Genus-one string amplitudes from CFT

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Based on

- 1809.10670
- 1706.02388
- 1705.02318
- 1612.03891

with (subsets of) Aharony, Alday and Perlmutter

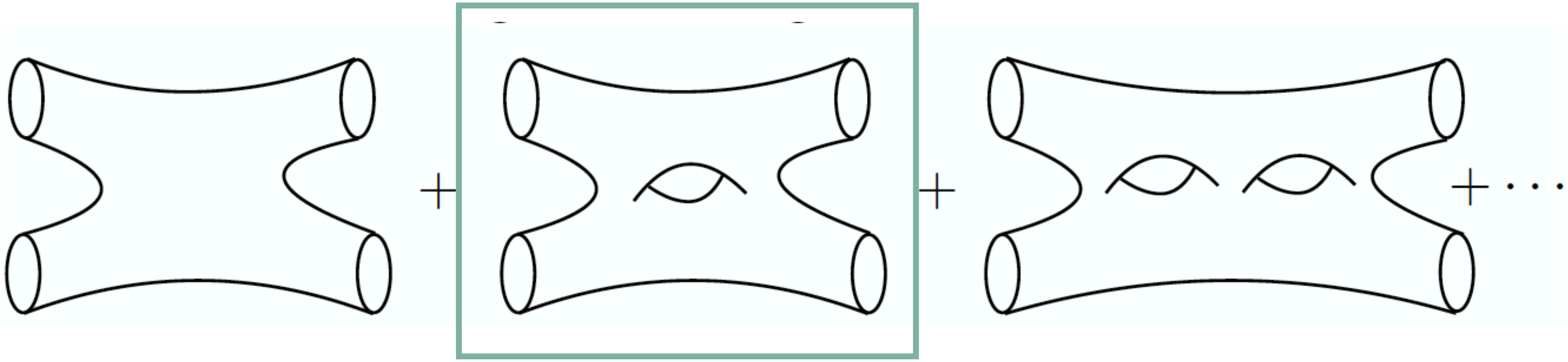
PROGRAM: study scattering amplitudes in AdS spaces, using CFT techniques applied to dual correlators.

Motivation

- Scattering amplitudes might uncover hidden structures of a theory, not apparent from its lagrangian formulation
- Using traditional techniques, scattering amplitudes in curved space are hard to study, already at genus zero
- Understand how much of the full structure of string amplitudes can be recovered/ predicted from the dual CFT, using only symmetries
- Precision test of AdS/CFT correspondence, away from the planar limit and from protected observables.

This talk

- I will discuss a setup to compute/constrain genus-one four graviton amplitude in string theory on $AdS_5 \times S^5$ in a low energy expansion.



$$A_{ST}^{\text{genus-one}}(\alpha', s, t, u) = A_{\text{sugra}}^{\text{one-loop}}(s, t, u) + \alpha'^3 A^{(1)}(s, t, u) + \dots$$

Setup

type IIB superstring theory

on $AdS_5 \times S^5$

AdS/CFT correspondence

four dimensional $\mathcal{N}=4$ SYM

with SU(N) gauge group

scattering amplitudes of gravitons

correlators of $T_{\mu\nu}$

Take both N and λ large: supergravity approximation

g_s genus expansion $g_s^2 \sim \frac{1}{N^2}$

$\alpha' = \frac{R^2}{\sqrt{\lambda}}$ higher derivative expansion $N = \frac{\lambda}{g_{YM}^2}$

Spectrum at strong coupling

- Protected (1/2 BPS) single trace operators

$$\mathcal{O}_p \sim \mathrm{Tr}(\phi_{i_1} \dots \phi_{i_p})$$

$$\Delta = p \longrightarrow$$

$p = 2$ STRESS TENSOR

$p > 2$ KALUZA KLEIN MODES

- Unprotected single trace operators

$$\Delta \sim \lambda^{1/4}$$

- Double trace operators

$$\mathcal{O}_{p_1} \Box^n \partial^\ell \mathcal{O}_{p_2}$$

$$\Delta = p_1 + p_2 + 2n + \ell + \frac{\gamma^{(1)}(\lambda)}{N^2} + \frac{\gamma^{(2)}(\lambda)}{N^4} + \dots$$

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LOOP CORRECTIONS TO SUGRA

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STRINGY CORRECTIONS TO SUGRA

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LOOP CORRECTIONS TO SUGRA

Expansions

$$\boxed{\langle \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^4 x_{34}^4}}$$

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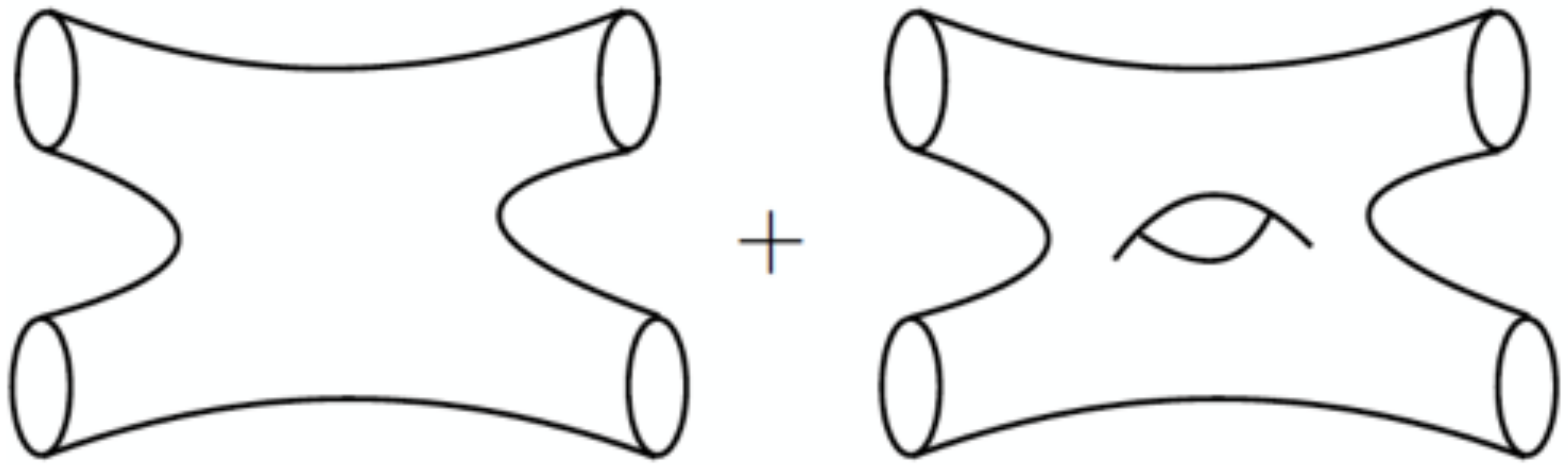
Large N expansion

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v,\lambda) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v,\lambda) + \dots$$

Genus expansion



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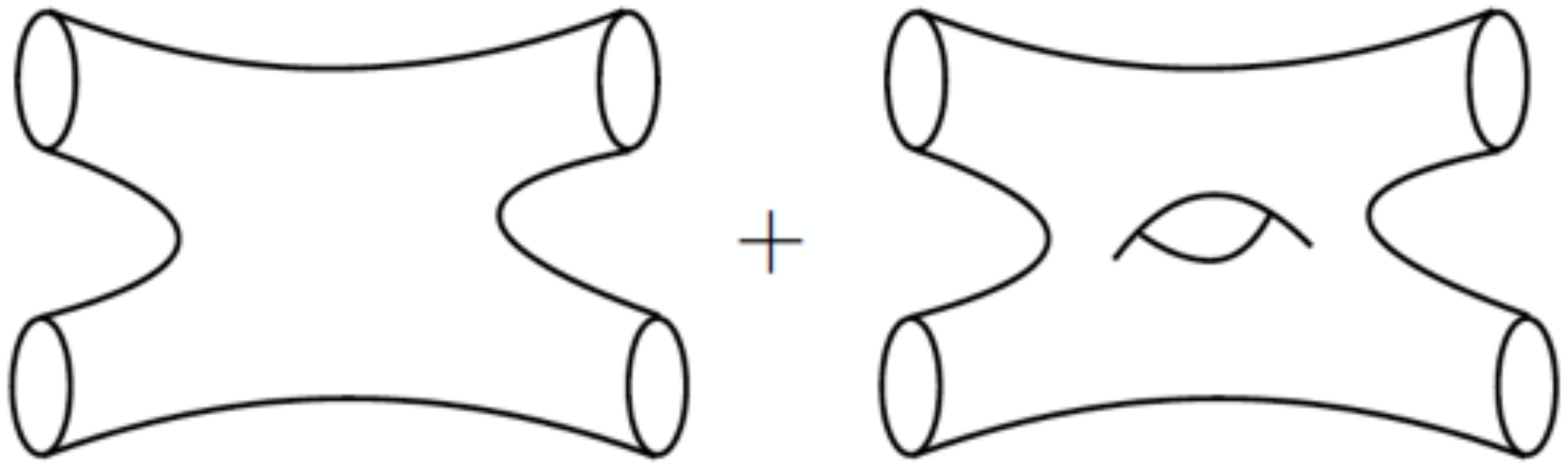
Large λ expansion

$$\mathcal{G}^{(1,1)}(u,v) + \frac{\mathcal{G}^{(1,2)}(u,v)}{\lambda^{\frac{3}{2}}} + \dots \qquad \mathcal{G}^{(2,1)}(u,v) + \frac{\mathcal{G}^{(2,2)}(u,v)}{\lambda^{\frac{3}{2}}} + \dots$$

Genus expansion

α' expansion



OPE decomposition

$$\mathcal{G}(u,v) = \underset{\substack{\downarrow\\ \text{identity}}}{1} + \sum_{\Delta,\ell} \underset{\substack{\downarrow\\ \text{3 pt functions}}}{a_{\Delta,\ell}} u^{\frac{\Delta-\ell}{2}} \underset{\substack{\downarrow\\ \text{conformal blocks}}}{g_{\Delta,\ell}(u,v)}$$

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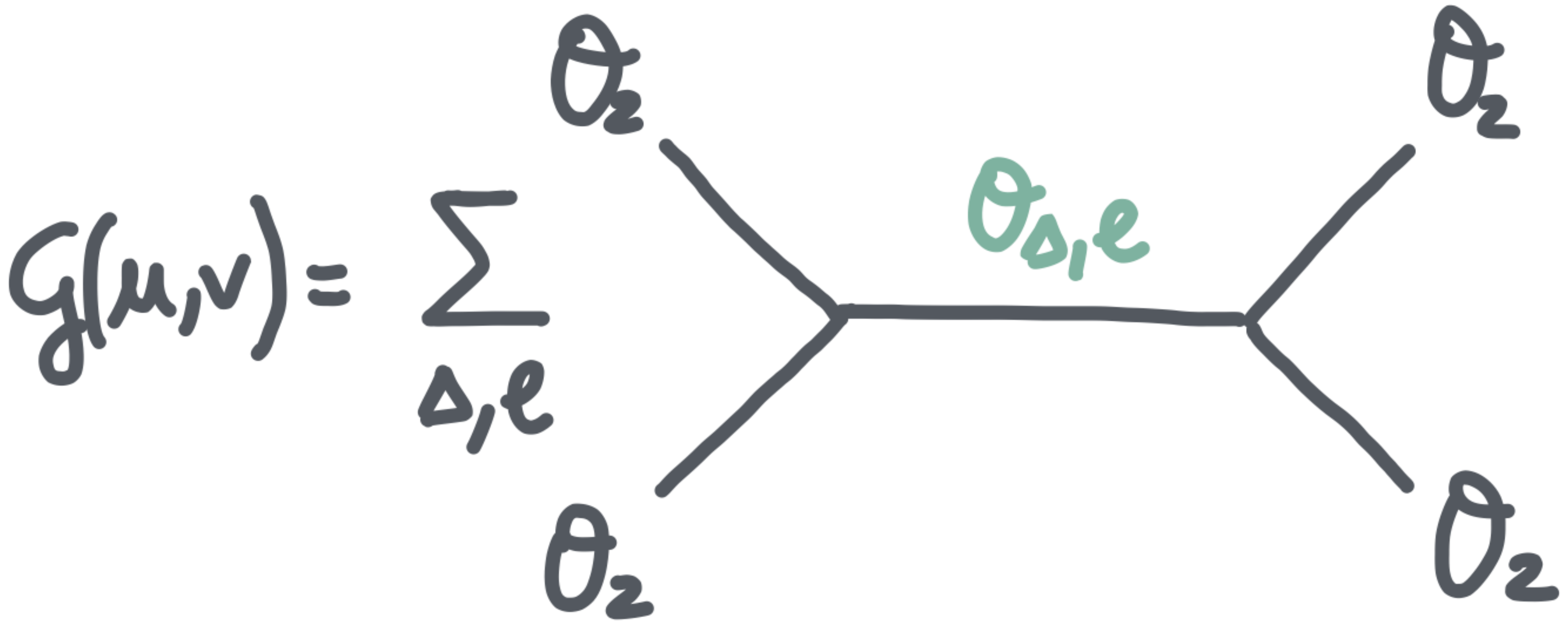
OPE decomposition



In this regime $\mathcal{O}_{\Delta,\ell}$ are :

- protected single traces
- unprotected double trace

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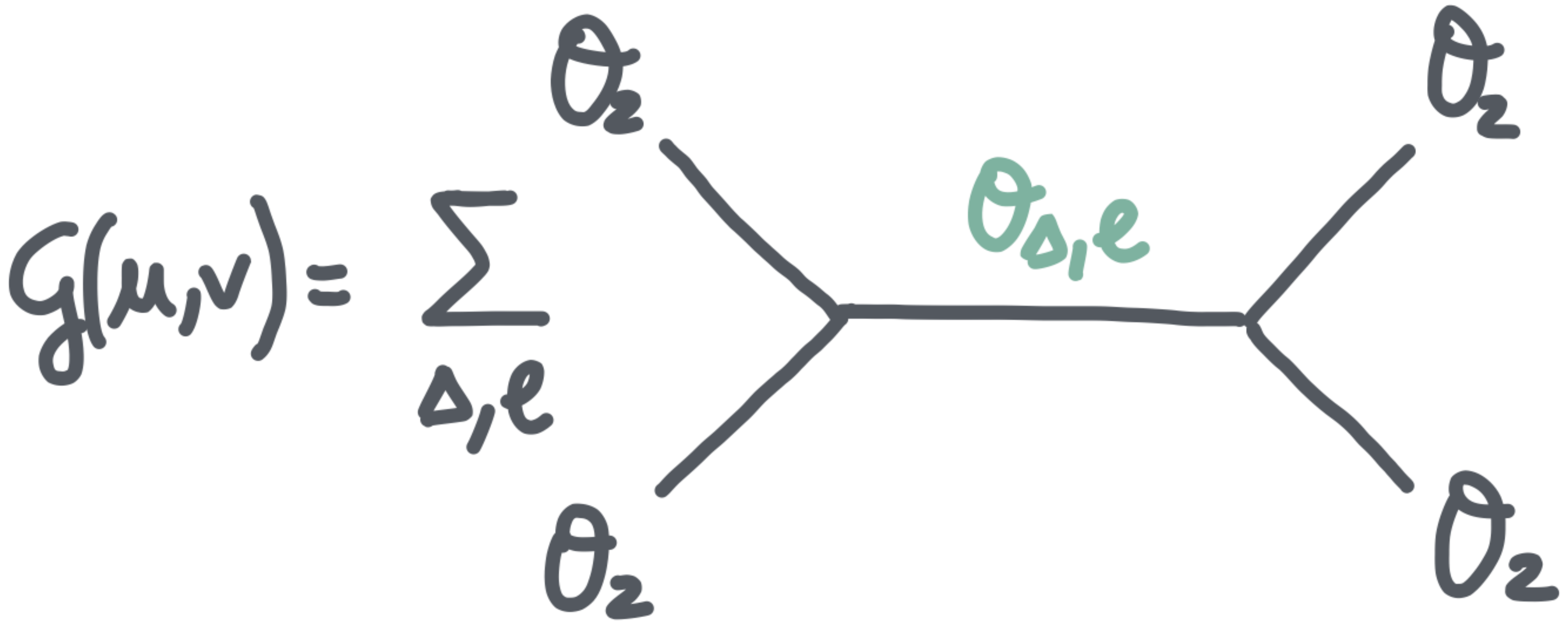

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Crossing symmetry

$$\mathcal{G}(u,v) = \left(\frac{u}{v}\right)^2 \mathcal{G}(v,u)$$

Analytic bootstrap

- As $v \to 0$ the correlator develops singularities.
- The whole OPE data/correlator can be reconstructed from such singularities

$$\mathrm{Sing}[\mathcal{G}(u,v)] \to \{a_{n,\ell}, \gamma_{n,\ell}\} \to \mathcal{G}(u,v)$$

- The structure of the singularities comes from crossing symmetry and the OPE content.

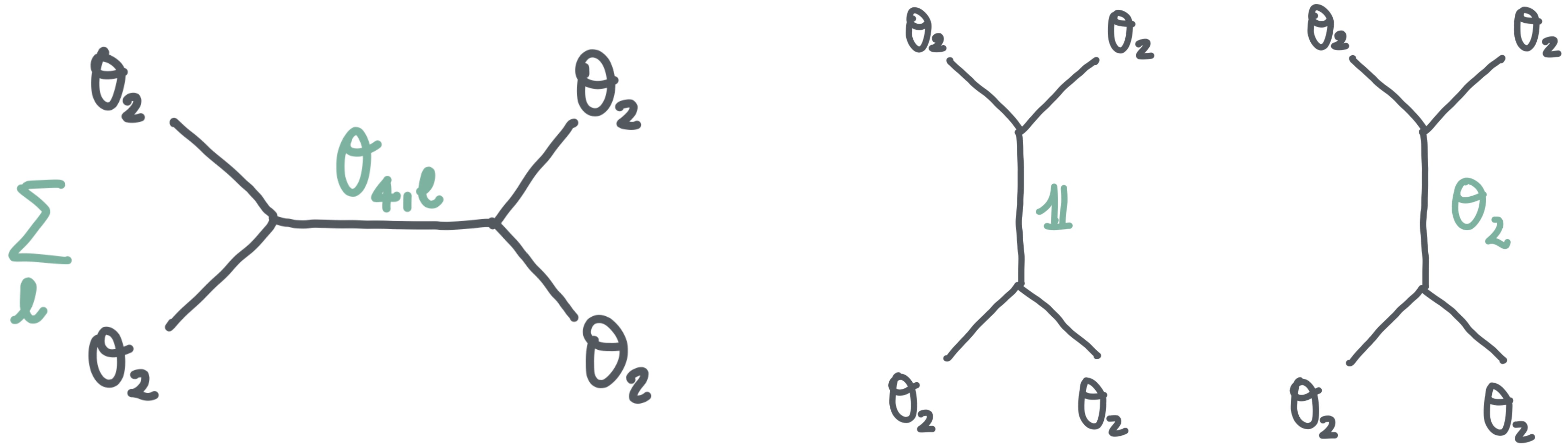
Fitzpatrick, Kaplan,Poland, Simmons-Duffin Komargodski, Zhiboedov

Alday Caron-Huot

Divergences, leading order

$$\sum_{\Delta,\ell} a_{\Delta,\ell} u^{\frac{\Delta-\ell}{2}} g_{\Delta,\ell}(u,v) = \left(\frac{u}{v}\right)^2 \Big(\underset{\text{identity}}{\underset{\downarrow}{1}} + \underset{\text{1/2 BPS op p=2}}{\underset{\downarrow}{a_{2,0}\, v\, g_{2,0}(v,u)}} + \underset{\text{double traces}}{\underset{\downarrow}{a_{4,0}\, v^2 g_{4,0}(v,u)}} + \dots\Big)$$

For $v \to 0$



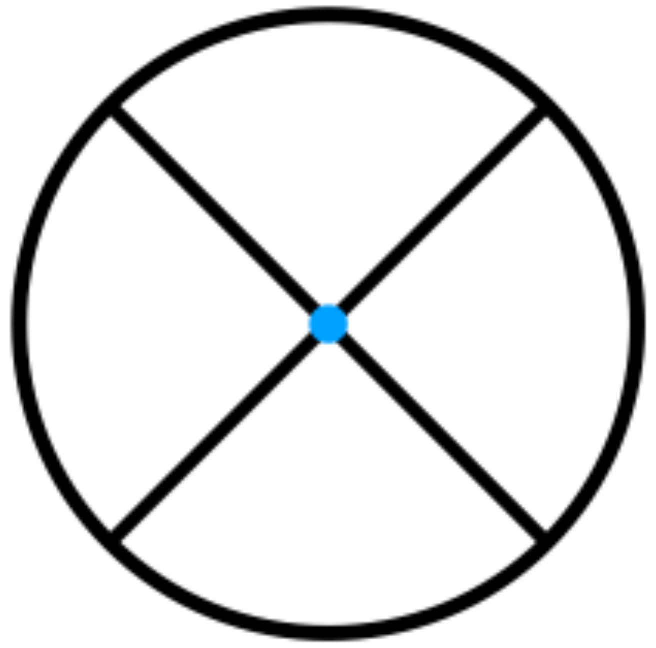
- Double trace operators (infinitely many) with large spin

⟷

- Low twist operators are associated to power-law divergences

Divergences

- Matching the divergences allows fixing the OPE data, up to finite spin ambiguities.
- These ambiguities correspond to crossing symmetric solutions without divergences. For such solutions $\gamma_{n,\ell}, a_{n,\ell} \neq 0$ for $\ell = 0, 2, \ldots L$
- They correspond to quartic vertices in AdS



Heemskerk, Penedones, Polchinski, Sully

- In addition to power law singularities , there are logarithmic singularities ($\sim \log^2 v$)
- Equivalently, the whole correlator can be reconstructed from the double discontinuity (dDisc). The same type of ambiguities arise also in this approach.

Caron-Huot

Strategy

- Expand the OPE data for large N

$$a_{n,\ell} = a^{(0)}_{n,\ell} + \frac{a^{(1)}_{n,\ell}(\lambda)}{N^2} + \frac{a^{(2)}_{n,\ell}(\lambda)}{N^4} + \dots$$

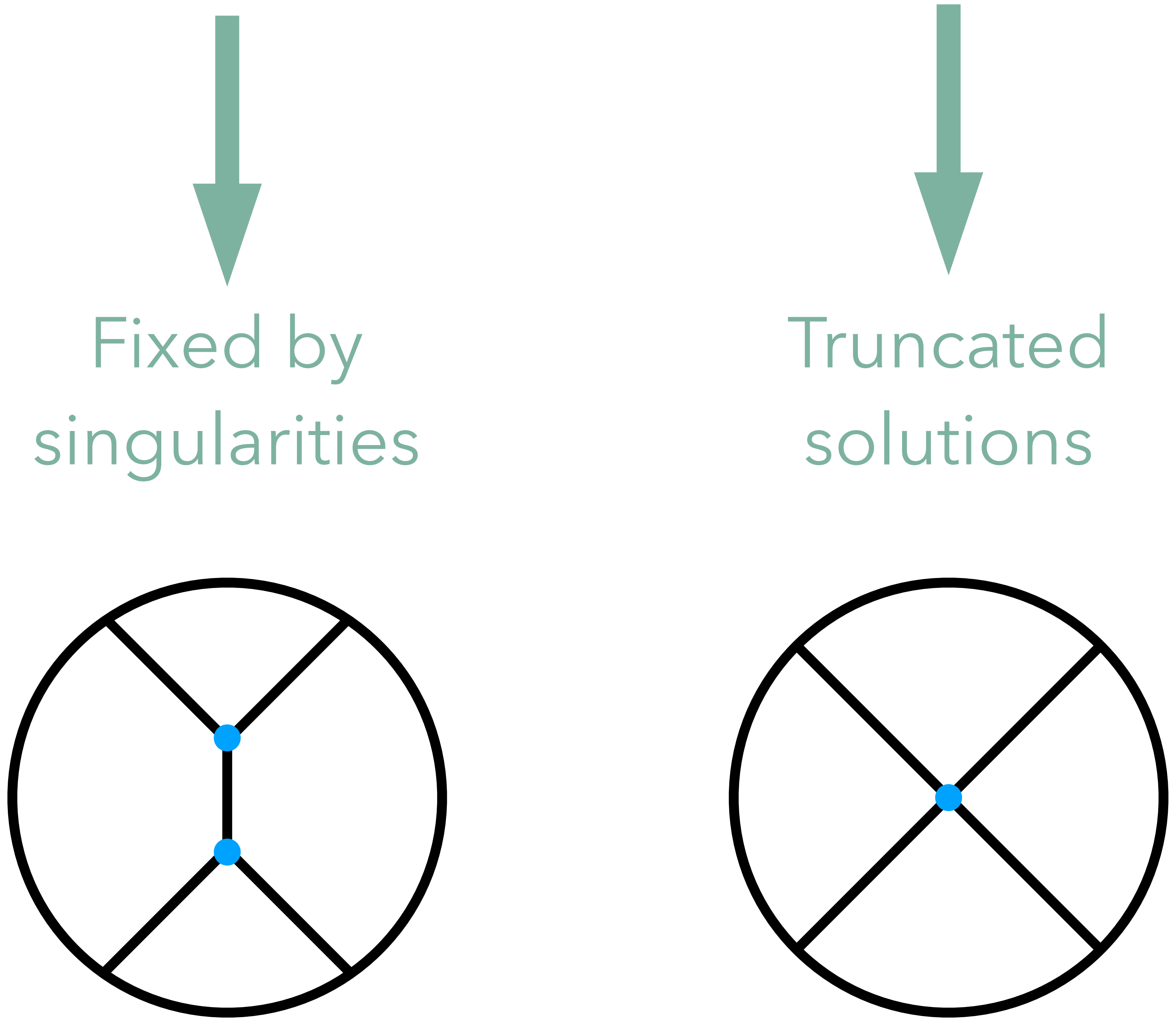
$$\Delta = 4 + 2n + \ell + \frac{\gamma^{(1)}_{n,\ell}(\lambda)}{N^2} + \frac{\gamma^{(2)}_{n,\ell}(\lambda)}{N^4} + \dots$$

- Use crossing symmetry to fix the OPE data by requiring the proper singularity structure.

- Up to order N^{-2} there are power law singularities and we can compute $a^{(0)}_{n,\ell}$ and $\gamma^{(1)}_{n,\ell}$

Sugra+strings

$$\gamma^{(1)}(\lambda) = \gamma^{(1)}_{sugra} + \frac{\gamma^{(1)}_{truncated}}{\lambda^{3/2}} + \frac{\gamma^{(1)}_{truncated}}{\lambda^{5/2}} + \dots$$



Fixed by singularities

Truncated solutions

How to go to one loop

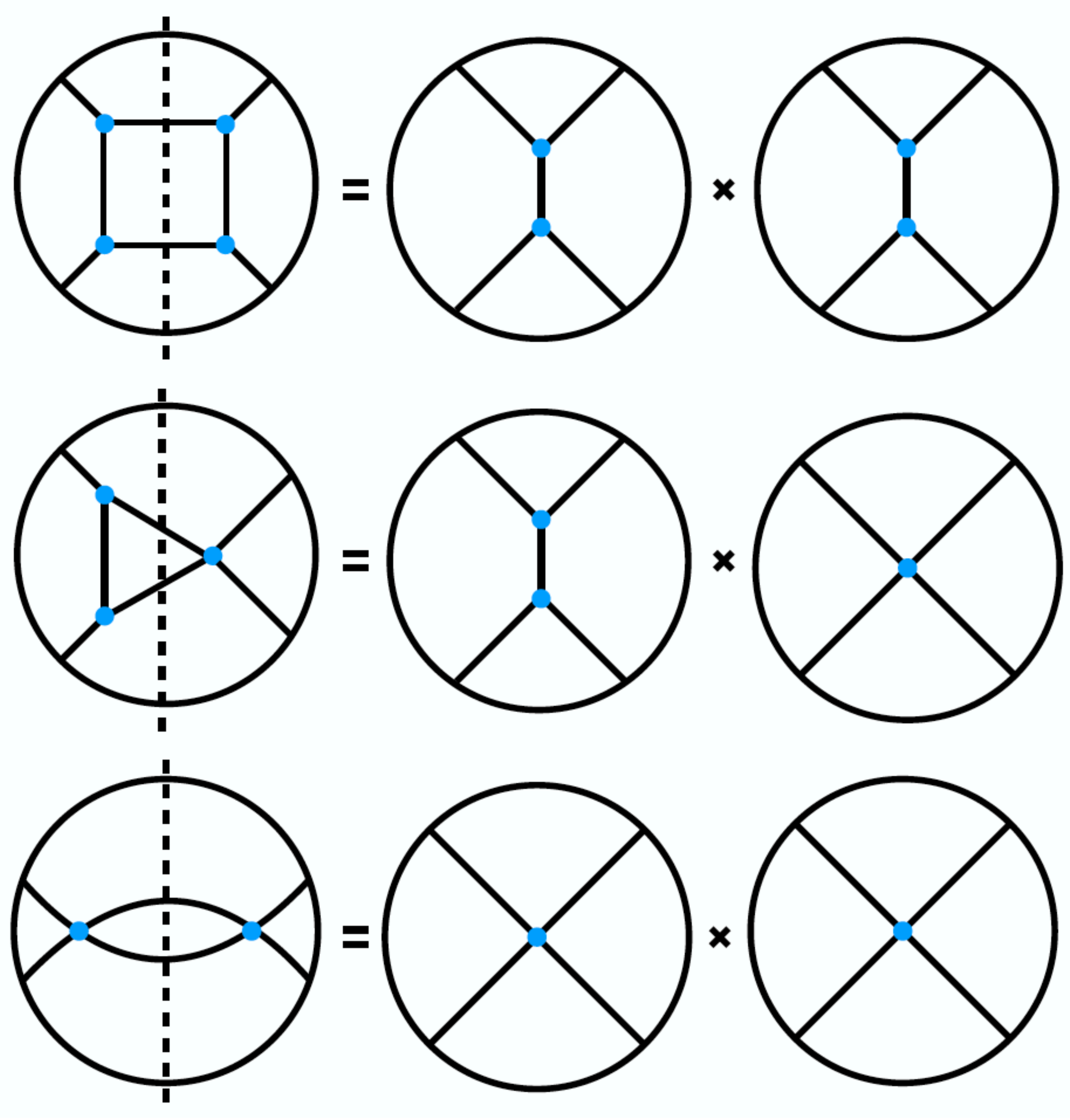
- At order N^{-4} there is a term of this form:

$$\left(\frac{u}{v}\right)^2 \sum_{n,\ell} a^{(0)}_{n,\ell} v^{2+n+\frac{\gamma^{(1)}_{n,\ell}}{N^2}+\frac{\gamma^{(2)}_{n,\ell}}{N^4}} g_{n,\ell}(v,u) \sim \frac{\boxed{\log^2 v}}{N^4} \sum_{n,\ell} a^{(0)}_{n,\ell} (\gamma^{(1)}_{n,\ell})^2 v^{2+n} g_{n,\ell}(v,u)$$

- The coefficient in front of the singularity is given by $a^{(0)}_{n,\ell}(\gamma^{(1)}_{n,\ell})^2$

- From this information it is possible to compute $\gamma^{(2)}_{n,\ell}$ and the full correlator.

$$\left(\gamma_{n,\ell}^{(1)}\right)^2 = \left(\gamma_{sugra}^{(1)} + \gamma_{truncated}^{(1)}\right)^2 = \gamma_{sugra}^{(1)} \times \gamma_{sugra}^{(1)} + \gamma_{sugra}^{(1)} \times \gamma_{truncated}^{(1)} + \gamma_{truncated}^{(1)} \times \gamma_{truncated}^{(1)}$$



$$\frac{1}{\lambda^{\frac{3}{2}}}$$

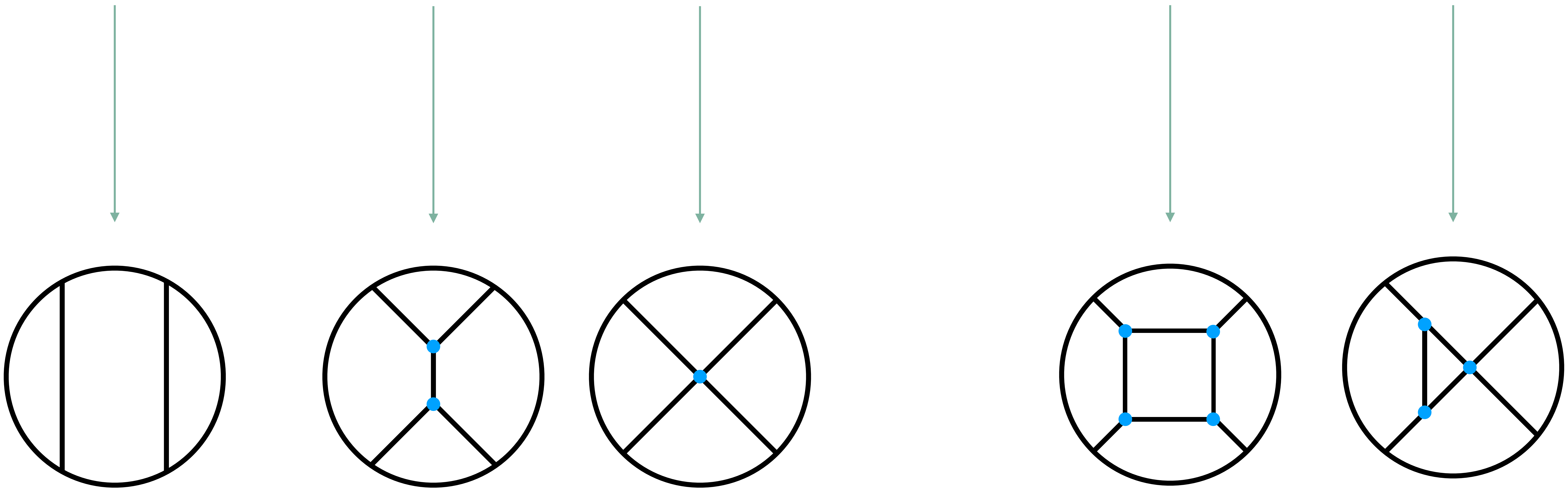
$$\frac{1}{\lambda^3}$$

Aprile, Drummond, Heslop, Paul

Alday, Caron-Huot

Final results

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\left(\mathcal{G}^{(1)}(u,v)_{sugra} + \frac{\mathcal{G}^{(1)}(u,v)_{string}}{\lambda^{\frac{3}{2}}} + \dots\right) + \frac{1}{N^4}\left(\mathcal{G}^{(2)}_{sugra}(u,v) + \frac{\mathcal{G}^{(2)}_{string}(u,v)}{\lambda^{\frac{3}{2}}} + \dots\right)$$



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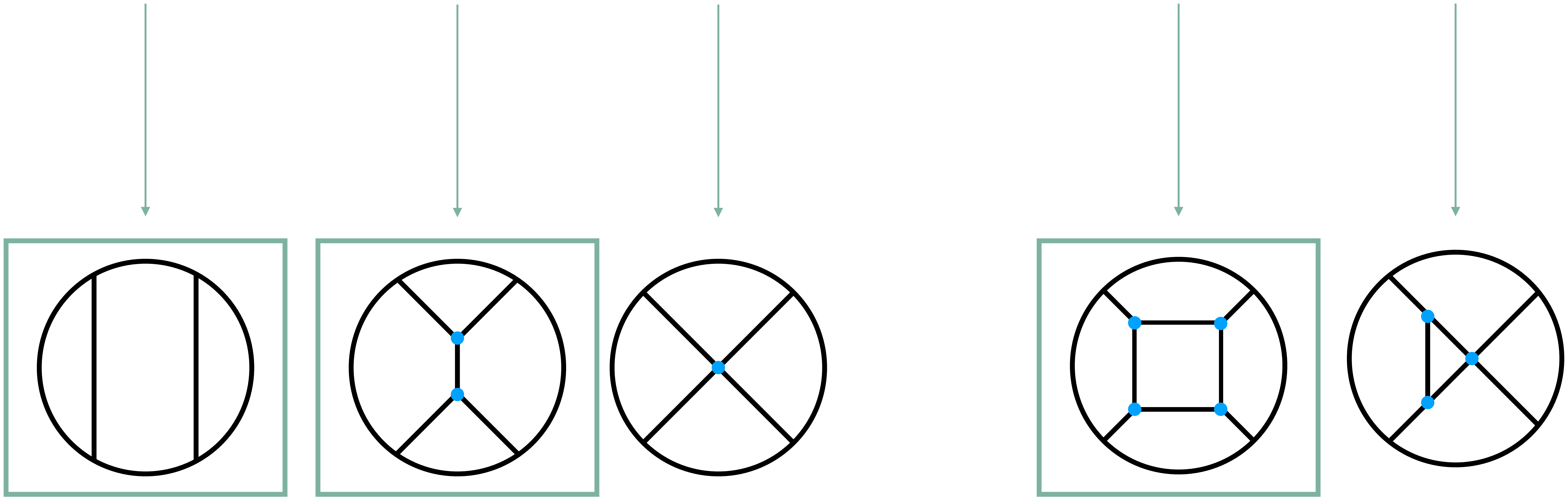


Completely fixed by the singularities arising from protected operators in the OPE

Mellin amplitudes

- String amplitudes on $AdS_5 \times S^5$ are given by the Mellin transform M(s,t,u)

Mack
Penedones
Rastelli,Zhou

$$\mathcal{G}(u',v) = \int_{-i\infty}^{i\infty} ds\, dt\, u'^s v^t M(s,t,u) \Gamma^2(2-s)\Gamma^2(2-t)\Gamma^2(2-u)$$

with s+t+u=2.

- At genus zero

$$M^{(0)}(s,t,u) = \frac{1}{(s-1)(t-1)(u-1)} + (\alpha')^3 a + (\alpha')^5 \left(b\left(s^2+t^2+u^2\right)+b_1\right) + ..$$

polynomials
in s,t,u

unknown

SUGRA $\qquad \mathcal{R}^4 \qquad \partial^4\mathcal{R}^4$

see also Drummond, Nandan, Paul, Rigatos

Flat space limit

- Take the flat space limit $s, t, u \to \infty, \quad \alpha' \to 0 \;\text{ with }\; s\alpha', \, t\alpha', \, u\alpha' \;\text{fixed}$

$$M^{(0)}(s,t,u) \to \tilde{M}^{(0)}(s,t,u)$$

SHAPIRO-VIRASORO
AMPLITUDE

- The matching with the Shapiro Virasoro amplitude fixes some coefficients:

$$M^{(0)}(s,t,u) = \frac{1}{(s-1)(t-1)(u-1)} + (\alpha')^3 \boxed{a} + (\alpha')^5 \left(\boxed{b}\left(s^2+t^2+u^2\right) + b_1\right) + \dots$$

$\downarrow$

curvature
corrections

see also
Binder,Chester,Pufu, Wang

- The same can be done for the genus one amplitude

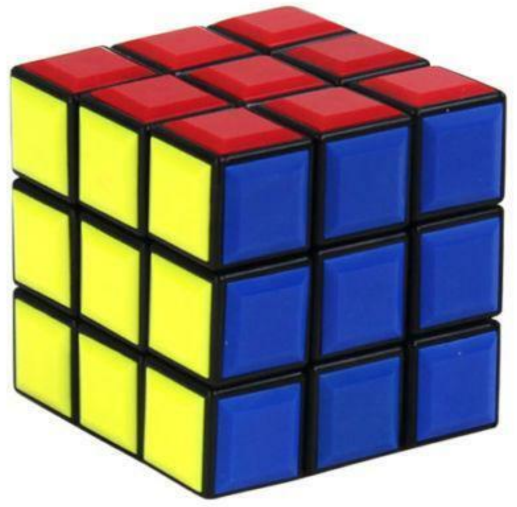
$$M^{(1)}(s,t,u) \to \quad \tilde{M}^{(1)}(s,t,u)$$

Non-analytic part of type IIB genus one string
ampl in flat space

Green,Russo,Vanhove

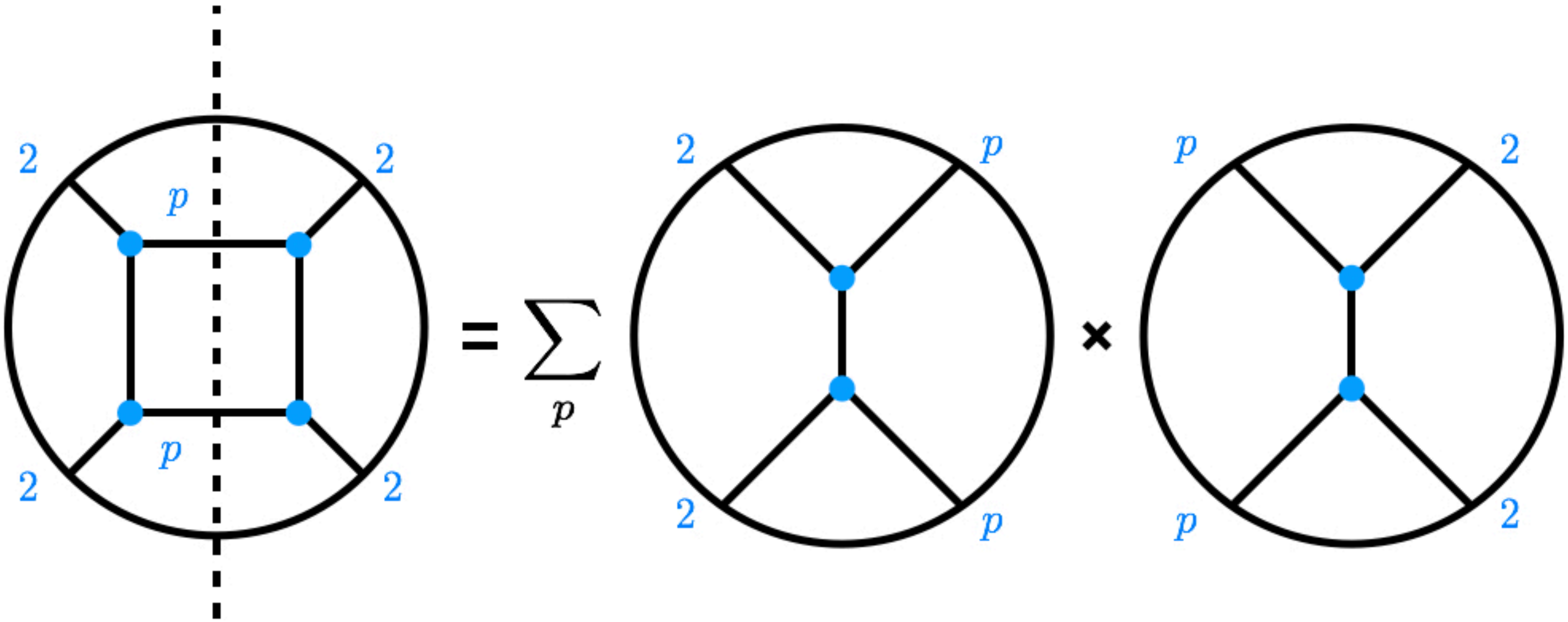
- The genus one matching puts constraints on the curvature corrections $b_1, \dots$

Conclusions



- I discussed how to construct genus one string amplitudes on curved space, studying the dual correlator.
- The matching with the flat space limit agrees with known results and give predictions to curvature corrections.

- Vertex operator
- Higher genus
- Heavy states
- Fixing ambiguities (numerical bootstrap?)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