AdS black holes and Cardy limits

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Statistical approach to black hole thermodynamics

- 5d BPS BHs from D1-D5-P. Cardy formula of 2d CFT [Strominger, Vafa] (1996) ……

$$Z(\tau) \sim \mathrm{Tr}\left[e^{2\pi i \tau L_0}\right] \sim \exp\left[\frac{\pi i c}{12\tau}\right] \quad \text{at } \tau \to i0^+$$

$$e^{S(P,c)} = \oint d\tau Z(\tau) e^{-2\pi i \tau P} \sim \exp\left[2\pi\sqrt{\frac{cP}{6}}\right] \quad \text{at } P \gg c \qquad c \sim Q_1 Q_5$$

- Derives Bekenstein-Hawking entropy of these BH's in the Cardy regime.
- BH's with AdS_3 near-horizon factors studied this way, using <u>QFTs for particular BH's</u>.

- Some black holes are simple, but some are complicated:
- single-center, multi-center, black rings, … : some BH's are dominant in non-Cardy regime.

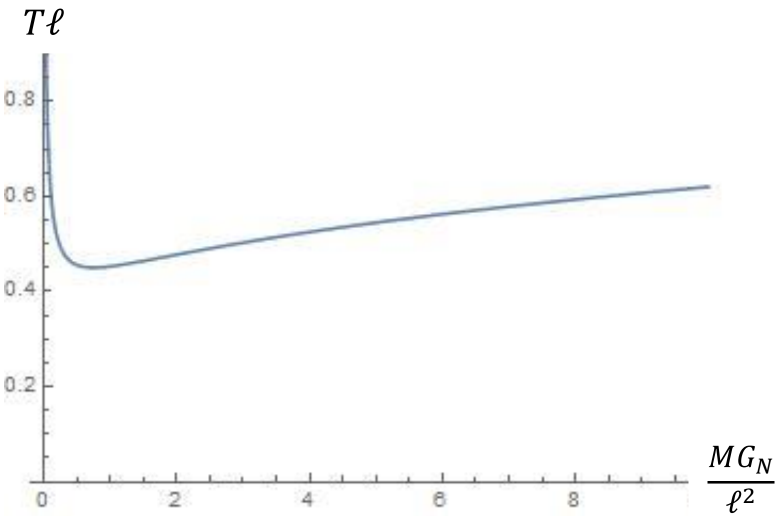
- Today's talk:
- Systematic description of BH's in a given system: AdS_{D+1} black holes from $CFT_{D>2}$.
- With <u>QFT for the whole quantum gravity</u>, rather than particular black holes.
- Establish a version of "Cardy formula" for the indices of $SCFT_D$ in $D > 2$.
- Count known BH's. (Also predict new BH's: mostly skipped today, with limited time)

Black holes in AdS/CFT

- Schwarzschild black holes in global AdS_5:
- small BH & large BH branches (ℓ: AdS radius)

$$T = \frac{r_+}{\pi\ell^2} + \frac{1}{2\pi r_+} \qquad r_+^2 = -\frac{\ell^2}{2} + \ell\sqrt{\frac{\ell^2}{4} + \omega M}$$

$$\omega \equiv \frac{16\pi G_N}{3\mathrm{vol}(S^3)}$$



- Hawking-Page transition (1983): at $T = \frac{3}{2\pi\ell}$
- Low T : gas of gravitons in AdS. Doesn't see $1/\mathrm{G_N} \sim N^2$ so that $F \sim O(N^0)$
- High T : large AdS black holes ($F_{BH} = -T\log Z_{BH} < 0$). Sees N^2.

- CFT dual (on $S^3 \times R$): confinement-deconfinement transition [Witten] (1998)
- Confined phase: $F \sim O(N^0)$, glue-balls (& mesons, etc.)
- Deconfined phase: $F \sim O(N^2)$ of gluons (& quarks)

- Weak coupling study [Aharony, Marsano, Minwalla, Papadodimas, Raamsdonk] (2003)
- Today's goal: quantitative strong coupling study of BPS BH's in $AdS_5 \times S^5$.

Supersymmetric black holes & QFT

[Gutowski,Reall] [Chong,Cvetic,Lu,Pope] [Kunduri,Lucietti,Reall] (2004-2006)

- Preserves 2 real SUSY (1/16-BPS): $SO(6)$ charges Q_I on S^5 & $SO(4)$ spins J_i on AdS_5 .
- Mass (= energy) determined by these 5 charges: $M\ell = Q_1 + Q_2 + Q_3 + J_1 + J_2$
- Known solutions have a charge relation: Solutions come w/ 4 independent parameters.

- Hairy BH's. No charge relations [Markeviciute,Santos] [Bhattacharyya,Minwalla,Papadodimas]
- The general set of BPS black holes in $AdS_5 \times S^5$ is (probably) unknown.

- BPS states at strong coupling: "index" on $S^3 \times R$. Coupling independent.

  [Romelsberger] [Kinney,Maldacena,Minwalla,Raju] ≡ [KMMR] (2005)

$$Z(\Delta_I, \omega_i) = \mathrm{Tr}\left[(-1)^F e^{-\sum_{I=1}^{3}\Delta_I Q_I - \sum_{i=1}^{2}\omega_i J_i}\right]$$
$$\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 0 \pmod{4\pi i}$$

⟷

$$\mathrm{Tr}\left[e^{-\sum_{I=1}^{3}\Delta_I Q_I - \sum_{i=1}^{2}\omega_i J_i}\right]$$
$$\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i \pmod{4\pi i}$$

Use $(-1)^{\mathrm{F}} = e^{-2\pi i Q_1}$ and shift $\Delta_1 \to \Delta_1 - 2\pi i$

- U(N) group:

$$Z = \frac{1}{N!}\int \prod_{a=1}^{N}\frac{d\alpha_a}{2\pi}\prod_{a<b}\left(2\sin\frac{\alpha_{ab}}{2}\right)^2 \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(1 - \frac{\prod_{I=1}^{3} 2\sinh\frac{n\Delta_I}{2}}{2\sinh\frac{n\omega_1}{2}\cdot 2\sinh\frac{n\omega_2}{2}}\right)\sum_{a,b=1}^{N} e^{in\alpha_{ab}}\right]$$

Or the version with $\Delta_1 \to \Delta_1 - 2\pi i$ shift

Large N index

- Questions: Does the low "temperature" index agree w/ that of gravitons in $AdS_5 \times S^5$?

  Does the index undergo deconfinement transition at higher "temperature"?

  (Low / high "temperature" ~ large/small Δ_I, ω_i (all positive).)

- Large N matrix integral → eigenvalue distribution:

$$\rho(\theta) = \frac{1}{2\pi} + \frac{1}{2\pi}\sum_{n=1}^{\infty}\left[\rho_n e^{in\theta} + \rho_{-n}e^{-in\theta}\right] \quad , \quad \rho_{-n} = \rho_n^*$$

$$\int_0^{2\pi} d\theta \rho(\theta) = 1$$

$$\rho(\theta) \geq 0$$

[Sundborg] [Aharony,Marsano,Minwalla,Papadodimas,Raamsdonk] [KMMR]

$$Z = \int \prod_{n=1}^{\infty}[d\rho_n d\rho_{-n}] \exp\left[-N^2 \sum_{n=1}^{\infty}\frac{1}{n}\rho_n\rho_{-n}\frac{\prod_I(1-e^{-n\Delta_I})}{\prod_i(1-e^{-n\omega_i})}\right]$$

- "Low T": Uniform distribution $\rho(\theta) = 1/2\pi$. "Confining phase." $f(\Delta_I, \omega_i) \equiv \frac{\prod_{I=1}^{3}(1-e^{-\Delta_I})}{\prod_{i=1}^{2}(1-e^{-\omega_i})}$ > 0.
- Agrees w/ BPS gravitons on [KMMR] (2005).

$$Z_{N\to\infty} = \prod_{n=1}^{\infty} f(n\Delta_I, n\omega_i)^{-1} = Z_{\text{gravitons}}$$

- Does the index deconfine at high enough T?

- Apparently, no, as $f > 0$ always (at real fugacities).
- So the index doesn't seem to deconfine, not seeing a free energy at $\log Z \sim N^2$.

Deconfinement & BH's from index?

- It has been speculated that $(-1)^F$ plays certain bad roles in the index.

- To see why, consider unrefined index $Z(x) = \sum_j \Omega_j x^j$ (where $j \equiv 6(Q+J)$). $e^{-\omega} = x^3,\ e^{-\Delta} = x^2$

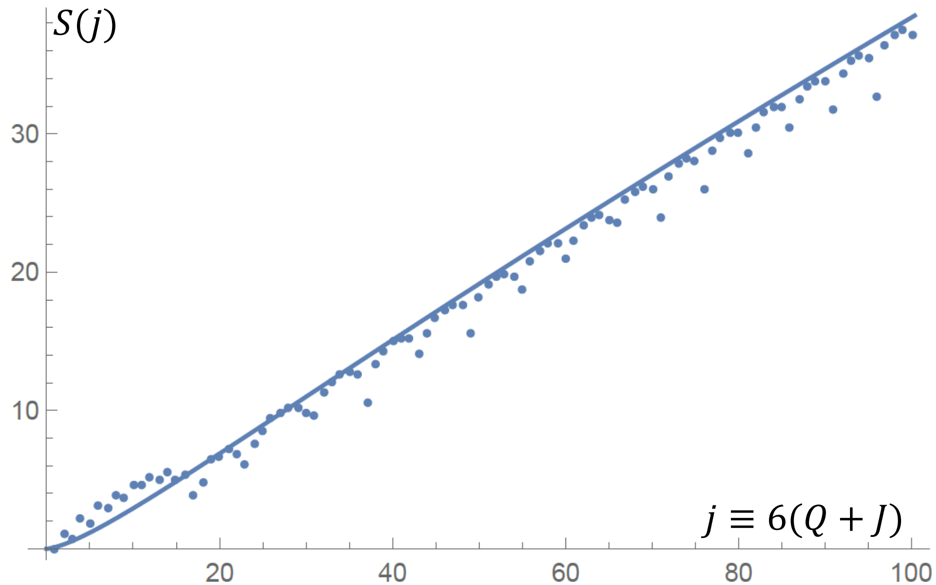
- E.g. U(5) index ($N^2 = 25 \gg 1 \ldots ?$): $\Delta_1 = \Delta_2 = \Delta_3 \equiv \Delta,\ \omega_1 = \omega_2 \equiv \omega$

$$\begin{aligned} Z \;=\; & 1 + 3x^2 - 2x^3 + 9x^4 - 6x^5 + 21x^6 - 18x^7 + 48x^8 - 42x^9 + 99x^{10} - 96x^{11} + 172x^{12} - 156x^{13} + 252x^{14} - 160x^{15} \\ & + 195x^{16} + 48x^{17} - 127x^{18} + 612x^{19} - 783x^{20} + 1258x^{21} - 948x^{22} + 450x^{23} + 1921x^{24} - 5430x^{25} + 11793x^{26} \\ & - 18812x^{27} + 26379x^{28} - 27750x^{29} + 17809x^{30} + 15648x^{31} - 78324x^{32} + 175030x^{33} - 285576x^{34} + 366024x^{35} \\ & - 323807x^{36} + 38856x^{37} + 624894x^{38} - 1718016x^{39} + 3094992x^{40} - 4226862x^{41} + 4098270x^{42} - 1210728x^{43} \\ & - 5968935x^{44} + 18061488x^{45} - 33152565x^{46} + 44941584x^{47} - 41448422x^{48} + 6241896x^{49} + 75761478x^{50} \\ & - 205993284x^{51} + 354209109x^{52} - 440168670x^{53} + 328572109x^{54} + 142704804x^{55} - 1079522706x^{56} \\ & + 2385844062x^{57} - 3584202447x^{58} + 3694263972x^{59} - 1331772481x^{60} - 4771857420x^{61} + 14697077445x^{62} \\ & - 25833114276x^{63} + 31549909440x^{64} - 21264664440x^{65} - 16439430686x^{66} + 86286819246x^{67} - 174750537792x^{68} \\ & + 238416590234x^{69} - 201108631665x^{70} + \mathcal{O}(x^{71}) \end{aligned}$$

- Ω_j grows at large j, but signs alternate.

- $S_j \equiv \log|\Omega_j| \sim N^2$ at $j \sim N^2$.

Line: $S_{BH}(j)$ of known black holes, inserting $N^2 \to 25$
dots: $S_j = \log|\Omega_j|$ from the index



- Higher ranks, orders in x under investigation

[Agarwal, Joonho Kim, SK, Nahmgoong] (in progress)

Idea for analytic approaches

- Large charge $j \sim O(N^2)$ approximation of $\Omega_j = \frac{1}{2\pi i}\oint \frac{dx}{x^{j+1}} Z(x)$
- Saddle pt. calculus: Legendre transform is insensitive to changing j by a quantum

- Can we get macroscopic entropies with wild ± 1 oscillation from phase factor?

  Namely, something like $\Omega_j \sim e^{S(j,x_*)} = e^{i\, Im[S(j,x_*)]}\, e^{Re[S(j,x_*)]} \ldots$?

- To seek for this possibility, one should turn on complex fugacities.

- In a different perspective, the fugacity phase is to be tuned,
- attempting to tame rapid oscillations between +/- signs at nearby charge,
- or to maximally obstruct cancelations (smearing) of nearby B/F.

- This is a discussion for microcanonical ensemble, but it should also have impacts on the grand canonical ensemble in the “thermodynamic limit”

Evidence: instability of confining saddle pt.

- Reconsider the large N index: Again, unrefined as $\Delta_1 = \Delta_2 = \Delta_3 \equiv \Delta,\ \omega_1 = \omega_2 \equiv \omega$

  $e^{-\omega} = x^3,\ e^{-\Delta} = x^2$

- The index w/ $x \to x\, e^{i\phi}$

$$Z = \int \prod_{n=1}^{\infty} [d\rho_n d\rho_{-n}] \exp\left[-N^2 \sum_{n=1}^{\infty} \frac{f(x^n)}{n} \rho_n \rho_{-n}\right] \qquad f(x) = \frac{(1-x^2)^3}{(1-x^3)^2}$$

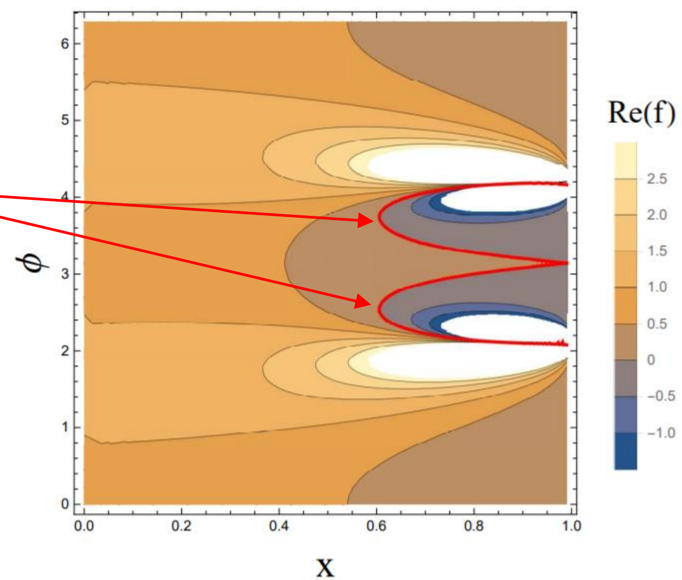
- Dial ϕ : $\rho_1 = 0$ is locally unstable if $Re\left[f\left(xe^{i\phi}\right)\right] < 0$.

- $Re[f(xe^{i\phi})]$: $\dfrac{(1-x^2)(1+x^2-2x\cos\phi)^2\,(2x(2+5x^2+2x^4)\cos\phi + (1+x^2)(1+4x^2+x^4+3x^2\cos(2\phi)))}{(1+x^6-2x^3\cos(3\phi))^2}$

- red curve: $Re\left[f\left(xe^{i\phi}\right)\right] = 0$. Lowest fugacity for instability

$$x_H = \sqrt{\frac{\sqrt{3}-1}{2}} \approx 0.605 \qquad \cos\phi = -\frac{1}{2x_H}$$

Here!



- It sounds unnatural if the large N saddle point calculus doesn't take advantage of this window of instability.
- So we interpret it as an upper bound for deconfinement.

[Choi, J. Kim, SK, Nahmgoong-2] ≡ [CKKN-2] (2018)

Cardy limit

- "high T limit" : $J_i \gg N^2 (\gg 1)$, $|\omega_i| \ll 1$. Similar to 2d, $P \gg c (\gg 1)$, $\tau \to i0^+$.

- Studied in [Di Pietro, Komargodski] [Ardehali], but only at real fugacities.
- Should also take $Re(\Delta_I) \to 0^+$, but generically keep finite $\mathrm{Im}(\Delta_I) \sim O(1)$:

$$\mathrm{Tr}\left[e^{-\sum_{I=1}^{3}\Delta_I Q_I - \sum_{i=1}^{2}\omega_i J_i}\right] \qquad \Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i \pmod{4\pi i}$$

- The matrix integral becomes:

$$Z \sim \frac{1}{N!}\oint \prod_{a=1}^{N}\frac{d\alpha_a}{2\pi}\exp\left[-\frac{1}{\omega_1\omega_2}\sum_{a\neq b}\sum_{s_1,s_2,s_3=\pm 1} s_1 s_2 s_3 \mathrm{Li}_3\left(-e^{\frac{s_I\Delta_I}{2}}e^{i\alpha_{ab}}\right)\right] \qquad \mathrm{Li}_3(x) \equiv \sum_{n=1}^{\infty}\frac{x^n}{n^3}$$

- "Maximally deconfining" saddle point $\alpha_1 = \alpha_2 = \cdots = \alpha_N$ is most dominant.

  [CKKN-1] (2018) [Honda] [Ardehali] [J. Kim,SK,Song] [Cabo Bizet,Cassani,Martelli,Murthy] (2019)
- This is natural, since quarks/gluons are effectively massless at high T limit.
- True for "all" SCFTs w/ $N = 1$ SUSY (i.e., checked for numerous examples) [J. Kim, SK, Song]

- Final result: [CKKN-1]

Use: $\mathrm{Li}_3(-e^x) - \mathrm{Li}_3(-e^{-x}) = -\frac{x^3}{6} - \frac{\pi^2 x}{6}$

$$\log Z \sim -\frac{N^2}{\omega_1\omega_2}\sum_{s_1 s_2 s_3 = +1}\left[\mathrm{Li}_3\left(-e^{\frac{s_I\Delta_I}{2}}\right) - \mathrm{Li}_3\left(-e^{-\frac{s_I\Delta_I}{2}}\right)\right] \xrightarrow{-\pi < \mathrm{Im}(x) < \pi} \log Z \sim \frac{N^2\Delta_1\Delta_2\Delta_3}{2\omega_1\omega_2}$$

Counting (large) black holes

- Further take large N & compute entropy: Legendre transform at macroscopic charge

$$S(\Delta_I, \omega_i; Q_I, J_i) = \frac{N^2}{2}\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} + \sum_{I=1}^{3} Q_I\Delta_I + \sum_{i=1}^{2} J_i\omega_i \qquad \Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i$$

- Discussed in the context of BH solutions [Hosseini, Hristov, Zaffaroni] (2017)

- Multiple solutions: $S_*(Q_I, J_i)$ is in general complex. Take the most "dominant" one.
- $e^{i\, Im(S_*)}$: Imitates $\pm$ sign alternations, as macroscopic charges change by basic quanta.
- More precisely, ∃ c.c. saddle point: $\sim e^{Re(S)} \cos[Im(s) + \cdots]$
- $Re(S_*)$: Lower bound of entropy. We'll count known BH's by finding $Re(S) = S_{BH}$.
- Values of Δ_I's: E.g. at $Q_1 = Q_2 = Q_3$, one finds $\Delta_1 = \Delta_2 = \Delta_3 = 2\pi i/3$ in the Cardy limit.
- $-1 = e^{\Delta_1+\Delta_2+\Delta_3}$ from $(-1)^F$ is distributed equally to Δ_I's at the BH saddle point.

- Known BPS BH's satisfy a charge relation: Impose this relation by hand. [CKKN-1]

$$S(Q_I, J_i) = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_1 + J_2)}$$

→ known expression for S_{BH} [SK, K.Lee] (2006)

$$= 2\pi\sqrt{\frac{Q_1Q_2Q_3 + \frac{N^2}{2}J_1J_2}{\frac{N^2}{2} + Q_1 + Q_2 + Q_3}}$$

Compatibility of two expressions: charge relation of known BH's

Cardy limits & BH's for M2 / M5 CFTs

- 3d SCFT on N M2s: [Choi, Hwang, SK] (to appear)

- Holonomy integral & monopole sum $Z = \sum_{m_1,\cdots,m_N=-\infty}^{\infty} \oint \prod_{a=1}^{N} \frac{d\alpha_a}{2\pi} Z_{\text{1-loop}}(\alpha_a, m_a, \Delta_I, \omega)$
- Cardy & large N: monopole condensation breaks U(N), spreading over a range $\sim N^{1/2}/\omega$
- mechanism of d.o.f. reduction: $N^2 \to N^{3/2}$
- Counts entropy of BPS BH's in $AdS_4 \times S^7$: $\log Z \sim -i\frac{4\sqrt{2}N^{\frac{3}{2}}\sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}}{3\omega}$ $\quad \sum_{I=1}^{4}\Delta_I - \omega = 2\pi i$
- Also explored a finite N version of $N^{3/2}$.

- 6d SCFTs on N M5's: from 't Hooft anomalies

- Cardy limit of $\log Z[S^{2n-1} \times S^1]$: effective action of background fields on S^{2n-1}
- Leading terms of indices come from Chern-Simons terms on S^{2n-1} [CKKN-1]
- CS coefficients from anomalies [Jain et.al.] (2013) [Jensen,Loganayagam,Yarom] (2013)
- N M5's index:

[CKKN-1] [Nahmgoong] (to appear)

$$\log Z \sim \boxed{-\frac{N^3}{24}\frac{\Delta_1^2\Delta_2^2}{\omega_1\omega_2\omega_3}} - \frac{N((\Delta_1+\Delta_2)^2+4\pi^2)((\Delta_1-\Delta_2)^2+4\pi^2)}{192\omega_1\omega_2\omega_3} + \frac{N(\Delta_1^2+\Delta_2^2-4\pi^2)(\omega_1^2+\omega_2^2+\omega_3^2)}{96\omega_1\omega_2\omega_3} + \mathcal{O}(\log\omega)$$

- Large N: counts BPS BH's in $AdS_7 \times S^4$ [CKKN-1] [Hosseini, Hristov, Zaffaroni]

Conclusion & comments

- Index of $SCFT_D$ sees BPS AdS_{D+1} black holes.
- Known large BH's are statistically counted in the Cardy limit.
- Non-Cardy regime studied assuming "Bethe root ↔ large N saddle pt." relation [Benini,Milan]

- Cardy limit of 5d SCFT: $\log Z \sim N^{5/2}$. Counts BPS BHs in $AdS_6 \times S^4/Z_2$ [Choi,SK] [CHKN]

- In certain regimes, dominant saddle points can be yet unknown **new BH's**.
- Hawking-Page transition & further conjectures on new black holes [CKKN-2]
- 1/8-BPS sector of $N = 4$ SYM ("Macdonald index" [Gadde,Rastelli,Razamat,Yan]): Known BH's don't exist in this sector, while we find new BH-like saddle points from QFT. [CKKN-1]

- More to be done (only a tiny & partial list)
- Large N saddle point analysis in non-Cardy regime. Hawking-Page transition.
- Construction of new 1/16-BPS operators [Berkooz,Reichmann,Simon] [Chang, Yin]…
- New BPS black holes: either more dominant or subdominant than known ones
- Better intuitions on hairy black holes? Apply more numerical GR techniques…?