Jackiw-Teitelboim dynamics of entanglement in 2D boundary CFT

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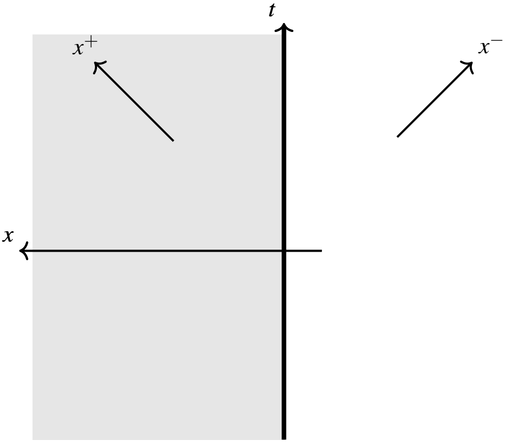
Based on 1808.10431, with Herman Verlinde
and 1808.05583

Strings 2019

bCFT$_2$ on $ds^2 = -dt^2 + dx^2 = -dx^+dx^-$, $x \geq 0$ in state $|0\rangle$



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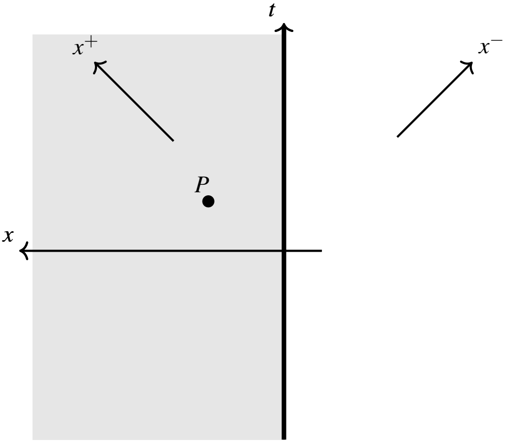

point P at (x^+, x^-)

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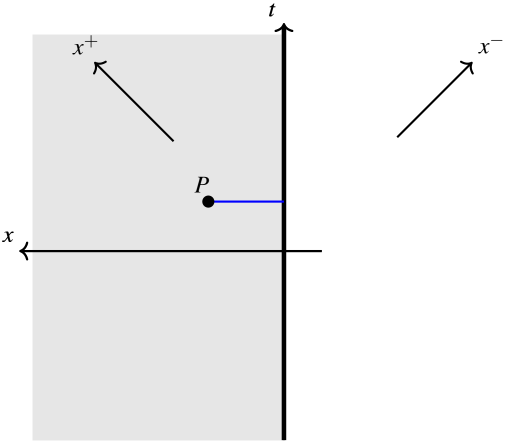

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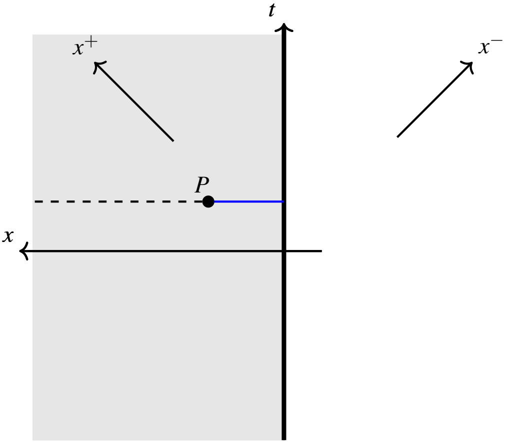

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$$S = -\mathrm{tr}(\rho \log \rho)$$
$$= \frac{c}{6} \log \frac{x^+ - x^-}{2\delta}$$

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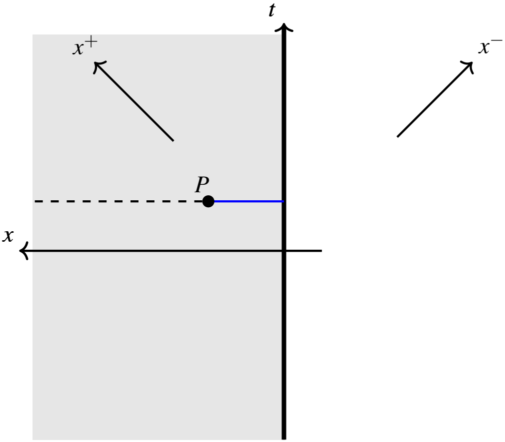

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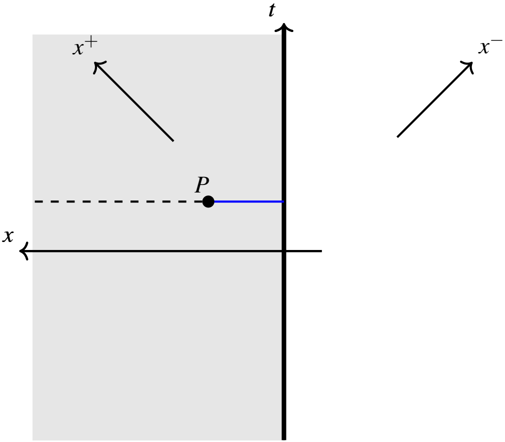

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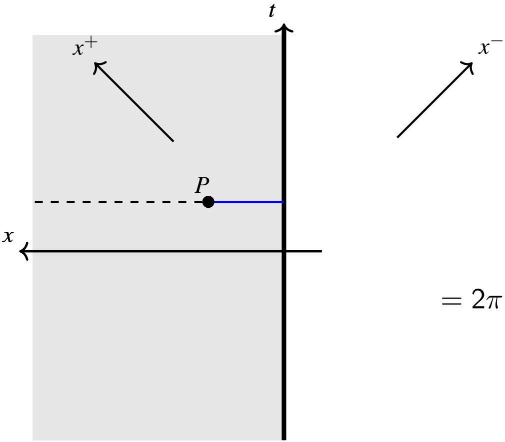

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$$K = -\log \rho$$
$$= 2\pi \int_{x^-}^{x^+} ds \frac{(s - x^-)(x^+ - s)}{x^+ - x^-} T_{++}(s)$$

local field $K(x^+, x^-)$ $\Rightarrow$ dynamics?

$$\partial_+\partial_- K = -\frac{2e^{-\frac{12}{c}S}}{\delta^2} K$$
$$\frac{12}{c}\partial_\pm S \partial_\pm K + \partial_\pm^2 K = 2\pi T_{\pm\pm}$$

Entanglement dynamics identities

$$\partial_+\partial_-\left(\frac{12}{c}S\right) = \frac{2}{\delta^2}e^{-\frac{12}{c}S}$$

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Answer: Jackiw-Teitelboim (JT) gravity coupled to the bCFT

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Theory of entanglement dynamics of a given bCFT is obtained by coupling bCFT_2 to AdS_2 JT gravity

$$I_{\text{ent dyn of bCFT}} = I_{JT,grav} + I_{bCFT}$$

Interpretation of the Schwarzian in the entanglement dynamics JT theory?

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Interpretation of Schwarzian requires entanglement dynamics interpretation of Φ_0

Entanglement dynamics interpretation of Φ_0

$$\Phi_0 = -\frac{\Phi_b}{4}\partial_x\omega = \frac{3\Phi_b}{c}\partial_x S = -\frac{\Phi_b}{\epsilon}\frac{3}{c}\left(S(x) - S(x-\epsilon)\right)$$

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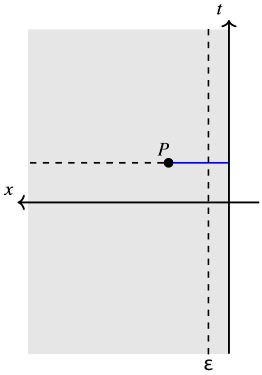

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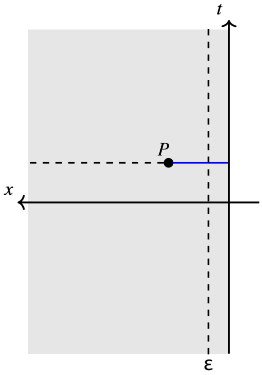

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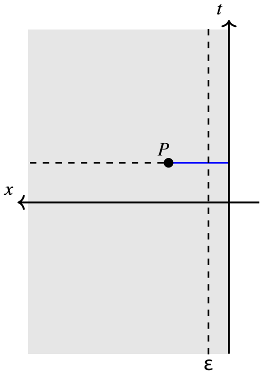

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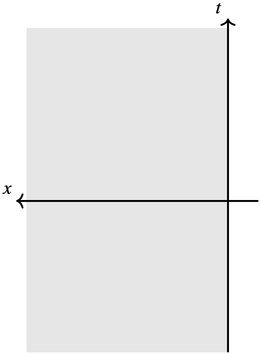
The Schwarzian describes entanglement renormalization in the bCFT

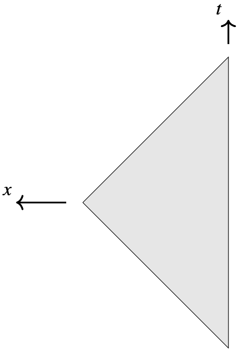
$$I^{\epsilon}_{bCFT} = I^{\epsilon\to 0}_{bCFT} + \frac{c\epsilon}{3}\int \{t,u\} du$$

Given bCFT_2



$$ds^2 = -dt^2 + dx^2$$

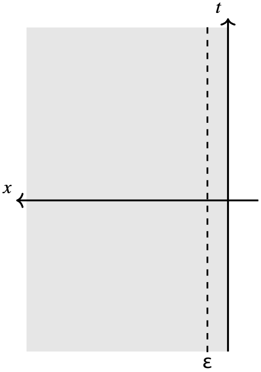
Entanglement dynamics of bCFT_2



$$ds^2 = \frac{-dt^2 + dx^2}{x^2}$$

Entanglement renormalization in given $bCFT_2$

cfr e.g. cMERA



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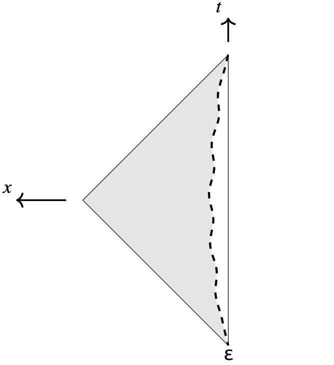
Boundary dynamics in entanglement dynamics of $bCFT_2$

described by Schwarzian QM



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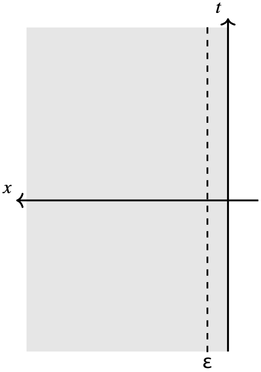

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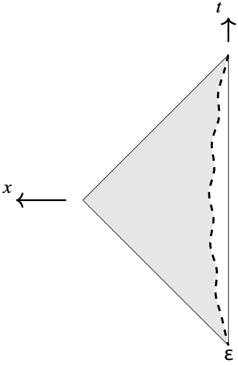

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Thank you