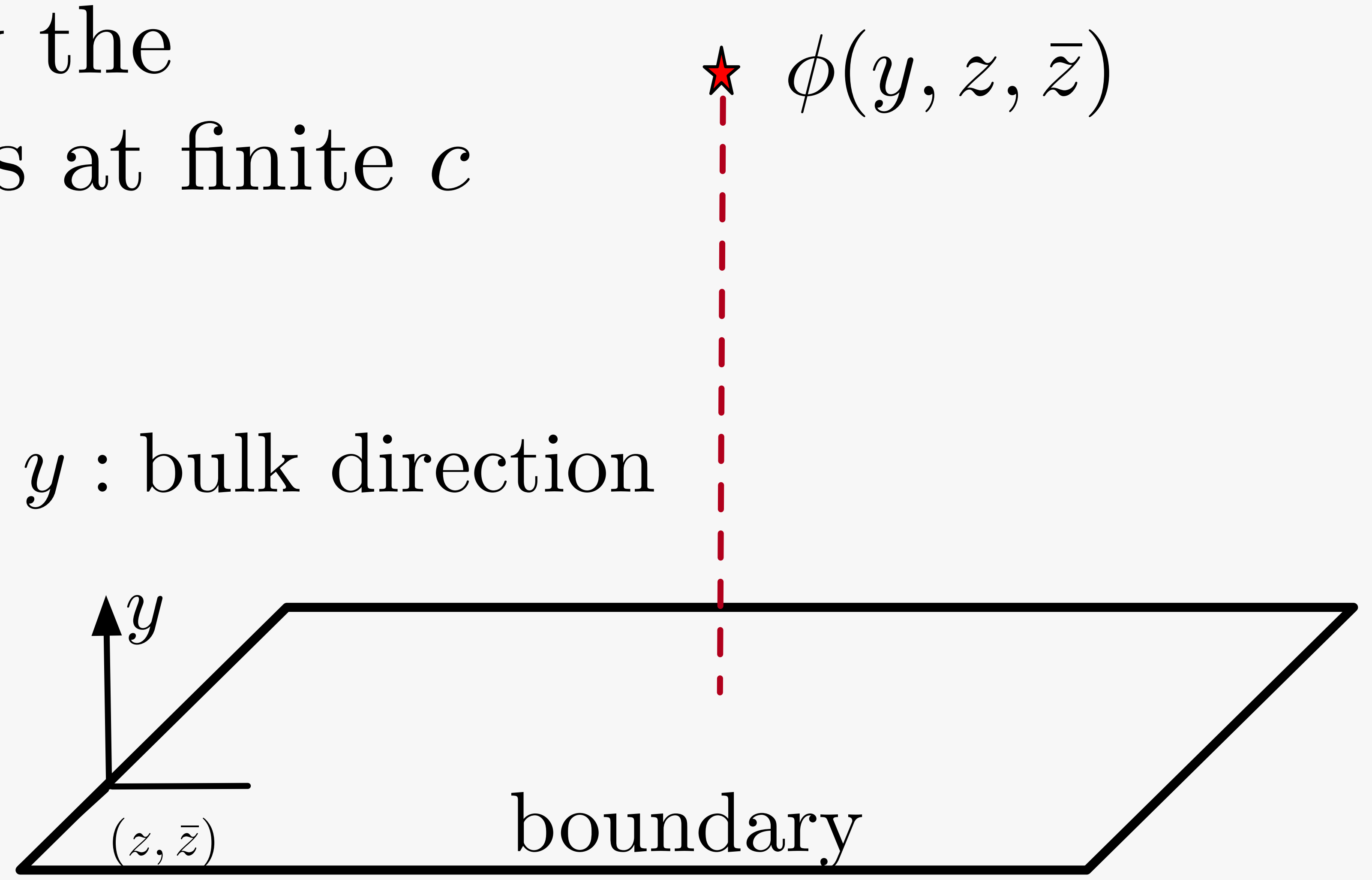

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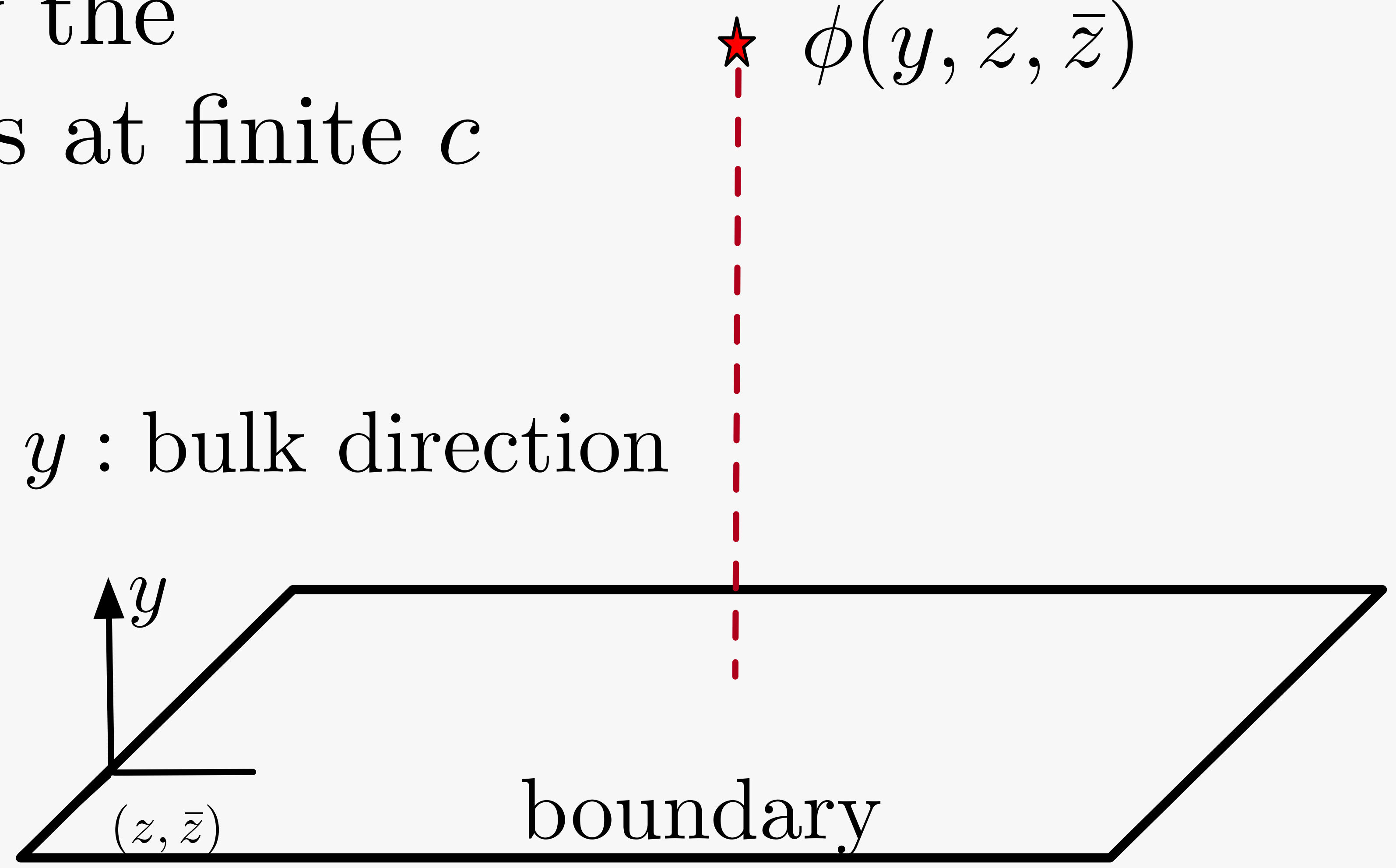

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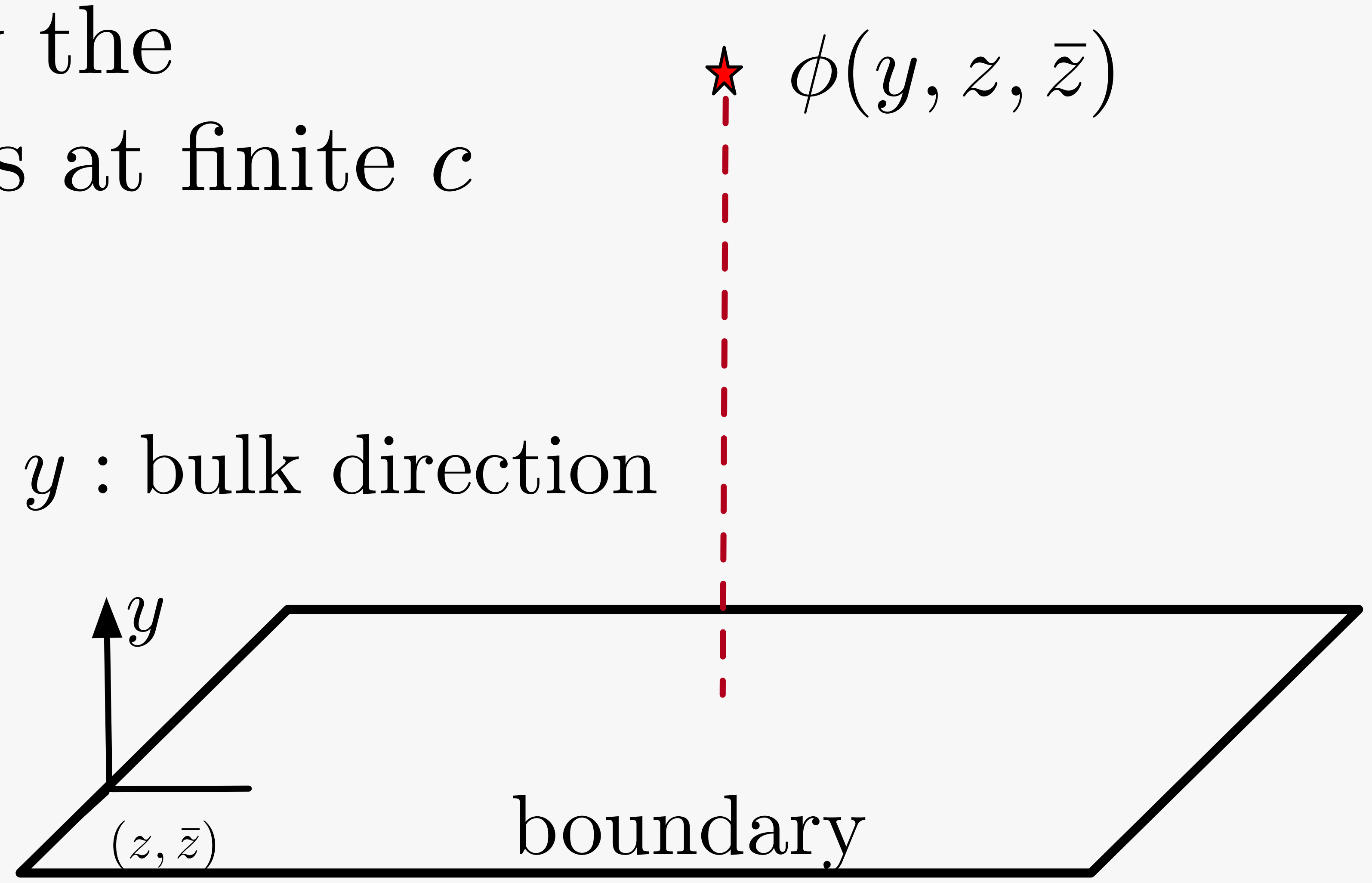

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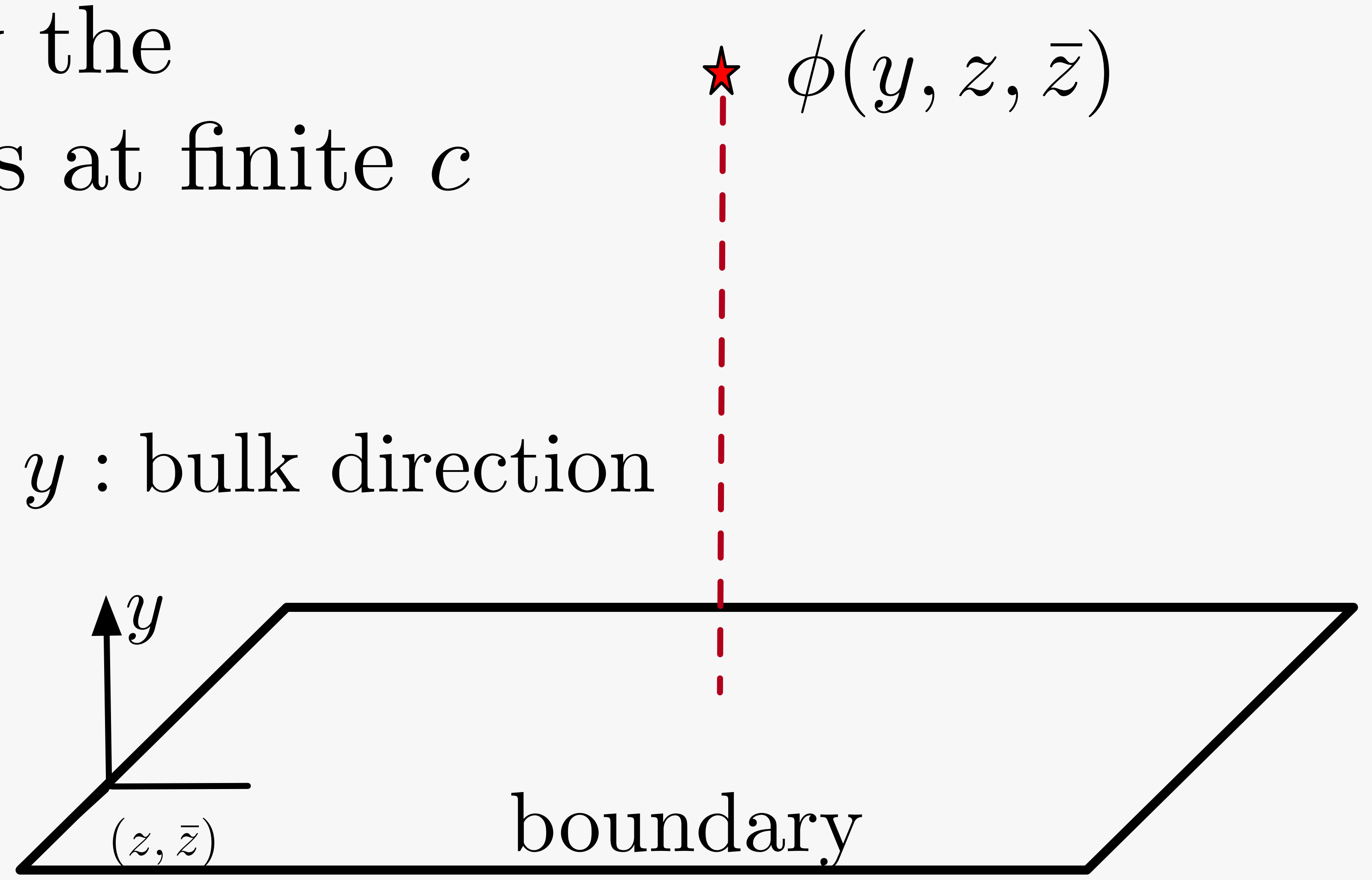

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See 1905.00015 for generalizations of our bulk reconstruction proposal!

Consistency Checks with Bulk Perturbative Gravity

- Using the CFT definition of the bulk proto-field, we can compute correlators like

$$\langle \phi \mathcal{O} T \rangle \quad \langle \phi \mathcal{O} T \cdots T \rangle \quad \langle \phi \phi \rangle \quad \langle \mathcal{O}_H | \phi_L \mathcal{O}_L | \mathcal{O}_H \rangle$$

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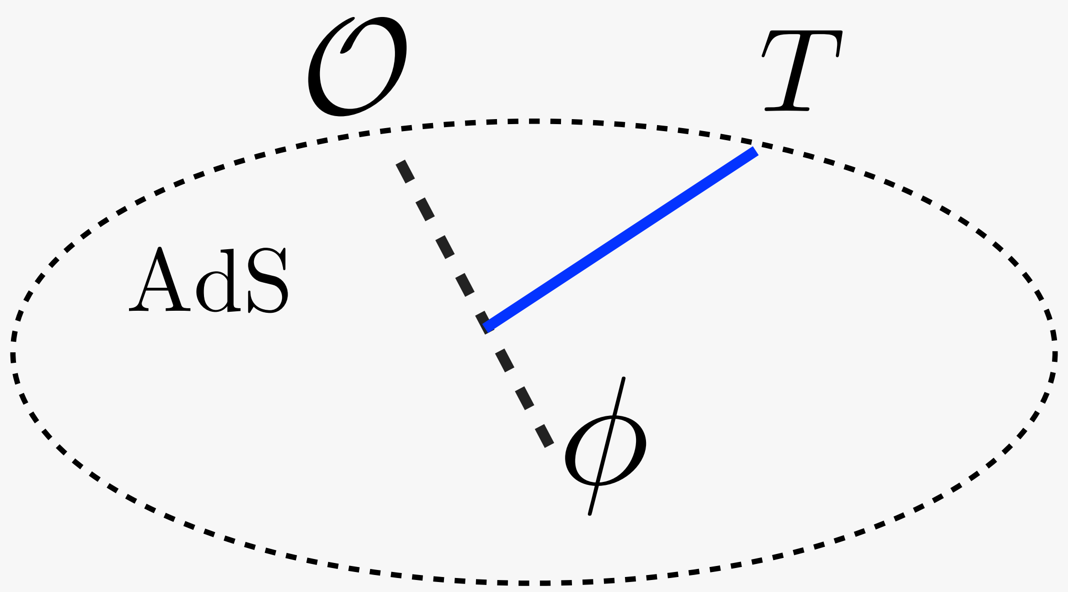

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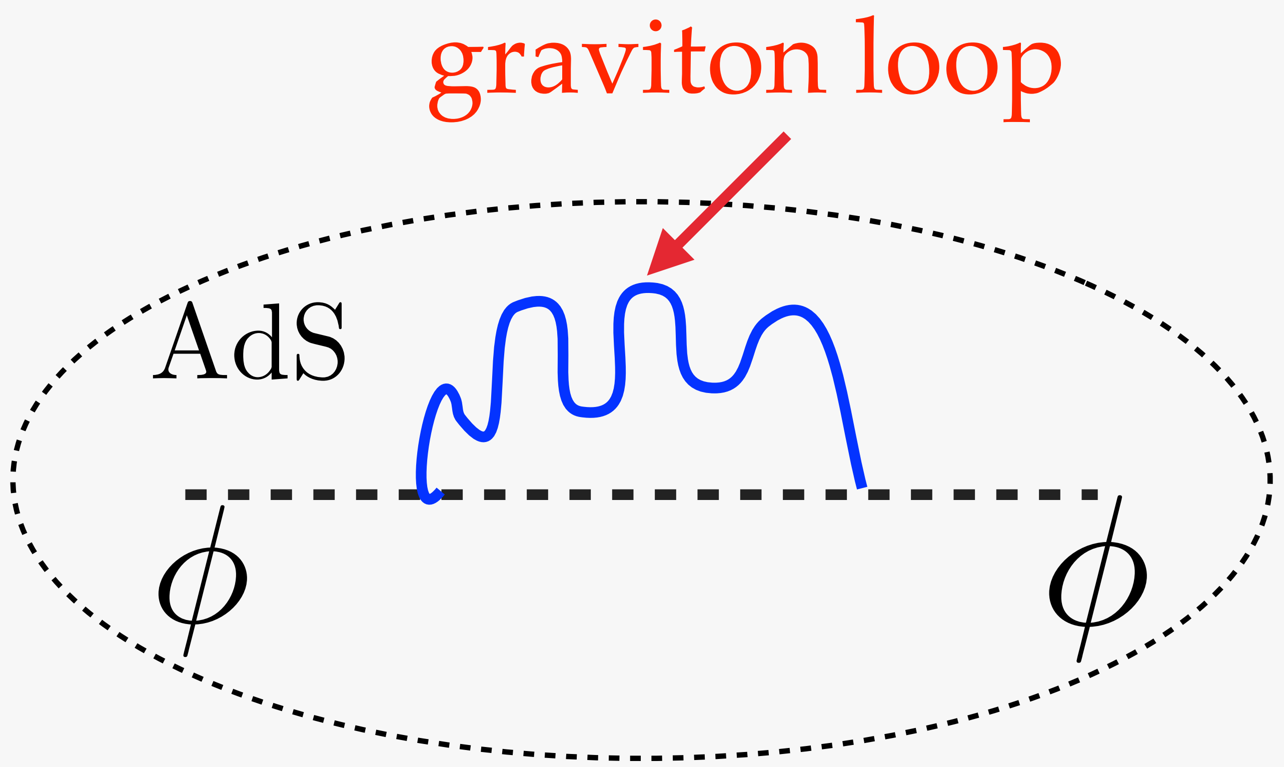

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$\mathcal{O}(1/c)$ checks

3: one-graviton-loop correction to the bulk-boundary propagator in a BTZ black hole background ✓

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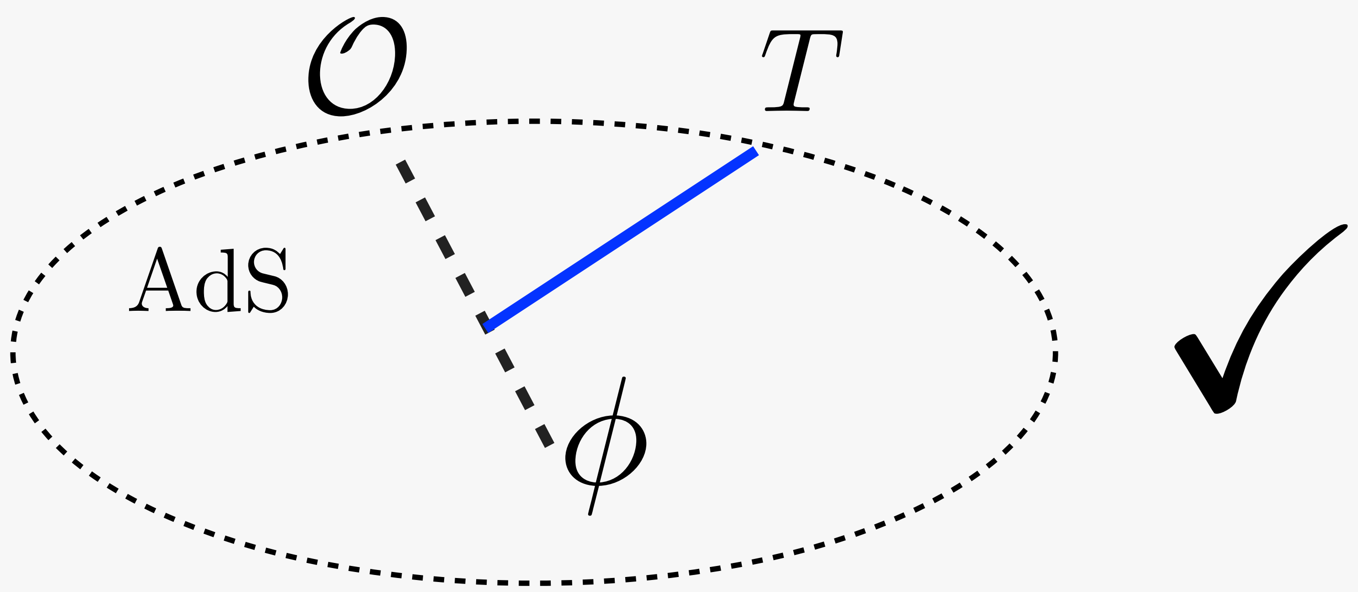

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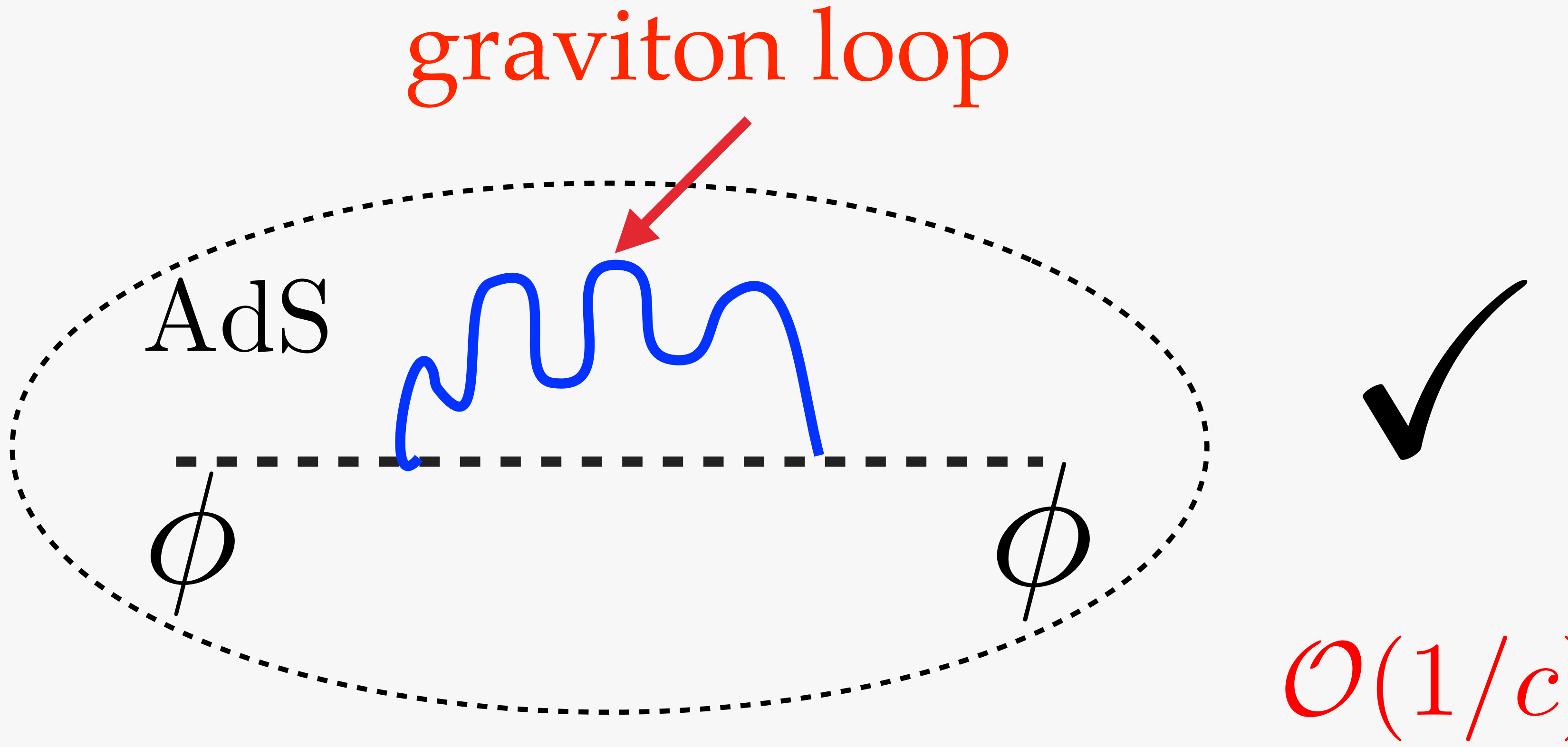

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These provide non-trivial checks of our CFT definition of a bulk scalar field interacting with gravity!

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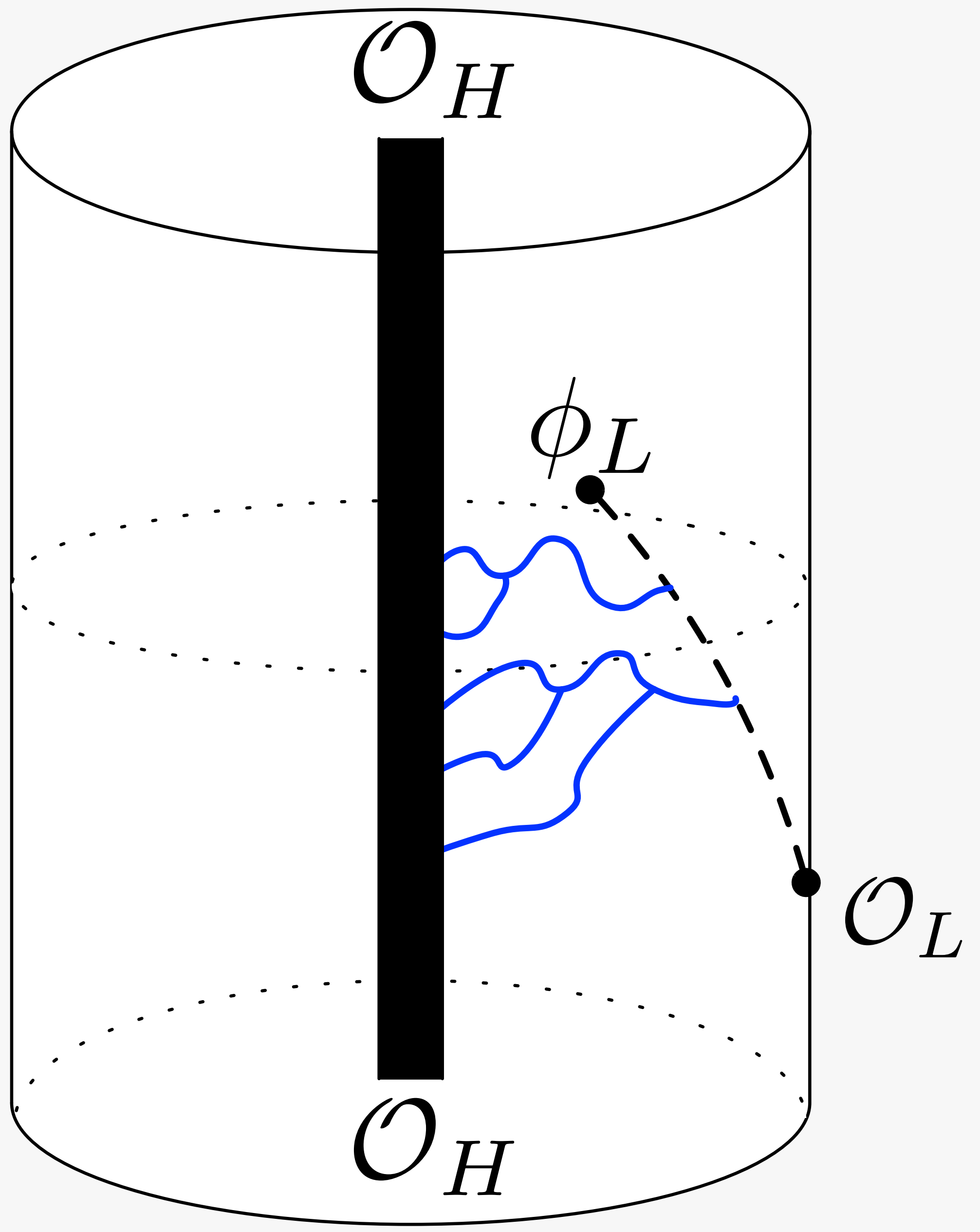
1810.02436 HC, Fitzpatrick, Kaplan, Li



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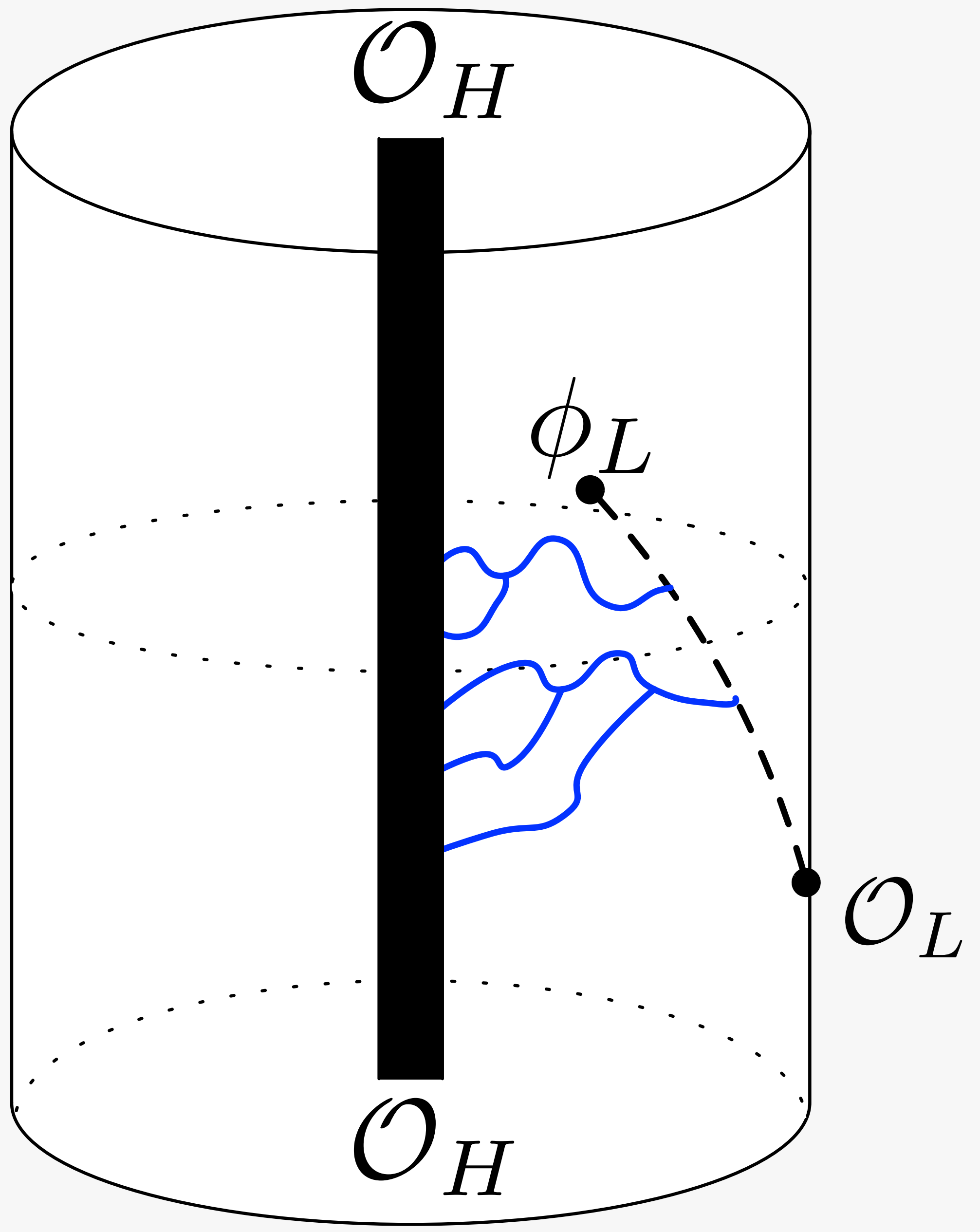
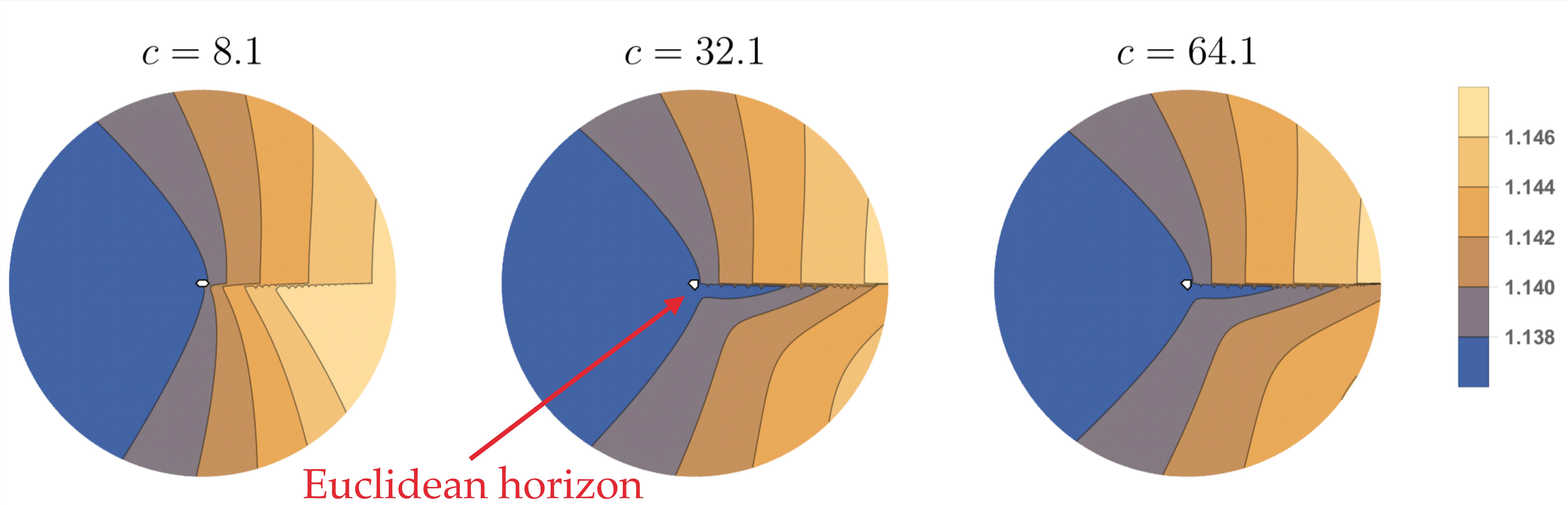
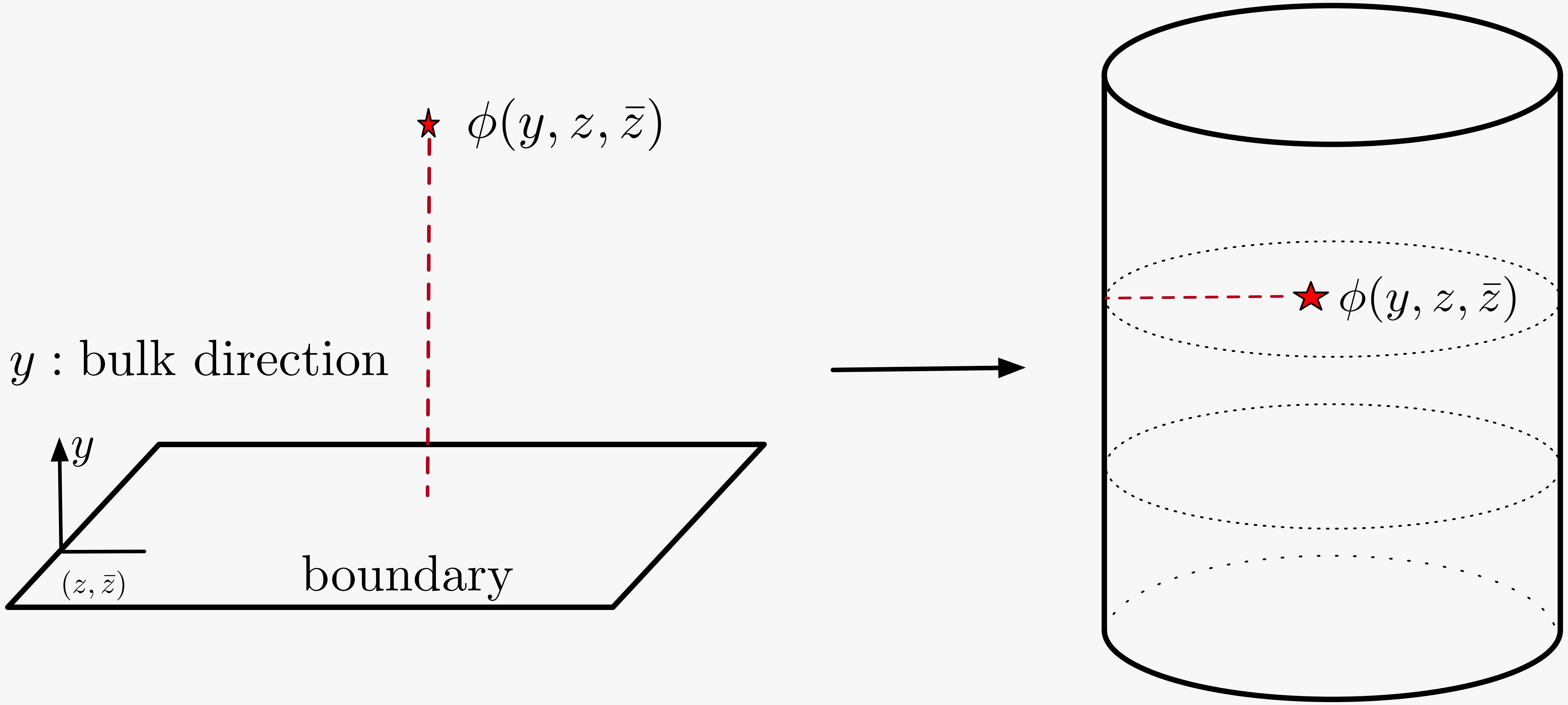




Thank you!

More General Bulk Proto-fields

◉ can be generalized to general boundary geometry, general gravitational dressings



1905.00015 HC, Kaplan, Sharma

◉ can be generalized to bulk operators in U(1) Chern-Simons theory with general Wilson lines

$$\text{bulk primary condition:} \;\; J_m \phi(y,0,0) = 0, \qquad \text{for } m \geq 1$$